Received April 29, 2017, accepted May 25, 2017, date of publication June 5, 2017, date of current version July 24, 2017.

Digital Object Identifier 10.1109/ACCESS.2017.2711488

Dynamic Output Feedback Fuzzy Control of Large-Scale Nonlinear Networked Systems: A Two-Channel Triggering Approach

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This work was supported in part by the Natural Science Foundation of Fujian Province, China, under Grant 2017J01781, in part by the National Natural Science Foundation of China under Grant 61603221, in part by the Natural Science Foundation of Shandong Province under Grant ZR2016FB11, in part by the China Postdoctoral Science Foundation under Grant 2017M610437, in part by the Special Foundation for Postdoctoral Science Foundation of Shandong Province under Grant 201601014, in part by the Applied Research Project for Postdoctoral Researchers of Qingdao under Grant 2016109, in part by the Research Fund for the Taishan Scholar Project of Shandong Province of China, and in part by the Open Fund Project of Fujian Provincial Key Laboratory of Information Processing and Intelligent Control, Minjiang University, under Grant MJUKF201731.

ABSTRACT Decentralized output-feedback (DOF) event-triggering control for large-scale nonlinear networked systems is examined in this paper. The Takagi–Sugeno model is applied to describe each nonlinear subsystem, where it shares the communication information through networks. A DOF control scheme with event-triggering is proposed, where two event-triggering mechanisms are placed in the sensor and in the actuator, respectively. Our goal is to design the DOF controller, which not only guarantees the stability of closed-loop control system but also reduces the data communication in the sensor-to-controller channels and controller-to-actuator channels. First, a novel model transformation is presented, where the closed-loop control system is reconstructed as a constant-delay system with extra feedback interconnections. By introducing a relaxing Lyapunov–Krasovskii functional combining with the scaled small gain theorem, the co-design consisting of the controller gains, event-triggered parameter, and sampled period is derived in the form of linear matrix inequality. The effectiveness of the proposed method is validated via a numerical example.

INDEX TERMS Large-scale fuzzy systems, two-channel triggering, decentralized output-feedback control, co-design.

I. INTRODUCTION

With the prompt development of digital technology, communication networks are usually used instead of point-to-point connections as they bring prominent advantages, such as reduced weight, low cost, power requirements, and simple installation [1], [2]. Unfortunately, some imperfections induced by communication networks, such as packet dropouts, quantization errors, and time delays, can degrade significantly the closed-loop control performance and may even result in instability [3]–[5]. Recently, network-based control system (NCS) has received considerable attention, and a large number of results have been published for studying these imperfections, see [6]–[8], and the references therein. It should be pointing out that the analyzing or designing NCS often achieves tradeoffs among the imperfections induced by networks. More specifically, sending larger information-packets will alleviate packet dropouts and quantization errors but typically bring in longer transmission time [9], [10]. In this way, one important issue for the application of NCS is to identify methods or techniques in order to utilize the limited network bandwidth effectively.

For the application of NCS, control tasks consisting of sampling, quantizing, sending plant outputs, and computing, execute control inputs, are often carried out by computers [11]. To deal with the control tasks, the previous time-triggered control often results in congestion or collision or longer listening time in the limited network bandwidth as in which the control task is processed in a periodic form. Recently, there is an increasing interest in the event-triggering control that aims at the reduction of information transmissions. The basic principle of event-triggering control is to send feedback signals in terms of a specified threshold [12]–[18]. To date, there exist several different aliases for the event-based control, such as self-triggered feedback [13], event-triggered feedback [14]–[16], state-triggered feedback [17], interrupt-based feedback [18]. More recently, the event-triggering control has been developed for T-S fuzzy networked systems [19]–[22]. To mention a few, the problem of event-triggering filter for T-S fuzzy systems was investigated in [19]. The design result on event-triggered fault detection was considered for T-S fuzzy networked systems by using SOS solvers [20]. The co-design problem with event-triggered state-feedback control for a class of T-S fuzzy sampled-data systems was reported in [21]. In [22], the problem of fuzzy dynamic output-feedback stabilization for a class of discrete-time T-S fuzzy systems was studied under an event-triggered scheme. We are aware of few attempts making on decentralized event-triggering control for large-scale fuzzy networked systems, which motivates us for this study.

In this paper, the problem of decentralized event-triggered output feedback control is studied for a class of large-scale nonlinear network systems, where each nonlinear subsystem is represented by a T-S model, and exchanges their information through networks. Our goal is to design a decentralized DOF controller with event-triggering, which guarantees the stability of the closed-loop control system and reduces the information communication in both the S-C channels and C-A channels. Firstly, by using the input delay approach, the control system with sampled-data measurement is reconstructed as a continuous-time system with time-varying delay. Then, we model the time-varying delay and event-triggered counterpart as disturbances. By hauling out these disturbances, the control system is reformulated as a constant-delay system with extra inputs and outputs. In terms of the new model, and introducing a relaxing LKF combined with the advantage of SSG theorem, the co-design consisting of the DOF controller gains, event-triggering parameter, and sampling period for the considered system is derive in the form of LMIs. Finally, the advantage of the proposed method is validated by a numerical example.

In summary, the main contributions are as below: i) As the first attempt, the problem of decentralized event-triggered DOF control with interconnections is investigated for large-scale T-S fuzzy systems; ii) Two event-triggering mechanisms (ETMs) placed respectively in the sensor and in the actuator are proposed, such that the data communication is reduced in the S-C channels and C-A channels; iii) A relaxing LKF combined with SSG method is introduced, and the less conservative results on the DOF event-triggering controller design are derived in terms of LMIs comparing with the method proposed in [23].

Notations. $\Re^n$ denotes the Euclidean space with n -dimensions. $\Re^{n\times m}$ is the set of $n\times m$ matrices. $P > 0\,(\geq 0)$ is positive definite (positive semidefinite). Sym$\{A\}$ denotes $A + A^T$. $\mathbf{I}_n$ and $0_{m\times n}$ denote the $n \times n$ identity matrix and $m \times n$ zero matrix, respectively. $\mathbb{N}$ represents the sets of positive integers. The subscripts n and $n \times m$ are omitted when the size is not relevant or can be determined from the context. $A \in \Re^{n\times n}$, A^{-1} and A^T are the inverse and transpose of the matrix A, respectively. diag$\{\cdots\}$ is a block-diagonal matrix. $l_2[0, \infty)$ refers to the space of square-summable infinite vector sequences over $[0, \infty)$. The notation $\|\cdot\|$ is the Euclidean vector norm, and $\|\cdot\|_2$ represents the usual $l_2[0, \infty)$ norm. The notation $\star$ indicates the symmetric term.

II. PROBLEM FORMULATION

This paper considers a continuous-time large-scale system containing N nonlinear subsystems, where T-S model can be applied to represent the i-th nonlinear subsystem as below:

Plant Rule $\mathscr{R}_i^l$: **IF** $\zeta_{i1}(t)$ is $\mathscr{F}_{i1}^l$ and $\zeta_{i2}(t)$ is $\mathscr{F}_{i2}^l$ and $\cdots$ and $\zeta_{ig}(t)$ is $\mathscr{F}_{ig}^l$, **THEN**

$$\begin{cases} \dot{x}_i(t) = A_{il}x_i\,(t) + B_{il}\hat{u}_i(t_k^i) + \sum\limits_{\substack{j=1\\ j\neq i}}^{N} \bar{A}_{ijl}x_j(t) \\ y_i(t) = C_i x_i(t), l \in \mathscr{L}_i := \{1, 2, \ldots, r_i\}, t \in [t_k, t_{k+1}) \end{cases} \tag{1}$$

where $i \in \mathcal{N} := \{1, 2, \ldots, N\}$, $\mathscr{R}_i^l$ is the l -th fuzzy inference rule, r_i is the number of inference rules, $\mathscr{F}_{i\phi}^l$ $(\phi = 1, 2, \ldots, g)$ is the fuzzy set; $x_i(t) \in \Re^{n_{xi}}$, $\hat{u}_i(t_k^i) \in \Re^{n_{ui}}$, $y_i(t) \in \Re^{n_{yi}}$ are the system state, the control input applied to the system at the instant t_k^i, and the measured output, respectively; $\zeta_i(t) := [\zeta_{i1}(t), \zeta_{i2}(t), \ldots, \zeta_{ig}(t)]$ are some measurable variables; $\{A_{il}, B_{il}, C_i\}$ denotes the l -th local model, and $\bar{A}_{ijl}$ is the interconnection matrix between the i-th and k-th subsystem.

