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Energy-Efficient Power Allocation for MIMO-SVD Systems

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ABSTRACT In this paper, we address the problem of energy-efficient power allocation in MIMO systems. In fact, the widely adopted water-filling power allocation does not ensure the maximization of the energy efficiency (EE). Since the EE maximization is a non-convex problem, numerical methods based on fractional programming were introduced to find the optimal power solutions. In this paper, we present a novel and simple power allocation scheme based on the explicit expressions of the optimal power. We also present a low-complexity algorithm that complements the proposed scheme for low circuit-power regime. Furthermore, we analyze power-constrained and rate-constrained systems and present the corresponding optimal power control. In the numerical results, we show that the presented analytical expressions are accurate and that the algorithm converges within two iterations. We also show that as the number of antennas increases, the system becomes more energy-efficient. Also, a saturation of the EE is observed at high-power budget and low minimal rate regimes.

INDEX TERMS Energy-efficiency, MIMO systems, minimal rate constrained, optimal power allocation, power budget constraint.

I. INTRODUCTION

The energy consumption in wireless communication systems (WCS) has become a crucial concern in order to meet the requirements of the 5G [1]. In particular, this concern is important when designing battery-powered nodes such as mobile stations and sensor networks to enhance the operation lifetime [2]. In addition, the urge to adapt *green* wireless systems is imposed by environmental considerations. In fact, the CO_2 footprint of the information and communications technology (ICT) is around 2% of the global CO_2 emissions with $0.2-0.4\%$ produced by the wireless networks [3]. Also, given the fact that the 5G data rate is required to increase 1000 times [4] and that the number of connected WCS is estimated to reach 50 billions in 2020 [5], the power consumption is expected to increase 1000 times or even more, unless new energy-efficient approaches and techniques are considered.

During the last years, while designing the WCSs the aim was to maximize the performances related to the spectral efficiency (SE) [6]. Another design paradigm hat emerged recently consists of maximizing the energy efficiency (EE) describing the transmission efficiency of the given frequency bandwidth [7]. Consequently, new WCS are designed to be efficient in terms of power consumption which means that the EE metric could be the main performance yardstick [8] instead of the SE [9].

In the literature, the EE started to attract the attention from an information theoretical point of view through the minimum energy per information bit (J/bit) [10]. Afterward, the focus was on the EE metric defined as the energy cost of the achievable rate [11]. In addition, some studies focused on the modeling of the power consumed by the RF circuit called the circuit-power [12], [13]. In the single-input and single-output (SISO) context, the EE maximization was performed, in many works, using the numerical fractional programming not resulting in explicit power expressions [13], [14]. In [15], an explicit expression of the power maximizing the EE was introduced for SISO orthogonal frequency-division multiple access (OFDMA) systems without considering multiple antenna transmission.

In more advanced systems, multiple-input and multiple-output (MIMO) transmission is considered to be a promising technology for the next generation cellular networks. In particular, the beamforming technique, or the MIMO-SVD based on the singular value decomposition (SVD) is widely used to achieve higher performance [16]. This technique transforms the MIMO channel into independent parallel channels to send multiple data streams offering significant improvement in terms of spectral efficiency. Therefore, studying the energy efficiency of the MIMO-SVD is also important. However, maximizing the EE in the MIMO context is not straightforward due to the non-convexity of the optimization problem. Therefore, numerical algorithms were presented to solve this type of problems. In previous works, the EE was optimized using numerical fractional programming methods that transforms the fractional objective function into a subtractive one [17]–[22]. In [19], for instance, the EE was maximized in a broadcast MIMO context using an iterative water-filling algorithm. While this method solves the problem numerically, it does not provide explicit expression of the optimal power profile since it fails to provide explicit expressions of the power [23].

To the best of our knowledge, an explicit expressions of the optimal power for MIMO-SVD systems were not presented before. Although in [24] the authors give the power of MIMO-SVD systems, the energy-per-bit was maximized which gives a different power control than the EE. In addition, the presented power control is based on the water filling which is used to maximize the spectral efficiency. From another side, in [25] and [26], the power allocation scheme that maximizes the energy efficiency a point to point scenario with a single antenna at each terminal is presented. When scaling up the number of antennas at each terminal, the power allocation scheme in [25] and [26] is no longer valid and cannot be used to extend to the case of multiple antennas. However, in [27], this scheme was used in MIMO CR framework by considering the sum energy efficiency of single parallel channels of the MIMO link. In our current work, we presented a more general power scheme involving multiple antennas at the terminals and maximizing the global energy efficiency defined as the total rate over the total consumed power.

In this paper, we compute the MIMO-SVD power expression that maximizes the EE and the corresponding maximum EE. Then, we present a simple algorithm that complements our results at low circuit-power regime. We also present the optimal power control when the system is limited by a certain power budget or required to achieve a certain minimal rate.

Specifically, our key contributions are summarized as follows:

- we derive novel explicit expression of the power that maximizes the non-convex EE of MIMO systems,
- we propose a simple iterative algorithm that complements the analytical power solutions at low values of circuit-power,
- we extend our analysis to power-constrained systems with power budget and minimal rate constraints and provide the corresponding optimal power allocation.

The rest of this paper is organized as follows. In Section II, the system model is presented. In Section III, the EE power allocation of unconstrained systems is computed. In Section IV, the EE power of constrained system is derived. Numerical results are presented in Section V. Finally, the conclusion of this paper is in Section VI.

II. SYSTEM MODEL

We consider a point-to-point MIMO communication link with N_t and N_r antennas at the transmitter and the receiver sides, respectively. The complex channel gain matrix is denoted by $\boldsymbol{H} \in \mathbb{C}^{N_r \times N_t}$ and the received signal is

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{y} \in \mathbb{C}^{N_r \times 1}$, $\boldsymbol{x} \in \mathbb{C}^{N_t \times 1}$ is the complex Gaussian transmitted signal and $\boldsymbol{n} \in \mathbb{C}^{N_t \times 1}$ is a circularly symmetric white Gaussian noise with covariance $\mathbb{E}[\boldsymbol{n}\boldsymbol{n}^H] = \boldsymbol{I}_{N_t}$, where $\mathbb{E}[\cdot]$ and $\cdot^H$ are the expectation and the Hermitian operators, respectively. We denote by $\boldsymbol{P} = \mathbb{E}[\boldsymbol{x}\boldsymbol{x}^H]$, the transmit signal covariance matrix.

