IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received April 28, 2017, accepted May 18, 2017, date of publication May 31, 2017, date of current version July 7, 2017.

Digital Object Identifier 10.1109/ACCESS.2017.2710142

Three Dimensional Comprehensive Analytical Solutions for Locating Sources of Sensor Networks in Unknown Velocity Mining System

LONGJUN DONG[1], (Member, IEEE), WEIWEI SHU[1], XIBING LI[1], GUANGJIE HAN[2,3], (Member, IEEE), AND WEI ZOU[1]

[1]School of Resources and Safety Engineering, Central South University, Changsha 410083, China
[2]Department of Information and Communication System, Hohai University, Changzhou 213022, China
[3]State Key Laboratory of Acoustics, Institute of Acoustics, Chinese Academy of Sciences, Beijing 100190, China

Corresponding author: Longjun Dong (lj.dong@csu.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 51504288 and Grant 41630642, in part by the National Basic Research Program of China under Grant 2015CB060200, in part by the China Postdoctoral Science Foundation under Grant 2015M570688 and Grant 2016T90639, in part by the Young Elite Scientists Sponsorship Program by CAST under Grant YESS20160175, in part by the Innovation-Driven Project of Central South University under Grant 2016CXS001, and in part by the Open fund of the State Key Laboratory of Acoustics under Grant SKLA201706.

ABSTRACT The accuracy of localization methods based on the arrival time difference is usually affected by the iterative algorithm, the initial value, and the pre-measured wave velocity. The analytical solutions are non-unique because of the square root operations in the calculating process for source coordinates. To solve these significant problems, the nonlinear equations were simplified to the linear equations. An analytical localization method without the pre-measured velocity or the square root operations was developed. The explicit formulas for analytical solutions were resolved for the six sensors network. The source coordinates can be solved for real time by substituting the arrival times and coordinates of sensors. Focusing on the practical engineering where the number of sensors is greater than six, the comprehensive analytical solutions were proposed on account of sensor networks, which formed through the combination of different sensors, and the logistic probability density function. The blasting tests in two mines verified its effectiveness and accuracy. Results show that the locating accuracy of three dimensional comprehensive analytical solutions is superior to the traditional methods. The assumed examples proved that the proposed method performs well under different scales of arrival time errors. This proposed method highlights four advantages: without iterative algorithm, without pre-measured velocity, without initial value, and without square root operations.

INDEX TERMS Localization method, microseismic/AE sources, sensor networks, unknown velocity system.

I. INTRODUCTION

Nowadays, the localization methods of arrival time difference are widely applied in the fields of aerospace, navigation, structural health, speaker location, underground tunnel, deep mining, and seismology [1]–[6]. It is known to all that the locating accuracy is under the influence of localization methods. Aiming at solving this significant problem, many researchers have discussed and developed numerous related localization methods mainly including the iterative methods and the analytical methods [7]–[17].

The iterative localization method is one of the most popular methods in recent years. The current source locating methods mostly cite from seismology in which the classic linear localization method of Geiger has been applied widely. Lienert *et al.* [18] developed the hypocenter algorithm based on the thoughts of Geiger. Aki and Lee [19] and Aki *et al.* [20] proposed the joint inversion theory of three dimensional velocity structure and seismic source by gridding the lateral nonuniform velocity in the interior earth. Tarantola and Valette [21] presented the general solution of the nonlinear inverse problem with a finite number of parameters. According to the Bayesian methodology, Matsu'ura [22] gave the estimation of hypocenter location with origin time eliminated. By taking arrival time difference as the dependent variable, Dong *et al.* [23] proposed the localization method without pre-measured velocity for microseismic sources.

Compared to the traditional methods measuring the velocity in advance, this method eliminates the effects of temporal and spatial errors for wave velocity, which is more feasible for practical applications. However, there are still some factors that restrict the precision of the iterative methods. Based on the summary and discussion of the iterative localization methods, Dong [24] concluded the disadvantages that it is difficult to obtain a unique solution. In addition, the accuracy of iterative solutions is dependent on the locating algorithm heavily.

It is interesting to note that the locating error caused by the iterative methods can be eliminated effectively through the analytical methods. For the microseismic monitoring networks of special geometric shapes, Dong *et al.* [25], Dong and Li [26], and Li and Dong [27] developed explicit formulas to resolve the accurate analytical solutions for cuboid and cube sensor networks to avoid the iterative error. However, the geometric shapes of monitoring networks can hardly be special in the practical layout, the analytical formulas for special shapes will lose efficacy. Obviously, the works for exploring analytical solutions under all kinds of random monitoring networks are urgent and important. Smith and Abel [28] derived three noniterative locating formulas from linear least square by minimizing the ''equation error'' and compared the statistical performances of the three methods. Chan and Ho [29] developed a localization method by applying the spherical interpolation method and the least square method. Duraiswami *et al.* [30] proposed a three dimensional localization method which is similar to the maximum likelihood estimation. Mellen *et al.* [31] presented the closed-form solution using arrival time difference when the number of sensors is greater than 3. Li and Dong [32] compared the nonlinear multi-robe localization method and the analytical method, as well as discussing their own advantages and disadvantages. Ge [33], [34] summarized the current main iterative methods and analytical methods. Most of these methods calculate the source coordinates with pre-measured velocity, whereas the real-time velocity of the specific path is usually unknown in fact. It is exactly the reason that causes the locating error in most situations of practical engineering.

The defects of most analytical localization methods can be summarized and explained with three aspects. First, the wave velocity is determined beforehand rather than real-time in most analytical methods. As shown in the Fig.1, the propagation medium in the mining system such as rock is usually of anisotropy, which means that the pre-measured velocity values of the specific paths will not equal to the velocity values of the other paths. Moreover, it is nearly impossible to measure the velocity values in all the directions in the practical engineering. Therefore, locating sources with pre-measured average velocity will cause inevitable error. Second, the analytical solutions are non-unique or inexistent because the solution formulas are obtained based on the quadratic equations. Third, some actual solutions are ignored due to the square root operations. In consequence, these problems lead to enormous locating error in the microseismic/AE source locating system and seriously restrict the development of source locating technology.

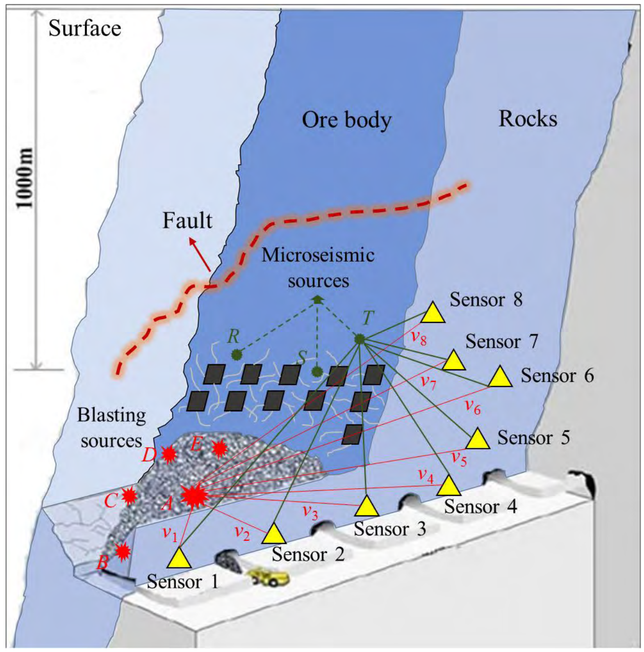

FIGURE 1. Upper graph shows an example for the propagation paths of *P*-wave with blasting sources and microseismic sources located at different coordinates. In the traditional methods such as STT and STD methods, the average velocity value of blasting source *A* is equal to the average value of v_1 to v_8, which is calculated through their own distances and travel times. However, the average velocity values of other blasting sources *B, C, D,* and *E* will not equal to the source *A* due to the anisotropy of rock. Similarly, the authentic velocity value of a microseismic source is not equal to the velocity value used in the localization process. Thus, it is inaccurate that locating sources using the pre-measured average velocity value of the blasting sources, where the velocity paths are different from the sources to be located.

To solve the above problems, the three dimensional comprehensive analytical solutions (TDCAS) without pre-measured velocity under random sensor networks is proposed. Furthermore, engineering practices are combined to verify its effectiveness and accuracy. It is proved that TDCAS-PDF also owns the ability to locate source coordinates under different scales of arrival time errors through the assumed examples.

II. COMPREHENSIVE LOCALIZATION METHOD

A. ANALYTICAL SOLUTIONS FOR SIX SENSORS

The coordinates of the microseismic/AE source and the 6 sensors are assumed as $P(x, y, z)$ and $S_i(x_i, y_i, z_i)$ $(i = 1, 2, 3, 4, 5, 6)$ respectively. The governing equation for the coordinates of the microseismic/AE source is shown below:

$$(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2 = v^2(t_i - t_0)^2 \tag{1}$$

where t_0 is the trigger time of the microseismic/AE source. t_i is the arrival time corresponding to the sensor S_i. The average velocity of *P*-wave is represented as v.