Based on fuzzy blending, the i-th overall fuzzy model is given by

$$\begin{cases} \dot{x}_i(t) = A_i(\mu_i)x_i\,(t) + B_i(\mu_i)\hat{u}_i(t_k^i) + \sum\limits_{\substack{j=1\\ j\neq i}}^{N} \bar{A}_{ij}(\mu_i)x_j(t) \\ y_i(t) = C_i x_i(t), t \in [t_k, t_{k+1}), i \in \mathcal{N} \end{cases} \tag{2}$$

where

$$\begin{aligned} A_i(\mu_i) &:= \sum_{l=1}^{r_i} \mu_{il}A_{il}, B_i(\mu_i) := \sum_{l=1}^{r_i} \mu_{il}B_{il}, \\ \bar{A}_{ij}(\mu_i) &:= \sum_{l=1}^{r_i} \mu_{il}\bar{A}_{ijl}. \end{aligned} \tag{3}$$

Remark 1: This paper considers the larger-scale system with nonlinear interconnection $\bar{A}_{ijl}$ in (1) instead of the ones with linear interconnection $\bar{A}_{ij}$ proposed in [23]. It is noted that more challenges will be induced into the control of large-scale fuzzy systems when considering nonlinear interconnection.

Before moving on, we require firstly the following assumptions.

Assumption 1: The sampler is clock-driven in each subsystem. Let h_i denotes the upper bound of sampling intervals, it has

$$t_{k+1}^i - t_k^i \le h_i, k \in \mathbb{N} \tag{4}$$

where $h_i > 0$.

Assumption 2: For each subsystem, we assume that the S-C and C-A channels are closed via networks.

Assumption 3: The zero-order-hold is event-driven, thus the latest sampling data are held until the next transmitting ones come.

It is noted that in networked control systems, the traditionally time-triggered control is unfavourable because of the limited bandwidth. Here, inspired in [14], two ETMs will be used to reduce data transmissions via networks. One is located in the sensor system that it determines when the system output should be transmitted in the S-C channels. Another is put into the controller system that it determines when the control input should be transmitted in the C-A channels. In the way, both the measured output and control input involve in the sampled-data measurements and event-triggered control. Therefore, the data transmission can be reduced in both the S-C and C-A channels. Thus, a decentralized DOF fuzzy event-triggering controller is proposed as below:

Controller Rule $\mathscr{R}_i^l$: **IF** $\zeta_{i1}(t)$ is $\mathscr{F}_{i1}^l$ and $\zeta_{i2}(t)$ is $\mathscr{F}_{i2}^l$ and $\cdots$ and $\zeta_{ig}(t)$ is $\mathscr{F}_{ig}^l$, **THEN**

$$\begin{cases} \dot{x}_{ci}(t) = A_{c1il}x_{ci}(t) + A_{c2il}x_{ci}\left(t_k^i\right) + B_{cil}\hat{y}_i(t_k^i) \\ u_i(t) = C_{ci}x_{ci}(t), t \in [t_k, t_{k+1}), i \in \mathcal{N} \end{cases} \tag{5}$$

where $x_{ci}(t) \in \Re^{n_{xi}}$ is the controller state, $\hat{y}_i(t_k^i)$ is the measured output applied to the controller at the instant t_k^i, and $\{A_{c1il}, A_{c2il}, B_{cil}, C_{ci}\}$, $l \in \mathscr{L}_i$, $i \in \mathcal{N}$ are designed controller gains with compatible dimensions.

Similarly, the overall DOF fuzzy event-triggering controller can be obtained as below,

$$\begin{cases} \dot{x}_{ci}(t) = A_{c1i}(\mu_i)x_{ci}(t) + A_{c2i}(\mu_i)x_{ci}\left(t_k^i\right) + B_{ci}(\mu_i)\hat{y}_i(t_k^i) \\ u_i(t) = C_{ci}x_{ci}(t), t \in [t_k, t_{k+1}), i \in \mathcal{N} \end{cases} \tag{6}$$

where

$$A_{c1i}(\mu_i) := \sum_{l=1}^{r_i} \mu_{il}A_{cil}, A_{c2i}(\mu_i) := \sum_{l=1}^{r_i} \mu_{il}A_{c2il},$$
$$B_{ci}(\mu_i) := \sum_{l=1}^{r_i} \mu_{il}B_{cil}. \tag{7}$$

For the implementation of the controllers given by (6), a solution is proposed in Fig. 1, where $\hat{y}_i\left(t_k^i\right)$ denotes the latest measurement output transmitting successfully to the controller; $\hat{u}_i\left(t_k^i\right)$ denotes the latest control input transmitting successfully to the actuator; SP^s and SP^c are the samplers in the sensor and in the controller, respectively; BF^s and BF^c are buffers in the sensor and in the controller, respectively;

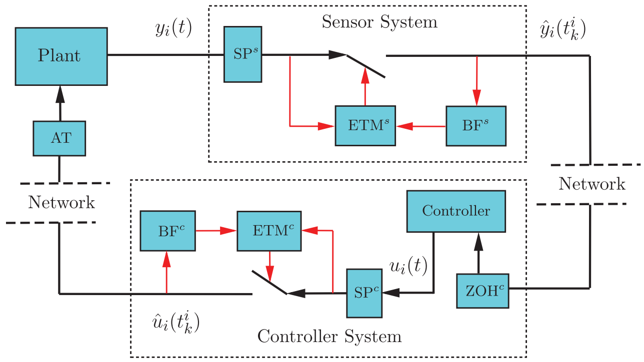

FIGURE 1. **An event-triggered DOF control scheme.**

ZOH^c is zero-order hold in the controller; AT is the actuator. As shown in Figure 1, the sensor system consists of an SP^s, an BF^s and an ETM^s, the controller system consists of an ZOH^c, and SP^c, an BF^c and an ETM^c. At each sampling instant, the ETM^s and ETM^c can be determined respectively when the measurement output and control input should be transmitted. Hence, the solution leads to a reduction of communication in both the S-C and C-A channels.

To implement the presented solution, the referred two ETMs can be represented as

$$\text{ETM}^s\text{: } y_i\left(t_k^i\right) \text{ is sent} \Leftrightarrow \left\| y_i\left(t_k^i\right) - \hat{y}_i\left(t_{k-1}^i\right)\right\| > \sigma_{yi}\left\|\hat{y}_i\left(t_k^i\right)\right\|, \tag{8}$$

$$\text{ETM}^c\text{: } u_i\left(t_k^i\right) \text{ is sent} \Leftrightarrow \left\| u_i\left(t_k^i\right) - \hat{u}_i\left(t_{k-1}^i\right)\right\| > \sigma_{ui}\left\|\hat{u}_i\left(t_k^i\right)\right\|, \tag{9}$$

where $\{\sigma_{yi}, \sigma_{ui}\}$ are two appropriate positive scalars.

Based on the above operation, an event-triggering proposal can be given as:

$$\hat{y}_i\left(t_k^i\right) = \begin{cases} y_i\left(t_k^i\right), & \text{when } \left\|y_i\left(t_k^i\right) - \hat{y}_i\left(t_{k-1}^i\right)\right\| > \sigma_{yi}\left\|y_i\left(t_k^i\right)\right\|, \\ \hat{y}_i\left(t_{k-1}^i\right), & \text{when } \left\|y_i\left(t_k^i\right) - \hat{y}_i\left(t_{k-1}^i\right)\right\| \le \sigma_{yi}\left\|y_i\left(t_k^i\right)\right\|, \end{cases} \tag{10}$$

$$\hat{u}_i\left(t_k^i\right) = \begin{cases} u_i\left(t_k^i\right), & \text{when } \left\|u_i\left(t_k^i\right) - \hat{u}_i\left(t_{k-1}^i\right)\right\| > \sigma_{ui}\left\|u_i\left(t_k^i\right)\right\|, \\ \hat{u}_i\left(t_{k-1}^i\right), & \text{when } \left\|u_i\left(t_k^i\right) - \hat{u}_i\left(t_{k-1}^i\right)\right\| \le \sigma_{ui}\left\|u_i\left(t_k^i\right)\right\|. \end{cases} \tag{11}$$

It follows from (2) and (6) that the resulting closed-loop control system is described by

$$\begin{cases} \dot{x}_i(t) = A_i(\mu_i)x_i(t) + B_i(\mu_i)\hat{u}_i(t_k^i) + \sum_{j=1, j\neq i}^{N} \bar{A}_{ij}(\mu_i)x_j(t) \\ \dot{x}_{ci}(t) = A_{c1i}(\mu_i)x_{ci}(t) + A_{c2i}(\mu_i)x_{ci}\left(t_k^i\right) \\ \qquad + B_{ci}(\mu_i)\hat{y}_i(t_k^i), t \in [t_k, t_{k+1}), i \in \mathcal{N}. \end{cases} \tag{12}$$

III. MAIN RESULTS

This section firstly proposes a novel model transformation, where the closed-loop control system is reconstructed as a constant time-delay system with extra feedback interconnections. Next, based on the new model, a relaxing LKF combined with the SSG theorem is imported, the stability analysis and co-design for the considered system will be presented, respectively. It will be shown that the DOF controller gains, event-triggered parameter, and sampling period can be obtained concurrently via solving a set of LMIs.