We also denote by P_c the circuit-power of each antenna and its corresponding RF chain. P_c includes the consumption of the RF chain components that are independent of the transmit power, i.e., analog to digital converters (ADC), filters, mixers, amplifiers, etc [26], [28]. The circuit-power P_c can be modeled in several ways as described in [29]. In our framework, we assume that P_c and the transmit power are decoupled so that P_c is constant. Without loss of generality, we assume that the circuit-powers of the RF chains are all equal to P_c.

We denote by P_{peak} the available power budget. When the transmit power is limited we introduce the constraint $\mathrm{Tr}\,(\boldsymbol{P}) \leq P_{peak}$ where $\mathrm{Tr}\,(\boldsymbol{A}) = \sum_j A(j,j)$ is the trace of the matrix $\boldsymbol{A}$. In addition, we denote by R_{min}, the minimal acceptable rate need for the transmission [20].

In a MIMO transmission with transmit channel side information, the SVD decomposes the MIMO channel into independent parallel SISO channels that are equivalent to real channels. When the number of antennas at the transmitter and at the receiver are different, the SVD produces N parallel channels where $N = \min\{N_t, N_r\}$. Each parallel channel is characterized by a singular value denoted λ_i where the singular values are in descending order. Each λ_i has a particular distribution that depends on its order among the other singular values and on their total number [30]. For a given singular value λ_i, we denote by $\gamma_i = \lambda_i^2$. In the case of MIMO-SVD, the power matrix $\boldsymbol{P}$ is diagonal with values denoted as P_i, $i \in 1, \ldots, N$.

The instantaneous MIMO-SVD spectral efficiency (SE) is defined as the sum of the partial spectral efficiencies corresponding to the different parallel channels, i.e.,

$$SE = \sum_{i=1}^{N} \log_2 (1 + \gamma_i P_i) \quad (bps/Hz). \tag{2}$$

The energy efficiency is defined to measure the cost of this achievable rate in terms of energy. In the multi-dimensional problems in general, there are three definitions of the EE as shown in [31]. The first is the global EE (GEE) defined as the global achievable rate over the total consumed power, characterizing the system as a single unit [15]. The second is the weighted sum of the partial EE's denoted by Sum-EE that aims to give priority to certain parts of the system [27]. The third is the product of the partial EE's denoted Prod-EE and aims to ensure fairness among all system's components.

In the MIMO framework, the Sum-EE and Prod-EE are not widely adopted since, in general, in the power allocation procedure, there is no priority or fairness among the antennas. Hence, we focus on the GEE that we denote by EE in rest of the paper, for simplicity. The expression of the EE is given by

$$EE = \frac{\sum_{i=1}^{N} \log_2 (1 + \gamma_i P_i)}{\sum_{i=1}^{N} (P_c + P_i)} \; (bits/J/Hz). \tag{3}$$

Our objective is to provide an energy-efficient power allocation (EEPA) scheme that maximizes the EE of power budget and rate constrained systems. We aim to explicitly find expressions of the optimal power of this problem, instead of the numerical fractional programming methods. However, due to the non-convexity of the problem, we first start, in Section III, by solving the unconstrained problem, i.e., $P_{peak} \rightarrow \infty$ and $R_{min} \rightarrow 0$. Then, in Section IV, we present the solution of the power budget and minimal rate constrained systems, i.e., $P_{peak} < \infty$ and $R_{min} > 0$.

III. UNCONSTRAINED SYSTEMS EE POWER ALLOCATION

The objective of maximizing the unconstrained EE is to find explicit expression of the power as a preliminarily result. Next, this expression is used to determine the power allocation for constrained systems in Section IV.

We first note that, for $i \in 1, \ldots, N$, the EE is a positive function of P_i and is not multimodal since it is a fraction of a logarithmic function over a linear function of P_i. In addition, we have $\lim_{P_i \to 0} EE = \lim_{P_i \to \infty} EE = 0 \; \forall i$. Hence, $\lim_{\boldsymbol{P} \to 0} EE = \lim_{\boldsymbol{P} \to \infty} EE = 0$, where $\lim_{\boldsymbol{P} \to 0}$ is defined as the limit to zero of all $\boldsymbol{P}$ components. Consequently, $EE(\boldsymbol{P})$ has a global and unique maxima [21].

Our objective is to find an explicit expression of the optimal power that maximizes the EE based on finding the root of its derivative which yields the unique and global maxima. For a given parallel channel i, the EE is a continuous and differentiable function for all values of $P_i \geq 0$. Also, since it has a global maxima, this maxima corresponds necessarily to a root of the first-order derivative which is computed as

$$\frac{\partial EE}{\partial P_i} = \frac{\gamma_i}{\log(2)(1 + \gamma_i P_i) \sum_{j=1}^{N} (P_c + P_j)} - \frac{\sum_{j=1}^{N} \log(1 + \gamma_j P_j)}{\log(2) \left(\sum_{j=1}^{N} (P_c + P_j) \right)^2}. \tag{4}$$

By equating this derivative to zero, the optimal power solutions denoted by P_i^* needs to satisfy the following condition

$$\frac{\gamma_i}{1 + \gamma_i P_i^*} = \frac{\sum_{j=1}^{N} \log(1 + \gamma_j P_j^*)}{\sum_{j=1}^{N} (P_c + P_j^*)}. \tag{5}$$

In order to find these solutions, we introduce the following lemma.

Lemma: For the non-negative real numbers a, b and c, a solution of the equation

$$\frac{a}{1 + ax} = \frac{\log(1 + ax) + b}{c + x}, \tag{6}$$

is given by

$$x = \frac{1}{a} \left(\exp\left(1 - b + W\left(\frac{ac - 1}{\exp(1 - b)} \right) \right) - 1 \right), \tag{7}$$

where $W(\cdot)$ is the main branch of the W-Lambert function defined on $[-\frac{1}{e}, \infty]$ [32].

Proof: The proof of the lemma is in Appendix A. ■

Note that the condition $\frac{ac-1}{\exp(1-b)} > -\frac{1}{e}$ i.e., $ac + e^{-b} > 1$ needs to be verified for the solution in (7) to be valid.

We use this lemma to solve (5) and find a preliminary expression of the optimal power in the following theorem.