From the point of space geometry, every equation from (1) represents a sphere with the center locating at the coordinates of own sensor. Any two spheres intersect to form a circle. The coordinates of source are located in these intersecting circles. By taking difference between the equation with $i = 1$ and the others ($i = 2, 3, 4, 5, 6$), we can obtain (2) as below:

$$2x(x_m - x_1) + 2y(y_m - y_1) + 2z(z_m - z_1) + 2v^2(t_m - t_1)t_0 + v^2(t_1^2 - t_m^2) = l_{m-1} \quad (2)$$

where: $m = (2, 3, 4, 5, 6)$, $l_{m-1} = (x_m^2 - x_1^2) + (y_m^2 - y_1^2) + (z_m^2 - z_1^2)$

By substituting V, S for v^2, Vt_0 respectively, (2) can be transformed into (3).

$$2x(x_m - x_1) + 2y(y_m - y_1) + 2z(z_m - z_1) + 2(t_m - t_1)S + V(t_1^2 - t_m^2) = l_{m-1} \quad (3)$$

Equation (3) can also rewrite as:

$$AS = B \quad (4)$$

where

$$A = \begin{bmatrix} 2(x_2-x_1) & 2(y_2-y_1) & 2(z_2-z_1) & 2(t_2-t_1) & 2(t_1^2-t_2^2) \\ 2(x_3-x_1) & 2(y_3-y_1) & 2(z_3-z_1) & 2(t_3-t_1) & 2(t_1^2-t_3^2) \\ 2(x_4-x_1) & 2(y_4-y_1) & 2(z_4-z_1) & 2(t_4-t_1) & 2(t_1^2-t_4^2) \\ 2(x_5-x_1) & 2(y_5-y_1) & 2(z_5-z_1) & 2(t_5-t_1) & 2(t_1^2-t_5^2) \\ 2(x_6-x_1) & 2(y_6-y_1) & 2(z_6-z_1) & 2(t_6-t_1) & 2(t_1^2-t_6^2) \end{bmatrix}$$

$$S = \begin{bmatrix} x \\ y \\ z \\ S \\ V \end{bmatrix}, \quad B = \begin{bmatrix} l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{bmatrix}$$

The important parameters including the coordinates of source (x, y, z), the average velocity v of P-wave, and the trigger time t_0 can be obtained easily through the proposed analytical method. Furthermore, there is no need to measure the velocity before monitoring. The form of calculating formulas is explicit. A set of unique coordinates for microseismic/AE sources can be determined by every 6 sensors.

B. ANALYTICAL SOLUTIONS FOR GREATER THAN SIX SENSORS

A unique solution can be obtained through the proposed analytical localization method with the coordinates and the arrival times of 6 sensors under the unknown velocity system. The sensor network is not only a critical factor to the locating accuracy [35]–[37], but also the basis for accurately locating and real-time identifying sources in the practical applications of engineering projects [38]–[42]. In fact, it is common that greater than 6 sensors are used to improve the locating accuracy in the practical engineering. Therefore, it is a vital problem to take full advantage of the remaining sensors.

As described in the previous section, every 6 sensors can make up a set of sensor network to calculate the source coordinates. It is feasible to obtain C_m^6 groups of analytical solutions in the locating system with m triggered sensors.

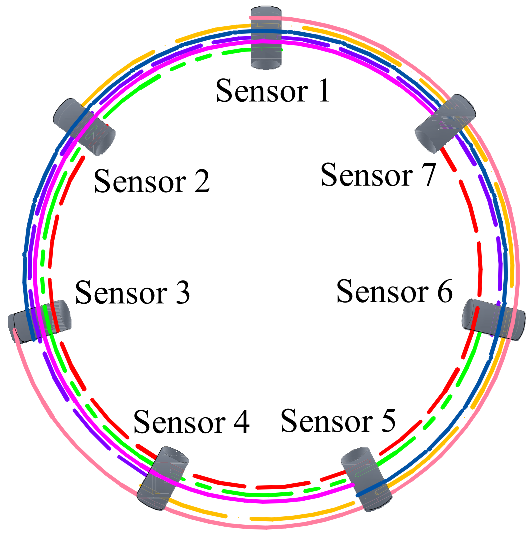

FIGURE 2. Upper graph exhibits an example for the formation of sensor networks, where *m* is assumed as 7. Every 6 triggered sensors can constitute a set of sensor network and 7 kinds of networks are represented by curves with different colors.

Thus, the number of analytical solutions is exactly equal to the number of sensor networks, which is equal to C_m^6. Fig.2 shows the example for the formation of sensor networks, where the parameter m is assumed as seven. All the C_m^6 groups of analytical solutions obtained from their own sensor networks can be regard as the same when the propagation medium is homogeneous and the arrival time error is inexistent. However, the propagation medium such as rock is hardly ever homogeneous, a comprehensive analytical localization method can be applied to determine the reasonable and reliable source coordinates. The detailed steps are stated below:

Firstly, the invalid sensors should be excluded. There are three main characteristics of the invalid sensors. One is that the recorded data are noise or the SNR is very low, another is the recorded data have no relation with the event to be located, the third one is locating result using the sensor S_x is a lot different from the others.

Secondly, all the analytical solutions will be analyzed statistically. 6 sensors are selected randomly from m triggered sensors to combine $n = C_m^6$ groups of analytical solutions. Numerous kinds of probability density functions are applied to fit the whole solutions. The coordinates of microseismic/AE source are exactly the abscissa corresponding to the maximum value of the probability density function which fits the data best.

After comparing and analyzing the commonly used more than 60 types of probability density functions, the logistic, normal, and generalized extreme value probability density functions are applied to fit the source coordinates due to their characteristics respectively. The normal distribution is the theoretic basis of many statistical methods. It is useful due to the central limit theorem. In addition, the analyzing

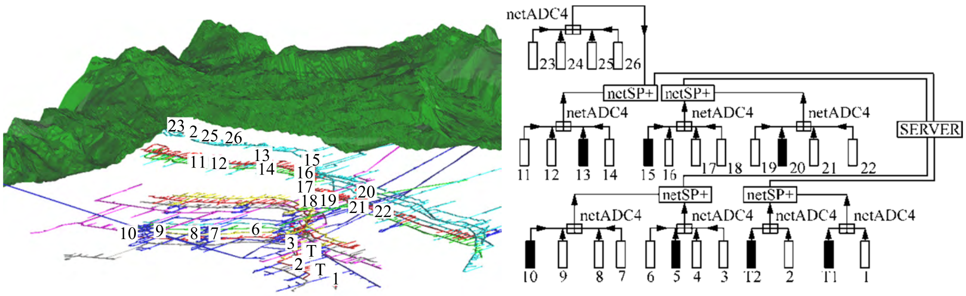

FIGURE 3. Here shows the established microseismic monitoring system in Yongshaba mine, where T1 as well as T2 are the three-component sensors and the remaining sensors are the single-component sensors. This system consists of sensors, data collectors, signal processors, underground data centers, communication cables and surface monitoring center.

variables usually approximately obey the normal distribution when the sample size is huge. Therefore, it is necessary to consider the normal distribution. Compared to the normal distribution, the logistic distribution has heavier tail, which often increases the robustness of analysis. The generalized extreme value distribution represents the Gumbel, Fréchet, and Weibull distributions with a uniform form. It is mostly used to describe the extreme variability of random variables and solve the stability problem.

The three functions are shown below, in which the function f represents the probability density function and the function F represents the cumulative distribution function.

$$f_L(x;\mu,s) = \frac{e^{-\frac{x-\mu}{s}}}{s\left(1+e^{-\frac{x-\mu}{s}}\right)^2} \tag{5}$$

$$F_L(x;\mu,s) = \frac{1}{1+e^{-\frac{x-\mu}{s}}} \tag{6}$$

In (5) and (6), x is the random variable, μ is the mean value, and s is a scale parameter proportional to the standard deviation. The fitting error decreases as the decrease of scale parameter s.

$$f_N(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{7}$$

$$F_N(x;\mu,\sigma) = \frac{1}{2}\left[1+erf\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right] \tag{8}$$

In (7) and (8), μ is the mean value of the normal distribution, and σ is the standard deviation. The fitting degree is better when the parameter σ is smaller.

$$f_G(x;\mu,\sigma,\xi) = \frac{1}{\sigma} t(x)^{\xi+1} e^{-t(x)} \tag{9}$$

$$F_G(x;\mu,\sigma,\xi) = e^{-t(x)} \tag{10}$$

where:

$$t(x) = \begin{cases} \left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi} & \xi \neq 0 \\ e^{-(x-\mu)/\sigma} & \xi = 0 \end{cases}$$

In (9) and (10), μ is the location parameter, σ is the scale parameter and ξ is the shape parameter. The possible source coordinates can be represented with μ.

As the probability density of source coordinate is considered in this method, that is the appearing frequency of different source coordinates. Therefore, the fitting results will not be affected seriously by a small amount of special values. A total of six blasting tests in two mines proved the accuracy and effectiveness of the proposed three dimensional comprehensive analytical and probability density function localization method.

III. RESULTS

A. YONGSHABA MINE

The ore body of Yongshaba mine is controlled by ten obvious faults and the stability of stope roof is poor. Some underground goafs are reserved due to the open stope mining methods in the early stage of mining engineering. As a result, the rockburst or rock instability for large areas may be induced by the local stress concentration. To avoid these destructive disasters, a 32-channel digital microseismic monitoring system was established. There are totally 26 single-component sensors and 2 three-component sensors distributed on the transport tunnels in 930, 1080, and 1120 levels to detect signals every day. Fig. 3 shows the layout of microseismic monitoring system and the relationship between geological structure. Fig. 4 shows the positions of sensors under the three dimensional structure of Yongshaba mine.