A. MODEL TRANSFORMATION

Here, we borrow the input delay approach from [25], the output and input with the sampled-data measurement are‘ reformulated as

$$\hat{y}_i(t_k^i) = \hat{y}_i(t-\eta_i(t)), \quad \hat{u}_i(t_k^i) = \hat{u}_i(t-\eta_i(t)), \tag{13}$$

where $\eta_i(t) = t - t_k^i$. It follows from Assumptions 1-3 that

$$0 \le \eta_i(t) < h_i, \quad t \in [t_k, t_{k+1}),\ k \in \mathbb{N}. \tag{14}$$

It is noted that in the majority of the proposed topologies the delay-dependent criteria are obtained by using the direct Lyapunov method. Another method names the input-output approach, which is based on indirect framework combined the model reformulation and the scaled small gain theorem, has been developed for a larger class of systems with time-delays [26], [30], [31]. This paper will formulate the two-channel event-triggering problem into the framework of input-output stability. To do so, we first model the event-triggered counterpart as disturbances [14], that is

$$e_{yi}(t) = \hat{y}_i(t-\eta_i(t)) - y_i(t-\eta_i(t)), \tag{15}$$
$$e_{ui}(t) = \hat{u}_i(t-\eta_i(t)) - u_i(t-\eta_i(t)). \tag{16}$$

Inspired in [26], we will approximate the uncertain term $\bar{x}_i(t-\eta_i(t))$ by $\bar{x}_i(t)$ and $\bar{x}_i(t-h_i)$, thus it yields

$$\begin{aligned}\frac{h_i}{2}e_{di}(t) &= \bar{x}_i(t-\eta_i(t)) - \frac{1}{2}\left[\bar{x}_i(t) + \bar{x}_i(t-h_i)\right] \\ &= \frac{1}{2}\int_{-h_i}^{-\eta_i(t)} \dot{\bar{x}}_i(t+\beta)\,d\beta - \frac{1}{2}\int_{-\eta_i(t)}^{0} \dot{\bar{x}}_i(t+\alpha)\,d\beta \\ &= \frac{1}{2}\int_{-h_i}^{0} \rho_i(\beta)\,\dot{\bar{x}}_i(t+\beta)\,d\beta, \end{aligned}\tag{17}$$

where

$$\rho_i(\beta) = \begin{cases} 1, & \text{if } \beta \le -\eta_i(t), \\ -1, & \text{if } \beta > -\eta_i(t). \end{cases}$$

Denoting that $\bar{x}_i(t) = \begin{bmatrix} x_i^T(t) & x_{ci}^T(t)\end{bmatrix}^T$, and substituting (13)-(17) into (12), the closed-loop control system is rewritten as the feedback interconnections:

$$\mathcal{R}_{i1}: \begin{cases} \dot{\bar{x}}_i(t) = \mathscr{A}_i(\mu_i)\bar{x}_i(t) + \frac{1}{2}\mathscr{A}_{di}(\mu_i)\bar{x}_i(t-h_i) \\ \qquad + R_1\sum\limits_{\substack{j=1\\ j\neq i}}^{N}\bar{A}_{ij}(\mu_i)x_j(t) + \frac{h_i}{2}\mathscr{A}_{di}(\mu_i)e_{di}(t) \\ \qquad + R_1B_i(\mu_i)e_{ui}(t) + R_2B_{ci}(\mu_i)e_{yi}(t), \\ \xi_{di}(t) = \dot{\bar{x}}_i(t), \\ \xi_{yi}(t) = \sigma_{yi}C_i\left[\frac{1}{2}x_i(t) + \frac{1}{2}x_i(t-h_i) + \frac{h_i}{2}e_{di}^{(1)}(t)\right], \\ \xi_{ui}(t) = \sigma_{ui}C_{ci}\left[\frac{1}{2}x_{ci}(t) + \frac{1}{2}x_{ci}(t-h_i) + \frac{h_i}{2}e_{di}^{(2)}(t)\right], \end{cases}$$

$$\mathcal{R}_{i2}: \begin{cases} e_{di}(t) = \Delta_{di}\xi_{di}(t), \\ e_{yi}(t) = \Delta_{ui}\xi_{yi}(t), \\ e_{ui}(t) = \Delta_{yi}\xi_{ui}(t), \end{cases} \tag{18}$$

where $i \in \mathcal{N}$, $\{\Delta_{yi}, \Delta_{ui}, \Delta_{di}\}$ denotes the uncertain operator in $\mathcal{R}_{i2}$, and

$$\begin{aligned} \mathscr{A}_i(\mu_i) &= \begin{bmatrix} A_i(\mu_i) & \frac{1}{2}B_i(\mu_i)C_{ci} \\ \frac{1}{2}B_{ci}(\mu_i)C_i & A_{c1i}(\mu_i) + \frac{1}{2}A_{c2i}(\mu_i) \end{bmatrix}, \\ \mathscr{A}_{di}(\mu_i) &= \begin{bmatrix} 0 & B_i(\mu_i)C_{ci} \\ B_{ci}(\mu_i)C_i & A_{c2i}(\mu_i) \end{bmatrix}, \\ R_1 &= \begin{bmatrix} \mathbf{I} & 0 \end{bmatrix}^T, \quad R_2 = \begin{bmatrix} 0 & \mathbf{I} \end{bmatrix}^T, \\ e_{di}(t) &= \begin{bmatrix} e_{di}^{(1)}(t) \\ e_{di}^{(2)}(t) \end{bmatrix}. \end{aligned}\tag{19}$$

In the light of the interconnected model in (18), the following lemma is provided:

Lemma 1: Consider the feedback interconnection with two subsystems $\mathcal{R}_{i1}$ and $\mathcal{R}_{i2}$ in (20), the uncertainty $\Delta_i : \xi_i(t) \longmapsto e_i(t)$ satisfies $\|\Delta_i\|_\infty \le 1$, where $\xi_i(t) = \begin{bmatrix} \xi_{id}^T(t) & \xi_{iy}^T(t) & \xi_{iu}^T(t)\end{bmatrix}^T$, $e_i(t) = \begin{bmatrix} e_{di}^T(t) & e_{yi}^T(t) & e_{ui}^T(t)\end{bmatrix}^T$, $\Delta_i = diag\{\Delta_{di}, \Delta_{yi}, \Delta_{ui}\}$.

Proof: It follows from the event-triggered proposal given in (10) and (11), one has

$$\begin{aligned} \|e_{yi}(t)\| &= \|\hat{y}_i(t-\eta_i(t)) - y_i(t-\eta_i(t))\| \\ &\le \sigma_{yi}\|y_i(t-\eta_i(t))\| \\ &= \sigma_{yi}\left\|C_i\left[\frac{1}{2}x_i(t) + \frac{1}{2}x_i(t-h_i) + \frac{h_i}{2}e_{di}^{(1)}(t)\right]\right\| \\ &= \|\xi_{yi}(t)\|, \end{aligned}\tag{20}$$

$$\begin{aligned} \|e_{ui}(t)\| &= \|\hat{u}_i(t-\eta_i(t)) - u_i(t-\eta_i(t))\| \\ &\le \sigma_{ui}\|u_i(t-\eta_i(t))\| \\ &= \sigma_{ui}\left\|C_{ci}\left[\frac{1}{2}x_{ci}(t) + \frac{1}{2}x_{ci}(t-h_i) + \frac{h_i}{2}e_{di}^{(2)}(t)\right]\right\| \\ &= \|\xi_{ui}(t)\|. \end{aligned}\tag{21}$$

In addition, consider zero-initial conditions and by using Jensen's inequality [29], it follows from (17) that

$$\begin{aligned}\int_0^t e_{di}^T(\alpha)\,e_{di}(\alpha)\,d\alpha &= \frac{1}{h_i^2}\int_0^t\left[\int_{-h_i}^0 \rho_i(\beta)\,\dot{\bar{x}}_i(\alpha+\beta)\,d\beta\right]^T(\star)\,d\alpha\\ &\le \frac{1}{h_i^2}\int_0^t\left[h_i\int_{-h_i}^0 \rho_i(\beta)\,\dot{\bar{x}}_i^T(\alpha+\beta)\,(\star)\,d\beta d\alpha\right]\\ &= \frac{1}{h_i}\int_{-h_i}^0\left[\int_0^t \dot{\bar{x}}_i^T(\alpha+\beta)\,\dot{\bar{x}}_i(\alpha+\beta)\,d\alpha\right]d\beta\\ &= \frac{1}{h_i}\int_{-h_i}^0\left[\int_\beta^{t+\beta} \dot{\bar{x}}_i^T(\alpha)\,\dot{\bar{x}}_i(\alpha)\,d\alpha\right]d\beta\\ &\le \frac{1}{h_i}\int_{-h_i}^0\left[\int_0^t \dot{\bar{x}}_i^T(\alpha)\,\dot{\bar{x}}_i(\alpha)\,d\alpha\right]d\beta\\ &= \int_0^t \dot{\bar{x}}_i^T(\alpha)\,\dot{\bar{x}}_i(\alpha)\,d\alpha\\ &= \int_0^t \xi_{di}^T(\alpha)\,\xi_{di}(\alpha)\,d\alpha. \qquad (22)\end{aligned}$$

It is straightforward to find from (20)-(22) that $\|\Delta_i\|_\infty \le 1$. Thus, completing this proof.

