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# Adaptive Legged Robots Through Exactly Constrained and Non-Redundant Design

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**ABSTRACT** This paper presents a novel strategy for designing passively adaptive, statically stable walking robots with full body mobility that are exactly constrained and non-redundantly actuated during stance. In general, fully mobile legged robots include a large number of actuated joints, giving them a wide range of controllable foot placements but resulting in overconstraint during stance, requiring kinematic redundancy and redundant control for effective locomotion. The proposed design strategy allows for the elimination of actuation redundancy, thus greatly reducing the weight and complexity of the legged robots obtained and allowing for simpler control schemes. Moreover, the underconstrained nature of the resulting robots during swing allows for passive adaptability to rough terrain without large contact forces. The strategy uses kinematic mobility analysis tools to synthesize leg topologies, underactuated robotics design approaches to effectively distribute actuation constraints, and elastic elements to influence nominal leg behavior. Several examples of legged robot designs using the suggested approach are thoroughly discussed and a proof-of-concept of a non-redundant walking robot is presented.

**INDEX TERMS** Legged locomotion, robot kinematics, self-adaptive mechanisms, mobile robots.

## I. INTRODUCTION

Legged locomotion has significant advantages over wheeled locomotion when presented with rough terrain, including obstacles presented by human environments such as stairs and curbs. By moving their feet around or over obstacles and placing them on discrete contact points, legged robots generate sets of stance patterns that are used to support and move the body during locomotion. Many multi-legged walking robots, e.g. robots that rely on kinetostatically stable rather than dynamically stable gaits, have often utilized highly-articulated and highly-actuated leg designs to enable arbitrary placement of each foot relative to the body. However, this can lead to issues of over-constraint when the robot is in contact with the ground if more constraints are added by the ground contacts than the number of degrees of freedom in the robot. Additionally, any legged robot with more than six actuators in a given stance leg set will necessarily be redundant, and even robots with fewer than six can be redundant depending on their configuration and kinematic topology.

Legged locomotion can be classified into a number of different gait types [1]. Many early robots, such as the Adaptive Suspension Vehicle [2], primarily implemented statically-stable walking gaits and often used legs with coupled joints to simplify control. Later robots such as LAURON II [3] and ROBOT III [4] independently actuated each of the joints of their legs to improve their mobility but encountered control difficulty due to over-constraint. Robots such as RHex [5] and Sprawlita [6] included passive compliant joints, allowing for faster, often dynamic, gaits but sacrificing ground clearance and posture control. More recently, robots such as BigDog [7] and StarlETH [8] have demonstrated impressive dynamic locomotion performance over rough terrain through the use of complex closed-loop feedback control with a large number of sensory inputs and redundant actuation. We believe that an adaptive kinetostatically stable walking robot could strike a balance between system complexity and locomotion performance over rough terrain, while avoiding stance over-constraint and redundancy. This approach aims to achieve design simplicity while exploiting the passive adaptability of the swing legs in the absence of full contact to traverse rough terrain.

Passive adaptability is one of the primary benefits of underactuation through differential mechanisms. It has been used in manipulators to greatly improve grasp acquisition and performance in unstructured environments with only open-loop control [9]. Such manipulators reconfigure in the presence of partial contact with a target object to accommodate its shape and position relative to the hand since they rely on multiple contacts to fully constrain them. Similarly, the under-constrained nature of the robot's legs during swing allow the legs to reconfigure to accommodate variations in ground height without active sensing or destabilizing reaction forces.