The data of three blasting tests in the Yongshaba mine were calculated to verify the proposed TDCAS-PDF. By substituting the coordinates and the arrival times of triggered sensors into (1)-(4), the source coordinates corresponding to different kinds of combinations can be obtained. Then, the logistic, normal, and generalized extreme value probability density functions are applied to fit the source coordinates. In the case of the event No.1, the fitting results of three probability density functions are compared in Table. 1. Obviously, the logistic probability density function shows the best fitting degree because the standard deviations for X, Y, and Z of logistic are the minimum among three functions. Therefore, the source coordinates will be selected according to the logistic probability density function. Fig. 5 to Fig. 7 show probability density functions and cumulative distribution functions

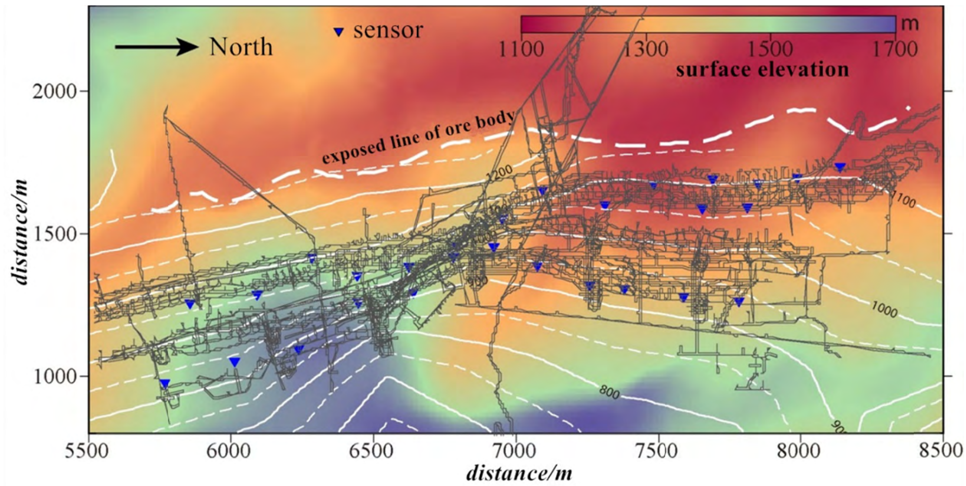

FIGURE 4. The positions of sensors and the main structure of Yongshaba mine are shown in the three dimensional cloud map. It is clear to see that the elevation of mine surface is more than 1500m. The current mining level is about 800m and the relative mining depth reaches 700m.

TABLE 1. Comparison results of the standard deviation error in the fitting for event No.1.

Probability density function	*X*		*Y*		*Z*	
	Std. Err.	Rank	Std. Err.	Rank	Std. Err.	Rank
Logistic	1.56	1	2.73	1	2.99	1
Normal	5.44	2	5.56	2	6.24	3
Generalized Extreme Value	8.63	3	5.94	3	5.88	2

TABLE 2. Location results using TDCAS-PDF in the Yongshaba mine.

Event No.	TDCAS-PDF			Blasting Coordinates			Error
	X/m	*Y*/m	*Z*/m	*X*/m	*Y*/m	*Z*/m	D^1/m
1	2996250	381183	1009	2996224	381194	1014	28.7
2	2997250	381546	1058	2997278	381590	1053	52.4
3	2997780	381631	1082	2997760	381683	1107	61.1
Average Value							47.4

D^1 is the absolute distance error (the distances between blasting coordinates and locating coordinates)

of the logistic, normal, and generalized extreme value for the events No.1 to No.3 in the Yongshaba mine respectively. The abscissa corresponding to the maximum value of the logistic probability density function is exactly the locating coordinate. The locating results are listed in the Table. 2. It is clear to see the absolute distance errors of the events No.1 to No.3 are 28.7m, 52.4m, and 61.1m respectively, that can commonly meet the requirements of engineering application in mines.

B. DONGGUASHAN MINE

The ore body and main surrounding rock of Dongguashan mine belong to hard rock, where exists the possibility of rockburst in the deep mining process [43]. The microseismic monitoring system was established to ensure the safety mining in the condition of multiple underground goafs. There are 12 single-component sensors working in the 514 and 558 levels, and other 6 single-component sensors are distributed in the 630 level.

The data of three blasting tests were calculated according to the proposed TDCAS-PDF [23]. Similarly, it is feasible to obtain the source coordinates by applying the three probability density functions. Fig. 8 to Fig. 10 show probability density functions and cumulative distribution functions of the logistic, normal, and generalized extreme value for the events No.4 to No.6 in the Dongguashan mine respectively. Also, the locating results and the absolute distance errors are listed in the Table. 3. The absolute distance errors of the

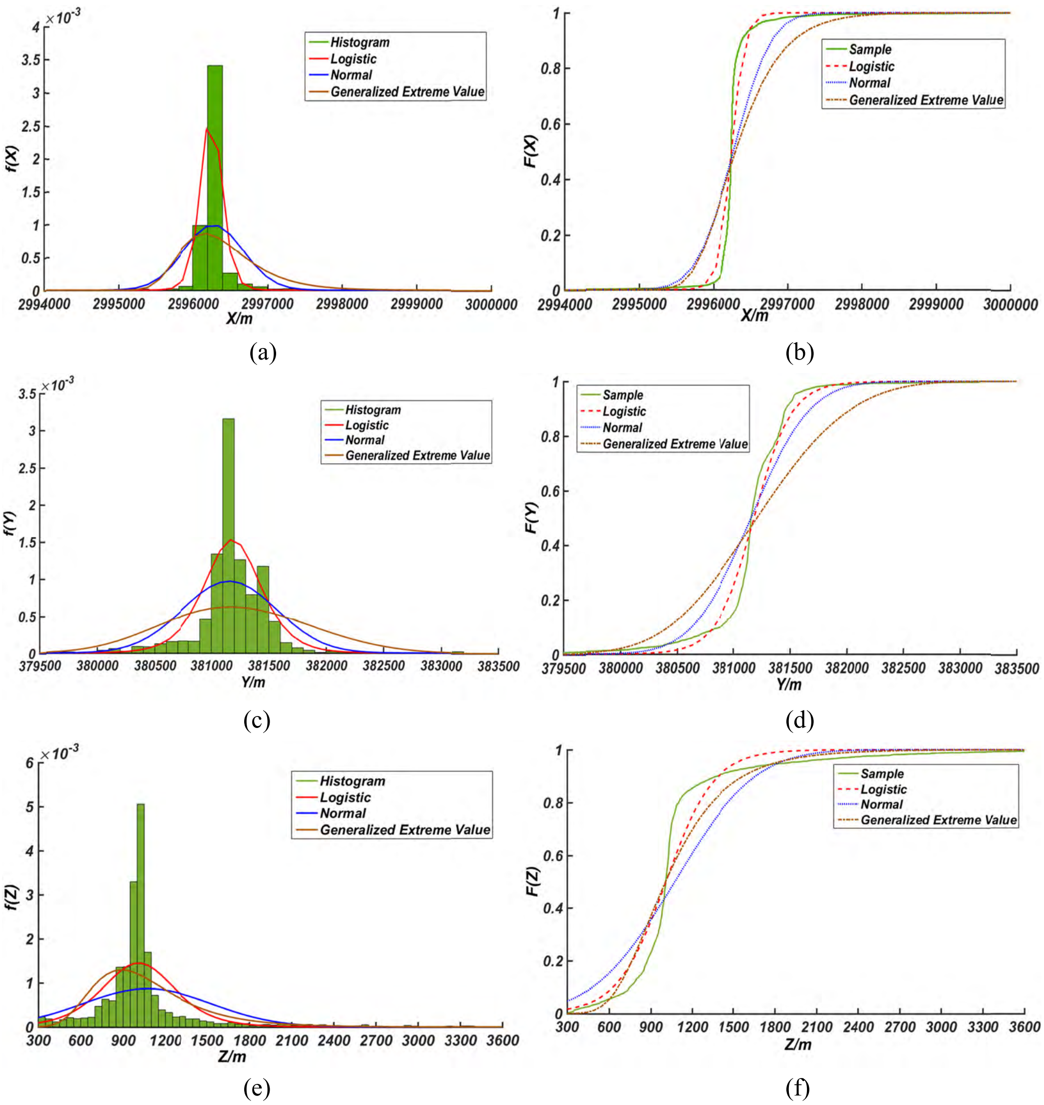

FIGURE 5. Upper graphs show the fitting results for the event No.1, where graphs (a) and (b) show the fitting results of the logistic, normal, and generalized extreme value distributions for the coordinate *X*. It is similar in graphs (c) and (d), as well as graphs (e) and (f) for the coordinate *Y* and *Z* respectively. The best fitting coordinates for the event No.1 are *X* = 2996250m, *Y* = 381183m, and *Z* = 1009m.

TABLE 3. Location results using TDCAS-PDF in the Dongguashan mine.

Event No.	TDCAS-PDF			Blasting Coordinates			Error
	X/m	*Y*/m	*Z*/m	*X*/m	*Y*/m	*Z*/m	*D*/m
4	84522.7	22556.0	-749.4	84528.4	22556.2	-753.2	6.85
5	84483.9	22571.0	-799.1	84479.0	22570.0	-814.4	16.09
6	84356.1	22685.8	-776.6	84359.0	22673.0	-795.5	23.01
Average Value							15.32

events No.4 to No.6 are 6.85m, 16.09m, and 23.01m respectively, which are less than the errors of STT method (12.90m, 16.17m, 24.90m) and TT method (10.99m, 20.32m, 32.43m). At the same time, the errors of the events No.4 and No.6 are also less than the STD method (12.93m, 14.84m, 24.90m). The absolute distance error of the event No.4 is less than the error of TD method (10.93m, 8.40m, 11.14m) [23].