Remark 2: Based on the input delay method, it is easy to see that the large-scale systems with event-triggering control can be reconstructed into the ones with time-varying delay. Note that the approximated method in (17) can be easily developed for the control systems with time-varying delay.

B. STABILITY ANALYSIS

We firstly introduce the following LKF:

$$V(t) = \sum_{i=1}^N V_i(t), \qquad (23)$$

with

$$\begin{aligned}V_i(t) &= \bar{x}_i^T(t)\,P_i\bar{x}_i(t) + \int_{t-h_i}^t \bar{x}_i^T(\alpha)\,Q_i\bar{x}_i(\alpha)\,d\alpha\\ &\quad + h_i\int_{-h_i}^0\int_{t+\beta}^t \dot{\bar{x}}_i^T(\alpha)\,Z_i\dot{\bar{x}}_i(\alpha)\,d\alpha d\beta, \qquad (24)\end{aligned}$$

where $\{P_i, Q_i, Z_i\} \in \Re^{2n_{xi}\times 2n_{xi}}$, $i \in \mathcal{N}$ are symmetric matrices, and $P_i > 0$, $Z_i > 0$.

It should be noticed that the matrix Q_i in (24) is not necessary to be positive definite [27]. Here, the following lemma is used to ensure $V(t) > 0$,

Lemma 2 [24]: Given the LKF in (23), then $V(t) \ge \eta\|\bar{x}(t)\|^2$, where $\bar{x}(t) = \left[\bar{x}_1^T(t), \bar{x}_2^T(t), \cdots, \bar{x}_N^T(t)\right]^T$, $\eta > 0$, if there exist positive-definite symmetric matrices $\{P_i, Z_i\} \in \Re^{2n_{xi}\times 2n_{xi}}$, and symmetric matrix $Q_i \in \Re^{2n_{xi}\times 2n_{xi}}$, such that for all $i \in \mathcal{N}$ the following LMIs hold:

$$\begin{bmatrix}\frac{1}{h_i}P_i + Z_i & -Z_i\\ \star & Q_i + Z_i\end{bmatrix} > 0. \qquad (25)$$

Based on the new model in (18) and the LKF in (23), the stability of the control system given in (12) can be verified by the following theorem.

Theorem 1: Consider the large-scale fuzzy system in (2) and the decentralized DOF event-triggering controller in (6), the stability of the control systems in (12) can be ensured, if there exist positive-definite symmetric matrices $\{P_i, Z_i, M_i\} \in \Re^{2n_{xi}\times 2n_{xi}}$, symmetric matrices $Q_i \in \Re^{2n_{xi}\times 2n_{xi}}$, matrix multipliers $\mathcal{G}_i \in \Re^{(8n_{xi}+n_{yi}+n_{ui})\times 2n_{xi}}$, and positive scalars $\{\sigma_{ui}, \sigma_{yi}, \epsilon_{i1}, \epsilon_{i2}\}$, $\varepsilon_{ij}(\mu_i) \le \varepsilon_0$, such that for all $i \in \mathcal{N}$ the following matrix inequalities hold:

$$\begin{bmatrix}\frac{1}{h_i}P_i + Z_i & -Z_i\\ \star & Q_i + Z_i\end{bmatrix} > 0, \qquad (26)$$

$$\begin{bmatrix}\Sigma_i(\mu_i) & \mathcal{G}_iR_1\mathscr{A}_{ij,j\neq i}(\mu_i)\\ \star & -\mathscr{E}_{ij,j\neq i}(\mu_i)\end{bmatrix} < 0, \qquad (27)$$

where

$$\begin{aligned}\Sigma_i(\mu_i) &= \Theta_i + \sigma_{yi}^2\epsilon_{i1}\Omega_1^T\Omega_1 + \sigma_{ui}^2\epsilon_{i2}\Omega_2^T\Omega_2\\ &\qquad + \mathrm{Sym}\{\mathcal{G}_i\mathbb{A}_i(\mu_i)\},\\ \Theta_i &= \begin{bmatrix}h_i^2Z_i + M_i & P_i & 0 & 0\\ \star & \Theta_{i(1)} & Z_i & 0\\ \star & \star & -Q_i - Z_i & 0\\ \star & \star & \star & \Theta_{i(2)}\end{bmatrix},\\ \Theta_{i(1)} &= Q_i - Z_i + \varepsilon_0(N-1)R_1R_1^T,\\ \Theta_{i(2)} &= \mathrm{diag}\{-M_i, -\epsilon_{i1}\mathbf{I}, -\epsilon_{i2}\mathbf{I}\},\\ \Omega_1 &= \begin{bmatrix}0 & \Omega_{11} & \Omega_{11} & \Omega_{12} & 0 & 0\end{bmatrix},\\ \Omega_{11} &= \begin{bmatrix}\frac{1}{2}C_i & 0\end{bmatrix},\quad \Omega_{12} = \begin{bmatrix}\frac{h_i}{2}C_i & 0\end{bmatrix},\\ \Omega_2 &= \begin{bmatrix}0 & \Omega_{21} & \Omega_{21} & \Omega_{22} & 0 & 0\end{bmatrix},\\ \Omega_{21} &= \begin{bmatrix}0 & \frac{1}{2}C_{ci}\end{bmatrix},\quad \Omega_{22} = \begin{bmatrix}0 & \frac{h_i}{2}C_{ci}\end{bmatrix},\\ \mathbb{A}_i(\mu_i) &= \Big[-\mathbf{I} \quad \mathscr{A}_i(\mu_i) \quad \frac{1}{2}\mathscr{A}_{di}(\mu_i)\\ &\qquad \frac{h_i}{2}\mathscr{A}_{di}(\mu_i) \quad R_1B_i(\mu_i) \quad R_2B_{ci}(\hat{\mu}_i)\Big],\\ \mathscr{A}_i(\mu_i) &= \begin{bmatrix}A_i(\mu_i) & \frac{1}{2}B_i(\mu_i)C_{ci}\\ \frac{1}{2}B_{ci}(\mu_i)C_i & A_{c1i}(\mu_i) + \frac{1}{2}A_{c2i}(\mu_i)\end{bmatrix},\\ \mathscr{A}_{di}(\mu_i) &= \begin{bmatrix}0 & B_i(\mu_i)C_{ci}\\ B_{ci}(\mu_i)C_i & A_{c2i}(\mu_i)\end{bmatrix},\\ \mathscr{A}_{ij,j\neq i}(\mu_i) &= \underbrace{\left[\bar{A}_{i1}(\mu_i)\cdots\bar{A}_{ij,j\neq i}(\mu_i)\cdots\bar{A}_{iN}(\mu_i)\right]}_{N-1},\\ \mathscr{E}_{ij,j\neq i}(\mu_i) &= \mathit{diag}\underbrace{\left\{\Lambda_{i1}(\mu_i)\cdots\Lambda_{ij}(\mu_i)\cdots\Lambda_{iN}(\mu_i)\right\}}_{N-1},\\ \Lambda_{ij}(\mu_i) &= \varepsilon_{ij,j\neq i}(\mu_i)\mathbf{I}_{n_{xi}},\\ R_1 &= \begin{bmatrix}\mathbf{I} & 0\end{bmatrix}^T, R_2 = \begin{bmatrix}0 & \mathbf{I}\end{bmatrix}^T. \qquad (28)\end{aligned}$$