Theorem 1: The power maximizing $EE = \frac{\sum_{i=1}^{N} \log_2(1+\gamma_i P_i)}{\sum_{i=1}^{N} (P_c+P_i)}$, is given, for $i \in 1, \ldots, N$, by

$$P_{EE,i} = \left[\frac{1}{\gamma_i} \left(\exp\left(1 - SR_i + W\left(\frac{\gamma_i SP_i - 1}{e^{1-SR_i}} \right) \right) - 1 \right) \right]^+, \tag{8}$$

where $[\cdot]^+ = \max\{\cdot, 0\}$ and SR_i and SP_i are the sum rate and sum power excluding the i-th channel defined by

$$SR_i = \sum_{j=1, j\neq i}^{N} \log\left(1 + \gamma_j P_{EE,j}\right) \text{ and } SP_i = NP_c + \sum_{j=1, j\neq i}^{N} P_{EE,j}. \tag{9}$$

Proof: It follows by applying the Lemma to solve (5), for $x = P_{EE,i}$, $a = \gamma_i$, $b = SR_i$ and $c = SP_i$. ■

Since, a unique root of first-order derivative is found for each i-th channel, the maxima of the EE corresponds necessarily to these roots. However, the powers $P_{EE,i}$'s are interdependent since SR_i and SP_i include the $P_{EE,j}$'s, for $j \neq i$. In other words, to allocate a power to the i-th parallel channel, we need to know the allocated powers of all the other parallel channels that depend also the i-th parallel channel's power. Hence, we present an analytical approach to solve this interdependence.

$$E_{\gamma_1}^{(N)} = \exp\left(1 - \frac{1}{N}\sum_{j=1}^{N}\log(\frac{\gamma_j}{\gamma_1}) + W\left(\frac{\gamma_1(P_c + \frac{1}{N}\sum_{j=1}^{N}(\frac{1}{\gamma_1} - \frac{1}{\gamma_j})) - 1}{e^{1-\frac{1}{N}\sum_{j=1}^{N}\log(\frac{\gamma_j}{\gamma_1})}}\right)\right) - 1 \tag{15}$$

A. ANALYTICAL APPROACH

In order to avoid the interdependence, we rewrite the N optimality conditions from (5). We then present a new equation relating the powers P_i and P_j, $i, j = 1, \ldots, N, i \neq j$, as follows,

$$\frac{\gamma_1}{1+\gamma_1 P_1} = \frac{\gamma_2}{1+\gamma_2 P_2} = \cdots = \frac{\gamma_N}{1+\gamma_N P_N}. \tag{10}$$

Hence, $\forall\, i, j \in 1, \ldots, N$, and $i \neq j$, we obtain

$$P_j = P_i + \frac{1}{\gamma_i} - \frac{1}{\gamma_j}, \tag{11}$$

$$\Rightarrow \log(1+\gamma_j P_j) = \log(1+\gamma_i P_i) + \log(\frac{\gamma_j}{\gamma_i}). \tag{12}$$

Note that even for $j = i$, the previous expressions are still valid. Hence, for $i \in 1, \ldots, N$, the optimality condition can be written as

$$\frac{\gamma_i}{1+\gamma_i P_i} = \frac{\log(1+\gamma_i P_i) + \frac{1}{N}\sum_{j=1}^{N}\log(\frac{\gamma_j}{\gamma_i})}{P_c + P_i + \frac{1}{N}\sum_{j=1,}^{N}(\frac{1}{\gamma_i} - \frac{1}{\gamma_j})}. \tag{13}$$

From (13) and (5), we obtain new expressions of SR_i and SP_i as following

$$SR_i = \frac{1}{N}\sum_{j=1}^{N}\log(\frac{\gamma_j}{\gamma_i}) \quad \text{and} \quad SP_i = P_c + \frac{1}{N}\sum_{j=1}^{N}(\frac{1}{\gamma_i} - \frac{1}{\gamma_j}). \tag{14}$$

These new expressions of SP_i and SR_i are different from (9) and are independent from the power solutions of the other parallel channels. By implementing these expressions in (8), we can directly determine the explicit expressions of the optimal power solutions that maximize the EE. First, we compute P_1^* such as $P_1^* = \frac{1}{\gamma_1}[E_{\gamma_1}^{(N)}]^+$, where $E_{\gamma_1}^{(N)}$ is given by (15) at the top of this page. Then, we compute the rest of the power solutions using (11).

In summary, the explicit expressions of the optimal power solutions and the corresponding EE are given by

$$P_1^* = \frac{1}{\gamma_1}[E_{\gamma_1}^{(N)}]^+ \quad \text{and } P_j^* = [P_1^* + \frac{1}{\gamma_1} - \frac{1}{\gamma_j}]^+, \quad \text{for } 2 \leq j \leq N \tag{16}$$

$$EE^* = \frac{\gamma_1}{\log(2)(1+[E_{\gamma_1}^{(N)}]^+)} \tag{17}$$

Note that these expressions are simple to derive. In particular, the value of P_1^* is directly computed using the available channel gains without the need of the powers allocated to other parallel channels. The rest of the power solutions, P_j^*, $j = 2, \cdots, N$ are straightforward computed using the value of P_1^*.

B. ANALYTICAL APPROACH LIMITATION

Although the expressions in (14) present non-dependent expressions of the power solutions, the condition $ac + e^{-b} > 1$ in (7) might not be satisfied, especially for low values of P_c. In fact, with the expressions of SR_i and SP_i in (14), the condition $ac+e^{-b}>1$ means that

$$\gamma_i(P_c + \frac{1}{N}\sum_{j=1}^{N}(\frac{1}{\gamma_i} - \frac{1}{\gamma_j})) + \exp(-\frac{1}{N}\sum_{j=1}^{N}\log(\frac{\gamma_j}{\gamma_i})) > 1.$$

In Appendix B, we showed that this condition is only verified when $P_C > AM - GM$ where AM and GM are the arithmetic and geometric means of the inverse of the channel gains, respectively, and are given by

$$AM = \frac{1}{N}\sum_{j=1}^{N}\frac{1}{\gamma_j} \quad \text{and} \quad GM = \sqrt[N]{\prod_{j=1}^{N}\frac{1}{\gamma_j}}. \tag{18}$$

Due to the *inequality of arithmetic and geometric means* that states the $AM \geq GM$ [33], there are cases where P_c will be below $AM-GM$ and the expressions of SR_i and SP_i in (14) are not valid. In that case, the expressions in (9) need to be used instead. In the latter case, even with no transmission, i.e., $P_{EE,i} = 0\ \forall i$, the term $e^{-b} = e^0 = 1$. Hence, the condition $ac + e^{-b} > 1$ is always true. Hence, in order to complement our analytical results when $P_C < AM - GM$, we propose a low-complexity algorithm (**Algorithm 1** below) to solve the EE maximization using the expressions in (9).

