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Warranty Cost Modeling and Warranty Length Optimization Under Two Types of Failure and Combination Free Replacement and Pro-Rata Warranty

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ABSTRACT The product warranty has become an indispensable facet of business operations. Burn-in is effective at eliminating infant mortality and improving operational reliability levels for consumers. This paper considers the influence of different failure states and different phases of product reliability and warranty policies on warranty costs from predelivery inspection to the end of the warranty period. We then propose a comprehensive warranty cost model that considers burn-in, free replacement warranty and pro-rata warranty as three phases for repairable products presenting two types of failure (minimal and catastrophic failure) that involve minimal repair and replacement, respectively. Warranty costs are the result of a combination of the three phases, where two types of failure occur individually or simultaneously. Moreover, we developed a framework for the modeling process of warranty costs, and the effects of various parameters, such as the burn-in time, warranty period, and distribution function on warranty costs, were analyzed. Finally, a practical case was examined by using a warranty cost model, and through an after-sales service data analysis, we obtained the failure rate distribution and optimal warranty length by minimizing the average warranty cost, which can serve as a reference for manufacturers when developing warranty policies.

INDEX TERMS Warranty cost model, burn-in, free replacement warranty, pro-rata warranty.

I. INTRODUCTION

As assurance of a manufacturer to a buyer, warranties have an important influence on consumer purchasing decisions. A warranty is a contractual agreement between a buyer and manufacturer [1], [2]. However, warranty costs cannot be ignored. For example, warranty rate data for household appliances purchased from 2003 to 2014 show that warranty costs account for sales profits of 0.6%-12.4% (the American ''warranty week''). Higher warranty costs increase product prices, while decreasing sales lower warranty costs; in addition, infant mortality levels affect customer satisfaction and lead to intangible costs such as losses of reputation and customer loyalty [3]. Therefore, the development of warranty periods that not only meet the needs of consumers but also ensure product profitability is key to improving market competitiveness.

Basic FRW, PRW, rebate policy and cumulative warranties are commonly used warranty policies. A combination warranty policy combines several warranty policies. Combination policies of a renewed FRW and PRW are used frequently, as they offer high promotional value for sellers while ensuring adequate control over costs for both buyers and sellers in most applications [4], [5]. Blischke and Murthy [6] have proposed a variety of warranty policies and have summarized applications of mathematical models and statistical analyses in research on warranty lengths and costs.

Researchers have mainly considered the following four aspects in developing reasonable warranty period while reducing warranty costs according to warranty policies and reliability levels.

A. RELIABILITY

Warranty costs are dependent on product reliability levels, warranty terms, warranty servicing strategies and maintenance actions [6]. Murthy [7] reviewed a paper related to product reliability and warranty. Product reliability refers to availability, successful operation, performance and the absence of failures. Murthy [7], [8] explored the reliability of product development and design and analyzed existing research on reliability levels and warranties (flexible warranties, reliability models and warranty data). Hussain and Murthy [9], [10] studied optimal strategies for improving product reliability and warranty services by using redundancy technologies and product development. Blischke *et al.* [11] studied the reliability growth model and methods of warranty cost modeling based on life distribution patterns.

B. BURN-IN

Burn-in schemes are effective at reducing infant mortality levels and at improving operational reliability for consumers [3]. However, burn-in schemes involve additional costs, and costs increase with burn-in time. When the benefits derived exceed the cost, the burn-in is worthwhile [1]. The study of burn-in optimization typically occurs in combination with an examination of life distributions such as L-shape and bathtub failure rates. Therefore, in the study of burn-in and warranties, the most significant problem concerns determining the optimal burn-in period for which the criterion of minimum warranty costs and minimum failure rates should be applied. Shafiee *et al.* [13] discuss an optimal burn-in/preventive maintenance strategy for a product with the bathtub-shaped failure rate that minimizes the total expected warranty cost.

Wu and Clements-Croome [14] presented the relationship between burn-in and warranty costs in the dormant and operating states while considering two burn-in policies based on levels of product reliability. Nguyen and Murthy [15] explored product burn-in to reduce the number of defective products entering the market and optimized the burn-in time to achieve a trade-off between decreased warranty costs and increased manufacturing cost. Sheu and Chien [16] developed an optimal burn-in time and field-operation model by considering a repairable product sold under warranty. Ye *et al.* [3] developed a bi-objective burn-in model to achieve a trade-off between costs (book and intangible costs) and performance measures. Shafiee and Zuo [17] developed an optimization model to evaluate the optimal burn-in time and warranty length of a product for the benefit of manufacturers. For more information, see Chou *et al.* (2007) [18], Jiang and Jardine (2007) [19], Yun *et al.* (2002) [12], Ye *et al.* (2012) [20], and Ye *et al.* (2013) [21].

C. WARRANTY COST MODELING

Many authors have developed cost models and warranty periods for burn-in and FRW/PRW policies. Yun *et al.* [12] developed the cost model and optimal burn-in time to minimize the mean cost of a product sold under a cumulative warranty. Cha *et al.* [22] considered two types of failure (minor and catastrophic failure) for a system while balancing the optimal burn-in time and the total expected warranty cost under the bathtub-shaped failure rate. Lengbamrung and Pongpullponsak [23] developed a cost model for calculating the optimal burn-in time and for minimizing the total warranty cost for products subjected to a renewable full service warranty. Ambad and Kulkarni [24] proposed an optimal warranty period model for analyzing the relationship between the MTBF and reliability and warranty costs based on early product reliability and warranty policies. Chou *et al.* [18] developed a cost model for burn-in time and fully renewing combination FRW/PRW and compared several types of warranty policies, but only for non-repairable products. For more information, see Nguyen and Murthy (1982) [15], Hong *et al.* (2007) [8], Chou *et al.* (2007) [18], Yeh *et al.* (2007) [25], Wu and Xie (2008) [26], and Shafiee and Zuo (2011) [17].

D. WARRANTY DATA COLLECTION AND ANALYSIS

Warranty data (i) provide useful information for evaluating field, inherent and design reliabilities [7], (ii) for predicting new product warranty claims and warranty costs and (iii) for developing appropriate warranties and maintenance policies.

Perstein *et al.* [27] analyzed the optimal burn-in time and warranty cost through Bayesian calculations under the assumption that the life distribution is a mixed exponential distribution. Huang *et al.* [28] established a burn-in test optimization model and a mixed competing failure model for products involving strophic and degradation failure and used the failure model to determine the mean residual life (MRL) value. Limon *et al.* [29] presented a means of estimating the product usage rate and proposed a reliability assessment approach that considers two different distributions for the remaining warranty period. Oh and Bai [30] estimated reliability and lifetime distributions by gathering additional field data after a warranty had expired. Jung *et al.* [31] put forward a periodic maintenance strategy that considers the limits of maintenance time and changes in warranty periods. The failure distributions of warranty data have also been considered in Suzuki (1987) [32], Chou and Tang (1992) [33], Lawless *et al.* (1995) [34], and Chukova *et al.* (2011) [35]. In addition, we highlight the work of Blischke *et al.* [36]. However, many burn-in and warranty cost models for products have assumed that products are unrepaired or that only one type of failure applies for a repaired product, and previous studies on warranty costs have focused primarily on a single FRW or PRW.