Proof: By taking the time derivative of $V(t)$, one has

$$\begin{aligned}\dot{V}_i(t) \leq\ & 2\bar{x}_i^T(t)P_i\dot{\bar{x}}_i(t) + \bar{x}_i^T(t)Q_i\bar{x}_i(t) \\ & - \bar{x}_i^T(t-h_i)Q_i\bar{x}_i(t-h_i) \\ & + h_i^2\dot{\bar{x}}_i^T(t)Z_i\dot{\bar{x}}_i(t) - h_i\int_{t-h_i}^{t}\dot{\bar{x}}_i^T(\alpha)Z_i\dot{\bar{x}}_i(\alpha)\,d\alpha. \end{aligned} \tag{29}$$

Based on Jensen's inequality [29], it yields that

$$\begin{aligned} -h_i&\int_{t-h_i}^{t}\dot{\bar{x}}_i(\alpha)Z_i\dot{\bar{x}}_i(\alpha)\,d\alpha \\ &\leq -\left[\int_{t-h_i}^{t}\dot{\bar{x}}_i(\alpha)\,d\alpha\right]^T Z_i\left[\int_{t-h_i}^{t}\dot{\bar{x}}_i(\alpha)\,d\alpha\right] \\ &= -\left(\bar{x}_i(t)-\bar{x}_i(t-h_i)\right)^T Z_i\left(\bar{x}_i(t)-\bar{x}_i(t-h_i)\right). \end{aligned} \tag{30}$$

Define the matrix multipliers $\mathcal{G}_i \in \Re^{(8n_{xi}+n_{yi}+n_{ui})\times 2n_{xi}}$, $i \in \mathcal{N}$, and it is easy to see from (18) that

$$\begin{aligned} 0 =& \sum_{i=1}^{N}2\chi_i^T(t)\mathcal{G}_i\mathbb{A}_i(\mu_i)\chi_i(t) \\ & + \sum_{i=1}^{N}2\chi_i^T(t)\mathcal{G}_iR_1\sum_{\substack{j=1\\ j\neq i}}^{N}\bar{A}_{ij}(\mu_i)x_j(t), \end{aligned} \tag{31}$$

where $\chi_i(t) = \begin{bmatrix}\dot{\bar{x}}_i^T(t) & \bar{x}_i^T(t) & \bar{x}_i^T(t-h_i) & e_{di}^T(t) & e_{yi}^T(t) & e_{ui}^T(t)\end{bmatrix}^T$, $\mathbb{A}_i(\mu_i)$ is defined in (28).

Note that

$$2\bar{x}^T\bar{y} \leq \kappa^{-1}\bar{x}^T\bar{x} + \kappa\bar{y}^T\bar{y}, \tag{32}$$

where $\bar{x}, \bar{y} \in \Re^n$ and scalar $\kappa > 0$.

Define the scalar parameters $0 < \varepsilon_{ij}(\mu_i) \leq \varepsilon_0$, where $\varepsilon_{ij}(\mu_i) := \sum_{l=1}^{r_i}\mu_{il}\varepsilon_{ijl}$, $i \in \mathcal{N}$, and by using the inequality (32), one has

$$\begin{aligned} \sum_{i=1}^{N}&2\chi_i^T(t)\mathcal{G}_iR_1\sum_{\substack{j=1\\ j\neq i}}^{N}\bar{A}_{ij}(\mu_i)x_j(t) \\ &\leq \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\chi_i^T(t)\mathcal{G}_iR_1\bar{A}_{ij}(\mu_i)\varepsilon_{ij}^{-1}(\mu_i)\bar{A}_{ij}^T(\mu_i)R_1^T\mathcal{G}_i^T\chi_i(t) \\ &\quad + \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\varepsilon_{ij}(\mu_i)x_j^T(t)x_j(t) \\ &\leq \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\chi_i^T(t)\mathcal{G}_iR_1\bar{A}_{ij}(\mu_i)\varepsilon_{ij}^{-1}(\mu_i)\bar{A}_{ij}^T(\mu_i)R_1^T\mathcal{G}_i^T\chi_i(t) \\ &\quad + \varepsilon_0(N-1)\sum_{i=1}^{N}x_i^T(t)x_i(t). \end{aligned} \tag{33}$$

Denote positive scalars ϵ_{i1} and ϵ_{i2}, and matrices $M_i = M_i^T > 0$, $i \in \mathcal{N}$, and consider the following index

$$\begin{aligned} J(t) =& \sum_{i=1}^{N}J_i(t) \\ =& \sum_{i=1}^{N}\int_0^\infty\Big\{\epsilon_{i1}\xi_{yi}^T(t)\xi_{yi}(t) - \epsilon_{i1}e_{yi}^T(t)e_{yi}(t) \\ & + \epsilon_{i2}\xi_{ui}^T(t)\xi_{ui}(t) - \epsilon_{i2}e_{ui}^T(t)e_{ui}(t) \\ & + \xi_{di}^T(t)M_i\xi_{di}(t) - e_{di}^T(t)M_ie_{di}(t)\Big\}\,dt. \end{aligned} \tag{34}$$

It is well-known that $V(0) = 0$ and $V(\infty) \geq 0$ under zero-initial conditions. Then, it follows from (29)-(34) that

$$\begin{aligned} J(t) \leq& J(t) + V(\infty) - V(0) \\ =& \sum_{i=1}^{N}\int_0^\infty\Big\{\dot{V}_i(t) + \epsilon_{i1}\xi_{yi}^T(t)\xi_{yi}(t) - \epsilon_{i1}e_{yi}^T(t)e_{yi}(t) \\ & + \epsilon_{i2}\xi_{ui}^T(t)\xi_{ui}(t) - \epsilon_{i2}e_{ui}^T(t)e_{ui}(t) \\ & + \xi_{di}^T(t)M_i\xi_{di}(t) - e_{di}^T(t)M_ie_{di}(t)\Big\}\,dt \\ \leq& \sum_{i=1}^{N}\int_0^\infty\chi_i^T(t)\Sigma_i(\mu_i)\chi_i(t)\,dt, \end{aligned} \tag{35}$$

with

$$\begin{aligned} \Sigma_i(\mu_i) =& \Theta_i + \sigma_{yi}^2\epsilon_{i1}\Omega_1^T\Omega_1 + \sigma_{ui}^2\epsilon_{i2}\Omega_2^T\Omega_2 \\ & + \mathrm{Sym}\{\mathcal{G}_i\mathbb{A}_i(\mu_i)\} \\ & + \sum_{\substack{j=1\\ j\neq i}}^{N}\varepsilon_{ij}^{-1}(\mu_i)\mathcal{G}_iR_1\bar{A}_{ij}(\mu_i)\bar{A}_{ij}^T(\mu_i)R_1^T\mathcal{G}_i^T, \end{aligned} \tag{36}$$

where all notations are given in (28).

By applying Schur complement to (27), we have $\Sigma_i(\mu_i) < 0$, which implies $\dot{V}(t) < 0$ and $J(t) < 0$. Together with the relationships in (20)-(22), and by using Lemma A1 (SSG theorem) in the Appendix, the stability of the control systems in (12) can be verified. Thus, the proof is completed.

C. CO-DESIGN

It should be pointed out that the stability results on (26) and (27) are nonlinear matrix inequalities. In this subsection, we will address the co-design for decentralized DOF event-triggering control of the presented large-scale fuzzy system. By specifying the multipliers $\mathcal{G}_i$ and using the matrix decomposition approach, the co-design is transformed in the form of LMIs as below.