As mentioned in the Fig. 1, the velocity value of *P*-wave is assumed as known, which needs to be measured beforehand in the traditional localization methods including the

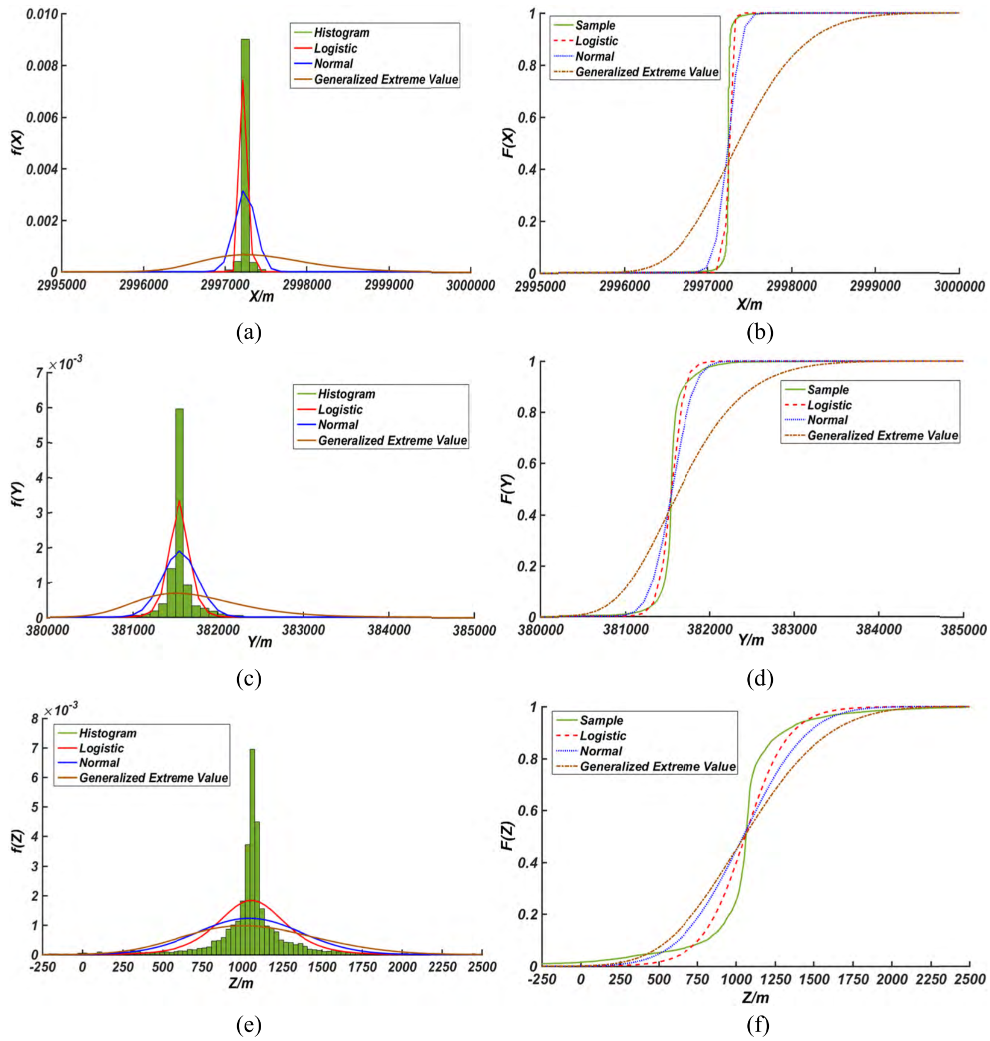

FIGURE 6. Upper graphs show the fitting results for the event No.2, where graphs (a) and (b) show the fitting results of the logistic, normal, and generalized extreme value distributions for the coordinate X. It is similar in graphs (c) and (d), as well as graphs (e) and (f) for the coordinate Y and Z respectively. The best fitting coordinates for the event No.2 are $X = 2997250$m, $Y = 381546$m, and $Z = 1058$m.

TABLE 4. Accurate arrival times of eight sensors triggered by four microseismic sources.

Sensors	Accurate arrival times of sensors triggered by *P*-wave/ms			
	P	*Q*	*S*	*T*
O	112.5995067	80.9919707	100.8019970	174.8570430
A	72.6253161	123.9263900	122.9943880	98.2505787
B	85.4992313	118.864396	125.4609370	115.7610930
C	120.8008440	72.3760424	103.8729196	184.7261870
D	59.7859293	70.7837130	51.1319635	89.2470626
E	146.3113410	194.5725390	179.7078140	106.9757798
F	152.5452390	191.5250500	181.3454540	123.1073490
G	75.3689298	60.3451559	57.7520220	108.4242926

STT method and the STD method. Thus, the locating accuracy will be affected seriously due to the difference between the authentic velocity value and the velocity value used in the calculating process. The dependent variables of the STT method and STD method are arrival time and arrival time difference respectively. However, the principle of the

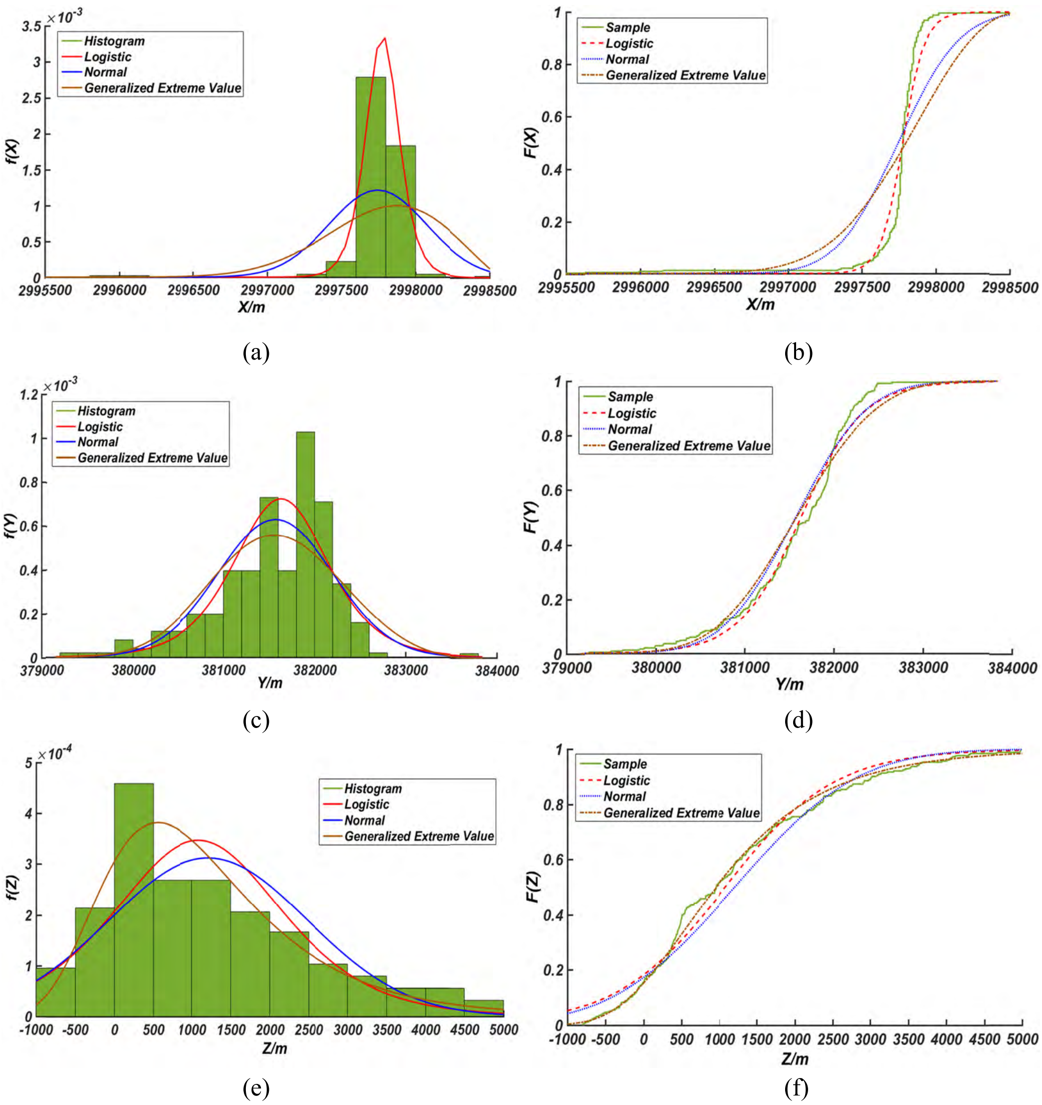

FIGURE 7. Upper graphs show the fitting results for the event No.3, where graphs (a) and (b) show the fitting results of the logistic, normal, and generalized extreme value distributions for the coordinate X. It is similar in graphs (c) and (d), as well as graphs (e) and (f) for the coordinate Y and Z respectively. The best fitting coordinates for the event No.3 are $X = 2997780$m, $Y = 381631$m, and $Z = 1082$m.

TABLE 5. Comparison results for P source under different scales of arrival time errors using two methods.