This paper considers the effects of different phases of product reliability and warranty policy-making from the production phase to the end of the warranty period. Most studies on the warranty cost model have considered only a single warranty policy or the failure state, thus failing to consider product burn-in schemes, combination warranty policies and multiple types of failure. Moreover, most studies have analyzed only total book costs and maintenance losses, while intangible costs such as losses of reputation and customer loyalty have not been fully considered.

We consider the burn-in time, the FRW period and the PRW period as the three phases of a comprehensive warranty policy under which the manufacturer is responsible for losses

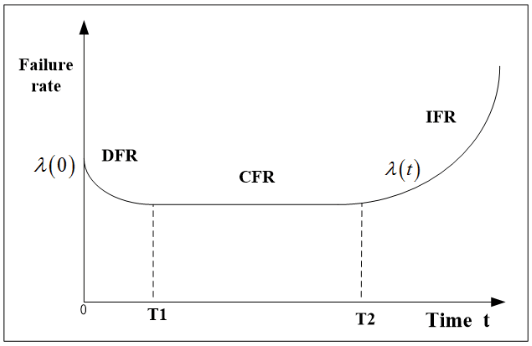


FIGURE 1. A typical bathtub-shaped failure rate.

incurred by product failures. We denote b as the burn-in time and W as the warranty length of a product. Renewing FRW: the seller agrees to offer free repairs or replacements for failed items from the point of initial purchase to the time point of W_1 for a repairable product where $W_1 < W$. PRW: the seller agrees to provide a prorated cost or prorated refund for failures in the time interval from W_1 to W. This practice is always used for a non-repairable product or combination warranty policy. Two types of failure are assumed to occur for repairable products: minimal (Type I failure) and catastrophic failure (Type II failure). Type I failure can be corrected through minimal repairs; Type II failure can be remedied only through a replacement or rebate. We assume that failure rates are constant over the three phases. When the Type I failure rate reaches a probability of p, the probability of Type II is $(1-p)$.

This paper first examines the frameworks and processes of warranty cost analyses that consider the effects of burn-in and failure types. The CWCM for repairable products presenting two types of failure under burn-in and FRW/PRW policies is then developed. The CWCM considers manufacturing, burn-in, warranty and intangible costs. Moreover, we analyzed warranty data on washing machine to obtain the failure rate distribution and optimal warranty length.

The rest of the paper is organized as follows. A three-phased framework for manufacturer costs is given in Section II. A product is assumed to present two types of failure and to be subject to a FRW/PRW policy. The CWCM is presented in Section III based on a three-phase analysis. A practical case is examined, and the optimal product warranty period is analyzed in Section IV. Some conclusions are provided in Section V.

II. THREE PHASES AND FAILURE CHARACTERIZATION

Most life distributions of new products conform to a bathtub curve [4] as shown in Fig. 1. Under the life distribution, the failure rate of a product has two change points (T_1, T_2), and the curve presents three stages [15].

$$\lambda(t) = \begin{cases} \textit{strictly decreases} & \text{if } 0 \le \mathrm{t} \le T_1, \\ \textit{constant} & \text{if } \mathrm{T}_1 < \mathrm{T} \le \mathrm{T}_2 \\ \textit{strictly increases} & \text{if } \mathrm{t} > \mathrm{T}_2 \end{cases}$$

The time interval $[0, T_1]$ is the infant mortality stage with decreasing failure rate (DFR). Failures occurring in the testing and operating phases are random. The time interval $[T_1, T_2]$ is a useful life stage with a constant failure rate (CFR). The time interval $[T_2, t]$ is an aging stage with an increasing failure rate (IFR). Failure periods occurring during the burn-in and warranty periods have a significant effect on product costs.

Failure characteristics of the product are described as follows [15] [36], where X denotes the failure time of a new product without burn-in.

$F_t(x) = F(t+x) - F(t)/1 - F(t)$: The residual life at the time t;

$\lambda(\mathrm{t}) = -dR(t)/R(t)\,dt$: The relation function of $\lambda(t)$ and $R(t)$.

Two types of failure are assumed to occur for repairable products. Before the product is released for sale, the burn-in procedure can scrap products for which Type II failures occur during the burn-in time $(0, b)$, and Type I failures can be repaired in the time interval $(0, b)$ to improve reliability levels when the failure rate decreases. To distribute failure characteristics of the two types of failure, the following parameters are defined as follows [15], [36], where a random variable Y denotes the time of Type II failure without burn-in.

$G(t)$: The cumulative distribution function of Y;

$\overline{G}(t) = 1 - G(t)$: The reliability function of Y;

$g(t) = -(d(\overline{G}(t)/dt)$: The density function of Y;

$h(t) = (1-p)\lambda(x)$: The failure rate of Y;

After the burn-in time b:

Y_{b}: The time to the first Type II failure;

$Y_{\mathrm{b}+W_1}$: The time to the first Type II failure after a burn-in time b and warranty time W_1;

$G_{\mathrm{b}}(t) = [G(b+t) - G(b)]/\overline{G}(b)$: The cumulative distribution function of Y_{b};

$g_{\mathrm{b}}(t) = g(b+t)/\overline{G}(b)$: The density function of Y_{b};

$\lambda_{\mathrm{b}}(t) = \lambda(b+t)$: The failure rate after the burn-in.

In this section, a framework that includes three phases for manufacturer costs is developed whereby intangible costs for products are considered.

1) Burn-in phase: When no failure occurs during the burn-in period, the product is released to sell. When a Type II failure occurs, the failed product is eliminated. For each Type I failure that occurs, minimal repairs are done, and the burn-in testing period is extended to time b.

2) FRW phase: When no failure occurs over the time interval $[0, W_1]$, there are no manufacturing costs, and consumers are satisfied. When a Type II failure occurs, the manufacturer supports product and warranty renewal. For a Type I failure, the manufacturer supplies minimal repairs free of charge to the customer.

