

Received April 5, 2017, accepted May 4, 2017, date of publication May 17, 2017, date of current version June 27, 2017.

Digital Object Identifier 10.1109/ACCESS.2017.2705120

A Probabilistic Mechanism Design for Online Auctions

JIE ZHANG^{1,2,4,5}, (Member, IEEE), LINJING LI¹, (Member, IEEE),
AND FEI-YUE WANG^{1,2,3,4}, (Fellow, IEEE)

¹State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China

²Institute of Parallel Economics, Qingdao Academy of Intelligent Industries, Shandong 266000, China

³Research Center for Computational Experiments and Parallel Systems Technology, National University of Defense Technology, Changsha 410073, China

⁴School of Economics and Management, University of Chinese Academy of Sciences, Beijing 100049, China

⁵School of Computer and Control Engineering, University of Chinese Academy of Sciences, Beijing 100049, China

Corresponding author: Fei-Yue Wang (feiyue@iee.org)

This work was supported by the National Natural Science Foundation of China under Contract 71402178, Contract 61233001, Contract 61374105, Contract 61533019, Contract 71232006, Contract 71202169, and Contract 71472174.

ABSTRACT Recently, there has been a rapid growth of the online auctions in e-commerce platforms, in which small and/or medium-sized enterprises (SMEs) heavily depend on the advertising systems. In this paper, we design flexible mechanisms to reduce the competition of SMEs without affecting competitive large companies in order to maximize the profit of e-commerce platform and to keep the ecosystem healthy. A probabilistic pricing mechanism design approach is investigated for online auctions. Utilizing this approach, we introduce the notation of *simple mechanisms* as a tool for designing new mechanisms. Based on a simple and a classical, the proposed mechanism probabilistic mechanisms are designed and their properties are analyzed. Furthermore, we devise two mechanism design algorithms for different application scenarios. Experiments are presented to demonstrate the flexibility and the effectiveness of the proposed probabilistic mechanism design approach.

INDEX TERMS Mechanism design, online auctions, randomized mechanisms, e-commerce, computational experiments, probabilistic mechanism design.

I. INTRODUCTION

According to a report of eMarketer¹ in 2016, the Chinese leading e-commerce platform Alibaba group generates 60% of online advertisements in China. Compared with the traditional online advertising services provided by search engines (e.g. Google AdWords [1]), online advertising auctions in e-commerce platforms (e.g. Alibaba, Amazon and eBay) have customers with much bigger purchase potential and well analyzed buying habits. Most of them are multi-sided platforms [2]. The e-commerce platform sells impressions to the advertisers, and the advertisers sell products to the customers on the platforms. Since the order of products in natural search results is very much related to the sales volume, there is very little opportunity to have effective natural impressions for SMEs. Consequently, the competition of

auctions in e-commerce platforms is intense and important. However, the auction mechanisms such as the generalized first-price [3], the generalized second-price [4], [5], and the VCG [6]–[8] have been designed according to the classical auction theory emphasizing competition resulting in the dropout of SMEs unless their budgets can afford large bids. Therefore, highly competitive advertising auctions will reduce the diversity of products of the e-commerce platforms and the attractiveness to the customers from the long term.

In order to keep the entire e-commerce industry growing it is crucial to maintain a large basis of participants regardless of their sales by reducing the competition of SMEs while encouraging the competition of large enterprises. A reasonable approach to deal with this issue is to adopt randomization [9], [10] in determining the winner of the auction and her charge. Normally, in a standard single item auction mechanism, the object will always be awarded to the bidder with the highest bid. This is not guaranteed in nonstandard mechanism [11] such as randomized mechanisms [10], [12].

¹eMarketer. *Programmatic Ad Spending in China Is Growing Rapidly*, February, 3 2016. <http://www.emarketer.com/Article/Programmatic-Ad-Spending-China-Growing-Rapidly/1013542>

However, since most auctions utilized by e-commerce platforms currently are not truthful,² (e.g. generalized second price), inappropriate randomization not only break the equilibriums of the original mechanism and make the advertising system unstable, but also hurt the interests of large enterprises and the e-commerce platform.

The objective of our mechanism design problem consist of two main parts: the winning rate of the bidders with low valuations (i.e. the SMEs) and the profit of the platform. We also try to reduce the social loss. In this paper, we propose a probabilistic approach to design balanced auctions for the e-commerce platform by introducing randomization. Moreover, we develop a method to select the collection of mechanisms in order to make sure rational bidders do not have to change their bidding strategies when adopting the probabilistic mechanisms. The contributions of this paper can be summarized as follows: (1) A probabilistic price mechanism design approach is proposed for online auctions. (2) The notation of *simple mechanisms* is introduced such that the probabilistic combination of a classical and a simple mechanism will keep the original equilibriums. (3) Two kinds of probabilistic price mechanisms are developed for e-commerce platforms.

The rest of this paper is organized as follows. Next section reviews related works. Section III describes our mechanism design problem and gives a preliminary analysis to classical first-price and second-price auctions. We will find it is very important for e-commerce platforms to consider the winning rate of bidders with low valuations. The probabilistic mechanism design approach is presented and used to develop particular mechanisms in Section IV. Based on the results of Section IV, Section V offers instantiated algorithms and reports computational experiments to evaluate the proposed probabilistic mechanisms for long-running auctions. Section VI concludes the paper.

II. RELATED WORK

Auction has been widely regarded to be effective and efficient for scarce resource allocation. There is a long history which can be traced back even to the age of the ancient Babylon [11]. Traditionally, it is employed mainly to sell valuable goods like antiquities, artifacts, jewelry, etc. Later it is applied to determine the allocation and price of the property right of land, mine, and state-owned enterprise, even to more abstract rights such as radio spectrum operation [13] and greenhouse gas emission allowance.³ Recently, Internet became fantastic for selling both tangible (e.g., ebay [14]) and virtual goods (e.g., Google AdWords [1], [3], ridesharing [15], and crowdsourcing [16]). Online auction became one of the most successful sectors of the Internet industry and it has triggered a new wave of research on auction theory.

²A truthful auction encourages bidders to bid their true valuations. However, the non-truthful GSP mechanism is much easier to understand for a non-professional advertiser, and is employed by many e-commerce platforms.

³The EU Emissions Trading System. http://ec.europa.eu/clima/policies/ets/index_en.htm

From the point of view of scarce good allocation, efficiency is the central issue discussed in most of the classical researches based on the seminal paper of Vickrey [6]. In that paper, Vickrey showed the equivalence of the first-price and second-price auctions in term of expected profit. Later, Myerson [17] proved this property in a more general setting and it is now widely referred as the revenue equivalence theorem (RET) in the auction related literatures. In the same paper, the optimal auction mechanism for single good is derived based on the RET. Many other researchers have extended these classical results in different directions, such as multiple goods, risk aversion (for both seller and bidder sides), bidder value correlation and affiliation, bidder asymmetry, non-commitment, deadlines and so forth [18]–[23].

Randomized mechanisms are well known for assignment problems. As a generalization of deterministic mechanisms, random serial dictatorship mechanism [9] and probabilistic serial mechanism [24] were introduced by the randomization of the ordering process. The idea is to regard each object as a continuum of probability shares [25]. Conitzer and Sandholm [10] modeled mechanism design as an optimization problem to find a randomized mechanism with a probability distribution over the outcome set in order to maximize the auctioneer's objective. In their definition of randomized mechanism with payments, the outcome is randomized and the payment selection function is deterministic. In a subsequent research [26], self-interested automated mechanism was designed to maximize the profit of the seller. Randomization is also employed for double auctions [27]. After the bids are submitted, they use the Trade reduction (TR) mechanism with probability p , and the VCG mechanism with probability $1-p$. In [12], bid-independent auctions are introduced for analyzing randomized truthful auctions. Note that, since the mechanisms considered in [12] and [27] are all truthful, the probabilistic combination will keep the equilibrium bidding strategies. However, if the auctions are not truthful, bidders need to investigate new strategies. In e-commerce platforms, we need to investigate randomized mechanisms without the truthful assumption.

III. NOTATIONS AND CURRENT PRACTICE

In this section we define our research problem and provide a preliminary analysis of classical first-price (FP) and second-price (SP) mechanisms [11]. We consider an auction with sealed price bids for N bidders with single object (e.g. a keyword) for a single-round sale, where $N \geq 2$.

(1) Bidder- i assigns a value of X_i to the object to represent the maximum amount of money⁴ she wants to pay for it. It is assumed that X_i , $i = 1, 2, \dots, N$, are independent and identically distributed random variables on the interval $B = (0, +\infty)$ according to a distribution function F .

⁴Besides the real valuation of the object by the bidder, budget [28], [29] is another critical factor affects the maximum amount of money. Generally, SMEs will have smaller X . We also assume the bidders are indistinguishable from the perspective of the e-commerce platform.

Bidder- i knows her actual valuation x_i , and others know the distribution. Bidder- i submits bid b_i to the e-commerce platform. $b = [b_1, \dots, b_N]^T \in B^N, x = [x_1, \dots, x_N]^T \in B^N$.

(2) The allocation rule is a function $L(b) : B^N \rightarrow \{0, 1\}^N$, showing that with the bidding vector b , the object is allocated to bidder- i if $L_i(b) = 1$ and $L_j(b) = 0, j \neq i$. In this case, we say bidder- i wins. We simplify the payment rule as a function $C(b) : B^N \rightarrow B^N$ to denote the cost of bidder- i to be $C_i(b)$. Note that, if bidder- i is charged by $C_i(\bar{b}) > x_i$, the allocation will fail. Thus, a mechanism $\mathcal{M} = (L, C)$ can be defined with an allocation rule L and a payment rule C . If both L and C are deterministic, the mechanism is deterministic.

Then, for a deterministic mechanism $\mathcal{M} = (L, C)$, the profit of the e-commerce platform is $\sum_{i=1}^N C_i(b)$. Suppose the auction is with individual rationality. Then, each bidder- i bids so as to maximize her expected payoff

$$\Pi_i(x, b) = x_i L_i(b) - C_i(b). \quad (1)$$

Define winning rate to be the probability a bidder wins in long-term. Since SMEs depend on the sponsored auction heavily, assuming a bidder cannot change the total clicks (which is not related to the advertising system very much) her impressions can be determined by her winning rate. Hence, she cannot obtain enough impressions to survive if she has a very small winning rate. We assume a bidder will have to leave the e-commerce platform if she has a very small winning rate less than a constant entrance threshold θ . For example, an advertiser with $\theta = 0.1\%$ may leave the e-commerce platform if she wins less than 100 bids after 100000 trials.