Arrival time error	TDCAS-PDF				TD method			
	X/m	Y/m	Z/m	D/m	X/m	Y/m	Z/m	D/m
5%	517.193	134.390	66.938	5.474	511.223	134.197	51.996	12.585
10%	515.926	133.264	67.449	6.498	506.833	130.590	41.743	24.306
15%	514.435	133.151	65.133	5.524	502.791	127.158	32.167	35.252
20%	513.009	133.457	61.963	5.537	499.061	123.885	23.001	45.674

two methods is similar, which is to obtain the best fitting degree by continuous approximation in the whole interval. Compared to the STT and STD methods, the advantage of the TT and TD methods lies in that the fitting process is performed without pre-measured velocity. Although the TT and TD methods eliminate the error of pre-measured velocity, the fitting process usually takes a lot of time to obtain the optimal solution, even the non-unique solutions, which is difficult to achieve the real-time localization.

After comparing and analyzing the lresults comprehensively, it can be concluded that the proposed TDCAS-PDF has a high locating accuracy, which is more accurate than the traditional localization methods. Furthermore, there is no need to solve the iterative solutions when applying the

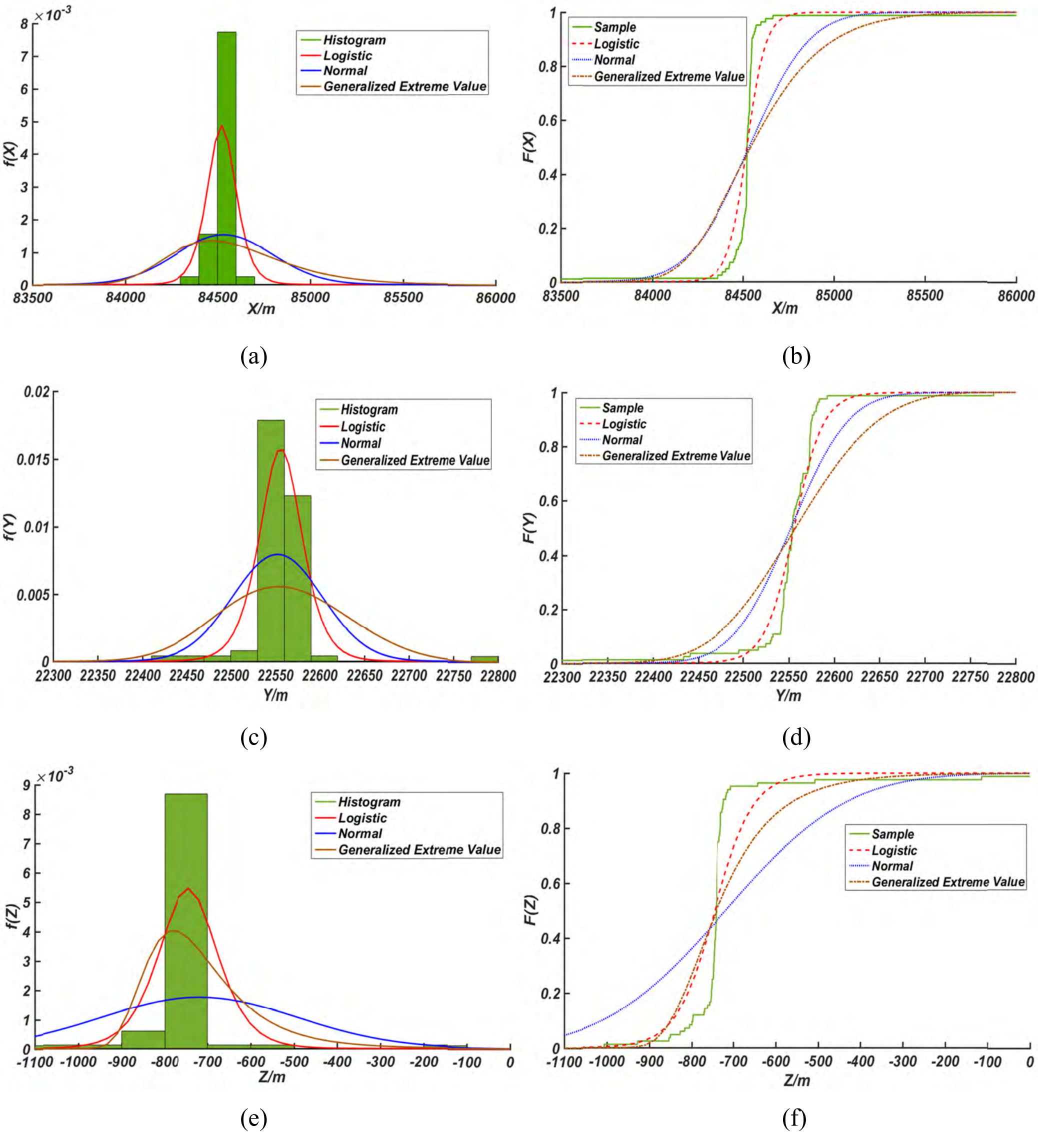

FIGURE 8. Upper graphs show the fitting results for the event No.4, where graphs (a) and (b) show the fitting results of the logistic, normal, and generalized extreme value distributions for the coordinate X. It is similar in graphs (c) and (d), as well as graphs (e) and (f) for the coordinate Y and Z respectively. The best fitting coordinates for the event No.4 are $X = 84522.7$m, $Y = 22556.0$m, and $Z = -749.4$m.

proposed TDCAS-PDF. The calculating formulas are explicit and the physical significance is clear, where the velocity of P-wave and the iterative algorithm make no influence to the source coordinates. All of these advantages make the TDCAS-PDF achieve the real-time locating and become more practical than traditional localization methods in the aspect of engineering applications.

IV. DISCUSSION

A. THE IMPORTANCE OF DETERMINATION AND OPTIMIZATION FOR SENSORS LAYOUT

1) THE POSITIONAL RELATIONSHIP BETWEEN THE MICROSEISMIC/AE SOURCE AND SENSORS

When the microseismic/AE source is close to one of the triggered sensors or at the same position, then the locating error will exceed the common value with simply using the arrival times received from 6 sensors network. In this situation, a locating system containing greater than 6 triggered sensors will be required to locate the authentic source coordinates. The solving steps can follow the method represented in the second section. If the distances from a microseismic/AE source to each sensor are equal or approximately equal, there is no solution by locating source coordinates with only 6 sensors. Therefore, the graphing method should be used to locate source coordinates.

2) THE POSITIONAL RELATIONSHIP BETWEEN THE TRIGGERED SENSORS

When two or several of the triggered sensors are at the same position or nearly the same, in that way only one sensor will be effective. If the distances between two or several sensors

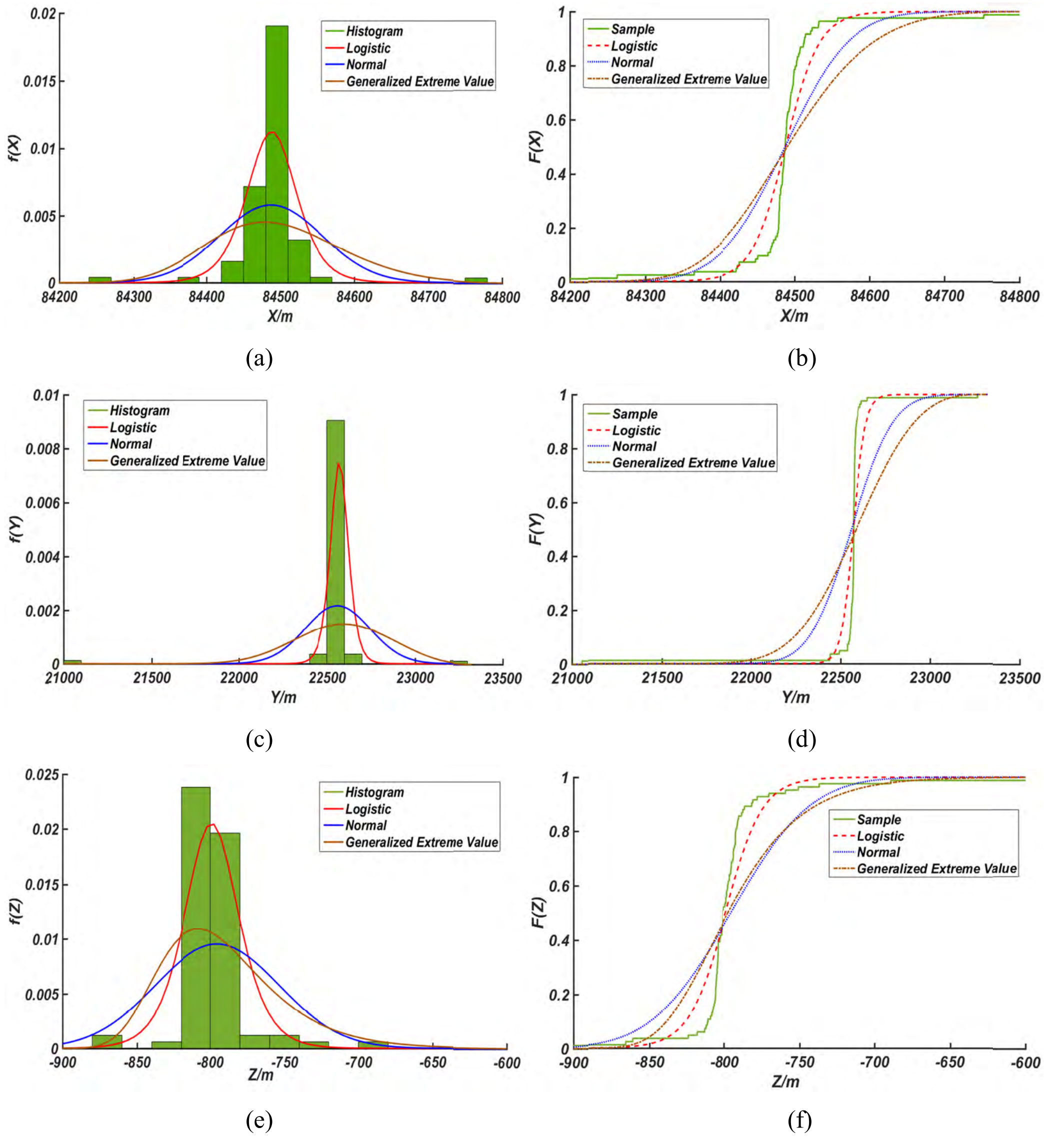

FIGURE 9. Upper graphs show the fitting results for the event No.5, where graphs (a) and (b) show the fitting results of the logistic, normal, and generalized extreme value distributions for the coordinate X. It is similar in graphs (c) and (d), as well as graphs (e) and (f) for the coordinate Y and Z respectively. The best fitting coordinates for the event No.5 are $X = 84483.9$m, $Y = 22571.0$m, and $Z = -799.1$m.

and the source are too large, then the locating accuracy will be reduced. At this moment, the analytical solution will be solved with enormous error when the number of sensors is greater than 6.