Theorem 2: Given the large-scale fuzzy system in (2). Then, a decentralized DOF fuzzy event-triggering controller in (6) exists, which ensures the stability of the control system (12), if there exist the positive-definite symmetric matrices $\{\bar{P}_i, \bar{Z}_i, \bar{M}_i\} \in \Re^{2n_{xi}\times 2n_{xi}}$, symmetric matrices $\bar{Q}_i, \in \Re^{2n_{xi}\times 2n_{xi}}$, matrices $\{H_i, V_{i1}, U_{i1}\} \in \Re^{n_{xi}\times n_{xi}}$, $\{\mathbb{A}_{c1il}, \mathbb{A}_{c2il}\} \in \Re^{n_{xi}\times n_{xi}}$, $\mathbb{B}_{cil} \in \Re^{n_{xi}\times n_{yi}}$, $\mathbb{C}_{ci} \in \Re^{n_{ui}\times n_{xi}}$, and positive scalars

$\{\sigma_{ui}, \sigma_{yi}, \epsilon_{i1}, \epsilon_{i2}, \varepsilon_{ijl}, \varepsilon_0\}$, $\varepsilon_{ijl} \leq \varepsilon_0$ such that for all $l \in \mathscr{L}_i, i \in \mathcal{N}$ the following LMIs hold:

$$\begin{bmatrix} \frac{1}{h_i}\bar{P}_i + \bar{Z}_i & -\bar{Z}_i \\ \star & \bar{Q}_i + \bar{Z}_i \end{bmatrix} > 0, \tag{37}$$

$$\begin{bmatrix} -\frac{\varepsilon_0^{-1}}{N-1}\mathbf{I} & 0 & 0 & \bar{\Omega}_3 & 0 \\ \star & -\epsilon_{i2}^{-1}\mathbf{I} & 0 & \sigma_{ui}\bar{\Omega}_2 & 0 \\ \star & \star & -\epsilon_{i1}^{-1}\mathbf{I} & \sigma_{yi}\bar{\Omega}_1 & 0 \\ \star & \star & \star & \tilde{\Sigma}_{il} & \mathcal{E}\bar{G}_i\mathscr{A}_{ijl,j\neq i} \\ \star & \star & \star & \star & -\mathscr{E}_{ij,j\neq i} \end{bmatrix} < 0, \tag{38}$$

where

$$\begin{aligned}
\tilde{\Sigma}_{il} &= \tilde{\Theta}_i + \mathrm{Sym}\{\mathcal{E}\Pi_{il}\}, \\
\tilde{\Theta}_i &= \begin{bmatrix} h_i^2\bar{Z}_i + \bar{M}_i & \bar{P}_i & 0 & 0 \\ \star & \bar{Q}_i - \bar{Z}_i & \bar{Z}_i & 0 \\ \star & \star & -\bar{Q}_i - \bar{Z}_i & 0 \\ \star & \star & \star & \tilde{\Theta}_{i(1)} \end{bmatrix}, \\
\tilde{\Theta}_{i(1)} &= \mathrm{diag}\{-\bar{M}_i, -\epsilon_{i1}\mathbf{I}, -\epsilon_{i2}\mathbf{I}\}, \\
\Pi_{il} &= \begin{bmatrix} \Pi_{(1)} & \Pi_{(2)} & \Pi_{(3)} & \Pi_{(4)} & \Pi_{(5)} & \Pi_{(6)} \end{bmatrix}, \\
\Pi_{(1)} &= -\begin{bmatrix} U_{i1} & \mathbf{I} \\ H_i & V_{i1}^T \end{bmatrix}, \\
\Pi_{(2)} &= \begin{bmatrix} A_{il}U_{i1} + \frac{1}{2}B_{il}\mathbb{C}_{ci} & A_{il} \\ \mathbb{A}_{c1il} & V_{i1}^TA_{il} + \frac{1}{2}\mathbb{B}_{cil}C_i \end{bmatrix}, \\
\Pi_{(3)} &= \frac{1}{2}\begin{bmatrix} B_{il}\mathbb{C}_{ci} & 0 \\ \mathbb{A}_{c2il} & \mathbb{B}_{cil}C_i \end{bmatrix}, \\
\Pi_{(4)} &= \frac{h_i}{2}\begin{bmatrix} B_{il}\mathbb{C}_{ci} & 0 \\ \mathbb{A}_{c2il} & \mathbb{B}_{cil}C_i \end{bmatrix}, \\
\Pi_{(5)} &= \begin{bmatrix} B_{il} \\ V_{i1}^TB_{il} \end{bmatrix}, \quad \Pi_{(6)} = \begin{bmatrix} 0 \\ \mathbb{B}_{cil} \end{bmatrix}, \\
\mathcal{E} &= \begin{bmatrix} \mathbf{I} & \mathbf{I} & 0_{2n_{xi}\times(4n_{xi}+n_{yi}+n_{ui})} \end{bmatrix}^T, \quad \bar{G}_i = \begin{bmatrix} \mathbf{I} \\ V_{i1}^T \end{bmatrix}, \\
\bar{\Omega}_1 &= \begin{bmatrix} 0 & \bar{\Omega}_{11} & \bar{\Omega}_{11} & \bar{\Omega}_{12} & 0 & 0 \end{bmatrix}, \\
\bar{\Omega}_{11} &= \begin{bmatrix} \frac{1}{2}C_iU_{i1} & \frac{1}{2}C_i \end{bmatrix}, \quad \bar{\Omega}_{12} = \begin{bmatrix} \frac{h_i}{2}C_iU_{i1} & \frac{h_i}{2}C_i \end{bmatrix}, \\
\bar{\Omega}_2 &= \begin{bmatrix} 0 & \bar{\Omega}_{21} & \bar{\Omega}_{21} & \bar{\Omega}_{22} & 0 & 0 \end{bmatrix}, \\
\bar{\Omega}_{21} &= \begin{bmatrix} \frac{1}{2}\mathbb{C}_{ci} & 0 \end{bmatrix}, \quad \bar{\Omega}_{22} = \begin{bmatrix} \frac{h_i}{2}\mathbb{C}_{ci} & 0 \end{bmatrix}, \\
\bar{\Omega}_3 &= \begin{bmatrix} 0 & \begin{bmatrix} U_{i1} & \mathbf{I} \end{bmatrix} & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\mathscr{E}_{ijl,j\neq i} &= diag\underbrace{\{\varepsilon_{i1l}\mathbf{I}_{n_{xi}} \cdots \varepsilon_{ijl,j\neq i}\mathbf{I}_{n_{xi}} \cdots \varepsilon_{iNl}\mathbf{I}_{n_{xi}}\}}_{N-1}, \\
\mathscr{A}_{ijl,j\neq i} &= \underbrace{[\bar{A}_{i1l} \cdots \bar{A}_{ijl,j\neq i} \cdots \bar{A}_{iNl}]}_{N-1}.
\end{aligned} \tag{39}$$

Furthermore, the decentralized DOF fuzzy event-triggering controller gains are calculated as

$$\begin{cases} C_{ci} = \mathbb{C}_{ci}U_{i2}^{-1}, \\ B_{cil} = V_{i2}^{-T}\mathbb{B}_{cil}, \\ A_{c2il} = V_{i2}^{-T}\mathbb{A}_{c2il}U_{i2}^{-1} - B_{cil}C_iU_{i1}U_{i2}^{-1} - V_{i2}^{-T}V_{i1}^TB_{il}C_{ci}, \\ A_{c1il} = V_{i2}^{-T}\mathbb{A}_{c1il}U_{i2}^{-1} - V_{i2}^{-T}V_{i1}^TA_{il}U_{i1}U_{i2}^{-1} \\ \qquad - \frac{1}{2}B_{cil}C_iU_{i1}U_{i2}^{-1} - \frac{1}{2}V_{i2}^{-T}V_{i1}^TB_{il}C_{ci} - \frac{1}{2}A_{c2il}, \end{cases} \tag{40}$$

where $l \in \mathscr{L}_i$, $U_{i2} = V_{i2}^{-T}H_i - V_{i2}^{-T}V_{i1}^TU_{i1}$, $i \in \mathcal{N}$.

Proof: By applying Schur complement to (27), one gets

$$\begin{bmatrix} -\frac{\varepsilon_0^{-1}}{N-1}\mathbf{I} & 0 & 0 & \Omega_3 & 0 \\ \star & -\epsilon_{i2}^{-1}\mathbf{I} & 0 & \sigma_{ui}\Omega_2 & 0 \\ \star & \star & -\epsilon_{i1}^{-1}\mathbf{I} & \sigma_{yi}\Omega_1 & 0 \\ \star & \star & \star & \bar{\Sigma}_i(\mu_i) & \mathcal{G}_iR_1\mathscr{A}_{ij,j\neq i}(\mu_i) \\ \star & \star & \star & \star & -\mathscr{E}_{ij,j\neq i}(\mu_i) \end{bmatrix} < 0, \tag{41}$$