The classical FP and SP mechanisms are defined as:

Definition 1 (First Price Mechanism, FP [11]): First price auction $\mathcal{M}^I = (L^I, C^I)$ allocates the object to the bidder with highest bid, and charge her with the highest bid.

Definition 2 (Second Price Mechanism, SP [11]): Second price auction $\mathcal{M}^{II} = (L^{II}, C^{II})$ allocates the object to the bidder with highest bid, and charge her with the second highest bid.

We can also set a reserve price r . If the payment is less than r , we will charge the winner with r .

The strategy of a rational bidder i is a function $b_i = \beta_i(x_i) : B \rightarrow B$ from her valuation to the bid. We focus on the symmetric case of bidders, i.e., all the bidders employ the same strategy β in the game. According to the results in [11], it is a symmetric equilibrium strategy with FP to bid

$$\beta^I(x_i; r = 0) = x_i - \int_r^{x_i} \frac{F(\xi)^{N-1}}{F(x_i)^{N-1}} d\xi, \quad (2)$$

which is a monotonic function of the valuation x_i . With SP, it is a weakly dominate symmetric equilibrium strategy to bid

$$\beta^{II}(x_i; r = 0) = x_i, \quad (3)$$

which is also monotonic. Hence, suppose all the bidders employ the equilibrium strategy, the winning rate of bidder- i with either FP or SP mechanisms equals to the probability

that she has the highest valuation x_i :

$$p^A(x_i; r = 0) = F(x_i)^{N-1}, \quad (4)$$

where $A = I$ or II . We denote AP as FP or SP for short.

FP/SP and any standard single object auction mechanisms with allocation rule $L^I = L^{II}$ have an important characteristic in common: “first-price” wins. Thus, the winning rate of a rational bidder is positive correlated with her valuation, i.e. bidders with lower valuations will have lower winning rate. However, insufficient winning rate will hurt the enthusiasm of the bidders with low valuations (SMEs). Consider a mechanism \mathcal{M}^A . Assume bidder- i has relatively low valuation x_i with $\Pr(X \leq x_i) = F(x_i) = 1/5$, her winning rate $p^A(x_i; r = 0) = 1/5^{N-1}$ will be very small when N is large. In e-commerce platforms, less enterprises means less attractiveness. FP and SP will reduce the platform’s long-term profit. In this research, our task is designing mechanisms in order to increase the winning rate of a SME to at least θ if her original winning rate $p^A(x_i; r) < \theta$ and $x_i \geq r$.

Hence, there are four kinds of stakeholders. (1) Customers like more kinds of products to choose, hence more enterprises are desired in the platform. (2) SMEs need sufficient impressions and higher winning rates. (3) A large enterprise wants to maintain the high payoff. Hence, (4) the platform will design a mechanism to help SMEs without reducing too much profits of the platform and the large enterprises. A reasonable solution is to introduce randomization to the classical mechanisms giving consideration to the interests of stakeholders. In order to reduce the migration cost, the original equilibrium strategies of rational bidders should be the equilibrium of the new mechanisms.

IV. PROBABILISTIC MECHANISM DESIGN

In this section, we introduce a probabilistic approach to design mechanisms incorporating both the interests of the bidders with low valuations and the platform.

A. SIMPLE MECHANISMS

In order to utilize standard (“first-price” wins) mechanisms such as FP and SP to have higher profit, while increasing the winning rate of bidders with low valuations slightly, we introduce randomization to design mechanisms. Suppose we have a mechanism \mathcal{M}_1 (e.g., FP), we will find a mechanism \mathcal{M}_0 and a discrete probability distribution $\bar{\lambda} = (\lambda_1, \lambda_0)$ over the two mechanisms. The platform announces $\bar{\lambda}$ and two base mechanisms to all the bidders. First, ask the bidders to provide their bids $b = (b_1, \dots, b_N)$; And then choose a price mechanism randomly from the mechanisms according to the distribution $\bar{\lambda}$; Finally, select the winner and decides the price using the selected mechanism. Next, we focus on finding a mechanism \mathcal{M}_0 and the distribution such that the new mechanism will hold the same equilibrium strategies for the bidders with \mathcal{M}_1 .

Formally, we can define a new mechanism with existing mechanisms $\mathcal{M}_j, j = 1, 2, \dots, M$, and a discrete probability distribution $\bar{\lambda} = (\lambda_1, \dots, \lambda_M)$, with $\sum_{j=1}^M \lambda_j = 1, \lambda_j \geq 0$, for

$j = 1, \dots, M$. Then, mechanism $\mathcal{M} = (\mathcal{M}_1, \dots, \mathcal{M}_M; \bar{\lambda})$ is with $\Pr(\mathcal{M} = \mathcal{M}_j) = \lambda_j$, for $j = 1, \dots, M$.

After a bidding contract is established and the randomly selected price mechanisms is \mathcal{M}_j , the randomized mechanism \mathcal{M} acts in the same way as the selected mechanism keeping properties such as the winner of the auction and the payment. Hence, the linearity of the conditional expectation operator implies the following property.

Lemma 1: Consider mechanisms \mathcal{M}_j , $j = 1, \dots, M$, and assume that the bidders bid b_1, \dots, b_N . The expected payoff of bidder- i with respect to mechanism $\mathcal{M} = (\mathcal{M}_1, \dots, \mathcal{M}_M; \bar{\lambda})$ equals

$$E[\Pi_i(x, b)] = \sum_{j=1}^M \lambda_j E[\Pi_i^j(x, b)], \quad (5)$$

where Π_i^j is the payoff of bidder- i with mechanism j .

Based on the above proposition, we can estimate the equilibrium strategies and the expected payment for bidders with the new mechanism, hence the expected profit. Thus, given user valuation distribution F and proper base mechanisms \mathcal{M}_j , $j = 1, \dots, M$, we can design mechanisms by selecting parameters $\bar{\lambda}$. The proposed mechanism will maintain part of the properties of the original mechanisms and have new characteristics influenced by the parameters $\bar{\lambda}$.

Hence, we introduce a kind of mechanism without competition to help SMEs. If both the expected values of the allocation rule and the payment rule are independent to the bids, we call it a *simple mechanism*. For example, one of the simplest simple mechanisms always allocates the object to bidder-1 and charge her with a constant price 0.

Following the definition of a simple mechanism, higher bid will not ensure the winning of a bid. Thus, we can combine a classical and a simple mechanism to increase the winning rate of SMEs. According to Lemma 1, we have the following theorem that if we combine an existing mechanism with a simple mechanism, the original equilibrium is also an equilibrium of the new mechanism.

Theorem 1: Consider mechanism $\mathcal{M} = (\mathcal{M}_1, \mathcal{M}_2; \bar{\lambda})$, where \mathcal{M}_1 is a simple mechanism. Suppose, for any N bidders, \mathcal{M}_2 leads to an equilibrium $b = (b_1, \dots, b_N)$, then b is also an equilibrium of \mathcal{M} .

The proof of Theorem 1 can be found in the appendix section. Considering the probabilistic combination of a standard mechanism (e.g. FP or SP) and a simple mechanism, the new mechanism will keep the equilibrium of the original mechanism. This property ensures that rational bidders do not have to change her bid while $\bar{\lambda}$ is changing. Thus, we can utilize Theorem 1 to design mechanisms based on existing mechanisms (truthful or not) to reduce the migration cost and acquire better performance. Moreover, we can adjust $\bar{\lambda}$ during the long-term operation of the auction without influencing the equilibrium. Next, we employ the above probabilistic mechanism design approach to define the

Equal-Possible mechanism without competition, i.e. a simple mechanism.⁵

Definition 3 (Equal-Possible Mechanism, EP): Let \mathcal{M}_j , $j = 1, \dots, N$ be deterministic mechanisms with $L_j(b) = 1$, and $C_j(b) = r$ such that mechanism \mathcal{M}_j will always let bidder- j win, and charge her for r . Let $\bar{\lambda} = (1/N, \dots, 1/N)$, then we call the probabilistic combination $\mathcal{M}^0 = (\mathcal{M}_1, \dots, \mathcal{M}_N; \bar{\lambda})$ an equal-possible (EP) mechanism.

EP mechanism \mathcal{M}^0 is quite different with the ‘‘first-price-win’’ mechanisms such as FP and SP. Most of the bidders ($x_i \geq r$) will have the same winning rate,

$$p^0(x_i; r) = \frac{1}{N} \cdot 1 + \frac{1}{N} \cdot 0 + \frac{1}{N} \cdot 0 + \dots = \frac{1}{N}. \quad (6)$$

Thus, EP can be much more friendly to bidders with low valuations (SMEs) compared with the competitive FP and SP mechanisms, since those bidders are at a disadvantage in the competition. Note that, if a rational bidder’s valuation $x_i < r$, she will not pay for her winning bid, and has 0 payoff. Hence, r is actually a reserve price of the proposed EP Mechanism. With the purpose to reduce the difficulty for SMEs, we would always assume that the reserve price r is with a very small value. Next, we will present a brief analysis of the EP mechanism.

A remarkable feature of EP is that any bidding vector $b = (b_1, \dots, b_N)$ can be the equilibrium of the game. This is because for any bidder, changing her bid will not increase her payoff, her winning rate or any mathematical characteristic in the meaning of the expected valuation.