B. THE SIGNIFICANCE OF IDENTIFYING ACCURATE ARRIVAL TIME OF P-WAVE

In the spherical equation (1), the six circles should intersect at one point or form a concentrated area, where the center of this area is exactly the source location. The locating error is small when the arrival time error is small enough. If the arrival time error is large, then the concentrated area will be larger too, which makes it difficult to locate accurate source coordinates and causes greater error. In the worst situation, if the arrival time error is too large to calculate any intersection point, there will be no analytical solutions. Hence, identifying the arrival time of P-wave is really important for locating source coordinates.

However, the identifying error for arrival time of P-wave exists commonly in the practice, which is related to the experience of signal processing workers and the quality of signal. Although the iterative localization method can take full advantage of the numerous sensors to solve accurate source coordinates by continuous approximate the optimal value, the locating accuracy is still affected by the arrival time of P-wave. In some cases, the iterative calculating results will be a lot different only with arrival time errors of several milliseconds. On the contrary, the proposed TDCAS-PDF will

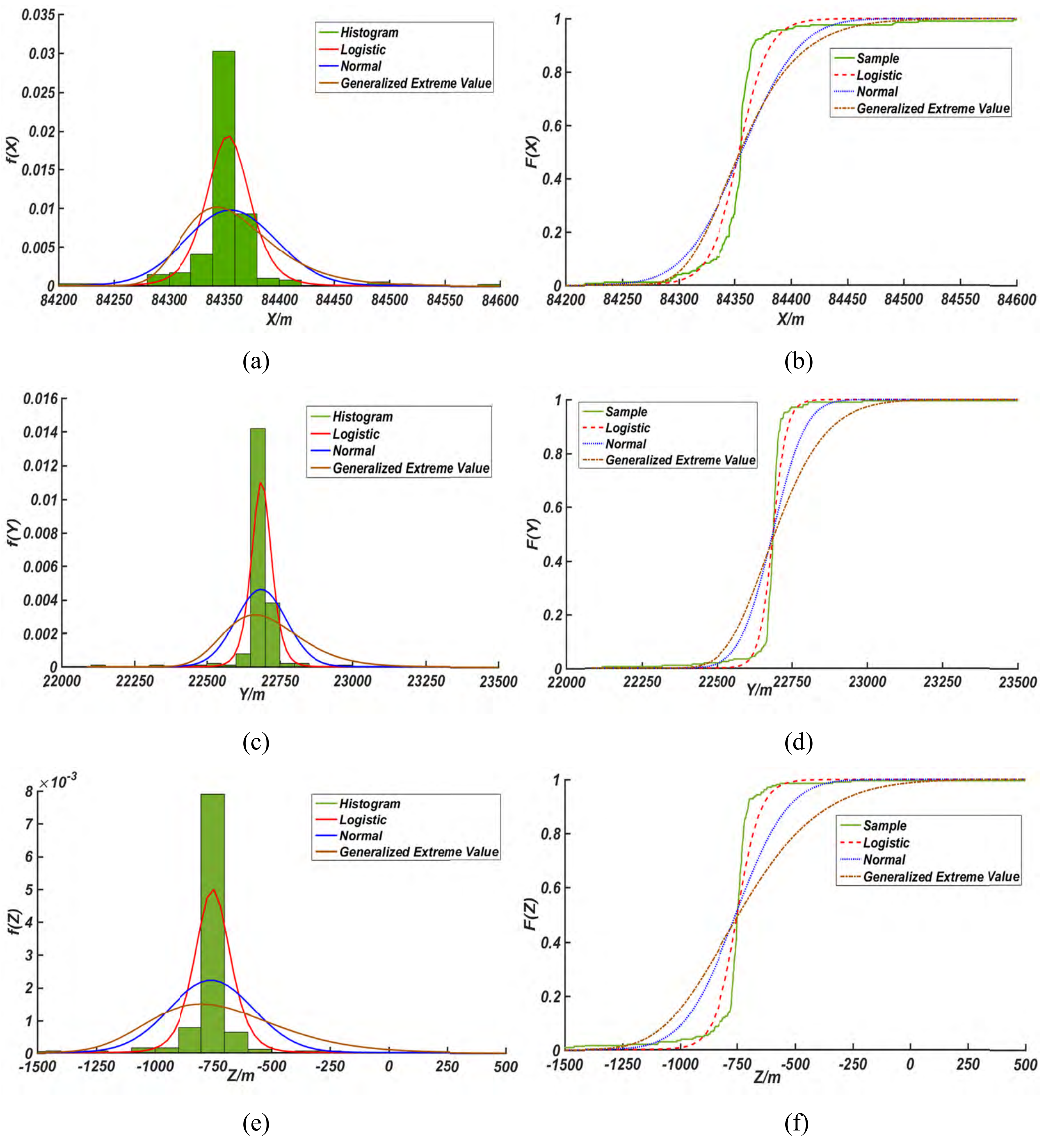

FIGURE 10. Upper graphs show the fitting results for the event No.6, where graphs (a) and (b) show the fitting results of the logistic, normal, and generalized extreme value distributions for the coordinate X. It is similar in graphs (c) and (d), as well as graphs (e) and (f) for the coordinate Y and Z respectively. The best fitting coordinates for the event No.6 are $X = 84356.1$m, $Y = 22685.8$m, and $Z = -776.6$m.

not be influenced clearly even under the large-scale arrival time errors. By making use of different sensor networks that combined with multiple sensors, the analytical solutions and statistical analysis are applied together to solve the optimal locating result. An example is established below to analyze the accuracy and reliability for an iterative method (the TD method) and the TDCAS-PDF under different scales of arrival time errors.

Fig. 11 shows an assumed locating system with a set of irregular sensor network. All the unit of these coordinates is meter. The sensors are distributed at the eight vertexes, which are O (0, 0, 0), A (800, 0, 0), B (800, 400, 0), C (0, 400, 0), D (400, 0, 240), E (1200, 0, 240), F (1200, 400, 240), and G (400, 400, 240) respectively. The sources P (516, 138, 63) and Q (260, 240, 98) locate inside the sensor network, while sources S (320, 180, 300) and T (745, 80, 450) locate outside the sensor network. The average velocity of P-wave in this medium is assumed as 5.4m/ms. It is assumed that the trigger time is 13ms in someday. The accurate arrival times of eight sensors triggered by four sources are listed in the Table. 4.

By adding more 5%, 10%, 15%, and 20% arrival time error to the farthest sensor away from the source by turns, which is the sensor F corresponds to the sources P and S, the sensor E corresponds to the source Q, and the sensor C corresponds to the source T specifically. The coordinates of these four sources under different arrival time errors can be solved with the TDCAS-PDF and the TD method. Table. 5 to Table. 8 list the coordinates and absolute distance errors of sources P,

TABLE 6. Comparison results for *Q* source under different scales of arrival time errors using two methods.

Arrival time error	TDCAS-PDF				TD method			
	X/m	*Y*/m	*Z*/m	*D*/m	*X*/m	*Y*/m	*Z*/m	*D*/m
5%	264.575	242.827	98.403	5.393	268.747	243.594	80.180	20.174
10%	266.529	237.234	92.800	8.793	276.463	247.122	64.430	38.062
15%	263.363	240.276	104.804	7.595	283.324	250.585	50.362	54.087
20%	267.124	239.578	98.159	7.138	289.470	253.978	37.684	68.570

TABLE 7. Comparison results for *S* source under different scales of arrival time errors using two methods.

Arrival time error	TDCAS-PDF				TD method			
	X/m	*Y*/m	*Z*/m	*D*/m	*X*/m	*Y*/m	*Z*/m	*D*/m
5%	330.812	179.386	306.918	12.850	325.085	176.983	266.244	34.270
10%	332.724	179.512	294.088	14.039	329.814	174.184	237.568	63.466
15%	334.466	179.790	304.262	15.082	334.163	171.391	212.941	88.623
20%	335.973	180.084	292.746	17.543	338.126	168.534	191.552	110.549

TABLE 8. Comparison results for *T* source under different scales of arrival time errors using two methods.