C. NUMERICAL APPROACH: ITERATIVE ALGORITHM

In the algorithm, we start by giving initial values to the powers. Then, we update the values of SP_i, SR_i with respect to (9). Afterwards, we, iteratively, compute the P_i's then SP_i and SR_i $i \in 1, \ldots, N$, till reaching the maximum EE. We present the steps of our algorithm in **Algorithm 1**. The advantage of **Algorithm 1** is the use of the expressions in (9) that, as we will show in the numerical results, converges to the maximum EE within two iterations.

Consequently, our proposed EEPA is given by the analytical expression if $ac + e^{-b} > 1$ and **Algorithm 1** otherwise. To conclude, we present in Fig. 1, our proposed power allocation scheme for the unconstrained MIMO-SVD systems.

Algorithm 1 Iterative Algorithm for MIMO Power Allocation

t=0.
Initialize $P_{EE,i}^{(0)}, i = 1, \ldots, N$.
Find the initial $EE^{(0)}$ using (3).
repeat
  $t \leftarrow t + 1$.
  For i=1 to N.
    Compute $SP_i^{(t)}, SR_i^{(t)}$ using (9).
    Compute $P_{EE,i}^{(t)}$ using (8).
  End
  Compute the updated $EE^{(t)}$ using (3).
until $|EE^{(t)} - EE^{(t-1)}|^2 \leq \epsilon$ where $\epsilon > 0$.
The optimal solutions of the optimization problem are $P_{EE,i}^{(t)}, \forall i = 1, \cdots, N$.

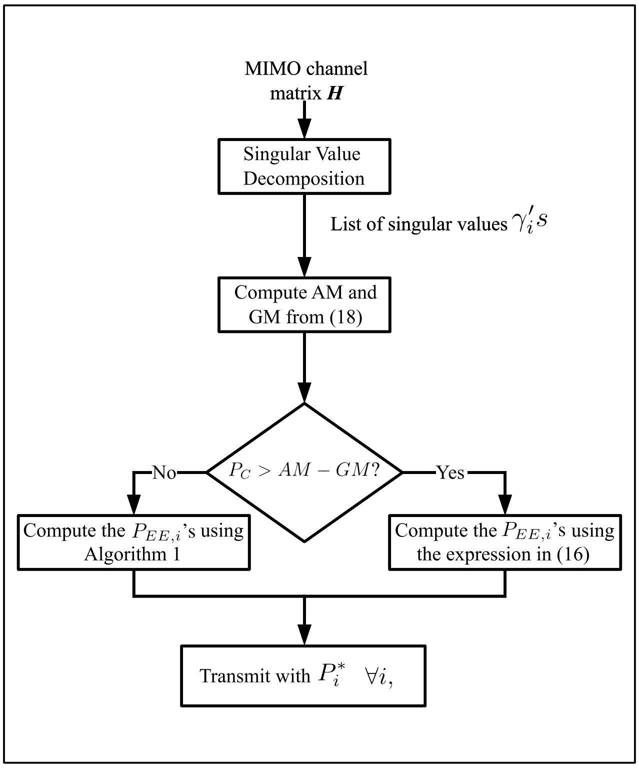

FIGURE 1. Illustration of the energy-efficient power allocation for the unconstrained MIMO-SVD systems.

IV. CONSTRAINED SYSTEMS EE POWER ALLOCATION

In the constrained problem, we consider two types of constraints: power budget and minimal rate constraints. In order to present the corresponding power solutions, we start by solving two sub-problems involving the EE as objective function and each constraint separately. Then, we present the solution of the EE maximization considering both constraints simultaneously.

A. POWER BUDGET CONSTRAINT

When the MIMO-SVD WCS's power is only constrained by power budget, P_{peak}, the optimization problem is given by

$$\max_{P_i \geq 0} \; EE = \frac{\sum_{i=1}^{N} \log_2(1 + \gamma_i P_i)}{\sum_{i=1}^{N} (P_c + P_i)}, \tag{19}$$

$$\text{subject to} \; \sum_{i=1}^{N} P_i \leq P_{peak}. \tag{20}$$

In order to solve the problem (19)-(20), we denote by $P_{\Sigma,N}^{EE}$ the sum of the optimal power solutions of the unconstrained problem derived in the previous Section, i.e., $P_{\Sigma,N}^{EE} = \sum_{i=1}^{N} P_{EE,i}$. Depending on the value of $P_{\Sigma,N}^{EE}$, we distinguish two cases:

1) CASE 2: $P_{peak} \geq P_{\Sigma,N}^{EE}$

In this region, the EEPA scheme presents feasible solution of the problem (19)-(20). Given the fact that the EEPA is the solution of the unconstrained problem, it represents the solution of this problem as well.

2) CASE 1: $P_{peak} < P_{\Sigma,N}^{EE}$

In this region, at each parallel channel the power is either equal or lower than $P_{EE,i}$ and the **Theorem 1** cannot be used. Since $EE(P_i)$ has a global and unique maxima at $P_{EE,i}$, the function $EE(P_i)$ is a strictly increasing function for $0 \leq P_i \leq P_{EE,i} \; \forall i$. Hence, to maximize the EE, that the power solutions need to be as close as possible to the $P_{EE,i}$'s. Meanwhile, since $\sum_{i=1}^{N} P_i$ is either equal or below P_{peak} and $P_{peak} \leq P_{\Sigma,N}^{EE}$, then we need to have $\sum_{i=1}^{N} P_i = P_{peak}$, since having lower sum of the power solutions will further decrease the EE. Consequently, the objective function in (19) can be written in this case as follows

$$EE = \frac{\sum_{i=1}^{N} \log_2(1 + \gamma_i P_i)}{NP_c + P_{peak}} = \frac{SE}{NP_c + P_{peak}}. \tag{21}$$

The solution of this objective function is the same that maximizes the SE which is given by the classical water-filling power allocation (WPA) [34]. We denote by P_{SE} the corresponding power solution which is given by

$$P_{SE,i} = \left[\frac{1}{\log(2)\mu} - \frac{1}{\lambda_i^2}\right]^+ \; \forall i, \tag{22}$$

where μ is the Lagrange multiplier associated with the power budget constraint in (20) and computed such that

$$\sum_{i=1}^{N} P_{SE,i} = \sum_{i=1}^{N} \left[\frac{1}{\log(2)\mu} - \frac{1}{\lambda_i^2}\right]^+ = P_{peak}. \tag{23}$$