The payoff of bidder- i is also independent with her bid b_i . If $x_i \geq r$, she has $1/N$ probability to get positive payoff

$$\Pi_i^0(x, b; r) = x_i - r. \quad (7)$$

The expected payment of bidder- i with $x_i \geq r$ is the product of her winning rate and the constant payment

$$m^0(x_i; r) = \frac{1}{N} \cdot r + \frac{1}{N} \cdot 0 + \frac{1}{N} \cdot 0 + \dots = \frac{r}{N}. \quad (8)$$

The expected payoff of bidder- i with $x_i \geq r$ is

$$\begin{aligned} \pi^0(x_i; r) &= \frac{1}{N} \cdot E[x_i - m^0(x_i)|x_i \geq r] + \frac{1}{N} \cdot 0 + \dots \\ &= \frac{1}{N} \cdot [1 - F(r)][x_i - r]. \end{aligned} \quad (9)$$

Thus, the expected profit of the seller is

$$\begin{aligned} R^0(r) &= N \cdot E[m^0(v)] = N \cdot \int_0^\infty m^0(x)f(x) dx \\ &= N \cdot \int_r^\infty \frac{r}{N} f(x) dx = r[1 - F(r)]. \end{aligned} \quad (10)$$

⁵Simple mechanisms can be useful in practice. By introducing additional parameters to the pricing rules, we can design simple mechanisms easily to adapt different scenarios. For example, in online auction settings, we can employ the advertising quality score q_i as the mechanism distribution in Definition 3, i.e. $\bar{\lambda} = (q_1/\sum_j q_j, \dots, q_N/\sum_j q_j)$. The proposed simple mechanism will give advantage to bidders with higher quality score.

Since FP and SP mechanisms will sell the object with much higher price than the reserve price with a big chance, with the same reserve price r , the expected profit of the seller with respect to the EP mechanism is much smaller than the FP and SP mechanisms. However, EP can be beneficial to the e-commerce platform from the long term with carefully designed probability distribution.

B. PROBABILISTIC MECHANISMS

In the following, we will take the EP mechanism and the classical FP or SP as base mechanisms, and try to find proper distribution $\bar{\lambda}$ over the mechanisms to balance the interest of the bidders with low valuations (SMEs) and the seller (e-commerce platform) to develop probabilistic mechanisms for e-commerce platforms.

Definition 4 (Probabilistic First Price Mechanism, pFP): Let \mathcal{M}^I be the FP mechanism, and \mathcal{M}^0 be the EP mechanism. For $\bar{\lambda} = (1 - \lambda, \lambda)$, $\lambda \in (0, 1)$, we define the probabilistic first price (pFP) mechanism as $\mathcal{M}^{pI} = \{\mathcal{M}^I, \mathcal{M}^0; \bar{\lambda}\}$.

Definition 5 (Probabilistic Second Price Mechanism, pSP): Let \mathcal{M}^{II} be the SP mechanism, and \mathcal{M}^0 be the EP mechanism. For $\bar{\lambda} = (1 - \lambda, \lambda)$, $\lambda \in (0, 1)$, we define the probabilistic second price (pSP) mechanism as $\mathcal{M}^{pII} = \{\mathcal{M}^{II}, \mathcal{M}^0; \bar{\lambda}\}$.

We use pAP to denote probabilistic first-price mechanism (pFP) \mathcal{M}^{pI} or probabilistic second-price mechanism (pSP) \mathcal{M}^{pII} . Denote $\mathcal{M}^{pA}(r_1, r_2, \lambda)$ for short with $r_1, r_2 \in B$ the reserve prices of the AP and the EP respectively, $r_2 \leq r_1$, and $\bar{\lambda} = (1 - \lambda, \lambda)$.

Utilizing Theorem 1, the symmetric equilibrium strategy of the rational bidders can be obtained. Then we can estimate the expected payoff of bidders, the winning rate of bidders, and the expected profit of the e-commerce platform with the parameters r_1, r_2 and $\bar{\lambda} = (1 - \lambda, \lambda)$.

Suppose the user valuation distribution is F , we employ the probabilistic mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$. Bidder- i with valuation $x_i \geq r_2$ has λ/N probability to get positive payoff from EP,

$$\Pi_i^{pA}(x, b; r_1, r_2, \lambda) = x_i - r_2.$$

If $x_i \geq r_1$, she has $1 - \lambda$ probability to get payoff from AP

$$\Pi_i^{pA}(x, b; r_1, r_2, \lambda) = x_i L_i^A(b) - C_i^A(b).$$

According to Theorem 1, if $x_i \geq r_1$, we have the symmetric equilibrium bidding strategy of \mathcal{M}^{pI} ,

$$\beta^{pI}(x_i; r_1, r_2, \lambda) = \beta^I(x_i; r_1). \tag{11}$$

The symmetric equilibrium bidding strategy of \mathcal{M}^{pII} is

$$\beta^{pII}(x_i; r_1, r_2, \lambda) = \beta^{II}(x_i; r_1). \tag{12}$$

A rational bidder with valuation $x_i < r_2$ will never have a chance to win. Bidding truthfully $b_i = x_i$ is an equilibrium strategy of bidder- i with valuation $r_2 \leq x_i < r_1$, since lower bid (less than r_2) will cause him lose the λ/N probability

to win. And, if she reduce her bid to $r_2 \leq b_i < x_i$, she will not have higher expected payoff or winning rate.

With both probabilistic mechanisms pAP, the winning rate of bidder- i is

$$\begin{aligned} p^{pA}(x_i; r_1, r_2, \lambda) &= p^0(x_i; r_2)\lambda + p^A(x_i; r_1)(1 - \lambda) \\ &= \begin{cases} \lambda/N + (1 - \lambda)F(x_i)^{N-1}, & x_i \in [r_1, \infty), \\ \lambda/N, & x_i \in [r_2, r_1), \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \tag{13}$$

Her expected payoff is

$$\begin{aligned} \pi^{pA}(x_i; r_1, r_2, \lambda) &= \lambda\pi^0(x_i; r_2) + (1 - \lambda)\pi^A(x_i; r_1) \\ &= \begin{cases} \lambda/N[x - r_2][1 - F(r_2)] \\ \quad + (1 - \lambda)[\xi G(\xi)]_{r_1}^x - \int_{r_1}^x yg(y)dy, & x_i \in [r_1, \infty), \\ \lambda/N[x - r_2][1 - F(r_2)], & x_i \in [r_2, r_1), \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \tag{14}$$

where $g(y) := (N - 1)F(y)^{N-2}\dot{F}(y)$, $G(y) := F(y)^{N-1}$ is the distribution of Y_1^{N-1} (the highest value among the $N - 1$ remaining bidders).

Then, the expected profit of the e-commerce platform is

$$\begin{aligned} R^{pA}(r_1, r_2, \lambda) &= R^0(r_2)\lambda + R^A(r_1)(1 - \lambda) \\ &= \lambda r_2[1 - F(r_2)] + (1 - \lambda)N \left[r_1[1 - F(r_1)]F(r_1)^{N-1} \right. \\ &\quad \left. + \int_{r_1}^{\infty} y[1 - F(y)]g(y) dy \right]. \end{aligned} \tag{15}$$

Now, we have constructed two families of probabilistic pricing mechanisms without the ‘‘first-price-win’’ allocation rule. The equilibrium strategies are the same as the classical FP or SP mechanisms if we don not change the original reserve price r_1 . The proposed mechanisms consist of the following three parameters.

The competitive reserve price r_1 , i.e. the reserve price of the original AP. Increasing the reserve price (smaller than the optimal reserve price \bar{r} as [11]) of AP mechanisms raise the expected profit. However, it also comes with some drawbacks. Firstly, it may have a detrimental effect on efficiency; Secondly, it introduces deadweight social loss; What’s more important, in real-world auctions, higher reserve price will exclude SMEs, and is not healthy for long-term selling. Hence, in most of the search auctions, the reserve price has a small value.

The subsidy reserve price r_2 , i.e. the reserve price of EP. From the intuitively point of view, r_2 should have a small value, since EP is employed to raise the winning rate of SMEs. Rational bidders with valuation smaller than r_2 will not have the chance to win. Hence, r_2 is the reserve price of the probabilistic mechanisms. However, when $r_2 \leq x < r_1$, the bidder can still have positive winning rate. Thus, with the help of subsidy reserve price, the e-commerce platform can

increase the competitive reserve price r_1 and decrease r_2 to get higher profit.

The mechanism distribution $\bar{\lambda}$ can be used to adjust the level of competition of the probabilistic mechanism. For the extreme cases, when $\bar{\lambda} \rightarrow (1, 0)$, the probabilistic mechanism is actually an AP mechanism, which has the highest competition level; when $\bar{\lambda} \rightarrow (0, 1)$, it degenerate to an EP mechanism without competition.

C. DESIGN OF PARAMETERS

In this section, we investigate the properties of the proposed probabilistic mechanisms with different parameters, and try to find a way to balance the interests of the bidders with different valuation and the e-commerce platform.

The invariance of the symmetric equilibrium of the mechanisms can be guaranteed by Theorem 1. Other major performance indexes of a probabilistic mechanism for e-commerce platforms are the winning rate of bidders with low valuations (SMEs), the expected payoff of bidders and the expected profit of the e-commerce platform. In the following, we will discuss these performance indexes respectively.

1) WINNING RATE

The subsidy reserve price r_2 is a direct factor of winning rate of SMEs, since a rational bidder with valuation $x_i < r_2$ has no chance to win a bid with $b_i \leq x_i$. Suppose the base-line is AP mechanism $\mathcal{M}^A(r)$ with a small reserve price r . The smallest positive winning rate of bidders is $p^A(x_i; r) = F(r)^{N-1}$, $x_i = r$. In contrast, $\mathcal{M}^{pA}(r_1, r_2, \lambda)$ will guarantee that the winning rate of bidder- i is more than λ/N if $x_i \geq r_2$, and zero otherwise. It means that $r_2 > r$ is not acceptable, since the new mechanism fail to increase the winning rate of SMEs with $r \leq x_i < r_2$.

Assume $r = r_2$. We will adjust the parameters r_1, r_2 , and λ to analysis their effects on different performance indexes, and then compare the probabilistic mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$ with the base-line $\mathcal{M}^A(r_2)$.

a: EFFECT OF λ

In order to analyze the effect of λ on the winning rate, we consider the partial derivative of the winning rate according to Eq.(13),

$$\frac{\partial p^{pA}}{\partial \lambda}(x_i; r_1, r_2, \lambda) = \begin{cases} 1/N - F(x_i)^{N-1}, & x_i \in [r_1, \infty), \\ 1/N, & x_i \in [r_2, r_1), \\ 0, & \text{otherwise.} \end{cases} \quad (16)$$

Define $\xi_1 \in B$ to satisfy

$$1/N - F(\xi_1)^{N-1} = 0, \quad (17)$$

$F(\xi_1) = (1/N)^{1/(N-1)}$. It can be verified that $(1/N)^{1/(N-1)}$ is monotonous with respect to $N \geq 2$, hence $F(\xi_1) \geq (1/2)^{1/(2-1)} = 1/2$, which means ξ_1 is with a relatively high value. According to the distribution, the valuations of half of the bidders are less than ξ_1 .