Arrival time error	TDCAS-PDF				TD method			
	X/m	*Y*/m	*Z*/m	*D*/m	*X*/m	*Y*/m	*Z*/m	*D*/m
5%	741.737	83.452	452.808	5.518	747.583	75.603	452.911	5.872
10%	742.048	82.807	452.859	4.977	749.887	71.558	455.687	11.291
15%	743.694	85.246	455.232	7.523	751.956	67.818	458.334	16.317
20%	746.044	85.514	456.624	8.682	753.824	64.344	460.860	20.998

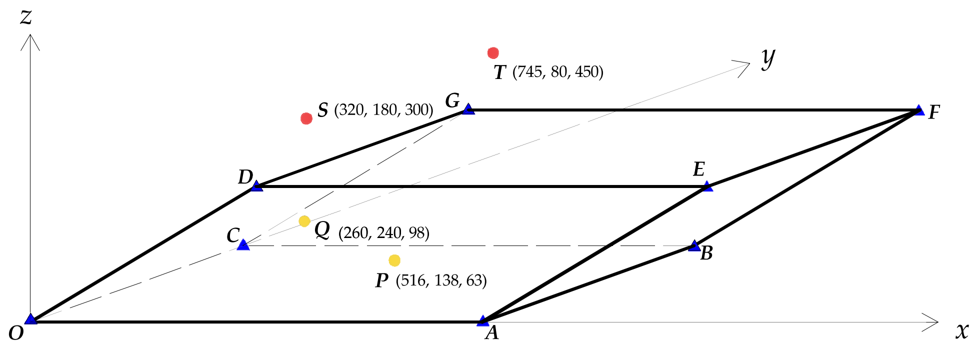

FIGURE 11. Upper graph shows an assumed sensor network which is used to locate the source coordinates. The sensors are distributed at the eight vertexes, which are *O* (0, 0, 0), *A* (800, 0, 0), *B* (800, 400, 0), *C* (0, 400, 0), *D* (400, 0, 240), *E* (1200, 0, 240), *F* (1200, 400, 240), and *G* (400, 400, 240) respectively. The sources *P* (516, 138, 63) and *Q* (260, 240, 98) locate inside the sensor network, while the sources *S* (320, 180, 300) and *T* (745, 80, 450) locate outside the sensor network.

Q, *S*, and *T* calculated by two methods under four scales of arrival time errors.

The results show that the absolute distance errors of the TDCAS-PDF for all the sources are less than the TD method under the same scale of arrival time error. Also, the absolute distance errors of the TDCAS-PDF are within the range which can meet the practical application. With the increasing of error scale, there is no obvious increase of the absolute distance error in the TDCAS-PDF. Nevertheless, the absolute distance errors increase rapidly as the arrival time errors become larger in the TD method. From the Fig. 12, it can be seen clearly that the growth rate of the TD method is great faster than the TDCAS-PDF. Therefore, we can believe that the proposed TDCAS-PDF is more suitable and reliable

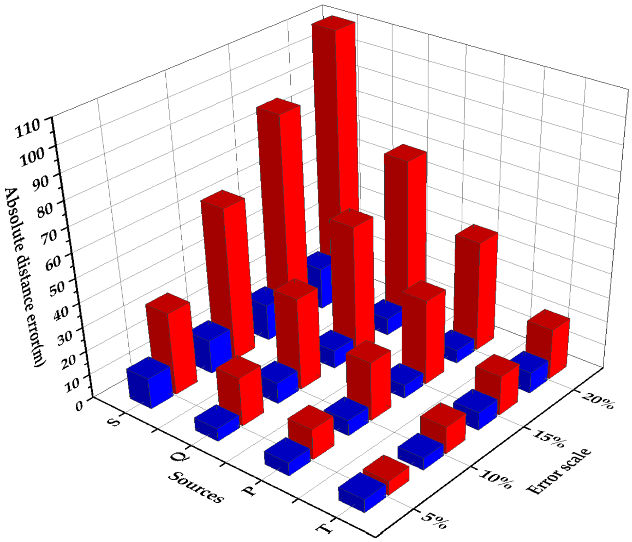

FIGURE 12. The three dimensional error analysis graph is shown above. The absolute distance errors of four sources under four scales of arrival time errors are represented by prisms with different heights. The blue prisms represent the absolute distance errors of the TDCAS-PDF and the red prisms represent the absolute distance errors of the TD method. It can be seen that the TDCAS-PDF is obviously superior to the TD method with arrival time errors.

for engineering practice, where the arrival time error exists commonly.

V. CONCLUSION

To eliminate the influence of wave velocity, iterative method, initial value, and non-unique solutions on the locating accuracy, the governing equations for locating the three dimensional microseismic/AE source in unknown velocity system are established to solve the accurate analytical solutions. The source coordinates can be determined for the 6 sensors network. The equations for calculating source coordinates are linear. Compared to the existing three dimensional analytical solutions, the advantages in locating source coordinates can be concluded as: no square root operation, no virtual root, and without pre-measured wave velocity. When the number of sensors is greater than 6, numerous sensor networks (a set of sensor network consists of every 6 sensors) are formed through the combination of triggered sensors. By applying logistic probability density function to the whole analytical solutions, the proposed localization method is verified with the data of six blasting tests in two different mines. The results show that the locating accuracy is significantly improved. In addition, the assumed examples with a random sensor network are established to discuss the locating accuracy under different scales of arrival time errors. It is obviously that the proposed TDCAS-PDF performs well under kinds of arrival time errors, which is more suitable and reliable for the practical engineering. In summary, the calculating formulas as well as the physical significance of the TDCAS-PDF are explicit and clear, where the source coordinates will not be affected by the velocity of P-wave, the iterative algorithm, and the arrival time error. It is more efficient than the traditional methods and the TT method in the aspect of engineering application, which is a beneficial complement for the locating theory of microseismic/AE source.

REFERENCES

[1] K. L. Rens, T. J. Wipf, and F. W. Klaiber, "Review of nondestructive evaluation techniques of civil infrastructure," *J. Perform. Constructed Facilities*, vol. 11, no. 4, pp. 152–160, Nov. 1997.
[2] H. Liu and E. Milios, "Acoustic positioning using multiple microphone arrays," *J. Acoust. Soc. Amer.*, vol. 117, no. 5, pp. 2772–2782, Apr. 2005.
[3] T. Kundu, "Acoustic source localization," *Ultrasonics*, vol. 54, no. 1, pp. 25–38, Jan. 2014.
[4] W. D. Ortlepp and T. R. Stacey, "Rockburst mechanisms in tunnels and shafts," *Tunnelling Underground Space Technol.*, vol. 9, no. 1, pp. 59–65, Jan. 1994.
[5] A. J. Mendecki, Ed., *Seismic Monitoring in Mines*, 1st ed. London, U.K.: Springer, 1996, pp. 220–245.
[6] A. Håkansson, D. Torrent, F. Cervera, and J. Sánchez-Dehesa, "Directional acoustic source by scattering acoustical elements," *Appl. Phys. Lett.*, vol. 90, no. 22, p. 224107, Jun. 2007.
[7] R. O. Schmidt, "A new approach to geometry of range difference location," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-8, no. 6, pp. 821–835, Nov. 1972.
[8] M. P. Collins and R. M. Belchamber, "Acoustic emission source locating using simplex optimization," *J. Acoust. Emission*, vol. 9, no. 4, pp. 271–276, 1990.
[9] E. Landis, C. Ouyang, and S. P. Shah, "Acoustic emission source locations in concrete," in *Proc. Mech. Comput. 1990's Beyond*, Columbus, OH, USA, 1991, pp. 1051–1055.
[10] M. S. Brandstein, J. E. Adcock, and H. F. Silverman, "A closed-form location estimator for use with room environment microphone arrays," *IEEE Trans. Speech Audio Process.*, vol. 5, no. 1, pp. 45–50, Jan. 1997.
[11] L. Dong, X. Li, and G. Xie, "An analytical solution for acoustic emission source location for known P wave velocity system," *Math. Probl. Eng.*, vol. 2014, Mar. 2014, Art. no. 290686.
[12] M. Ge, H. R. Hardy, Jr., H. Wang, and J. Wang, "Development of a simple mechanical impact system as a non-explosive seismic source," *Geotech. Geol. Eng.*, vol. 29, no. 1, pp. 137–142, Jan. 2011.
[13] J. C. Chen, R. E. Hudson, and K. Yao, "Maximum-likelihood source localization and unknown sensor location estimation for wideband signals in the near-field," *IEEE Trans. Signal Process.*, vol. 50, no. 8, pp. 1843–1854, Aug. 2002.
[14] B. L. N. Kennett, K. Marson-Pidgeon, and M. S. Sambridge, "Seismic source characterization using a neighbourhood algorithm," *Geophys. Res. Lett.*, vol. 27, no. 20, pp. 3401–3404, Oct. 2000.
[15] P. Nivesrangsan, J. A. Steel, and R. L. Reuben, "Source location of acoustic emission in diesel engines," *Mech. Syst. Signal Process.*, vol. 21, no. 2, pp. 1103–1114, Feb. 2007.
[16] F. Waldhauser and W. L. Ellsworth, "A double-difference earthquake location algorithm: Method and application to the northern Hayward fault, California," *Bull. Seismol. Soc. Amer.*, vol. 90, no. 6, pp. 1353–1368, Dec. 2000.
[17] M. Knapmeyer, "Location of seismic events using inaccurate data from very sparse networks," *Geophys. J. Int.*, vol. 175, no. 3, pp. 975–991, Dec. 2008.
[18] B. R. Lienert, E. Berg, and L. N. Frazer, "HYPOCENTER: An earthquake location method using centered, scaled, and adaptively damped least squares," *Bull. Seismol. Soc. Amer.*, vol. 76, no. 3, pp. 771–783, Jun. 1986.
[19] K. Aki and W. H. K. Lee, "Determination of three-dimensional velocity anomalies under a seismic array using first P arrival times from local earthquakes: 1. A homogeneous initial model," *J. Geophys. Res.*, vol. 81, no. 23, pp. 4381–4399, Aug. 1976.
[20] K. Aki, A. Christoffersson, and E. S. Husebye, "Determination of the three-dimensional seismic structure of the lithosphere," *J. Geophys. Res.*, vol. 82, no. 2, pp. 277–296, Jan. 1977.
[21] A. Tarantola and B. Valette, "Inverse problems—Quest for information," *J. Geophys.*, vol. 50, no. 2, pp. 159–170, Oct. 1982.
[22] M. Matsu'ura, "Bayesian estimation of hypocenter with origin time eliminated," *J. Phys. Earth*, vol. 32, no. 6, pp. 469–483, Jan. 1984.