3) PRW phase: When no failure that is the same as that of the FRW phase has occurred and when a Type II failure occurs over the time interval $[W_1, W]$, the manufacturer will provide a replacement by charging the customer a prorated cost. The rate is related to the remaining amount of warranty time. For each Type I failure that occurs, the

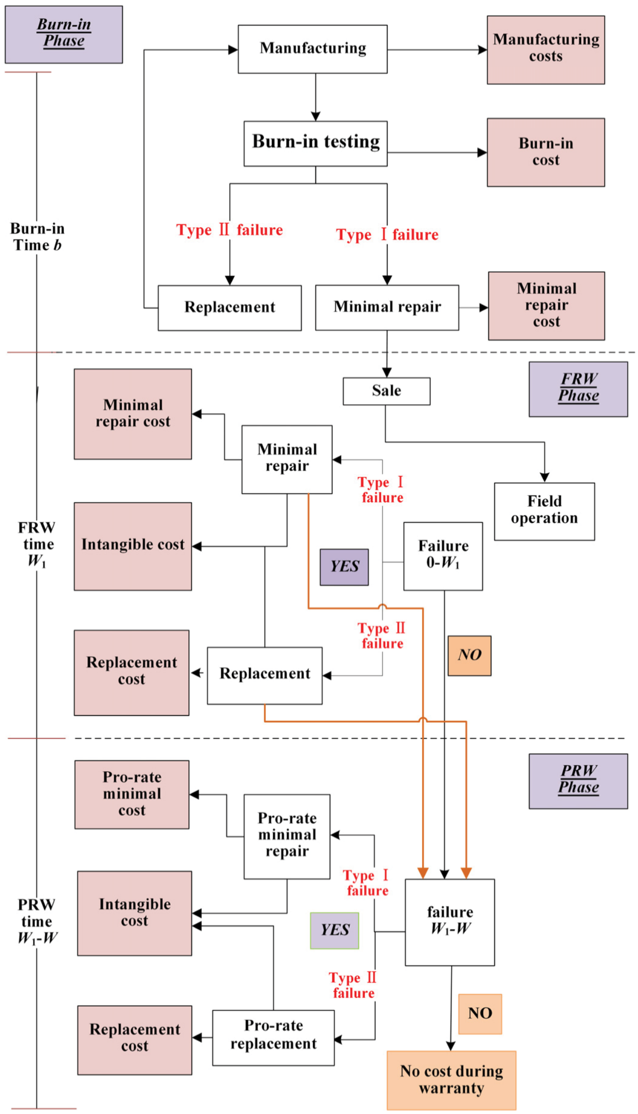


FIGURE 2. Framework of the cost of three phases.

manufacturer provides repairs by charging the consumer a prorated cost.

Information on the three phases is shown in Fig. 2.

III. THE COMPREHENSIVE WARRANTY COST MODEL

From the analysis and framework described above, the total cost to the manufacturer includes the following parts: 1, the manufacturing cost; 2, the burn-in cost and minimal repair cost incurred over the burn-in time; 3, free replacement or repair costs and intangible costs incurred over the time interval $[0, W_1]$; 4, prorated costs for replacements or repairs and intangible costs incurred over the time interval $[W_1, W]$. Before presenting the cost model, we list our assumptions as follows:

1. All failures are assumed to be detected instantly;
2. Repair and replacement times are negligibly short and can be ignored;
3. Environmental conditions of burn-in is similar to field conditions;
4. The costs of burn-in testing are in linear proportion to the burn-in time;
5. The probability of Type I failure p is constant;
6. The product failure time is independent.

A. TOTAL COSTS INCURRED DURING THE BURN-IN PHASE

Parameters of the cost model described in this section include the following:

C_0: costs of manufacturing per unit without a burn-in;

C_1: costs of a fixed burn-in set-up per unit;

C_2: costs of burn-in testing per unit of product per unit time;

C_3: the minimal repair cost per Type I failure;

$C(b)$: the total manufacturing and burn-in cost per unit;

$C_E(b)$: the expectation of $C(b)$;

η: the number of Type II failures made during the burn-in time;

$P(\eta)$: the probability distribution function of η;

Y_i: the time between the $(i)^{\text{th}}$ failure and $(i-1)^{\text{th}}$ failure $(i \geq 1)$;

η is also the number of replacements made until the first unit survives during the time interval $[0, b]$. Therefore, variable $(\eta + 1)$ obeys a geometric distribution. From characteristics of the distribution, $P(\eta)$ is given by:

$$P(\eta) = (G(b))^{\eta}\,\overline{G}(b), \quad \eta \geq 0 \tag{1}$$

$$E(\eta) = E(\eta+1) - 1 = \frac{1}{\overline{G}(b)} - 1 = \frac{G(b)}{\overline{G}(b)} \tag{2}$$

In the above case, when $\eta = 0$ and when the time of Type II failure for the first time is $Y_1 > b$, the random cost $C(b)$ is given by:

$$C(b) = C_0 + C_1 + C_2 b + C_3 \int_0^b p\lambda(u)\,du \tag{3}$$

When $\eta \geq 1$ and $Y_1 \leq b$, $C(b)$ is given by:

$$C(b) = \sum_{i=1}^{\eta}\left[C_0 + C_1 + C_2 Y_i + C_3 \int_0^{Y_i} p\lambda(u)\,du\right] + \left[C_0 + C_1 + C_2 b + C_3 \int_0^{b} p\lambda(u)\,du\right] \tag{4}$$

Then, according to Eqs. (3) and (4), the expectation for $C(b)$ is given by:

$$\begin{aligned} &E[C(b)] \\ &= E[C(b); Y_1 > b] + E[C(b); Y_1 \leq b] \\ &= G(b)\cdot E\left\{\sum_{i=1}^{\eta}\left[C_0 + C_1 + C_2 Y_i + C_3 \int_0^{Y_i} p\lambda(u)\,du\right]\right\} \\ &\quad + \left[C_0 + C_1 + C_2 b + C_3 \int_0^{b} p\lambda(u)\,du\right] \end{aligned} \tag{5}$$

As η is a stopping time with respect to Y_i, based on Wald's identity, we have:

$$E[C(b)] = G(b)\cdot E(\eta) \cdot E\left(C_0 + C_1 + C_2 Y_i + C_3 \int_0^{Y_i} p\lambda(u)\,du\right) + \left[C_0 + C_1 + C_2 b + C_3 \int_0^{b} p\lambda(u)\,du\right] \quad (6)$$

The expectation of $C(b)$ are written as follows:

$$C_E(b) = \left[\frac{1}{\overline{G}(b)} + \overline{G}(b) - 1\right](C_0 + C_1) + \left[\overline{G}(b)\cdot b + \int_0^{b} \overline{G}(u)du\right] C_2 + C_3 \cdot \frac{p}{1-p}\left[\frac{G^2(b)}{\overline{G}(b)} - \overline{G}(b)\ln \overline{G}(b)\right], \quad p \neq 1 \quad (7)$$

When $p = 1$, the failure in $[0, b]$ is only a minor failure that can be removed by repairs, and the cost under these conditions can be obtained from Eq. (3). When $p = 0$, the failure is only Type II, and the cost is as shown in Eq. (6). From the bathtub curve, the failure rate of the burn-in test is shown to be decreasing.

$$C_E(b)_{p=1} = C_0 + C_1 + C_2 b + C_3 E[\int_0^{b} \lambda(u)\,du] \quad (8)$$

For $0 < p \leq 1$, the derivative of $C(b)$ with respect to b can be derived as:

$$\frac{d[C_E(b)]}{db} = [2 - b\cdot h(b)]\left(C_2 - \frac{p}{1-p}C_3\right) + h(b)G(b)(C_0 + C_1) \quad (9)$$

Eqs. (6), (7), (8) and (9) show that the cost of a product is proportional to the burn-in time, which will increase the cost. However, the burn-in time can reduce the early failure rate of a product when the failure rate obeys a certain life distribution.