In summary, the constrained case optimal power is given by

$$P_i^* = \begin{cases} P_{EE,i} & \text{if } P_{peak} \geq \sum_{i=1}^{N} P_{EE,i}, \\ P_{SE,i} & \text{otherwise}. \end{cases} \quad (24)$$

B. MINIMAL RATE CONSTRAINT

Maximizing the EE given only a power budget constraint may not be adequate for systems requiring a certain data rate, it is of interest to include a minimal data rate constraint in order to guarantee conveying a decent rate as the optimal solution may always be geared toward "not transmit" case. Recall that the minimal data rate is denoted by R_{min}. Hence, the EE maximization problem with minimal rate constraint is given by

$$\max_{P_i \geq 0} EE = \frac{\sum_{i=1}^{N} \log_2(1+\gamma_i P_i)}{\sum_{i=1}^{N}(P_c + P_i)}, \quad (25)$$

$$\text{subject to } \sum_{i=1}^{N} \log_2(1+\gamma_i P_i) \geq R_{min}. \quad (26)$$

Before solving this problem, we first start by denoting by $R_{\Sigma,N}^{EE}$ the sum of the rates of the EEPA solutions, i.e.,

$$R_{\Sigma,N}^{EE} \triangleq \sum_{i=1}^{N} \log_2\left(1+\gamma_i P_{EE,i}\right). \quad (27)$$

The value of $R_{\Sigma,N}^{EE}$ is important to perform the analysis of the power allocation scheme. In fact, depending on the value of $R_{\Sigma,N}^{EE}$ compared to R_{min}, we distinguish two cases

1) CASE 1: $R_{\Sigma,N}^{EE} \geq R_{min}$

In this case, the EEPA solutions derived in the unconstrained part satisfy the rate constraint. Consequently, the EEPA scheme represents the solution of the problem in this case.

2) CASE 2 : $R_{\Sigma,N}^{EE} < R_{min}$

In this case, the EE power solutions are no longer feasible as the rate constraint is not respected. However, the solution of the problem should necessary satisfy the constraint with equality, i.e., $\sum_{i=1}^{N} \log_2(1+\gamma_i P_i) = R_{min}$. The reason behind this condition, similarly to the power budget constrained case, is that having higher power solutions than the EEPA will further decrease the EE. Consequently, the objective function in (25) can be rewritten as follows

$$EE = \frac{R_{min}}{P_c + \sum_{i=1}^{N} P_i}. \quad (28)$$

Given this new expression, maximizing the EE is equivalent to minimizing the denominator of (28). In other words, the new problem is given by

$$\min_{P_i \geq 0} \sum_{i=1}^{N} P_i, \quad (29)$$

$$\text{subject to } \sum_{i=1}^{N} \log_2(1+\gamma_i P_i) \geq R_{min}. \quad (30)$$

In this case, the objective function and the constraint are convex. Hence, we use the Lagrangian method [35], to solve this problem. We derive the Lagrangian function as follows

$$\mathcal{L} = \sum_{i=1}^{N} P_i - \nu \left(\sum_{i=1}^{N} \log_2(1+\gamma_i P_i) - R_{min} \right) \quad (31)$$

where ν is the Lagrange multiplier associated with the rate constraint (30). We compute the derivative of $\mathcal{L}$ with respect to P_i, $\forall i$, and we obtain the following necessary and sufficient condition, $\forall i$,

$$1 - \nu \frac{\gamma_i}{\log(2)(1+\gamma_i P_i)} = 0. \quad (32)$$

Hence, the corresponding power solution, denoted as power minimization (PM) scheme, is given by

$$P_{PM,i} = \left[\frac{\nu}{\log(2)} - \frac{1}{\gamma_i} \right]^+ \quad \forall n, \quad (33)$$

where ν is computed such that

$$\sum_{i=1}^{N} \log_2\left(1+\gamma_i P_{PM,i}\right) = R_{min}. \quad (34)$$

In summary, the solution of the problem (25)-(26) is given by

$$P_i^* = \begin{cases} P_{EE,i} & \text{if } R_{min} \leq R_{\Sigma,N}^{EE}, \\ P_{PM,i} & \text{otherwise}. \end{cases} \quad (35)$$

C. POWER BUDGET AND MINIMAL RATE CONSTRAINTS

We aim in this part to solve the complete EE maximization problem involving both power a budget and minimal rate constraints. The EE maximization problem is given by

$$\max_{P_i \geq 0} EE = \frac{\sum_{i=1}^{N} \log_2(1+\gamma_i P_i)}{\sum_{i=1}^{N}(P_c + P_i)}, \quad (36)$$

$$\text{subject to } \sum_{i=1}^{N} P_i \leq P_{peak}. \quad (37)$$

$$\text{and } \sum_{i=1}^{N} \log_2(1+\gamma_i P_i) \geq R_{min}. \quad (38)$$

We start by computing $P_{\Sigma,N}^{EE}$ and $R_{\Sigma,N}^{EE}$, then depending on the values of P_{peak} and R_{min}, we distinguish four cases

1) CASE 1: $R_{min} \leq R_{\Sigma,N}^{EE}$ and $P_{peak} \geq P_{\Sigma,N}^{EE}$

Since the EEPA solutions satisfy both constraints, the solution of the problem is given by the EEPA scheme described in Section III.

TABLE 1. Energy-efficient optimal power for MIMO systems with power budget and minimal rate constraints.