Hence, we set $r_1 \leq \xi_1$. The winning rate of a SME with $r_2 < x_i < \xi_1$ is increasing with respect to λ , and the winning rate of a large enterprise with $x_i > \xi_1$ is decreasing with respect to λ . Hence, we have the following proposition to describe the effect of λ on the winning rate of bidders.

Lemma 2: Consider mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$ and the base-line mechanism $\mathcal{M}^A(r_1)$. $r_2 \leq r_1 \leq \min\{\bar{r}, \xi_1\}$.

- (1) *The winning rate of a bidder with $x_i < \xi_1$ is increasing with respect to λ . Moreover, the winning rate satisfies $p^{pA}(x_i; r_1, r_2, \lambda) > p^A(x_i; r_1)$.*
- (2) *The winning rate of a bidder with $x_i > \xi_1$ is decreasing with respect to λ . Moreover, the winning rate satisfies $p^{pA}(x; r_1, r_2, \lambda) < p^A(x; r_1)$.*

Lemma 2 shows that, in order to increase the winning rate of SMEs, we can try to increase λ .

b: EFFECT OF RESERVE PRICES

We introduce the indicator $w(x_i; r_1, r_2, \lambda) := p^{pA}(x_i; r_1, r_2, \lambda) - p^A(x_i; r_2)$ of the difference between the winning rate of a bidder with valuation x_i considering the proposed mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$ and the base-line mechanism $\mathcal{M}^A(r_2)$. The two mechanisms have the same reserve price r_2 . We will find parameters to increase the winning rates of SMEs.

$$w(x_i; r_1, r_2, \lambda) = \begin{cases} \lambda/N - \lambda F(x_i)^{N-1}, & x_i \in [r_1, \infty), \\ \lambda/N - F(x_i)^{N-1}, & x_i \in [r_2, r_1), \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

Note that ξ_1 satisfies Eq.(17). Hence, if $r_1 > \xi_1$, $\xi_1 \in [r_2, r_1)$, and $w(\xi_1; r_1, r_2, \lambda) = 0$. Define $\xi_2 \in B$ to satisfy

$$\lambda/N - F(\xi_2)^{N-1} = 0, \quad (19)$$

$F(\xi_2) = (\lambda/N)^{1/(N-1)} < (1/N)^{1/(N-1)} = F(\xi_1)$. Since F is monotonous, we have $\xi_2 < \xi_1$ for any $0 < \lambda < 1$. When λ is very small with fixed N , ξ_2 can be small too. For example, when $N = 2$, $F(\xi_2) = \lambda/2 \ll 1/2$. For more competitive markets, N is larger, then ξ_2 can be with a higher value. For any fixed $\lambda > 0$, when $N \rightarrow \infty$, $\xi_2 \rightarrow \sup B$. Thus, we have the following Lemma to show the effects of the reserve prices on the winning rate of bidders.

Lemma 3: Consider mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$ and the base-line mechanism $\mathcal{M}^A(r_2)$. $r_2 \leq r_1 \leq \min\{\bar{r}, \xi_1\}$.

(1) *If $r_1 < \xi_2$ or equivalently,*

$$\lambda > NF(r_1)^{N-1}, \quad (20)$$

then $p^{pA}(x_i; r_1, r_2, \lambda) > p^A(x_i; r_2)$ for $r_2 < x_i < \xi_1$.

(2) *If $r_1 = \xi_2$ or equivalently,*

$$\lambda = NF(r_1)^{N-1}, \quad (21)$$

then $p^{pA}(x_i; r_1, r_2, \lambda) > p^A(x_i; r_2)$ for $r_2 < x_i < \xi_2$.

(3) *If $r_2 < \xi_2 < r_1$ or equivalently,*

$$\lambda > NF(r_2)^{N-1}, \quad (22)$$

$$\lambda < NF(r_1)^{N-1}, \quad (23)$$

then $p^{pA}(x_i; r_1, r_2, \lambda) > p^A(x_i; r_2)$ for $r_2 < x_i < \xi_2$.

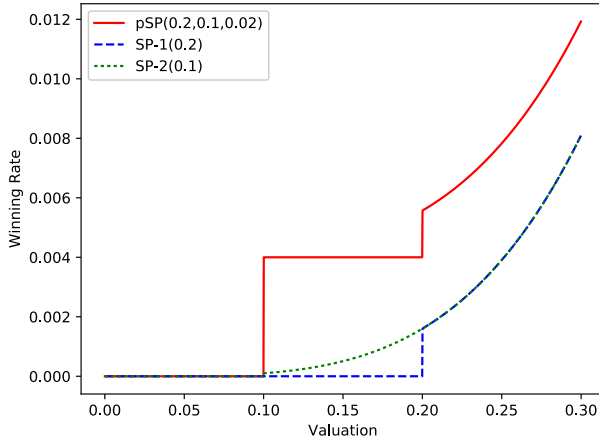


FIGURE 1. Case (1), $r_1 < \xi_2$. Higher winning rates for SMEs.

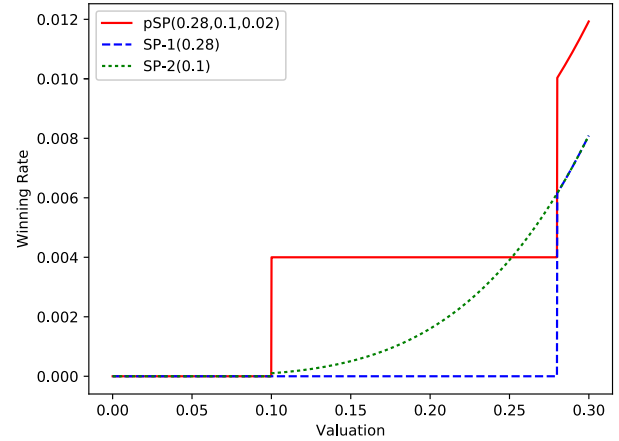


FIGURE 3. Case (3), $r_1 > \xi_2$. Higher winning rates for SMEs.

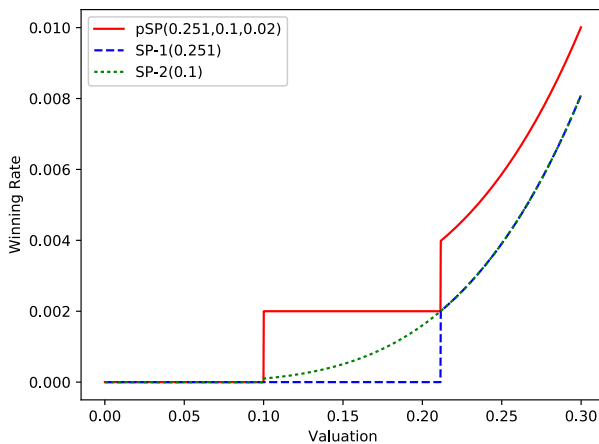


FIGURE 2. Case (2), $r_1 = \xi_2$. Higher winning rates for SMEs.

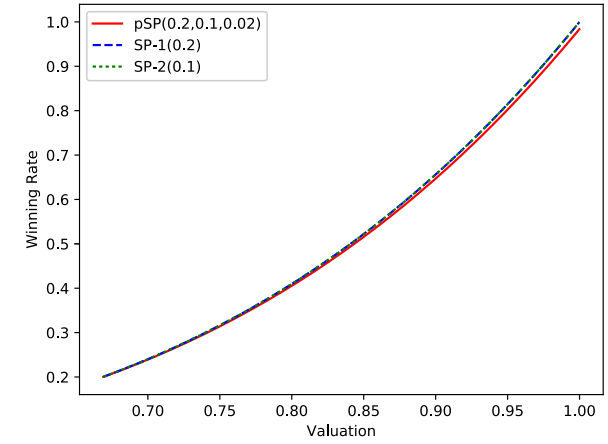


FIGURE 4. Lower winning rates for large enterprises.

The proof can be found in the appendix section. Lemma 3 shows that, there exists parameters r_1, r_2, λ such that the winning rate of SMEs ($p^{pA}(x_i; r_1, r_2, \lambda), r_2 < x_i < \xi_2$) can be improved. Note that, for highly competitive markets, ξ_2 can be very large, hence we can choose proper λ to meet the conditions Eq.(20), Eq.(21) and Eq.(22).

In Figure 1-3, we demonstrate the winning rates of bidders with different valuations. The red lines labeled “pSP” are with the probabilistic mechanism $\mathcal{M}^{pII}(r_1, r_2, \lambda)$. The blue lines labeled “SP-1” are with the SP mechanism $\mathcal{M}^{II}(r_1)$. And the green lines labeled “SP-2” are with the SP mechanism $\mathcal{M}^{II}(r_2)$. We consider the valuation uniform distributed over $[0, 1]$, $N = 5$, $r_2 = 0.1$, $\lambda = 0.02$. In this example, $\xi_1 = (1/5)^{1/4} \approx 0.669 > 1/2$ and $\xi_2 = (0.02/5)^{1/4} \approx 0.251 < 1/2$. We set r_1 to be 0.2, 0.251 and 0.28, respectively.

For Case (1) that $r_1 = 0.2 < \xi_2$, as is shown in Figure 1, bidders with low valuations ($0.1 < x_i < 0.3$ in the figure) will be benefited using the probabilistic mechanism. For Case (2) that $r_1 = \xi_2 = 0.251$, as is shown in Figure 2, bidders with low valuations will be benefited except the ones with $x_i = r_1$ ($0.1 < x_i < 0.251$ and $0.251 < x_i < 0.3$ in the figure). Actually, we can extend the range of valuations to

$B = (0, 1]$, and except for $x_i = r_1$ bidders with $0.1 < x_i < \xi_1 = 0.669$ will have increased winning rate. We name these two cases *SME-friendly* to indicate that almost all the bidders with low valuations have higher winning rates. SME-friendly mechanisms are with

$$\lambda \geq NF(r_1)^{N-1}. \quad (24)$$

For Case (3) that $r_1 = 0.28 > \xi_2$, as is shown in Figure 3, bidders with very low valuations ($0.1 < x_i < \xi_2 = 0.251$ in the figure) will be benefited, while the winning rates of bidders with medium valuations ($0.251 < x_i < r_1 = 0.28$ in the figure) will be reduced. However, if we select λ carefully, all the bidders with $x_i \geq r_2$ are with winning rates above the entrance threshold θ . We name Case (3) *semi SME-friendly*, since some of bidders with medium valuations will have lower winning rates compared with the original AP mechanism.