where

$$\begin{aligned}
\bar{\Sigma}_i(\mu_i) &= \bar{\Theta}_i + \mathrm{Sym}\{\mathcal{G}_i\mathbb{A}_i(\mu_i)\}, \\
\bar{\Theta}_i &= \begin{bmatrix} h_i^2Z_i + M_i & P_i & 0 & 0 \\ \star & Q_i - Z_i & Z_i & 0 \\ \star & \star & -Q_i - Z_i & 0 \\ \star & \star & \star & \bar{\Theta}_{i(1)} \end{bmatrix}, \\
\bar{\Theta}_{i(1)} &= \mathrm{diag}\{-M_i, -\epsilon_{i1}\mathbf{I}, -\epsilon_{i2}\mathbf{I}\}, \\
\Omega_1 &= \begin{bmatrix} 0 & \Omega_{11} & \Omega_{11} & \Omega_{12} & 0 & 0 \end{bmatrix}, \\
\Omega_{12} &= \begin{bmatrix} \frac{h_i}{2}C_i & 0 \end{bmatrix}, \quad \Omega_{11} = \begin{bmatrix} \frac{1}{2}C_i & 0 \end{bmatrix}, \\
\Omega_2 &= \begin{bmatrix} 0 & \Omega_{21} & \Omega_{21} & \Omega_{22} & 0 & 0 \end{bmatrix}, \\
\Omega_{21} &= \begin{bmatrix} 0 & \frac{1}{2}C_{ci} \end{bmatrix}, \quad \Omega_{22} = \begin{bmatrix} 0 & \frac{h_i}{2}C_{ci} \end{bmatrix}, \\
\Omega_3 &= \begin{bmatrix} 0 & \begin{bmatrix} \mathbf{I} & 0 \end{bmatrix} & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\mathbb{A}_i(\mu_i) &= \Big[-\mathbf{I} \quad \mathscr{A}_i(\mu_i) \quad \frac{1}{2}\mathscr{A}_{di}(\mu_i) \\
&\qquad \frac{h_i}{2}\mathscr{A}_{di}(\mu_i) \quad R_1B_i(\mu_i) \quad R_2B_{ci}(\mu_i) \Big], \\
\mathscr{A}_i(\mu_i) &= \begin{bmatrix} A_i(\mu_i) & \frac{1}{2}B_i(\mu_i)C_{ci} \\ \frac{1}{2}B_{ci}(\mu_i)C_i & A_{c1i}(\mu_i) + \frac{1}{2}A_{c2i}(\mu_i) \end{bmatrix}, \\
\mathscr{A}_{di}(\mu_i) &= \begin{bmatrix} 0 & B_i(\mu_i)C_{ci} \\ B_{ci}(\mu_i)C_i & A_{c2i}(\mu_i) \end{bmatrix}, \\
\mathscr{A}_{ij,j\neq i}(\mu_i) &= \underbrace{[\bar{A}_{i1}(\mu_i) \cdots \bar{A}_{ij,j\neq i}(\mu_i) \cdots \bar{A}_{iN}(\mu_i)]}_{N-1}, \\
\mathscr{E}_{ij,j\neq i}(\mu_i) &= \mathrm{diag}\underbrace{\{\Lambda_{i1}(\mu_i) \cdots \Lambda_{ij}(\mu_i) \cdots \Lambda_{iN}(\mu_i)\}}_{N-1}, \\
R_1 &= \begin{bmatrix} \mathbf{I} & 0 \end{bmatrix}^T, \quad R_2 = \begin{bmatrix} 0 & \mathbf{I} \end{bmatrix}^T, \\
\Lambda_{ij}(\mu_i) &= \varepsilon_{ij,j\neq i}(\mu_i)\mathbf{I}_{n_{xi}},
\end{aligned} \tag{42}$$

For simplification, the matrix multipliers $\mathcal{G}_i$ can be designated as

$$\mathcal{G}_i = \begin{bmatrix} G_i & G_i & 0_{2n_{xi}\times(4n_{xi}+n_{yi}+n_{ui})} \end{bmatrix}^T, i \in \mathcal{N} \tag{43}$$

where the matrices $G_i \in \Re^{2n_{xi}\times 2n_{xi}}$ are nonsingular.

Now, we further define [28]

$$\begin{cases} W_i = \begin{bmatrix} U_{i1} & \mathbf{I} \\ U_{i2} & 0 \end{bmatrix}, \ G_i = \begin{bmatrix} V_{i1} & \bullet \\ V_{i2} & \bullet \end{bmatrix}, \ G_i^{-1} = \begin{bmatrix} U_{i1} & \bullet \\ U_{i2} & \bullet \end{bmatrix}, \\ \Gamma_1 := \operatorname{diag}\left\{ \mathbf{I}_{n_{xi}} \quad \mathbf{I}_{n_{ui}} \quad \mathbf{I}_{n_{yi}} \quad W_i \quad W_i \quad W_i \quad W_i \right. \\ \qquad\qquad \left. \mathbf{I}_{n_{yi}} \quad \mathbf{I}_{n_{ui}} \quad \mathscr{E} \right\}, \\ \bar{P}_i = W_i^T P_i W_i, \quad \bar{Q}_i = W_i^T Q_i W_i, \\ \bar{Z}_i = W_i^T Z_i W_i, \quad \bar{M}_i = W_i^T M_i W_i, \end{cases} \tag{44}$$

where the matrix $W_i \in \Re^{2n_{xi}\times 2n_{xi}}$ is nonsingular; "$\bullet$" represents the elements satisfying $G_i G_i^{-1} = \mathbf{I}$.

By substituting the matrix multipliers $\mathcal{G}_i$ into (41), we realize a congruence transformation by Γ_1 in (44), and extract the fuzzy basis functions. The inequality in (38) can be directly derived. Finally, we perform a congruence transformation to (26) by $\Gamma_2 := \operatorname{diag}\{G_i, G_i\}$, the inequalities (37) can be obtained.

It is noted that the inequality

$$h_i^2 \bar{Z}_i + \bar{M}_i - \operatorname{Sym}\left\{ \begin{bmatrix} U_{i1} & \mathbf{I} \\ H_i & V_{i1}^T \end{bmatrix} \right\} < 0 \tag{45}$$

in (38) means that

$$\begin{cases} \operatorname{Sym}\{V_{i1}\} > 0 \\ \operatorname{Sym}\{U_{i1}\} > 0 \\ \operatorname{Sym}\{U_{i1}V_{i1}^T - V_{i1}^T U_{i1} + V_{i2}^T U_{i2}\} > 0 \end{cases} \tag{46}$$

holds. In this way, it can be seen that the matrices U_{2i} are nonsingular. Thus, the controller gains could be calculated by (40). In addition, these two facts, i.e., $\Pi_{(1)} = -W_i^T G_i W_i$ and the nonsingular $\Pi_{(1)}$, imply that the matrices G_i and W_i are nonsingular. Thus, completing this proof.

IV. SIMULATION EXAMPLES

Consider the large-scale fuzzy system as below:

Plant Rule $\mathscr{R}_i^l$: **IF** $\zeta_{i1}(t)$ is $\mathscr{F}_{i1}^l$, **THEN**

$$\begin{cases} \dot{x}_i(t) = A_{il}x_i(t) + B_{il}\hat{u}_i(t_k^i) + \sum\limits_{\substack{j=1 \\ j\neq i}}^{N} \bar{A}_{ijl}x_j(t) \\ y_i(t) = C_i x_i(t), l \in \{1, 2\}, t \in [t_k, t_{k+1}) \end{cases}$$

where

$$A_{11} = \begin{bmatrix} 0 & 1 \\ -0.8 & 0.6 \end{bmatrix}, \quad \bar{A}_{121} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix}$$
$$A_{12} = \begin{bmatrix} 0 & 0.9 \\ -1.3 & 0.7 \end{bmatrix}, \quad \bar{A}_{122} = \begin{bmatrix} 0 & 0 \\ 0.2 & 0 \end{bmatrix}$$
$$B_{11} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

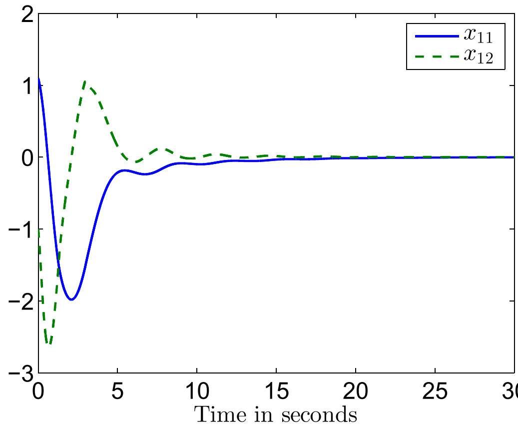

FIGURE 2. State responses of subsystem 1.

for the first subsystem, and

$$A_{21} = \begin{bmatrix} 0 & 1.1 \\ -1 & 0.6 \end{bmatrix}, \quad \bar{A}_{211} = \begin{bmatrix} 0 & 0 \\ 0.2 & 0 \end{bmatrix}$$
$$A_{22} = \begin{bmatrix} 0 & 0.9 \\ -1 & 0.5 \end{bmatrix}, \quad \bar{A}_{212} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix}$$
$$B_{21} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0 \\ 0.6 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

for the second subsystem.