$P_i^* =$	$P_{peak} \geq P_{\Sigma,N}^{EE}$	$P_{peak} < P_{\Sigma,N}^{EE}$
$R_{min} \leq R_{\Sigma,N}^{EE}$	$P_{EE,i}$	$P_{SE,i}$, if $R_{\Sigma,N}^{SE} \geq R_{min}$, Outage, otherwise
$R_{min} > R_{\Sigma,N}^{EE}$	$P_{PM,i}$, if $P_{\Sigma,N}^{PM} \leq P_{peak}$, Outage, otherwise	Outage

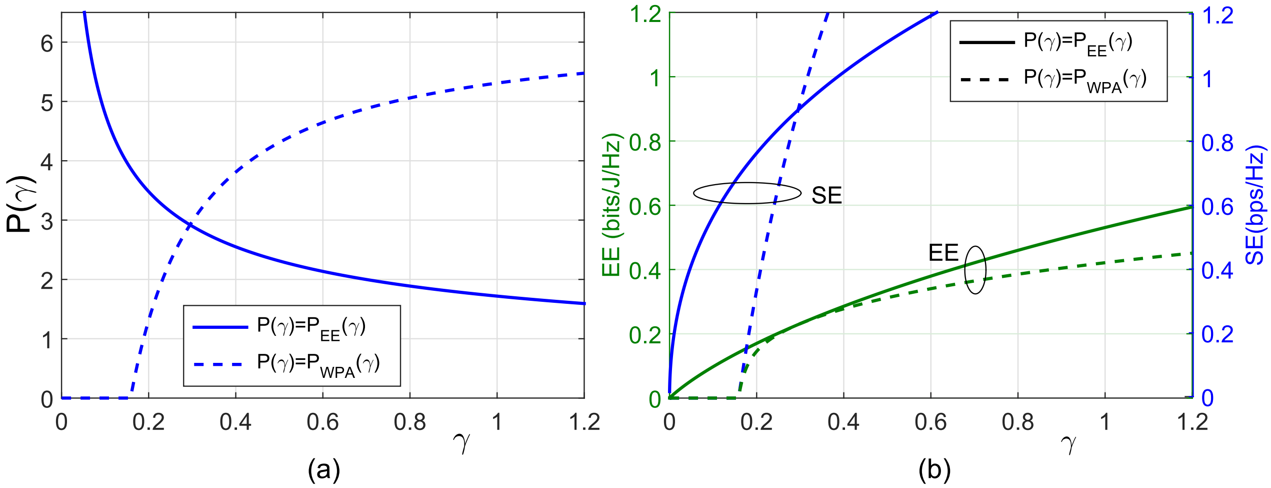

FIGURE 2. Comparison between energy-efficient and water-filling power allocations with $P_c = 1$ W and $P_{peak} = 6$ dB.

2) CASE 2: $R_{min} > R_{\Sigma,N}^{EE}$ and $P_{peak} < P_{\Sigma,N}^{EE}$

In this case, both constraints are not respected by the EEPA and a different power allocation scheme should be adopted. However, if the power solutions are higher than the EEPA, to meet the rate constraint, or lower than the EEPA, to meet the power budget constraint, there will be always one constraint that is not respected. Consequently, this problem does not have a solution and an outage is declared.

3) CASE 3: $R_{min} \leq R_{\Sigma,N}^{EE}$ and $P_{peak} < P_{\Sigma,N}^{EE}$

In this case, the EEPA scheme respects the rate constraint but not the power budget constraint. Hence, the WPA solution should be adopted to satisfy the power budget constraint. However, the WPA solution does not necessarily respect the rate constraint. For this reason, we denote by $R_{\Sigma,N}^{SE}$ the sum of the rates of the WPA scheme, i.e.,

$$R_{\Sigma,N}^{SE} \triangleq \sum_{i=1}^{N} \log_2\left(1 + \gamma_i P_{SE,i}\right). \tag{39}$$

Hence, if the WPA scheme satisfies the rate constant, i.e., $R_{\Sigma,N}^{SE} \geq R_{min}$ then the optimal power is the SE. Otherwise, an outage is declared.

4) CASE 4: $R_{min} > R_{\Sigma,N}^{EE}$ and $P_{peak} \geq P_{\Sigma,N}^{EE}$

In this case, the EEPA scheme respects the power budget constraint but not the rate constraint. Hence, the PM scheme in (33) should be adopted to satisfy the rate constraint. However, the PM solution does not necessarily respect the rate constraint. For this reason, we denote by $P_{\Sigma,N}^{PM}$ the sum of the PM power solutions, i.e.,

$$P_{\Sigma,N}^{PM} \triangleq \sum_{i=1}^{N} P_{PM,i}. \tag{40}$$

Hence, if $P_{\Sigma,N}^{PM} \leq P_{peak}$, then the solution is given by the PM scheme. Otherwise we declare an outage.

The previous four cases cover the solutions of the EE maximization problems when both power budget and minimal rate constraints are considered. These solutions are summarized in Table 1 with respect to all the cases discussed in this Section.

V. NUMERICAL RESULTS

We present the performance of the proposed power allocation scheme under Rayleigh fading channel, i.e., the elements of the channel matrix $\boldsymbol{H}$ are circularly symmetric complex Gaussian random variables with zero mean and unit variance. The performances are plotted by averaging 10^3 channels realizations.

A. UNCONSTRAINED EE RESULTS

In order to characterize the proposed energy-efficient power allocation against the classical water-filling power allocation, we compare in Fig. 2, the EEPA and the WPA schemes in terms of power and performance variations with the channel realization for one parallel channel denoted by γ. In Fig. 2.a,

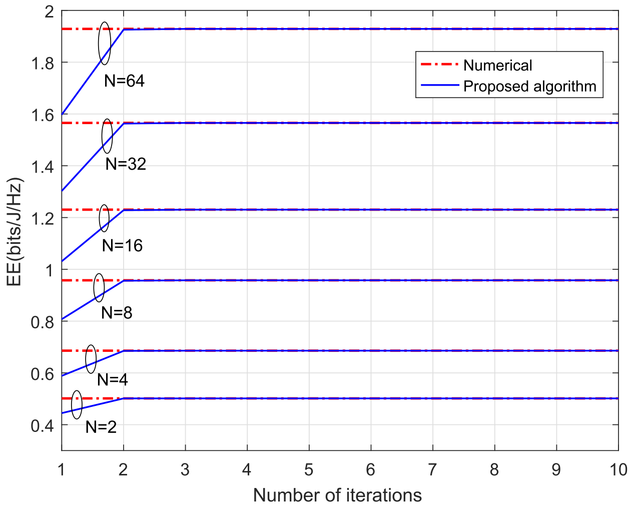

FIGURE 3. **Unconstrained EE as a function of the number of iterations of Algorithm 1 with different values of N and $P_c = 1$W.**