As is shown in Figure 4, the winning rates of bidders with high valuations ($x_i > \xi_1 = 0.669$ in the figure) will be slightly reduced. We also have the following theorem to show the effect of the probabilistic mechanism on large enterprises is limited.

Theorem 2: Consider mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$, and the base-line mechanisms $\mathcal{M}^A(r_1)$ and $\mathcal{M}^A(r_2)$. $r_2 \leq r_1 \leq \min\{\bar{r}, \xi_1\}$. The winning rate of bidder- i with $x_i > \xi_1$ satisfies

$$p^{pA}(x_i; r_1, r_2, \lambda) < p^A(x_i; r_1) = p^A(x_i; r_2)$$

$$p^{pA}(x_i; r_1, r_2, \lambda) \geq [1 - \lambda(1 - \frac{1}{N})]p^A(x_i; r_2).$$

The proof can be found in the appendix section. Theorem 2 shows that the reduction of winning rates of large enterprises can be acceptable, if λ is small.

2) PAYOFF OF BIDDERS

Although winning rate is very important for SMEs, the expected payoff is the critical performance index for most bidders. Hence, it is not acceptable to reduce the expected payoff of large enterprises significantly. Fortunately, we have the following proposition.

Theorem 3: Consider mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$, and the base-line mechanism $\mathcal{M}^A(r_1)$. $r_2 \leq r_1 \leq \min\{\bar{r}, \xi_1\}$. The expected payoff of bidder- i with $x_i \geq \xi_1$ satisfies

$$\pi^{pA}(x_i; r_1, r_2, \lambda) < \pi^A(x_i; r_1),$$

$$\pi^{pA}(x_i; r_1, r_2, \lambda) \geq (1 - \lambda)\pi^A(x_i; r_1).$$

The proof is straightforward according to Eq.(14). Theorem 3 shows that the reduction of a large enterprise’s expected payoff can be acceptable, if λ is small and $r_1 \approx r_2$.

3) PROFIT OF PLATFORM

Assuming all the bidders employ the symmetric equilibrium strategy, the expected profit of the e-commerce platform is one of the most important performance indexes of a pricing mechanism. Following Eq.(15), we have the following result.

Theorem 4: Consider mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$, and the base-line mechanisms $\mathcal{M}^A(r_1)$ and $\mathcal{M}^A(r_2)$. $r_2 \leq r_1 \leq \min\{\bar{r}, \xi_1\}$. The expected profit of the e-commerce platform satisfies

$$R^{pA}(r_1, r_2, \lambda) \geq (1 - \lambda)R^A(r_1) \geq (1 - \lambda)R^A(r_2).$$

Theorem 4 shows that there exists an acceptable lower bound of the expected profit of the e-commerce platform, if λ is small. In the following we will report our results for higher profit compared with the original AP mechanism. By calculating the partial derivatives of each parameters, we have the following properties.

Lemma 4: Consider mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$. $r_2 \leq r_1 \leq \min\{\bar{r}, \xi_1\}$. The expected profit of the e-commerce platform is decreasing with respect to λ , increasing with respect to r_1 , and increasing with respect to r_2 .

According Lemma 4, utilizing probabilistic mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$, the e-commerce platform can try to increase r_1 and r_2 or decrease λ to get higher profit. However, changing r_1, r_2 or λ will also influence performance indexes such as the winning rate of bidders with low valuations. Thus, we need to compare the probabilistic pricing mechanism with the base-line mechanism $\mathcal{M}^A(r_2)$ to find proper parameters for both the seller and the bidder sides.

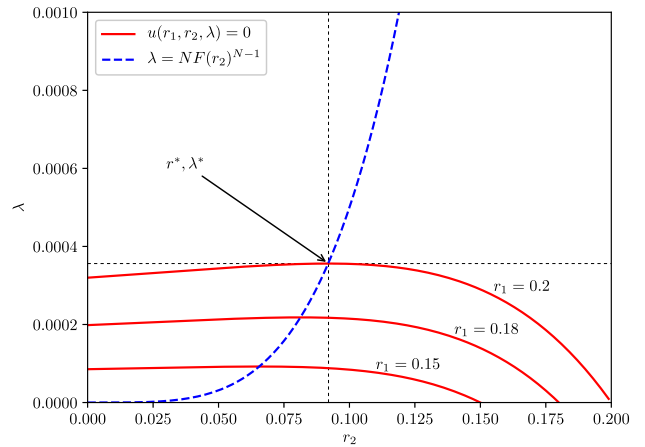


FIGURE 5. Contour-profit lines and semi SME-friendly condition.

We introduce the indicator $u(r_1, r_2, \lambda) = R^{pA}(r_1, r_2, \lambda) - R^A(r_2)$ of the difference between the expected profits with $\mathcal{M}^{pA}(r_1, r_2, \lambda)$ and $\mathcal{M}^A(r_2)$. As we can see, the two mechanisms have the same reserve price. We try to find a probabilistic mechanism such that $w(x_i; r_1, r_2, \lambda) \geq 0$ for small x_i to be SME-friendly or semi SME-friendly, while $u(r_1, r_2, \lambda) \geq 0$, i.e., to increase the winning rate of SMEs without loss of profit.

For any $r_1 \leq \bar{r}$, we define a *contour-profit line* of λ and r_2 , satisfying $u(r_1, r_2, \lambda) = 0$, or equivalently,

$$\lambda = \frac{R^A(r_1) - R^A(r_2)}{R^A(r_1) - R^0(r_2)}. \tag{25}$$

We have the following property.

Lemma 5: Consider mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$, and the base-line mechanism $\mathcal{M}^A(r_2)$. For $0 < r_1 \leq \min\{\bar{r}, \xi_1\}$, if $u(r_1, r^*, \lambda^*) = 0$, and

$$\frac{R^A(r_1) - R^A(r_2)}{R^A(r_1) - R^0(r_2)} \leq \frac{R^A(r_1) - R^A(r^*)}{R^A(r_1) - R^0(r^*)}, \quad \forall 0 \leq r_2 \leq r_1,$$

then,

$$\lambda^* = NF(r^*)^{N-1}. \tag{26}$$

The proof of Lemma 5 can be found in the appendix section. In Figure 5, there are three contour-profit lines with $r_1 = 0.15, 0.18$ and 0.20 respectively. The intersection points of Eq.(25) and Eq.(26) are $r_2 = r^*$ and $\lambda = \lambda^*$.

On one hand, according to Lemma 3, if we choose parameters in the left of the green line in Figure 5, it is semi SME-friendly. On the other hand, according to Lemma 4 and Lemma 5, if we have selected r_1 , and choose parameters below the corresponding red line in Figure 5, the expected profit of the e-commerce platform will be reduced compared with the base-line AP with the same reserve price r_2 . Hence, the existence of semi SME-friendly mechanisms without profit-loss can be find in the following theorem.

Theorem 5: Consider mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$, and the base-line mechanism $\mathcal{M}^A(r_2)$. If $r_2 \leq r_1 \leq \min\{\bar{r}, \xi_1\}$ and

$$NF(r_2)^{N-1} < \lambda \leq \frac{R^A(r_1) - R^A(r_2)}{R^A(r_1) - R^0(r_2)}, \tag{27}$$

the probabilistic mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$ is semi SME-friendly, and the expected profit of the e-commerce platform satisfies $R^{pA}(r_1, r_2, \lambda) \geq R^A(r_2)$.

The proof of Theorem 5 is straightforward after the proof of Lemma 3 and Lemma 5. We will present such mechanisms in Section V.

Suppose there exists r_1, r_2, λ , such that the probabilistic mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$ is SME-friendly without profit-loss. Then, according to Lemma 5,

$$\lambda \leq \sup_{r_2} \left\{ \frac{R^A(r_1) - R^A(r_2)}{R^A(r_1) - R^0(r_2)} \right\} = NF(r_2)^{N-1} \leq N(r_1)^{N-1}.$$

The last inequality leads to $r_1 = r_2$. However, if $r_1 = r_2$,

$$\lambda = \frac{R^A(r_1) - R^A(r_2)}{R^A(r_1) - R^0(r_2)} = 0.$$

Hence, there is no SME-friendly probabilistic mechanism without profit-loss.

Theorem 6: Consider mechanism $\mathcal{M}^{pA}(r_1, r_2, \lambda)$ and the base-line mechanism $\mathcal{M}^A(r_2)$. There is no probabilistic mechanism such that

- the e-commerce platform has higher expected profit, and
- the mechanism is SME-friendly.

D. PRINCIPLES OF PROBABILISTIC MECHANISM DESIGN

Now, we can employ the theorems in this section to design probabilistic mechanisms.

(1) According to Theorem 1, we can design probabilistic mechanisms by finding proper parameters r_1, r_2 and λ .

(2) Although the probabilistic mechanisms will bring dead-weight loss, Theorem 2, Theorem 3 and Theorem 4 guarantee that if λ is very small, the loss will be small too. We can design SME-friendly mechanisms with λ social loss compared with the original mechanism.

(3) Theorem 5 shows the existence of semi SME-friendly mechanism without profit-loss by construction. Hence, we can also design such mechanisms according to inequalities (27).

V. ALGORITHMS AND EXPERIMENTS

In this section, we employ the probabilistic approach to design mechanisms which ensure the bidders with low valuations can get higher winning rate compared with FP/SP mechanism. We also assume an advertiser will leave the e-commerce platform if her winning rate is lower than the entrance threshold $\theta = 0.1\%$. Consider an auction with $N = 5$ rational bidders whose valuations uniformly distributed in $[0, 1]$, and take SP $\mathcal{M}^H(r)$ as the base-line mechanism. If $r = 0.1$, the winning rate of bidder- i with $x_i = 0.1$ is 0.1^9 . Since $F(0.178)^{N-1} \approx \theta$, the winning rates of about 17.8% bidders are lower than the entrance threshold $\theta = 0.1\%$.