B. WARRANTY COST MODEL FOR THE RENEWING FRW PHASE

In this section, manufacturing provides total free replacements or repairs from the initial point of purchase to the time point of W_1. A warranty cost model for describing the FRW after burn-in testing is in turn developed. The warranty process is shown in Fig. 3.

Special parameters of the cost model in the section include the following:

C_4: additional costs of repair per unit, including transportation, administrative and labor costs;

C_5: additional costs of replacement per unit;

$C_{\mathrm{Lo}1}$: intangible costs of the Type I failure per unit for the time interval $[0, W_1]$; we assume that this is the linear function of the number of minimal repairs and that α is the proportionality coefficient;

$C_{\mathrm{LR}1}$: intangible costs of Type II failure per unit; α' is the proportionality coefficient of replacement time n and the intangible cost.

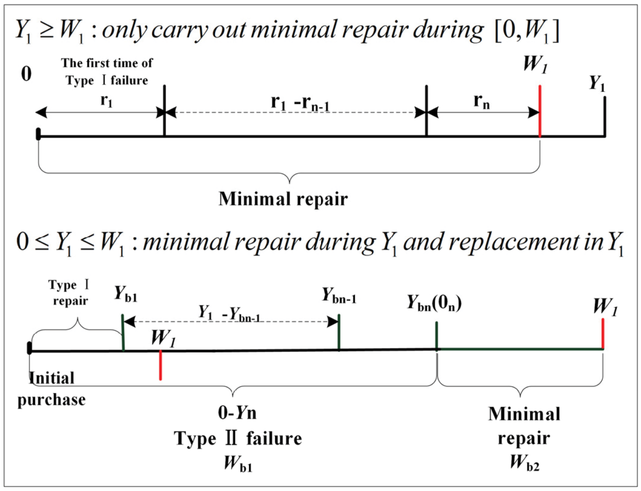


FIGURE 3. Renewing FRW policy in the time interval $[0, W_1]$.

$$\begin{cases} C_{Lo1} = \alpha \int_o^t p\lambda(t)\,dt \\ C_{LR1} = \alpha' n \end{cases} \quad (10)$$

Y_{bi}: the time between the $(i)^{\text{th}}$ failure and $(i\text{-}1)^{\text{th}}$ failure $(i \geq 1)$ during the time interval $[0, W_1]$;

W_{b1}: the total cost of replacement and repair before the last time Type II failure occurs;

W_{b2}: the minimal repair cost of the time interval $[0, W_1]$ after the last time Type II failure occurs;

$C_{bE}(W_1)$: the expected total cost of the time interval $[0, W_1]$.

We must consider two cases: (i) when $Y_{b1} > W_1$, the minimal repair cost $C_E^*(W_1)$ is given by:

$$C_b^*(W_1) = (C_3 + C_4 + \alpha)\int_0^{W_1} p\lambda(u+b)\,du \quad (11)$$

(ii) When $Y_{b1} \leq W_1$, costs $C_b(W_1)$ are divided between W_{b1} and W_{b2}.

Based on the above analysis, W_{b2} is easily acquired:

$$W_{b2} = C_b^*(W_1) \quad (12)$$

The total cost of replacements and repairs from the initial point of purchase to the time point Y_{bn}, when $Y_{bn} < W_1$, W_{b1} is given by:

$$W_{b1} = \sum_{i=1}^{n}[(C_3 + C_4 + \alpha)\cdot \int_0^{Y_{bi}} p\lambda(b+u)du + C_E(b) + C_5 + C_{LR1}] \quad (13)$$

Under this condition, the total warranty cost $C_b(W_1)$ of this phase is as follows:

$$C_b(W_1) = W_{b1} + W_{b2} \tag{14}$$

The expected cost $C_{bE}(W_1)$ of the Renewing FRW period is:

$$\begin{aligned} C_{bE}(W_1) &= \left[C_E(b) + C_5 + \frac{\alpha'}{\overline{G}_b(W_1)}\right] \cdot \frac{G_b^2(W_1)}{\overline{G}_b(W_1)} \\ &\quad + (C_3 + C_4 + \alpha) \cdot \frac{p}{1-p} \\ &\quad \cdot \left[\frac{G_b^2(W_1)}{\overline{G}_b(W_1)} - \overline{G}_b(W_1) Ln\overline{G}_b(W_1)\right], \quad p \neq 1 \end{aligned} \tag{15}$$

When $p = 1$, the product is only repaired as bad as old until a Type II failure occurs. The warranty period of the cost model is shown in Eq. (11). The expected warranty cost is given by:

$$\begin{aligned} C(W_1)_{P=1} &= E[(C_3 + C_4 + \alpha) \int_0^{W_1} p\lambda(u+b)\,du] \\ &= (C_3 + C_4 + \alpha) \int_0^{W_1} \lambda(u+b)\,du \end{aligned} \tag{16}$$

The derivative of $C_{bE}(W_1)$ with respect to b can be derived as:

$$\begin{aligned} &\frac{d[C_{bE}(W_1)]}{d(b)} \\ &= \left[(C_3 + C_4 + \alpha) \cdot \frac{p}{1-p} + \frac{d[C_E(b)]}{d(b)} + C_5 + \frac{\alpha'}{\overline{G}_b(W_1)}\right] \\ &\quad \cdot \left[\frac{2G_b(W_1) \cdot g_b(W_1)}{\overline{G}_b(W_1)} + \frac{G_b^2(W_1) \cdot g_b(W_1)}{\overline{G}_b^2(W_1)}\right] \\ &\quad - g_b(W_1)\left[1 + Ln\overline{G}_b(W_1)\right] \cdot (C_3 + C_4 + \alpha) \cdot \frac{p}{1-p}, \\ &\qquad p \neq 1 \end{aligned} \tag{17}$$

The model represents the relationship between average warranty costs and various parameters such as burn-in periods, warranty periods, and failure rates. From Eqs. (15), (16) and (17), the influence of the burn-in time on warranty costs is related to $g_b(W_1)$. For example, when $p = 0$, the increase in $C_{bE}(W_1)$ is correlated with $G_b(W_1)$.

C. WARRANTY COST MODEL OF THE PRW PHASE

In this section, the manufacturer provides prorated repairs when Type I failures occur during the time interval $[W_1, W]$. When a Type II failure occurs for the first time after the change point W_1, the manufacturer refunds a proportion of the sale price. A cost model for describing the third phase after burn-in testing and for FRW renewal is in turn developed. The warranty policy of this phase is shown in Fig. 4.