[23] L. Dong, X. Li, L. Tang, and F. Gong, "Mathematical functions and parameters for microseismic source location without pre-measuring speed," *Chin. J. Rock Mech. Eng.*, vol. 30, no. 10, pp. 2057–2067, Oct. 2011.

[24] L. J. Dong, "Methodologies and applications for precise location and realtime identification of microseismic sources in mines," Ph.D. dissertation, Dept. Mining Eng., Central South Univ., Changsha, China, 2013.

[25] L.-J. Dong, X.-B. Li, Z.-L. Zhou, G.-H. Chen, and J. Ma, "Three-dimensional analytical solution of acoustic emission source location for cuboid monitoring network without pre-measured wave velocity," *Trans. Nonferrous Met. Soc. China*, vol. 25, no. 1, pp. 293–302, Jan. 2015.

[26] L.-J. Dong and X.-B. Li, "Three-dimensional analytical solution of acoustic emission or microseismic source location under cube monitoring network," *Trans. Nonferrous Met. Soc. China*, vol. 22, no. 12, pp. 3087–3094, Dec. 2012.

[27] X. Li and L. Dong, "An efficient closed-form solution for acoustic emission source location in three-dimensional structures," *AIP Adv.*, vol. 4, no. 2, pp. 027110-1–027110-8, Feb. 2014.

[28] J. Smith and J. Abel, "Closed-form least-squares source location estimation from range-difference measurements," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 35, no. 12, pp. 1661–1669, Dec. 1987.

[29] Y. T. Chan and K. C. Ho, "An efficient closed-form localization solution from time difference of arrival measurements," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, Adelaide, SA, Australia, Apr. 1994, pp. II-393–II-396.

[30] R. Duraiswami, D. Zotkin, and L. Davis, "Exact solutions for the problem of source location from measured time differences of arrival," *J. Acoust. Soc. Amer.*, vol. 106, no. 4, p. 2277, Aug. 1999.

[31] G. Mellen, M. Pachter, and J. Raquet, "Closed-form solution for determining emitter location using time difference of arrival measurements," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 39, no. 3, pp. 1056–1058, Jul. 2003.

[32] X. Li and L. J. Dong, "Comparison of two methods in acoustic emission source location using four sensors without measuring sonic speed," *Sensor Lett.*, vol. 9, no. 5, pp. 2025–2029, Oct. 2011.

[33] M. Ge, "Analysis of source location algorithms—Part I: Overview and non-iterative methods," *J. Acoust. Emission*, vol. 21, pp. 14–28, Jan. 2003.

[34] M. Ge, "Analysis of source location algorithms—Part II: Iterative methods," *J. Acoust. Emission*, vol. 21, pp. 29–51, Jan. 2003.

[35] G. Han, X. Yang, L. Liu, W. Zhang, and M. Guizani, "A disaster management-oriented path planning for mobile anchor node-based localization in wireless sensor networks," *IEEE Trans. Emerg. Topics Comput.*, to be published.

[36] S. Xie and Y. Wang, "Construction of tree network with limited delivery latency in homogeneous wireless sensor networks," *Wireless Pers. Commun.*, vol. 78, no. 1, pp. 231–246, Sep. 2014.

[37] J. Shen, H.-W. Tan, J. Wang, J.-W. Wang, and S.-Y. Lee, "A novel routing protocol providing good transmission reliability in underwater sensor networks," *J. Internet Technol.*, vol. 16, no. 1, pp. 171–178, Jan. 2015.

[38] Y. Zhang, X. Sun, and B. Wang, "Efficient algorithm for K-barrier coverage based on integer linear programming," *China Commun.*, vol. 13, no. 7, pp. 16–23, Sep. 2016.

[39] G. Han, L. Liu, S. Chan, R. Yu, and Y. Yang, "HySense: A hybrid mobile crowdsensing framework for sensing opportunities compensation under dynamic coverage constraint," *IEEE Commun. Mag.*, vol. 55, no. 3, pp. 93–99, Mar. 2017.

[40] G. Han, L. Liu, J. Jiang, L. Shu, and G. Hancke, "Analysis of energy-efficient connected target coverage algorithms for industrial wireless sensor networks," *IEEE Trans. Ind. Informat.*, vol. 13, no. 1, pp. 135–143, Feb. 2017.

[41] L. Dong, J. Wesseloo, Y. Potvin, and X.-B. Li, "Discriminant models of blasts and seismic events in mine seismology," *Int. J. Rock Mech. Mining Sci.*, vol. 86, pp. 282–291, Jul. 2016.

[42] L. Dong, J. Wesseloo, Y. Potvin, and X. Li, "Discrimination of mine seismic events and blasts using the fisher classifier, naive Bayesian classifier and logistic regression," *Rock Mech. Rock Eng.*, vol. 49, no. 1, pp. 183–211, Jan. 2016.

[43] L. Tang, C. Yang, and C. Pan, "Optimization of microseismic monitoring network for large-scale deep well mining," *Chin. J. Rock Mech. Eng.*, vol. 25, no. 10, pp. 2036–2042, Oct. 2006.

LONGJUN DONG (M'16) received the Ph.D. degree from the School of Resources and Safety Engineering, Central South University, Changsha, China, in 2013. From 2012 to 2013, he was an Assistant Researcher with the Australia Center for Geomechanics, The University of Western Australia, Perth, Australia. His current research interests include computational methods in location and identification for shock sources, seismic signals, machine learning algorithms, and rock/mineral mechanics for mining science.

He is currently an Associate Professor with the School of Resource and Safety Engineering, Central South University. He is a member of ASCE and ISRM. He has served as a Reviewer of over 30 journals. He was selected for The Young Elite Scientists Sponsorship Program by the China Association for Science and Technology.He is invited to serve as the Editorial Board Member of *Scientific Reports*, the *Internal Journal of Distributed Sensor Networks*, and *Shock and Vibration*.

WEIWEI SHU received the B.Sc. degree in mining engineering from Central South University, Changsha, China, in 2016, where he is currently pursuing the M.Sc. degree. His research interests include rock mechanics, microseismic monitoring, and the localization method for shock sources.

XIBING LI received the Ph.D. degree in mining engineering from the Central South University of Technology, Changsha, China, in 1992. He was a Senior Visiting Scholar with the Rock Mechanics and Explosives Research Center, University of Missouri Rolla, from 1998 to 1999. He was also a Researcher with Nanyang Technological University, Singapore, from 1999 to 2001. He received four projects of National Science and Technology Awards, and 14 projects of Provincial/Ministerial Science and Technology Awards. He was selected for The National Science Fund for Distinguished Young Scholars and Chang Jiang Distinguished Professor. He received awards, including the National Award for Youth in Science and Technology and the National Excellent Scientific and Technological Worker.

Dr. Li has been a Committee Member with the Rock Dynamics of International Society for Rock Mechanics since 2008. He has also been the Vice President of the Chinese Society for Rock Mechanics and Engineering since 2016.

GUANGJIE HAN (S'01–M'05) received the Ph.D. degree from Northeastern University, Shenyang, China, in 2004. From 2004 to 2006, he was a Product Manager of ZTE Company. In 2008, he was a Post-Doctoral Researcher with the Department of Computer Science, Chonnam National University, Gwangju, South Korea. From 2010 to 2011, he was a Visiting Research Scholar with Osaka University, Suita, Japan. He is currently a Professor with the Department of Information and Communication System, Hohai University, Changzhou, China. He has authored over 230 papers published in related international conference proceedings and journals. He holds 100 patents. His current research interests include sensor networks, computer communications, mobile cloud computing, and multimedia communication and security.

Dr. Han is a member of the ACM. He received the ComManTel 2014, ComComAP 2014, Chinacom 2014, and Qshine 2016 Best Paper Awards. He has served as a Co-Chair for over 50 international conferences/workshops and a Technical Program Committee Member of over 150 conferences. He has served as a Reviewer of over 50 journals. He has served on the Editorial Boards of up to 14 international journals, including the IEEE Access, *Telecommunication Systems*, the *International Journal of Ad Hoc and Ubiquitous Computing*, the *Journal of Internet Technology*, and the *KSII Transactions on Internet and Information Systems*. He guest edited a number of special issues in the IEEE Journals and Magazines.

WEI ZOU received the B.Sc. degree in safety engineering from Central South University, Changsha, China, in 2016, where he is currently pursuing the M.Sc. degree. His research interests include the algorithms of acoustic emission and microseismic source localization, and the mechanical properties of rock true triaxial experiments.