The first mechanism is designed to be SME-friendly with profit-loss. It increases the winning rate (not less than θ) of SMEs, and decreases the expected profit of the

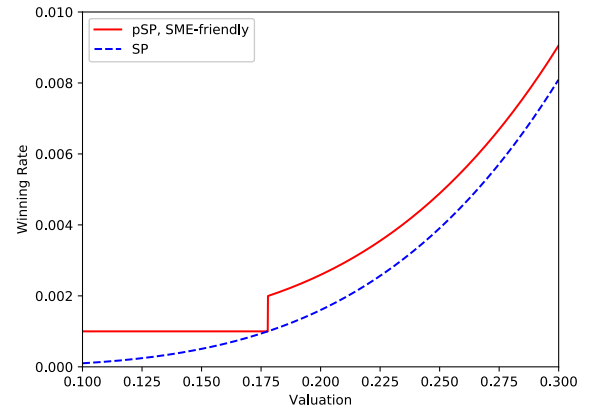


FIGURE 6. The winning rate over bidder's valuation. (Alg 1).

e-commerce platform. The second mechanism aims the SME-friendly property without loss of profit.

A. ALGORITHMS

1) SME-FRIENDLY MECHANISM WITH PROFIT-LOSS

Algorithm 1 proposed a SME-friendly probabilistic mechanism based on the base-line SP mechanism $\mathcal{M}^H(r)$, and try to reduce the loss of the seller.

We design the algorithm according to Lemma 3, and Theorem 2, 3, 4. If $F(r)^{N-1} < \theta$, for example $r = 0.1$, $\mathcal{M}^H(r)$ will turn about 17.8% advertisers off the platform. Our task is to increase the winning rate of bidders with low valuations to θ . Let $r_2 \leftarrow r$, $\lambda \leftarrow N\theta$, and $F(r_1) = (\lambda/N)^{1/(N-1)}$. The results are shown in Figure 6-7.

Algorithm 1 SME-Friendly Mechanism With Profit-Loss

Data: $\mathcal{M}^H(r), F, N, \theta$

Result: $\mathcal{M}^{pH}(r_1, r_2, \lambda)$

$r_2 \leftarrow r;$

$\lambda \leftarrow N\theta;$

$r_1 \leftarrow \text{FindRoot} \{F^{N-1}(r_1) = \theta, r_1\};$

Return $\mathcal{M}^{pH}(r_1, r_2, \lambda)$.

Since $r_1 = \xi_2$, the proposed mechanism is SME-friendly according to Lemma 3. As is shown in Figure 6, bidders with low valuations (e.g., $0.1 < x_i \leq 0.3$ in this figure) will have higher winning rates using Algorithm 1 (red line) compared with the base-line SP mechanism (blue line). The winning rates of bidders with $x_i > 0.1$ (90% advertisers compared with about 82.2% of SP) are higher than the entrance threshold $\theta = 0.1\%$.

Figure 7 presents the expected profit of the e-commerce platform with respect to different reserve prices r of the base-line SP mechanisms, $r \in [0, 0.178]$. As we can see, the expected profit of the e-commerce platform reduces slightly.

The proposed mechanism defined by Algorithm 1 is SME-friendly. Compared with the base-line SP mechanism, the proposed pSP keeps 9.5% more advertisers stay, causes less than 0.4841% profit loss of the platform and less than 0.161% payoff loss of large enterprises. It can be used as an

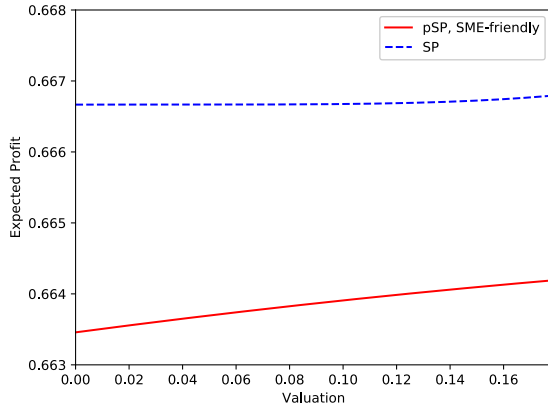


FIGURE 7. The expected profit over reserve price. (Alg 1).

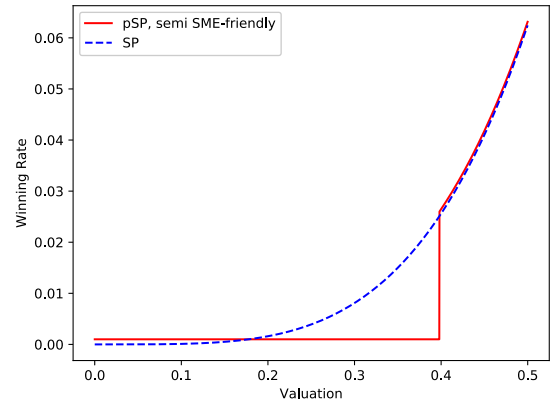


FIGURE 8. The winning rate over bidder's valuation. (Alg 2).

alternative to second price auctions for e-commerce platforms to grow the user group.

2) SEMI SME-FRIENDLY MECHANISM WITHOUT PROFIT-LOSS

Algorithm 1 reduces the profit of the e-commerce platform slightly. Algorithm 2 designs a semi SME-friendly probabilistic mechanism based on the base-line SP mechanism $\mathcal{M}^{\text{II}}(r)$, and keep the expected profit the same as the original one.

We designs the algorithm according to Theorem 5. If $F(r)^{N-1} < \theta$, set $r_2 \leftarrow r$, $\lambda \leftarrow N\theta$, and choose r_1 such that $u(r_1, r_2, \lambda) = 0$. According to Theorem 5, r_1, r_2, λ satisfy the inequalities (27). Hence it is SME-friendly without profit-loss. In the experiment, we set $r = r_2 = 0$. The results are in Figure 8.

Algorithm 2 Semi SME-Friendly Mechanism Without Profit-Loss

Data: $\mathcal{M}^{\text{II}}(r), F, N, \theta$

Result: $\mathcal{M}^{p\text{II}}(r_1, r_2, \lambda)$

$r_2 \leftarrow r$;

$\lambda \leftarrow N \cdot \theta$;

$r_1 \leftarrow \text{FindRoot}$

$\{\lambda R^0(r_2) + (1 - \lambda)R^{\text{II}}(r_1) = R^{\text{II}}(r_2), r_1\}$;

Return $\mathcal{M}^{p\text{II}}(r_1, r_2, \lambda)$.

As we can see from Figure 8, although the reserve price of SP is $r = 0$, bidders with $0 < x_i < 0.178$ do not have enough impressions. They will have higher winning rate using Algorithm 2. The winning rate of all the bidders are higher than $\theta = 0.1\%$. Moreover, the expected profits with the two mechanisms are the same. If the expected profit is a critical performance index of the e-commerce platform, the proposed mechanism helps SMEs with $0 < x_i < 0.178$ without any profit-loss, and reduces the winning rate of bidders with $0.178 < x_i < 0.396$.

This mechanism mainly aims demonstrating the probabilistic approach for budget limited platforms. According to Theorem 5, we can also design probabilistic mechanisms with higher expected profit compared with the base-line SP with

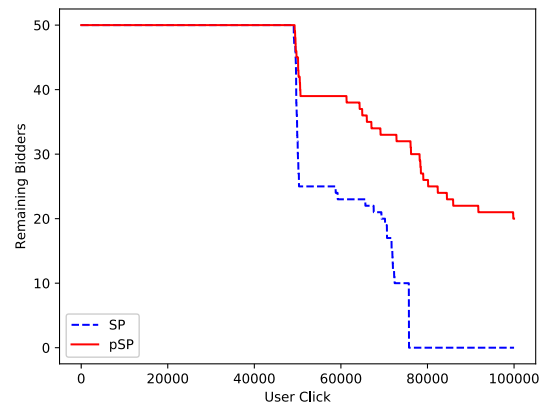


FIGURE 9. Remaining players comparison.

the same reserve price. However, this will impact the winning rate of medium valuation bidders, and is not suggested.

B. COMPUTATIONAL EXPERIMENTS

Next, we conduct computational experiments to evaluate the performance (profit of the e-commerce platform) of SP without reserve prices ($r_2 = 0$) and the corresponding probabilistic pSP mechanism according to Algorithm 1. Suppose there are 50 active advertisers in the platform. Averagely, $N = 10$ of them bid for an impression opportunity of a user-click. For simplicity, for each user-click, we choose 10 bidders randomly. Assume a bidder will employ the symmetric equilibrium strategy to the user-click, and she will leave the e-commerce platform if the average winning rate is less than $\theta = 0.1\%$ after her 10000 trials. We simulate for 100000 user-clicks, and present the results in Figure 9-10.

Since we want to keep the diversity of the e-commerce platform, the number of remaining players is a performance index in our computational experiments. In Figure 9, after about 49k trials, some bidders have accumulated more than 20000 bids, and starts to record the winning rates. Hence, we can see rapid drawdown with both mechanisms. Overall, pSP keeps more (20/50) bidders in the e-commerce platform, and outperforms SP (0/50).

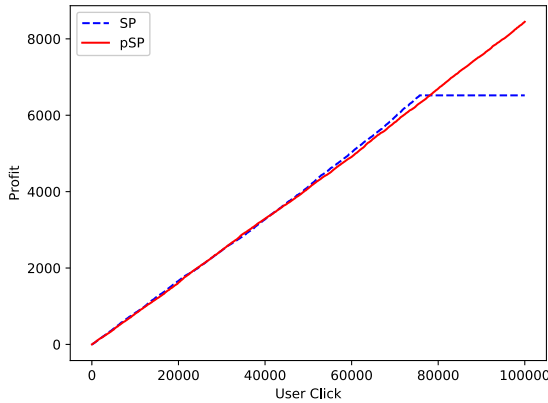


FIGURE 10. Profits comparison.

The profits of the e-commerce platform with SP and pSP are shown in Figure 10. In the first 49k trials, the profits of both mechanisms are close. Later, the profit of SP increases faster than pSP, since the dropout of SMEs. However, after about 75k trials, the remaining players of SP vanish because of the intense long-term competition. The proposed pSP mechanism outperforms SP with both performance indexes in the computational experiments. Different distributions of valuation and different base mechanisms (FP/pFP) lead to similar results.