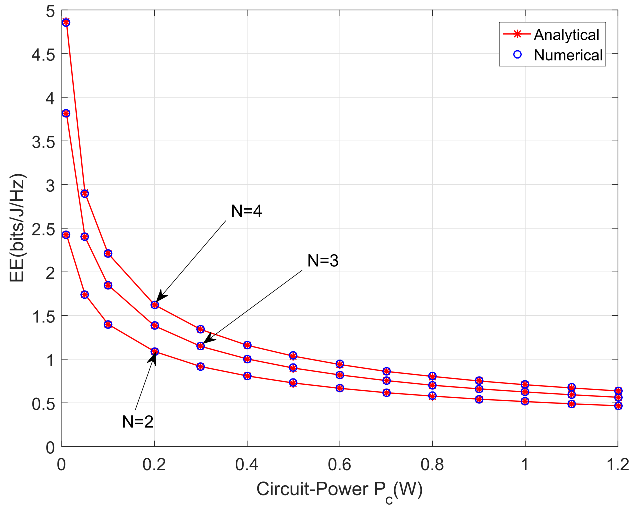

FIGURE 4. **Unconstrained EE as a function of P_c different values of N.**

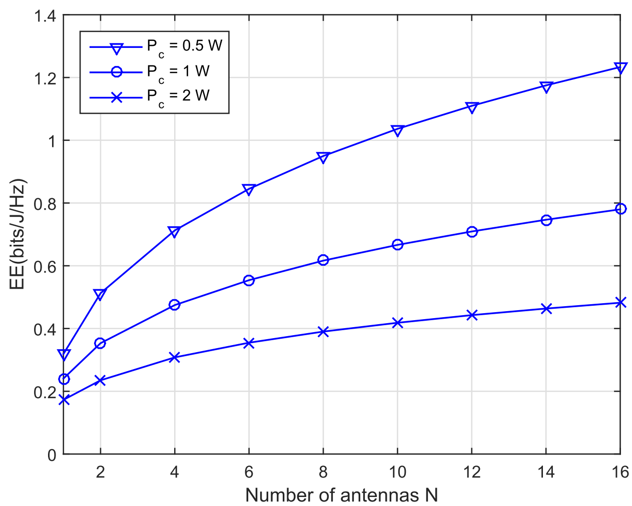

FIGURE 5. **Unconstrained EE as a function of N for different P_c.**

we show that variation of the EEPA with respect to γ is remarkably different from the variation of the WPA. In fact, the WPA allocates high power to high channel realizations to obtain a high global SE. However, the EEPA allocates low power to high channel realizations since by dividing the SE by low power will give high EE.

In Fig. 2.b, we plot the corresponding SE and EE as a function of γ. We show that for low channel gain, the EEPA performances in terms of SE and EE is higher than the WPA. However, as the channel realization values increase, the SE of the WPA outperforms the SE of EEPA which gives high global SE but low EE.

In Fig. 3, we aim to study the convergence speed of **Algorithm 1** by displaying the number of iteration needed to reach the maximal EE. We plot the EE resulted from the numerical simulations using MATLAB in comparison with the proposed algorithm for $P_c = 1$W which uses the expressions of SP_i and SR_i in (9). We show that the proposed iterative algorithm is accurate and converges rapidly as it needs only two iterations. In addition, regardless of the complexity of increasing the number of antennas from 2 to 64, the required number of iterations is at most two.

In Fig. 4, we highlight the effect of the circuit-power by plotting the EE as a function of P_c with a different number of antennas. From (3), we notice that the EE is a rational function with respect to P_c meaning that the curve of the EE is dictated by a hyperbola. Hence, as P_c increases, the EE decreases steeply for $P_c < 0.2$. However, for $P_c > 0.2$, the variation of EE is limited which shows that at relatively high values of P_c, the EE converges slowly to zero for $P_c \to \infty$.

In Fig. 5, we highlight the effect of multi-antennas on the EE. As can be seen in Fig. 5, the EE is increasing logarithmically as N increases. Although this result cannot be generalized to all cases, it gives insights that adopting an EEPA can result in an increasing EE as N increases. However, the EE gain with MIMO transmissions is relatively high compared to the SISO transmission. For example, the EE gain from 1 to 16 antennas is 179% for $P_c = 2$W and 286% for $P_c = 0.5$W. We also observe that having MIMO transmissions enhances the energy-efficiency of the WCS. In addition, for a given circuit-power per RF chain, P_c, having MIMO transmissions enhances the energy-efficiency of the WCS compared to a transmission with a single antenna. These results hold in the case of a pilot contamination free channel acquisition as shown in [36].

B. CONSTRAINED EE RESULTS

In Fig. 6, we plot the constrained EE performances as function of the power budget P_{peak}. We show that there are two regimes of the EE. The first regime is the "power limited" regime in which the EE varies with P_{peak} meaning that the power budget constraint is active. Clearly, this is the regime in which we have $P^{EE}_{\Sigma,N} \geq P_{peak}$, in our analysis. In fact, by studying the limits of the EE with respect to P_{peak}, we find

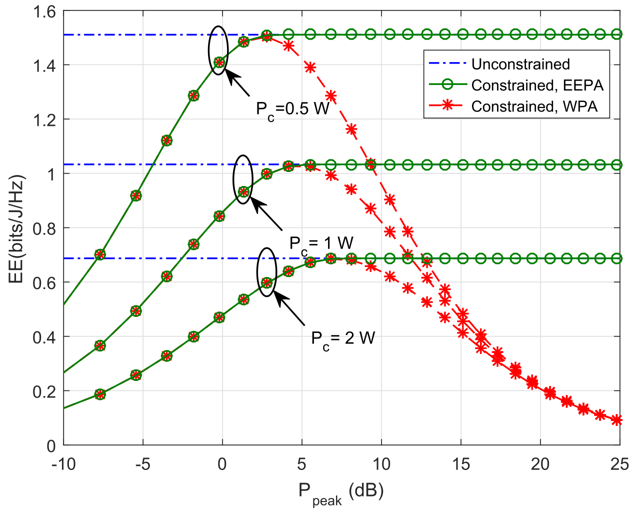

FIGURE 6. Constrained EE as a function of P_{peak} for different values of P_c with $N = 4$ with no minimal rate constraint.