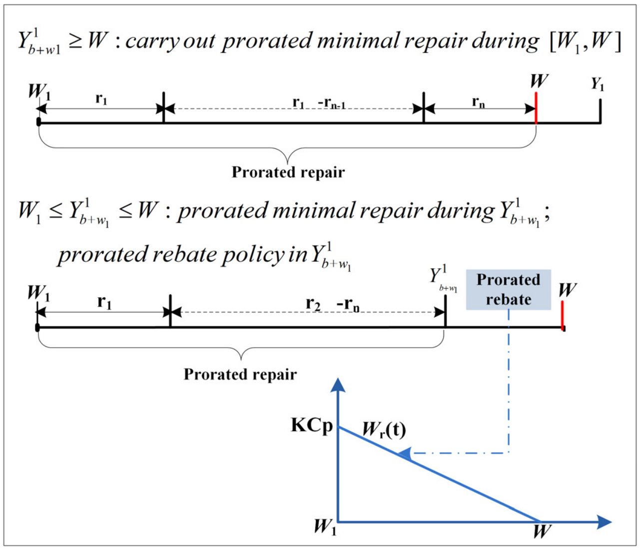


FIGURE 4. Prorated repair and rebate of the PRW policy in $[W_1, W]$.

Parameters of the cost model in this section include the following:

C_s: The sale price per unit;

$C_{bE}(W_p)$: The expected cost of the PRW period.

$Y^1_{b+w_1}$: The first time of Type II failure during the time interval $[W_1, W]$;

C_{Lo2}: Intangible per unit costs incurred during the PRW period, assuming that they are a linear function of the number of minimal repairs. β is the proportionality coefficient.

$$\begin{aligned} &Y^1_{b+w_1} \geq W : C_{Lo2} = \beta \int_0^t p\lambda(t)d \\ &Y^1_{b+w_1} < W : C_{Lo2} = \beta \int_0^t p\lambda(t)d + \delta \frac{W - Y^1_{b+w_1}}{W - W_1} \end{aligned} \tag{18}$$

$\mathfrak{R}$: The proportionality coefficient of the cost per unit of repair for the consumer and $\mathfrak{R} \in [0, 1]$;

W_{b3}: The total cost incurred during the *PRW* phase;

$W_r(x)$: The rebate when Type II failure occurs in interval $[W_1, W]$;

Nguyen and Murthy [15] assumed that:

$$W_r(t) = \begin{cases} kC_s(1 - \theta \dfrac{Y^1_{b+w_1} - W_1}{W - W_1}), & W_1 \leq Y^1_{b+w_1} \leq W \\ 0; & Y^1_{b+w_1} > W \end{cases} \tag{19}$$

where k is the factor of the sale price C_S, $k \in (0, 1]$. θ is the proportionality coefficient of the Type II failure period and warranty length, $\theta \in [0, 1]$.

When $Y^1_{b+w_1} \in [W_1, W]$, the cost model includes prorated minimal repair and the rebate costs, and the cost model W_{b3}

is given by:

$$W_{b3}^{1} = P\left(Y_{b+w_1}^{1} \le W\right) \cdot \left\{ \begin{array}{l} \left[(1-\Re)(C_3+C_4+\beta)\cdot \int_{W_1}^{Y_{b+w_1}^{1}} p\lambda(b+u)du \right] \\ + kC_s\left(1-\frac{\theta(W-Y_{b+w_1}^{1})}{W-W_1}\right) + \delta\frac{W-Y_{b+w_1}^{1}}{W-W_1} \end{array} \right\} \tag{20}$$

When $Y_{b+w_1}^{1} \ge W$: the cost model includes only the minimal repair cost and is given by:

$$W_{b3}^{2} = P\left(Y_{b+w_1}^{1} > W\right) \cdot \left[(1-\Re)(C_3+C_4+\beta)\cdot \int_{W_1}^{W} p\lambda(b+u)\,du \right] \tag{21}$$

Then:

$$W_{b3} = W_{b3}^{1} + W_{b3}^{2} \tag{22}$$

When $p \ne 1$, $W \ne W_1$, the expected costs incurred during the PRW period are:

$$\begin{aligned} E(W_{b3}) &= (1-\Re)(C_3+C_4+\beta)\cdot\frac{p}{1-p}\cdot G_{b+w_1}(W-W_1) \\ &+ kC_s\left[1-(1-\theta)\overline{G}_{b+w_1}(W-W_1) - \frac{\theta\int_{W_1}^{W}\overline{G}(b+t)dt}{(W-W_1)\overline{G}(b+W_1)}\right] \\ &+ (1-\delta)\overline{G}_{b+w_1}(W-W_1) + \frac{\delta\int_{W_1}^{W}\overline{G}(b+t)dt}{(W-W_1)\overline{G}(b+W_1)}, \quad p \ne 1 \end{aligned} \tag{23}$$

When $p = 1$, only a minimal failure occurs in this phase, and the expected warranty cost is:

$$E(W_{b3})_{p=1} = (1-\Re)(C_3+C_4+\beta)\cdot \ln\left[\frac{\overline{G}(W_1+b)}{\overline{G}(W+b)}\right] \tag{24}$$

The derivative of $C_{bE}(W_1)$ with respect to b ($p \ne 1$, $W \ne W_1$) can be derived as:

$$\begin{aligned} \frac{d[E(W_{b3})]}{d(b)} &= (1-\Re)(C_3+C_4+\beta)\cdot\frac{p}{1-p}\cdot g_{b+w_1}(W-W_1) \\ &- (1-\delta+kC_s\theta)\,g_{b+w_1}(W-W_1) + (\delta-kC_s\theta) \\ &\cdot \frac{\left[G_{b+w_1}(W-W_1)\cdot\overline{G}(b+W_1) - h(b+W_1)\int_{w_1}^{W}\overline{G}(b+t)dt\right]}{W-W_1} \end{aligned} \tag{25}$$

Thus, from Eqs. (23) and (25), the warranty cost is related to various parameters (W_1/W, W, b, p) and to the distribution function. Eq. (25) shows effects of the burn-in time on the PRW warranty cost. The effect is related not only to the life distribution but also to various parameters. To determine the optimal warranty length, an analysis of the influence of the W_1/W, W, b, p on warranty costs is required.

D. THE COMPREHENSIVE WARRANTY COST MODEL

From the analyses shown in parts A, B and C of this section, we can determine the total cost to the manufacturer of the three phases.

As each party cost is independent, from the equations shown in parts A, B and C, the total expected warranty cost C is:

$$C = C(b) + C_b(W_1) + W_{b3} \tag{26}$$

It is difficult to describe the comprehensive warranty cost with random variables. However, the expectation of C is often used.

From our phased analysis of warranty costs, the CWCM analysis can be divided into $p = 1$, $p = 0$ and $0 < p < 1$ based on the value of p.