C. DISCUSSION AND MANAGERIAL INSIGHTS

The winning rate of SMEs and the expected profit of the e-commerce platform are the two most important performance indexes of a pricing mechanisms for e-commerce platforms. In order to balance the two conflicting interests, only the reserve price can be adjusted in classical first-price and second-price mechanisms. This constraint is highly relaxed in our method by introducing additional parameters. In practice, the simple mechanisms can be regarded as fine tunings of the original one. The e-commerce platform can adopt the probabilistic mechanism equipped with proper parameters (e.g., with very small λ) to bring better performance for SMEs while still keeping some desired qualities of the original one.

The e-commerce platform can also design mechanisms through tuning variable λ and r_2 utilizing the probabilistic approach in more complex scenarios while not changing the equilibriums. Thus, it provides a practical approach for real-world applications, which are involved with highly dynamical and non-truthful mechanisms.

VI. CONCLUSIONS

In this paper, we propose a probabilistic approach to design advertising mechanisms for e-commerce platforms. Simple mechanisms are introduced as useful tools for designing probabilistic mechanisms to keep the original equilibrium. We designed two kinds of probabilistic mechanisms based on the classical first-price and second-price mechanisms. Properties of the proposed probabilistic mechanisms pFP and pSP are then investigated. Furthermore, we also implement two

algorithms to help design mechanisms for different application scenarios. Experiment results demonstrate the flexibility and performance of the proposed probabilistic approach for e-commerce platforms.

Another desirable property of the proposed probabilistic approach is that the new mechanism will not change the equilibrium when using simple mechanisms as the probabilistic combination. This can be useful if the e-commerce platform wants to change the mechanism without affecting the strategies of bidders. When building highly dynamical mechanisms, we can employ probabilistic mechanisms with sufficient numbers of parameters to get better performance ([30]) and do not have to concern the changing of equilibriums. Moreover, the proposed probabilistic approach for mechanism design is not limited for a generalization of first-price or second-price mechanism. Our analysis suggests new probabilistic mechanisms based on existence ones for multiple purpose.

APPENDIX

Proof of Theorem 1: Consider mechanism \mathcal{M}_2 , denote the equilibrium strategy of the bidders $\bar{\beta}$. Suppose all the bidders except bidder- i follow the original equilibrium strategy of mechanism \mathcal{M}_2 , then the expected payoff of bidder i when bidding the amount b_i is

$$\begin{aligned} E[\Pi_i(x, b)] &= \lambda_1 E[\Pi_i^1(x, b)] + \lambda_2 E[\Pi_i^2(x, b)] \\ &= \lambda_1 E[\Pi_i^1(x, \bar{\beta}(x_1), \dots, \bar{\beta}(x_{i-1}), b_i, \bar{\beta}(x_{i+1}), \dots, \bar{\beta}(x_N))] \\ &\quad + \lambda_2 E[\Pi_i^2(x, \bar{\beta}(x_1), \dots, \bar{\beta}(x_{i-1}), b_i, \bar{\beta}(x_{i+1}), \dots, \bar{\beta}(x_N))] \\ &\leq \lambda_1 E[\Pi_i^1(x, \bar{\beta}(x_1), \dots, \bar{\beta}(x_N))] \\ &\quad + \lambda_2 E[\Pi_i^2(x, \bar{\beta}(x_1), \dots, \bar{\beta}(x_N))]. \end{aligned}$$

The inequity holds, since a) the expected payoff of a simple mechanism can be determined by the allocation rule and the payment rule, and is independent with b , and b) $\bar{\beta}$ is an equilibrium strategy of \mathcal{M}_2 .

Hence, β is an equilibrium of the probabilistic pricing mechanism $(\mathcal{M}_1, \mathcal{M}_2, \bar{\lambda})$ for any $\bar{\lambda}$. The proof is completed. \square

Proof of Lemma 3: (1) Since $r_1 < \xi_1$, we have $F(r_1)^{N-1} < F(\xi_1)^{N-1} = \lambda/N$. Since $w(x)$ is decreasing when $r_2 < x < r_1$, for $r_2 < x < r_1$, $w(x; r_1, r_2, \lambda) > 0$. $w(\xi_1) = \lambda/N - \lambda F(\xi_1)^{N-1} = \lambda[1/N - F(\xi_1)^{N-1}] = 0$. Since $w(x)$ is decreasing when $x \geq r_1$, for $r_1 \leq \xi_1$, $w(x; r_1, r_2, \lambda) > 0$. The proof is completed for Case (1).

(2) Since $w(x)$ is decreasing when $r_2 < x < r_1$, and $w(r_1-) = w(\xi_2-) = \lambda/N - F(\xi_2-)^{N-1} = \lambda/N - F(\xi_2)^{N-1} = 0$, for $r_2 < x < \xi_2$, $w(x) > 0$. The proof is completed for Case (2). Moreover, for $r_2 = \xi_2 < x < \xi_1$, we also have $w(x) > 0$. Hence, in Case (2), $p^{pA}(x_i; r_1, r_2, \lambda) > p^A(x_i; r_2)$ for almost all the bidders with $r_2 < x < \xi_1$.

(3) Since $w(x)$ is decreasing when $r_2 < x < r_1$, $\xi_2 < r_1$, and $w(\xi_2-) = \lambda/N - F(\xi_2-)^{N-1} = \lambda/N - F(\xi_2)^{N-1} = 0$, for $r_2 < x < \xi_2$, $w(x) > 0$. The proof is completed

for Case (3). For the same reason, we know that if $\xi_2 < x < r_1$, $w(x) < 0$. If $r_1 < x < \xi_1$, $w(x) > 0$. \square

Proof of Theorem 2: (1) Since $w(x)$ is decreasing when $x > \xi_1 > r_2$, and $w(\xi_1) = \lambda/N - \lambda F(\xi_1)^{N-1} = \lambda[1/N - F(\xi_1)^{N-1}] = 0$, if $x > \xi_1$, $p^{p^A}(x_i; r_1, r_2, \lambda) < p^A(x; r_1)$.

(2) Since $p^A(x_i; r_2) > 0$,

$$\begin{aligned} \frac{p^{p^A}(x; r_1, r_2, \lambda)}{p^A(x; r_2)} &= \frac{\lambda/N + (1 - \lambda)F(x)^{N-1}}{F(x)^{N-1}} \\ &= \frac{\lambda}{NF(x)^{N-1}} + 1 - \lambda \leq \frac{\lambda}{N} + 1 - \lambda \\ &= 1 - \lambda(1 - \frac{1}{N}). \end{aligned}$$

The proof is completed. \square

Proof of Lemma 5: Define

$$\begin{aligned} h_1(r_1, r_2) &:= \frac{R^A(r_1) - R^A(r_2)}{R^A(r_1) - R^0(r_2)}, \\ h_2(r_2) &:= NF(r_2)^{N-1}. \end{aligned}$$

We will show that if r satisfies $\partial h_1/\partial r_2(r_1, r) = 0$, then $h_1(r_1, r_2) \leq h_1(r_1, r)$, and $h_1(r_1, r) = h_2(r)$.

$$\begin{aligned} \frac{\partial h_1}{\partial r_2}(r_1, r) &= \frac{-\dot{R}^A(r)[R^A(r_1) - R^0(r)] + \dot{R}^0(r)[R^A(r_1) - R^A(r)]}{[R^A(r_1) - R^0(r)]^2} \\ &= \left[-NF(r)^{N-1}[R^A(r_1) - R^0(r)] + [R^A(r_1) - R^A(r)] \right] \\ &\quad \cdot \frac{1 - F(r) - rf(r)}{[R^A(r_1) - R^0(r)]^2} \\ &= u(r_1, r, h_2(r)) \cdot \frac{1 - F(r) - rf(r)}{[R^A(r_1) - R^0(r)]^2}. \end{aligned}$$

Since $r \leq \bar{r}$, $1 - F(r) - rf(r) > 0$, $\partial h_1/\partial r_2(r_1, r) = 0$ is equivalent to

$$\begin{aligned} u(r_1, r, h_2(r)) &= 0, \\ h_2(r) &= \frac{R^A(r_1) - R^A(r)}{R^A(r_1) - R^0(r)} = h_1(r_1, r). \end{aligned}$$

Next, we will show that $u(r_1, r_2, \lambda)$ is decreasing with respect to r_2 along $\lambda = h_2(r_2)$. It can be verified that,

$$\begin{aligned} \frac{\partial u}{\partial \lambda}(r_1, r_2, \lambda) &= r_2[1 - F(r_2)] - r_1[1 - F(r_1)]F(r_1)^{N-1} \\ &\quad - \int_{r_1}^{\infty} y(1 - F(y))g(y) dy \leq 0, \end{aligned}$$

$$\frac{\partial u}{\partial r_2}(r_1, r_2, \lambda) = [\lambda - N \cdot F(r_2)^{N-1}][1 - F(r_2) - r_2f(r_2)].$$

Define $h_3(r_1, r_2) = u(r_1, r_2, h_2(r_2))$, it is decreasing with respect to r_2 , since

$$\begin{aligned} \frac{\partial h_3}{\partial r_2}(r_1, r_2) &= \frac{\partial u}{\partial r_2}(r_1, r_2, h_2(r_2)) + \frac{\partial u}{\partial \lambda}(r_1, r_2, h_2(r_2)) \frac{\partial h_2}{\partial r_2}(r_2) \\ &= 0 + \frac{\partial u}{\partial \lambda}(r_1, r_2, h_2(r_2))Ng(r_2) \leq 0. \end{aligned}$$

If $r_2 \leq r$, $h_3(r_1, r_2) \geq h_3(r_1, r)$. Hence,

$$\begin{aligned} u(r_1, r_2, h_2(r_2)) &\geq u(r_1, r, h_2(r)) = 0, \\ \frac{h_1}{r_2}(r_1, r_2) &= u(r_1, r_2, h_2(r_2)) \\ &\quad \cdot \frac{1 - F(r_2) - r_2f(r_2)}{[R^A(r_1) - R^0(r_2)]^2} \geq 0. \end{aligned}$$