It is straightforward to see that the open-loop system is unstable. Here, we assume that $h_i = 0.001$, $\sigma_{y1} = 0.08$, $\sigma_{u1} = 0.04$, $\sigma_{y2} = 0.08$, and $\sigma_{u2} = 0.07$. Note that the free-weighting matrices method proposed in [23] fails to obtain feasible solutions. However, given the scalars $\epsilon_{i1} = 10$, $\epsilon_{i2} = 10$, $\varepsilon_0 = 0.5$, and by applying Theorem 2, we indeed obtain a feasible solution:

$$A_{c111} = \begin{bmatrix} 4.5109 & 4.1842 \\ -4.1934 & -3.3908 \end{bmatrix},$$
$$A_{c211} = \begin{bmatrix} 0.5740 & 1.4881 \\ -12.5391 & -10.2282 \end{bmatrix},$$
$$A_{c112} = \begin{matrix} 2.0620 & 2.0962 \\ -2.5833 & -1.8043 \end{matrix},$$
$$A_{c212} = \begin{bmatrix} 2.0455 & 2.5539 \\ -11.2591 & -9.2347 \end{bmatrix},$$
$$B_{c11} = \begin{bmatrix} -5.6291 \\ 2.5372 \end{bmatrix}, \quad B_{c12} = \begin{bmatrix} -5.1681 \\ 2.4856 \end{bmatrix}$$
$$C_{c1} = \begin{bmatrix} 19.3201 & 14.9928 \end{bmatrix}$$

for the first subsystem, and

$$A_{c121} = \begin{bmatrix} 1.4503 & 1.5681 \\ 0.3649 & 0.2381 \end{bmatrix},$$
$$A_{c221} = \begin{bmatrix} -1.2089 & 0.2072 \\ -6.3197 & -4.9639 \end{bmatrix},$$
$$A_{c122} = \begin{matrix} -0.0343 & 0.4256 \\ 1.6731 & 1.2393 \end{matrix},$$

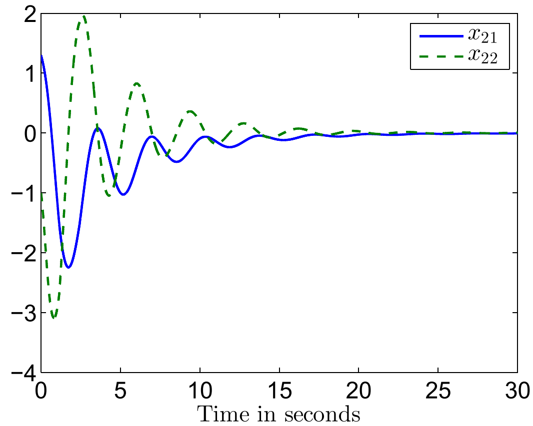

FIGURE 3. State responses of subsystem 2.

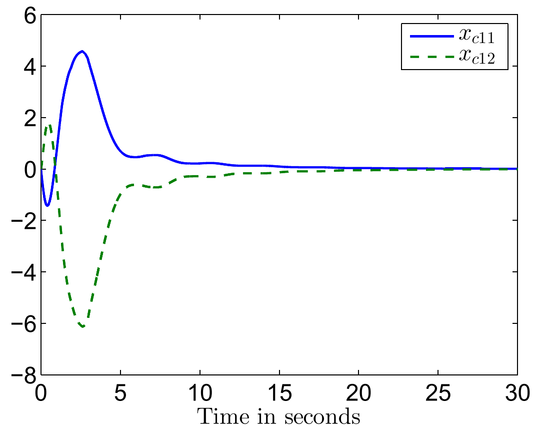

FIGURE 4. State responses of controller 1.

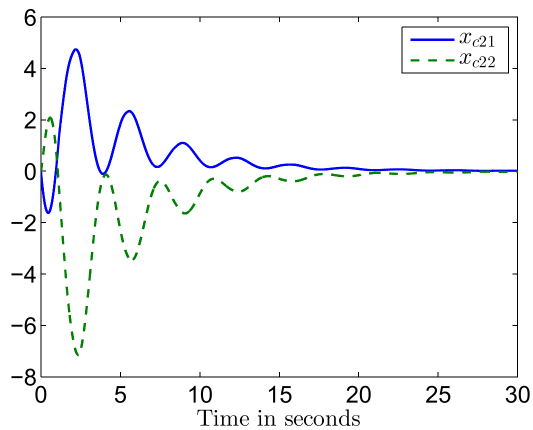

FIGURE 5. State responses of controller 2.

$$A_{c222} = \begin{bmatrix} 0.9037 & 1.5714 \\ -10.4058 & -7.7552 \end{bmatrix},$$
$$B_{c21} = \begin{bmatrix} -5.8172 \\ 3.1303 \end{bmatrix}, \quad B_{c22} = \begin{bmatrix} -5.2421 \\ 2.9892 \end{bmatrix}$$
$$C_{c2} = \begin{bmatrix} 7.6979 & 5.2961 \end{bmatrix}$$

for the second subsystem.

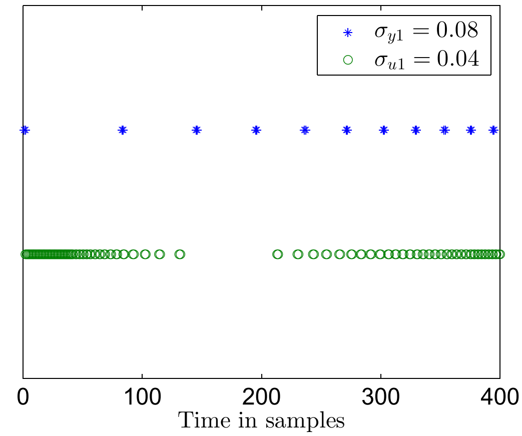

FIGURE 6. Triggering responses of control subsystem 1.

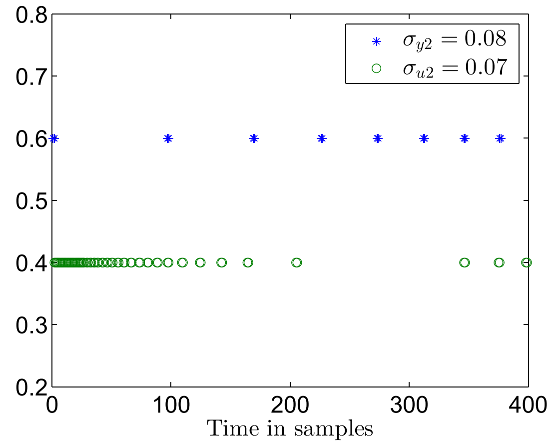

FIGURE 7. Triggering responses of control subsystem 2.

Based on the above solution and given the initial conditions $x_1(0) = [1.1, -1]^T$, $x_2(0) = [1.3, -1]^T$, Figs. 2-5 show that the state responses for the subsystems 1 and 2, and the controller 1 and 2 tend to zero. Fig. 6 shows that the number of data transmissions in the control system 1 reduces from 400 to 11 in the S-C channel, and from 400 to 78 in the C-A channel, when using event-triggering policy. Fig. 7 shows that the number of data transmissions in the control system 2 reduces from 400 to 8 in the S-C channel, and from 400 to 41 in the C-A channel.

V. CONCLUSIONS

This paper studied the problem of decentralized DOF event-triggering control for large-scale T-S fuzzy networked systems with nonlinear interconnections. An event-triggering control scheme was proposed here, and a decentralized DOF event-triggering controller was designed to reduce the communications in both the S-C and C-A channels. A novel model transformation was presented, and a relaxing LKF combined with SSG method was introduced, the co-design was implemented to obtain synchronously the DOF controller gains, event-triggered parameter, and sampled period.

Simulation results illustrate that the designed controller guarantees the stability of the control system and the reduction of data transmissions. Considering the asynchronous premise variables to the two-channel triggering scheme and applying these theoretical results to some real-world complicated systems, such as electric ground vehicles, will be part of our future research.

Appendix

Lemma A1 [30], [31]: Consider the interconnected system in (12) and assume that $\mathcal{R}_1$ is internally stable. The interconnected system in (12) with two subsystems $\mathcal{R}_1$ and $\mathcal{R}_2$ is robustly stable for all $\Delta \in \mathcal{M}$ if $\left\| T_e \circ \mathbf{G} \circ T_\eta^{-1} \right\|_\infty < 1$ holds for some matrices $\{T_e, T_\eta\} \in \mathcal{T}$ with $\mathcal{T} := \left\{ \{T_\eta, T_e\} \in \Re^{n_\eta \times n_\eta} \times \Re^{n_e \times n_e} : T_\eta, T_e \text{ are nonsingular}; \left\| T_\eta \circ \Delta \circ T_e^{-1} \right\|_\infty \le 1 \right\}$.

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