1) First, when $p = 1$, the repaired-product failure of the warranty period requires only minimal repairs. Based on Eqs. (8), (16) and (24), the warranty cost model for the three phases is easily obtained. The warranty cost model ($p = 1$) is:

$$\begin{aligned} C_E(b) &= C_E(b)_{p=1} + C(W_1)_{P=1} + E(W_{b3})_{p=1} \\ &= C_0 + C_1 + C_2 b + C_3\int_0^b \lambda(u)du + [C_3+C_4+\alpha] \\ &\cdot \int_0^{W_1}\lambda(u+b)\,du + (1-\Re)(C_3+C_4+\beta) \\ &\cdot \int_{W_1}^{W}\lambda(u+b)\,du \end{aligned} \tag{27}$$

2) When $0 < p < 1$, Type I and Type II failures occur in the three phases, and the warranty cost model is complex. Based on Eqs. (5), (15), (23) and (25), the expected warranty cost model of each component is given by:

$$\begin{aligned} E(C) &= E[C(b)] + E[C_b(W_1)] + E(W_{b3}) \\ &= C_E(b) + C_{bE}(W_1) + C_{bE}(w_p) \end{aligned} \tag{28}$$

3) When $p = 0$, only non-repaired failure occurs. In the analysis of warranty costs for non-repairable products and $G(t) = F(t)$, the expected warranty cost model of each component is given by:

$$\begin{aligned} E(C) &= E[C(b)] + E[C_b(W_1)] + E(W_{b3}) \\ &= \left\{\left[\frac{1}{\overline{F}(b)} + \overline{F}(b) - 1\right](C_0+C_1) + \left[\overline{F}(b)b + \int_0^b \overline{F}(u)du\right]C_2\right\} \end{aligned}$$

TABLE 1. Warranty data.

Region	PURCHASE DATE	Failure Data	Failure Component	Maintenance Measure	Service Mode	Selling Price	Traffic Expenses	Operating Cost	Others
Beijing	27/8/2013	11/9/2013	Computer board	Replace components	Room service	2000	7	30	20

$$\cdot\left(1+\frac{F_b^2(W_1)}{\overline{F}_b(W_1)}\right)+\left(C_5+\frac{\alpha'}{\overline{F}_b(W_1)}\right)\cdot\frac{F_b^2(W_1)}{\overline{F}_b(W_1)}$$
$$+kC_s\left[1-(1-\theta)\overline{F}_{b+w_1}(W-W_1)-\frac{\theta\int_{W_1}^{W}\overline{F}(b+t)dt}{(W-W_1)\overline{F}(b+W_1)}\right]$$
$$+(1-\delta)\overline{F}_{b+w_1}(W-W_1)+\frac{\delta\int_{W_1}^{W}\overline{F}(b+t)dt}{(W-W_1)\overline{F}(b+W_1)} \quad (29)$$

From the equation shown above, the following results can be observed. The failure time cumulative distribution of failure Type II ($G(t)$) is central to the comprehensive cost model, which includes p and the failure rate λ(t). Thus, the total cost is affected by intrinsic properties of the product. Then, the total cost is also affected by the burn-in time b and the warranty periods $[0, W_1]$ and $[W_1, W]$. Therefore, a cost analysis can be used to determine the optimal warranty length and burn-in time. From the comprehensive warranty cost model, a problem related to the optimal warranty length under a comprehensive warranty policy was analyzed.

IV. A NUMERICAL CASE STUDY

A. WARRANTY DATA ANALYSIS

In this study, warranty data were obtained on a certain model of washing machine developed by a well-known household appliance manufacturer based in China. The FRW warranty strategy is applied. The complete washing machine warranty period spans 1 year, and the warranty period of key components (PC board, driver board, display board, etc.) and the service life cover 3 and 5 years, respectively. Washing machine warranty data for July 2012 to May 2014 were collected and analyzed and are shown in Table I.

After processing the data, 19,159 unrelated values (e.g., consulting and marketing) were removed, and 111,061 valid values were obtained. On average, we obtained 4,829 values for each month, and the size of the sample used has statistical significance. The washing machine offers numerous additional components as part of a system. The component with the highest failure frequency and the failure mode are key factors that affect maintenance costs and reliability levels. To simplify the analysis, the main components with more failure times are analyzed in the paper. The failure time of main components of the washing machine is shown in Table II, and the failure frequency is presented in Fig. 5.

In total, 1,113,109 computer boards were produced during this period of time t based on our statistics. The failure value

TABLE 2. Failure times of main components of the washing machine.

Mode number	Failure component	Failure number
1	Computer board	19693
2	Driver board	10582
3	Complete machine	5398
4	Micro switch	5016
5	Damping clutch	2934
6	Water level switch	2681
7	Electric machinery	5557
8	Drain pipe	1336
9	Birthday products	1230
10	Power switch	922
11	Drain motor	854
12	Control panel film	732
13	Base Frame	698
14	Top lid	673
15	Control panel seat	664

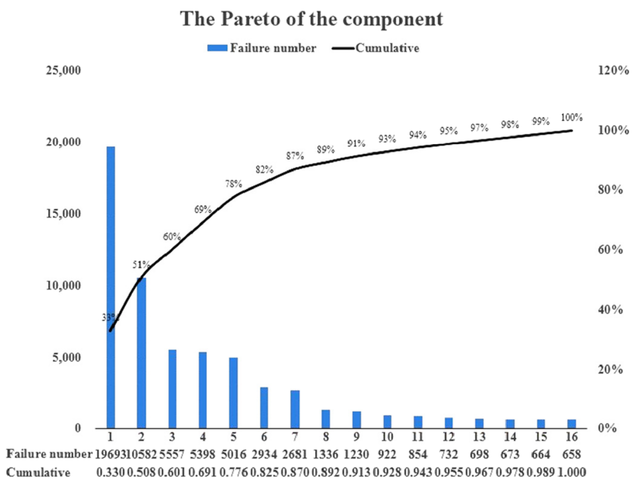


FIGURE 5. Failure frequency of main components of the washing machine.

is 19,693, and this accounts for 1.76% of total production. Most forms of computer board failure are not easy to detect or address (e.g., circuit board burning, line corrosion, keypad failures, etc.). Therefore, a failed product must be replaced after failure occurs, and we assume that the Type I failure rate occurs with a probability value of $p = 0$, that the burn-in time is 3 days (0.01 years), and that the computer board warranty period spans 3 years as a main component. Manufacturers offer refunds (the proportionality coefficient is 0.3) for component failures over 3-7 years.