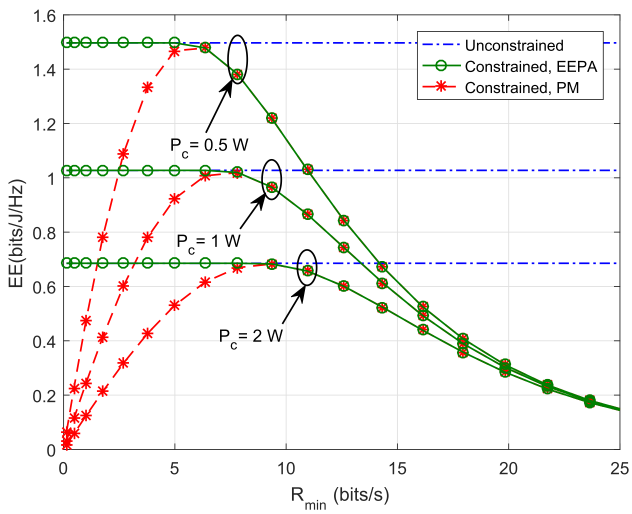

FIGURE 7. Constrained EE as a function of R_{min} for different values of P_c with $N = 4$ with no power budget constraint.

that when $P_{peak} \rightarrow 0$, the sum of the power solutions is smaller than the sum of the EEPA solutions. Hence, the EE varies and increases with P_{peak}. The second regime is the "saturated" regime in which the EE stagnates at a certain maximum level as a function of P_{peak}. This stagnation level corresponds to the optimal EE in the unconstrained case given by (17) and to the case in which we have $P^{EE}_{\Sigma,N} \leq P_{peak}$ meaning that the EE reaches its maximum and cannot further increase and that the power budget constraint is no longer active. Hence, the cases where $P_{peak} = P^{EE}_{\Sigma,N}$ present the limit between the two regimes and are easy to spot from the figure when the EEPA and the WPA schemes give different values of the EE. In addition, there is no need to transmit above $P^{EE}_{\Sigma,N}$ since the EE will not increase and the power is wasted.

In Fig. 7, we plot the EE as a function of the minimal rate R_{min} with both EEPA and PM schemes with no power budget constraint. We show that, for the EEPA scheme, there

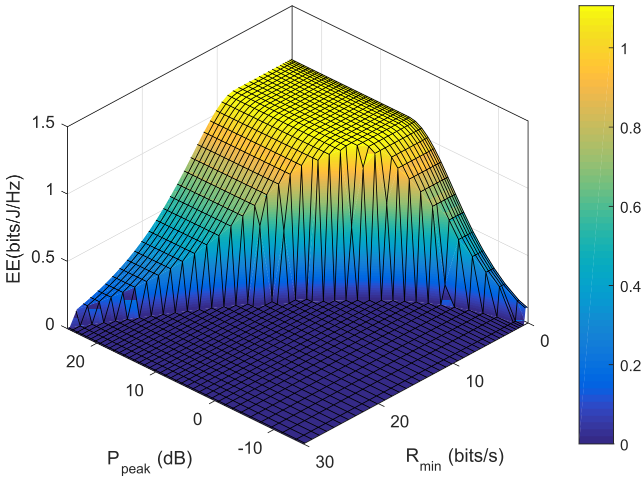

FIGURE 8. Illustration of the constrained EE as a function of P_{peak} and R_{min} for different values of $P_c = 1$ W with N= 4.

is also two regimes in which the EE is either saturated or variable with R_{min}. The saturated regime characterizes the regime of low values of R_{min}. In fact, in this regime, the rate constraint is not active as the EEPA solutions achieve rates higher than R_{min}. When R_{min} is high, given that there is no power limitation, the allocated power is high to meet the rate requirement which causes the drop of the EE for both EEPA and PM schemes. We also note that there are values of R_{min} that separate the two regimes and can be easily noted when the performances of the EEPA and the PM are different. These values vary between 6 to 9 bits/s depending on the values of P_c.

In Fig. 8, we present the variations of the EE as a function of both P_{peak} and R_{min}. We show that the EE stagnation region is clearly defined by a rectangle shape limited by high values of P_{peak} and low values of R_{min}. In addition, we clearly highlight the outage region in which the EE is equal to zero.

VI. CONCLUSION

In this paper, we studied the optimal power allocation that maximizes the energy efficiency. Unlike previous works that adopted numerical methods such as fractional programming, we presented explicit expressions of the optimal power and a corresponding iterative algorithm. We extended our results to power budget constrained systems. We showed that the proposed algorithm is accurate and converges to the optimal solution within two iterations. Furthermore, we show that the EE is improved when the number of antennas increases. We also distinguish a saturation regime of the EE when the system is power budget constrained.

APPENDIX A: PROOF OF THE LEMMA

The equality (6) can be written as

$$ax + ac = (1 + ax)\left(\log(1 + ax) + b\right)$$
$$\Rightarrow ac - 1 = (1 + ax)\left(\log(1 + ax) + b - 1\right)$$
$$\Rightarrow (ac - 1)e^{b-1} = (1 + ax)e^{b-1}\log\left((1 + ax)e^{b-1}\right)$$

$$\Rightarrow (ac-1)e^{b-1} = e^{\log(X)}\log(X)$$
$$(\text{where } X = (1+ax)e^{b-1})$$
$$\Rightarrow X = \exp\left(W\left(\frac{ac-1}{e^{1-b}}\right)\right)$$
$$\Rightarrow x = \frac{1}{a}\left(\exp\left(1-b+W\left(\frac{ac-1}{e^{1-b}}\right)\right)-1\right)$$

APPENDIX B : OBTAINING THE LOWER BOUND OF P_c

When the expressions of SR_i and SP_i in (14) are adopted, for a given $i \in 1, \ldots, N$, the condition $ac + e^{-b} > 1$ is written as

$$\gamma_i(P_c + \frac{1}{N}\sum_{j=1}^{N}(\frac{1}{\gamma_i} - \frac{1}{\gamma_j})) + \exp(-\frac{1}{N}\sum_{j=1}^{N}\log(\frac{\gamma_j}{\gamma_i})) > 1$$
$$\Rightarrow \gamma_i P_c + 1 - \frac{\gamma_i}{N}\sum_{j=1}^{N}\frac{1}{\gamma_j} + \exp(\frac{1}{N}\log(\prod_{j=1}^{N}\frac{\gamma_i}{\gamma_j})) > 1$$
$$\Rightarrow \gamma_i P_c > \frac{\gamma_i}{N}\sum_{j=1}^{N}\frac{1}{\gamma_j} - \gamma_i\sqrt[N]{\prod_{j=1}^{N}\frac{1}{\gamma_j}}$$
$$\Rightarrow P_c > \frac{1}{N}\sum_{j=1}^{N}\frac{1}{\gamma_j} - \sqrt[N]{\prod_{j=1}^{N}\frac{1}{\gamma_j}}$$
$$\Rightarrow P_c > AM - GM$$

where $AM = \frac{1}{N}\sum_{j=1}^{N}\frac{1}{\gamma_j}$ and $GM = \sqrt[N]{\prod_{j=1}^{N}\frac{1}{\gamma_j}}$.

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