If $r_2 \geq r$, $h_3(r_1, r_2) \leq h_3(r_1, r)$. Hence,

$$\begin{aligned} u(r_1, r_2, h_2(r_2)) &\leq u(r_1, r, h_2(r)) = 0, \\ \frac{h_1}{r_2}(r_1, r_2) &= u(r_1, r_2, h_2(r_2)) \\ &\quad \cdot \frac{1 - F(r_2) - r_2f(r_2)}{[R^A(r_1) - R^0(r_2)]^2} \leq 0. \end{aligned}$$

To conclude, $\partial h_1/\partial r_2$ is 0 when $r_2 = r$, and it is non-negative when $r_2 \leq r$, and non-positive when $r_2 \geq r$. Hence $r_2 = r$ maximize $h(r_1, r_2)$, and $h_1(r_1, r) = h_2(r)$. The proof is completed. \square

Proof of Theorem 5: Since $NF(r_2)^{N-1} < \lambda$, the mechanism is semi SME-friendly.

$$\begin{aligned} \frac{\partial u}{\partial \lambda}(r_1, r_2, \lambda) &= r_2[1 - F(r_2)] - r_1[1 - F(r_1)]F(r_1)^{N-1} \\ &\quad - \int_{r_1}^{\infty} y(1 - F(y))g(y) dy \leq 0. \end{aligned}$$

Hence, $u(r_1, r_2, \lambda) \geq u(r_1, r_2, h_1(r_1, r_2)) = 0$. Thus, $R^{p^A}(r_1, r_2, \lambda) \geq R^A(r_2)$. The proof is completed. \square

REFERENCES

- [1] S. Muthukrishnan, "Internet ad auctions: Insights and directions," in *Automata, Languages and Programming. ICALP 2008* (Lecture Notes in Computer Science), L. Aceto, I. Damgård, L. A. Goldberg, M. M. Halldórsson, A. Ingólfssdóttir, and I. Walukiewicz, vol. 5125. Berlin, Germany: Springer, 2008, pp. 14–23.
- [2] D. S. Evans, "The antitrust economics of multi-sided platform markets," *Yale J. Reg.*, vol. 20, p. 325, 2003.
- [3] B. J. Jansen and T. Mullen, "Sponsored search: An overview of the concept, history, and technology," *Int. J. Electron. Bus.*, vol. 6, no. 2, pp. 114–131, 2008.
- [4] B. Edelman, M. Ostrovsky, and M. Schwarz, "Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords," *Amer. Econ. Rev.*, vol. 97, no. 1, pp. 242–259, 2007.
- [5] H. R. Varian, "Online ad auctions," *Amer. Econ. Rev.*, vol. 99, no. 2, pp. 430–434, 2009.
- [6] W. Vickrey, "Counterspeculation, auctions, and competitive sealed tenders," *J. Finance*, vol. 16, no. 1, pp. 8–37, Mar. 1961.
- [7] E. H. Clarke, "Multipart pricing of public goods," *Public Choice*, vol. 11, no. 1, pp. 17–33, Sep. 1971.
- [8] T. Groves, "Incentives in teams," *Econometrica*, vol. 41, no. 4, pp. 617–631, Jul. 1973.
- [9] A. Abdulkadiroğlu and T. Sönmez, "Random serial dictatorship and the core from random endowments in house allocation problems," *Econometrica*, vol. 66, no. 3, pp. 689–701, May 1998.
- [10] V. Conitzer and T. Sandholm, "Complexity of mechanism design," in *Proc. 18th Conf. Uncertainty Artif. Intell.*, 2002, pp. 103–110.
- [11] V. Krishna, *Auction Theory*, 2nd ed. San Diego, CA, USA: Academic, 2010.
- [12] A. V. Goldberg, J. D. Hartline, A. R. Karlin, M. Saks, and A. Wright, "Competitive auctions," *Games Econ. Behavior*, vol. 55, no. 2, pp. 242–269, May 2006.
- [13] P. Milgrom, *Putting Auction Theory to Practice*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
- [14] R. J. Garratt, M. Walker, and J. Wooders, "Behavior in second-price auctions by highly experienced eBay buyers and sellers," *Experim. Econ.*, vol. 15, no. 1, pp. 44–57, Mar. 2012.

- [15] J. Zhang, D. Wen, and S. Zeng, "A discounted trade reduction mechanism for dynamic ridesharing pricing," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 6, pp. 1586–1595, Jun. 2016.
- [16] T. Luo, S. K. Das, H. P. Tan, and L. Xia, "Incentive mechanism design for crowdsourcing: An all-pay auction approach," *ACM Trans. Intell. Syst. Technol.*, vol. 7, no. 3, p. 35, 2016.
- [17] R. B. Myerson, "Optimal auction design," *Math. Oper. Res.*, vol. 6, no. 1, pp. 58–73, Feb. 1981.
- [18] E. Maskin and J. Riley, "Optimal multi-unit auctions," in *The Economics of Missing Markets, Information, and Games*, F. Hahn, Ed. London, U.K.: Oxford Univ. Press, 1989, pp. 312–335.
- [19] R. P. McAfee and J. McMillan, "Auctions and bidding," *J. Econ. Literature*, vol. 25, no. 2, pp. 699–738, Jun. 1987.
- [20] P. Klemperer, "Auction theory: A guide to the literature," *J. Econ. Surv.*, vol. 13, no. 3, pp. 227–286, Jul. 1999.
- [21] V. Skreta, "Optimal auction design under non-commitment," *J. Econ. Theory*, vol. 159, pp. 854–890, Sep. 2015.
- [22] J. D. Hartline and B. Lucier, "Non-optimal mechanism design," *Amer. Econ. Rev.*, vol. 105, no. 10, pp. 3102–3124, 2015.
- [23] K. Mierendorff, "Optimal dynamic mechanism design with deadlines," *J. Econ. Theory*, vol. 161, pp. 190–222, Jan. 2016.
- [24] A. Bogomolnaia and H. Moulin, "A new solution to the random assignment problem," *J. Econ. Theory*, vol. 100, no. 2, pp. 295–328, Oct. 2001.
- [25] F. Kojima and M. Manea, "Incentives in the probabilistic serial mechanism," *J. Econ. Theory*, vol. 145, no. 1, pp. 106–123, Jan. 2010.
- [26] V. Conitzer and T. Sandholm, "Self-interested automated mechanism design and implications for optimal combinatorial auctions," in *Proc. 5th ACM Conf. Electron. Commerce*, 2004, pp. 132–141.
- [27] M. Babaioff and N. Nisan, "Concurrent auctions across the supply chain," *J. Artif. Intell. Res.*, vol. 21, pp. 595–629, May 2004.
- [28] J.-P. Benoît and V. Krishna, "Multiple-object auctions with budget constrained bidders," *Rev. Econ. Stud.*, vol. 68, no. 1, pp. 155–179, 2001.
- [29] J. Zhang, Y. Yang, X. Li, R. Qin, and D. Zeng, "Dynamic dual adjustment of daily budgets and bids in sponsored search auctions," *Decision Support Syst.*, vol. 57, pp. 105–114, Jan. 2014.
- [30] F.-Y. Wang et al., "Where does AlphaGo go: From church-Turing thesis to AlphaGo thesis and beyond," *IEEE/CAA J. Autom. Sinica*, vol. 3, no. 2, pp. 113–120, Apr. 2016.



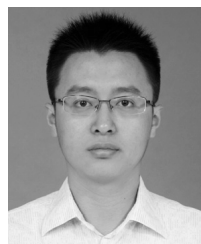
LINJING LI (M'15) received the B.S. and M.S. degrees in electrical engineering and automation from the Harbin Institute of Technology, Heilongjiang, China, and the Ph.D. degree in computer application from the Institute of Automation, Chinese Academy of Sciences, Beijing, China, in 2005, 2007, and 2001, respectively. He is an Associate Professor with the State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy



of Sciences. His current research interests include e-commerce and business intelligence.

FEI-YUE WANG (S'87–M'89–SM'94–F'03) received the Ph.D. degree in computer and systems engineering from Rensselaer Polytechnic Institute, Troy, New York, in 1990. He joined the University of Arizona in 1990 and he became a Professor and the Director of the Robotics and Automation Lab and Program in Advanced Research for Complex Systems. In 1999, he founded the Intelligent Control and Systems Engineering Center, Institute of Automation, Chinese Academy of Sciences (CAS), Beijing, China, under the support of the Outstanding Oversea Chinese Talents Program from the State Planning Council and 100 Talent Program from CAS, and in 2002, was appointed as the Director of the Key Laboratory of Complex Systems and Intelligence Science, CAS. In 2011, he became the State Specially Appointed Expert and the Director of the State Key Laboratory of Management and Control for Complex Systems.

His current research focuses on methods and applications for parallel systems, social computing, and knowledge automation. He was elected a Fellow of the IEEE, the INCOSE, the IFAC, the ASME, and the AAAS. In 2007, he received the Second Class National Prize in Natural Sciences of China and was awarded the Outstanding Scientist by the ACM for his work in intelligent control and social computing. He received the IEEE ITS Outstanding Application and Research Awards in 2009 and 2011, and the IEEE SMC Norbert Wiener Award in 2014. He was the Founding Editor-in-Chief of the *International Journal of Intelligent Control and Systems* (1995–2000), the Founding EiC of the *IEEE Intelligent Transportation Systems Magazine* (2006–2007), an EiC of the *IEEE Intelligent Systems* (2009–2012), and an EiC of the *IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS* (2009–2016). He is currently an EiC of *China's Journal of Command and Control*. Since 1997, he has served as the General or Program Chair of over 20 IEEE, INFORMS, ACM, and ASME conferences. He was the President of the IEEE ITS Society (2005–2007), Chinese Association for Science and Technology, USA, in 2005, the American Zhu Kezhen Education Foundation (2007–2008), and the Vice President of the ACM China Council (2010–2011). Since 2008, he has been the Vice President and Secretary General of the Chinese Association of Automation.



JIE ZHANG (M'16) received the B.Sc. degree in information and computing science from Tsinghua University in 2005, the M.Sc. degree in operations research and control theory from the Renmin University of China in 2009, and the Ph.D. degree in technology of computer application from the University of the Chinese Academy of Sciences in 2015. He is currently an Associate Professor with the State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing, China. His current research interests include mechanism design and parallel control.

...