B. WARRANTY DATA DISTRIBUTION EVALUATION

From the warranty data for each month from July 2012 to May 2014, the failure rate is obtained from the number of failures divided by the number of products that are

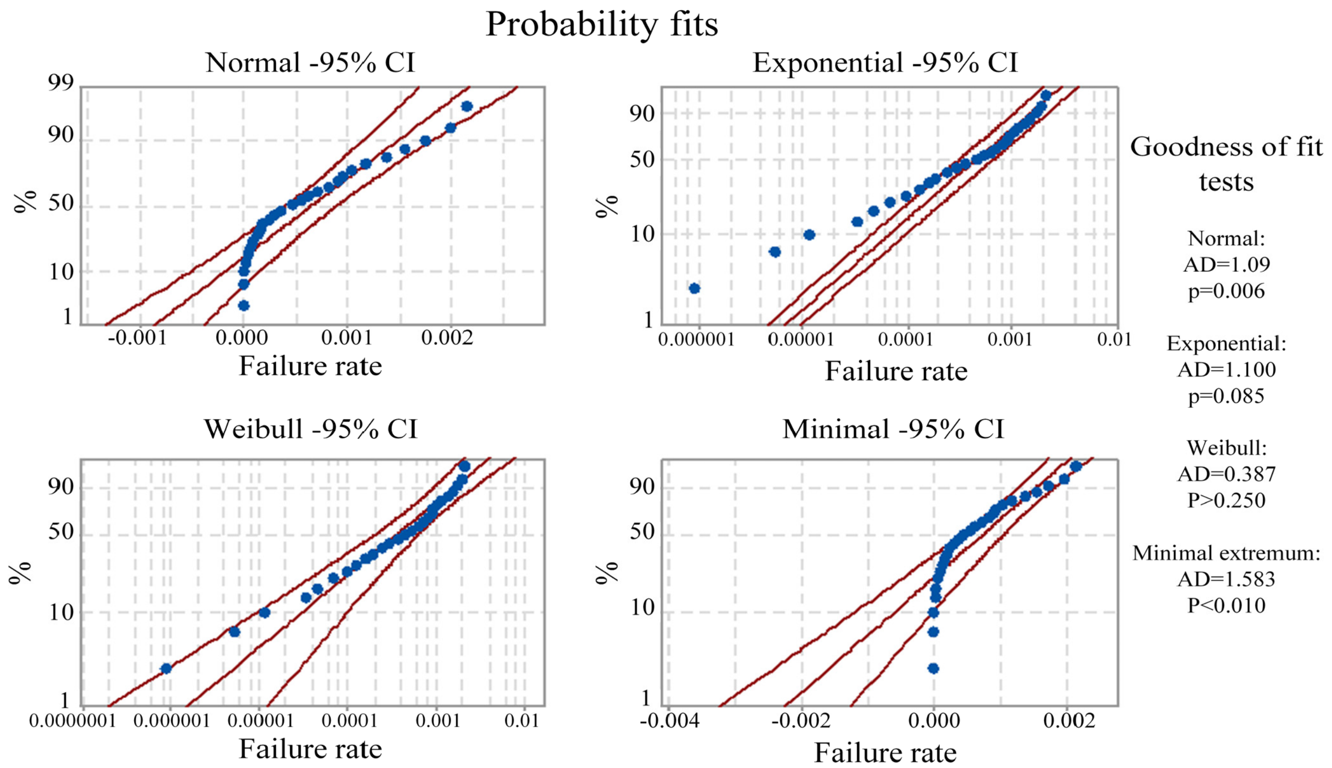
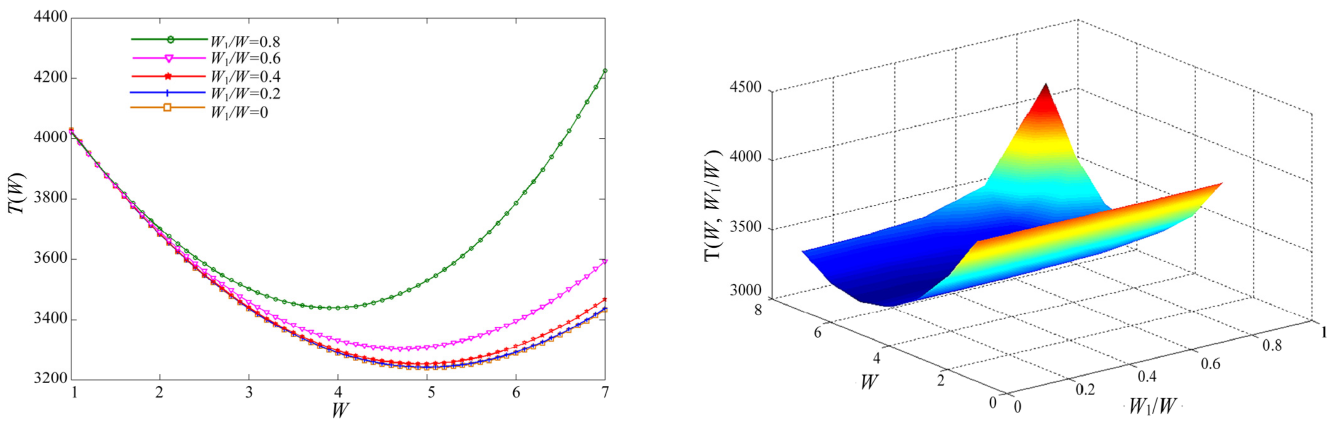


FIGURE 6. **Probability plots for failure rate distributions.**



(a) (b)

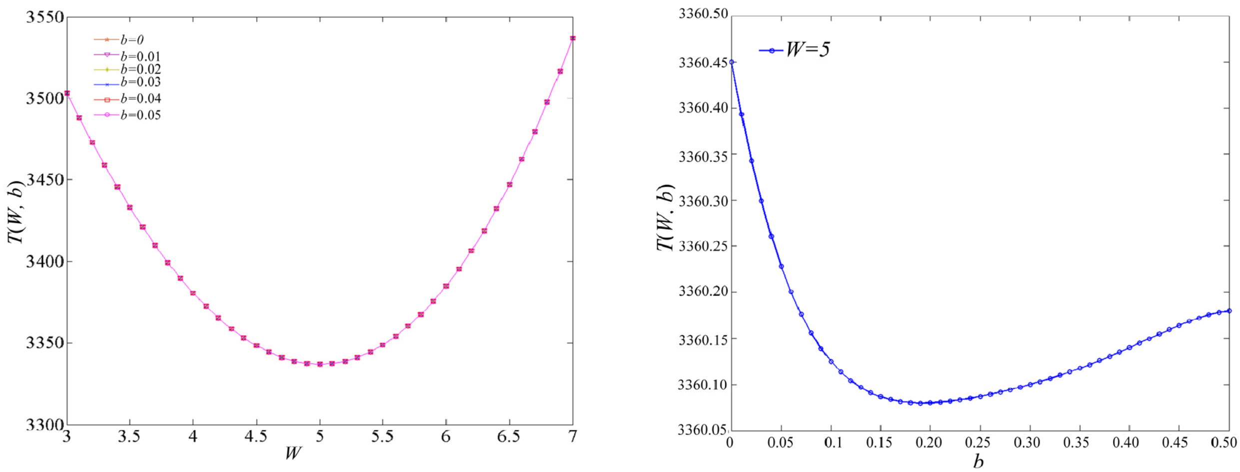
FIGURE 7. **Estimating the warranty cost ($b = 0.01$). (a) The relation between W and $T(W)$. (b) The relation between $T(W, W_1/W)$ and W_1/W.**

still in normal operation in the previous month. The use time and failure rate of the computer board are shown in Table III.

After fitting the failure rate data in Table III, the goodness-of-fit of the normal distribution, the exponential distribution, the Weibull distribution, and the minimum extreme value distribution (95% confidence interval) are derived as shown in Fig. 6. The collective failure rate provides a good fit to the Weibull distribution as specified by the smaller value of the Anderson-Darling (AD=0.387) metric.

By determining the failure probability distribution in [37], the shape parameter $\alpha = 16.3932$ and scale parameter $\beta = 2$ of the Weibull distribution are determined via the Weibull parameter estimation method. The cumulative failure distribution function of the computer board is as follows (t/month, T/year):

$$\begin{aligned} F(t) &= 1 - \exp[(-t/16.693)^2] \\ &= 1 - \exp[(-T^*12/16.693)^2] \end{aligned} \tag{30}$$



(a) (b)

FIGURE 8. Estimating the warranty cost ($W_1 = 3$). (a) The relation between W and $T(W, b)$. (b) The relation between b and $T(W, b)$.

TABLE 3. Use time and failure rate of the computer board.

Use Time / Month	Failure Number	λ	Use Time / Month	Failure Number	λ
0	1	8.98E-07	14	617	5.55E-04
1	6	5.39E-06	15	700	6.31E-04
2	13	1.17E-05	16	794	7.16E-04
3	37	3.32E-05	17	923	8.33E-04
4	53	4.76E-05	18	1011	9.13E-04
5	77	6.92E-05	19	1052	9.51E-04
6	112	1.01E-04	20	1171	1.06E-03
7	146	1.31E-04	21	1313	1.19E-03
8	178	1.6E-04	22	1531	1.39E-04
9	214	1.92E-04	23	1714	1.56E-03
10	278	2.5E-04	24	1931	1.76E-03
11	335	3.01E-04	25	2194	1.99E-03
12	412	3.71E-04	26	2364	2.16E-03
13	516	4.64E-04			

TABLE 4. Parameter values of the cost model.

Parameter	Value	Parameter	Value
b_0	35	p	0
b_1	10	1-p	1
K	29.6	α'	4
C_0	70	k	0.3
C_1	3	θ	0
C_2	10	δ	1
C_5	34.4	Cs	120

C. ANALYSIS OF WARRANTY COST AND OPTIMAL WARRANTY LENGTH

Based on our analysis of the failure data, 1) when the product fails, it will not fail again within a short period of time. Thus, the total number of failures is equal to the number of product failures. 2) The repair time is much shorter than the time of use and it can be disregarded. 3) The computer board is produced through the same production process, and the reliability value is similar, reflecting the hypothesis of the model shown in Section III. In this section, the expected total cost function is used as follows [38]:

$$T(W) = B(W) + K + C(W) = (b_0 - b_1 W)^2 + K + E(C) \tag{31}$$

where $T(W)$ is the expected total costs per unit; $C(W)$ is the expected warranty cost per unit; $B(W)$ is the function associated with warranty benefits; b_0 is the initial value of warranty benefits; b_1 is the proportionality coefficient of warranty length and the warranty costs; and K is the fixed management cost related to the warranty. From the warranty cost for the computer board, the mean value of traffic expenses is ¥8.2, and that of operating costs is ¥26.2. Therefore, the additional replacement cost is ¥34.4. From the washing machine warranty charge standard and market conditions, the maintenance cost of the computer board is ¥45, the sales price is ¥120, the manufacturing cost is ¥70, and the warranty management cost is ¥29.6. Parameter values of the cost model are shown in Table IV.

When $p = 0$, $W = 7$, $b = 0.01$ and $W_1 = 3$, the expected warranty cost $C_E = ¥34.68$ can be determined from Eqs. (5), (15), (23) and (28). The warranty cost derived from the warranty data is ¥35.394. The error between the model and actual value is 2.01%, validating the availability of the model shown in Section III.

Based on the model validation, the burn-in time $b = 0.01$ is assumed to be constant. Parameter values of the cost model are shown in Table IV. When the numerical calculation is used to estimate the optional warranty length, the corresponding relation between W, W_1 and $T(W, W_1/W)$ is as shown in Fig. 7. When $W_1 = 3$, the corresponding relation between W, b and $T(W, b)$ is as shown in Fig. 8.

This analysis implies that the minimum expected total cost $T(W)$ is achieved when $b = 0.01$, $W_1 = 0$, and $W = 5$. As is shown in Fig. 7. (a) and (b), the warranty cost shows a sharp increase when $W_1 \leq 3$, $W \geq 6$ and $W_1/W \geq 0.6$. Therefore, manufacturers should avoid these parameters when developing warranty periods.

As is shown in Fig. 8. (a) and (b), the minimum expected cost is achieved when $W = 5$ and $b = 0.02$. The estimated cost of these parameters is not particularly sensitive to the burn-in time. Above all, for this product, the optimal warranty period for the minimum expected warranty cost is $W^* = 5$, $W_1^* = 3$ and $b^* = 0.02$.

V. CONCLUSIONS

In the present paper, we present a comprehensive warranty cost model for repairable products presenting two types of failure under burn-in and FRW/PRW policies. The CWCM includes manufacturing costs, burn-in costs, warranty costs and intangible costs. Two types of failure (minimal failure and catastrophic failure) are assumed to occur through three phases (including burn-in/FRW/PRW). The framework for warranty costs is proposed for the analysis of product failure types of the three phases. Through the warranty cost model, the total expected cost to manufacturers is determined. The total cost is impacted by intrinsic properties of a product, burn-in times, warranty length and manufacturer costs per unit during the three phases. Manufacturers can reference functions listed in the paper when determining burn-in periods, warranty policies and prices of reparable products. Ultimately, we determined the failure rate distribution and optimal warranty length from the warranty cost model and from after-sales service data. In addition, the calculated optimal warranty length can guide the development of more reasonable warranty policies. The warranty cost undergoes a sharp increase when W_1 is too small, when W is too large and when the ratio of W_1 to W is too large. Therefore, manufacturers should avoid these parameters when determining warranty length.

This paper mainly focuses on products during warranty periods, while sales delays, which also have strong effects on product performance and on warranties, are not considered. The warranty cost model considers product use periods. However, it is not suited to rapid product updates. The study of warranty strategies is one-dimensional. The analytical method applied differs from two-dimensional numerical analysis and warranty data analysis methods, which we plan to discuss further.

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