Stance over-constraint complicates control due to the over-determined kinematic system. Additionally, the control of over-constrained legged robots can impose internal motions on the body of the robot that lead to a violation of the ground contact constraints, e.g. driving the legs into the ground and disrupting the body stability, or causing the feet to slip. Researchers have proposed control laws to address this problem, either by avoiding over-constrained motions (e.g. [10]), by using impedance control (e.g. [11]), or by learning low-impedance force profiles specific to the tasks performed (e.g. [12]). Although these methods have been shown to work under the right conditions, they rely on either low-impedance actuators or motors with high-impedance gear transmissions with high-fidelity output sensing, and end up being imperfectly implemented due to shortcomings in either/both. Alternatively, over-constraint has been addressed by adding additional passive degrees of freedom to the robot but this introduces kinematic redundancy to the design, increasing weight and power requirements.

In this paper, we present a systematic strategy for the design synthesis of kinetostatic walking robots that avoid kinematic redundancy in the stance phase while using the smallest number of actuators necessary for locomotion with full 6-DOF body mobility. We utilize kinematic mobility analysis tools traditionally used in parallel mechanisms research [13], [14] while building off of the work presented in [15], and also apply underactuated robotics design tools to effectively distribute actuation constraints, avoiding both over-constraint and undesired free motion. We call these systems *non-redundant walking robots.* They are the first systems designed to minimize kinematic and actuator redundancy while maintaining some minimal stance mobility. Such an approach would reduce the mechatronic and control complexity of the robots, thereby reducing their power consumption, weight, and cost, while at the same time achieving stable locomotion with full or partial postural control over the robot's body, allowing for effective traversal of rough terrain. Note that while full 6-DOF body mobility is not necessary for all locomotion tasks, we limit our scope to full-mobility robots for tractability and length and leave the design of lower-DOF robots for future work.

The rest of this paper is organized as follows. Section II presents a classification of legged robots developed over the last three decades based on controllable DOF's and stance actuators. Section III discusses a systematic analysis of legged robots' body mobility and a simple method for synthesizing robot topologies and leg designs. Section IV describes why fully-actuated legs result in over-constraint and presents strategies for reducing the number of actuators needed to control a robot in stance. A number of potential designs of exactly-constrained walking robots along with a proof-of-concept prototype are presented in Section V. Finally, we conclude in Section VI with a discussion of the proposed strategy and prospects for future research.

## II. LEGGED ROBOT CLASSIFICATION

Fig. 1 shows a scatter-plot categorizing legged robots developed over the last three decades based on the number of actuators used during the stance phase and the number of controllable body degrees of freedom (DOFs). The data was derived from papers describing 31 different legged robots from 1977-2013 [3], [5]–[8], [16]–[41]. The number of stance actuators was determined in several ways: if the robot was simply actuated, without any coupling between joints, the number of actuators per leg was simply multiplied by the number of legs in stance. If the robot had an actuator coupled to both the stance and swing legs, it was simply counted as one actuator. For all robots, particularly quadrupeds, even those whose gaits were primarily dynamic, we assumed the minimal stance set of three legs.

The number of controllable body DOF was determined by analyzing the robots using the Chebychev-Grübler–Kutzbach (CGK) criterion [42]. Those that included some compliant elements, e.g. RHex [5], were approximated as having additional passive joints before evaluating their mobility. Ground contact was treated as point contacts with friction [43], which are kinematically equivalent to a 3-DOF spherical joint and have been used to approximate similar contact cases in manipulation applications [44]. While varying the foot geometry or using a fixed ground constraint with a passive ankle joint can be used for different contact constraints, this assumption is maintained for the remainder of the paper. Robots that were technically over-constrained were qualitatively analyzed using published results to see how many controllable body DOFs existed in practice (e.g. propulsion and turning).

Fig. 1 shows that most existing robots fall into one of two categories. There are a large number of robots that are designed for dynamic locomotion with highly-articulated, full-mobility legs that leave them over-actuated from a static stance mobility perspective. At the same time, there are a number of lower-mobility robots that are exactly-actuated, or close to exactly-actuated, but lack the ability to independently locate and orient their bodies in space while in stance. This loss of controllability does not prevent these robots from serving as effective mobile platforms. However, for posture-sensitive applications or high-slope environments,

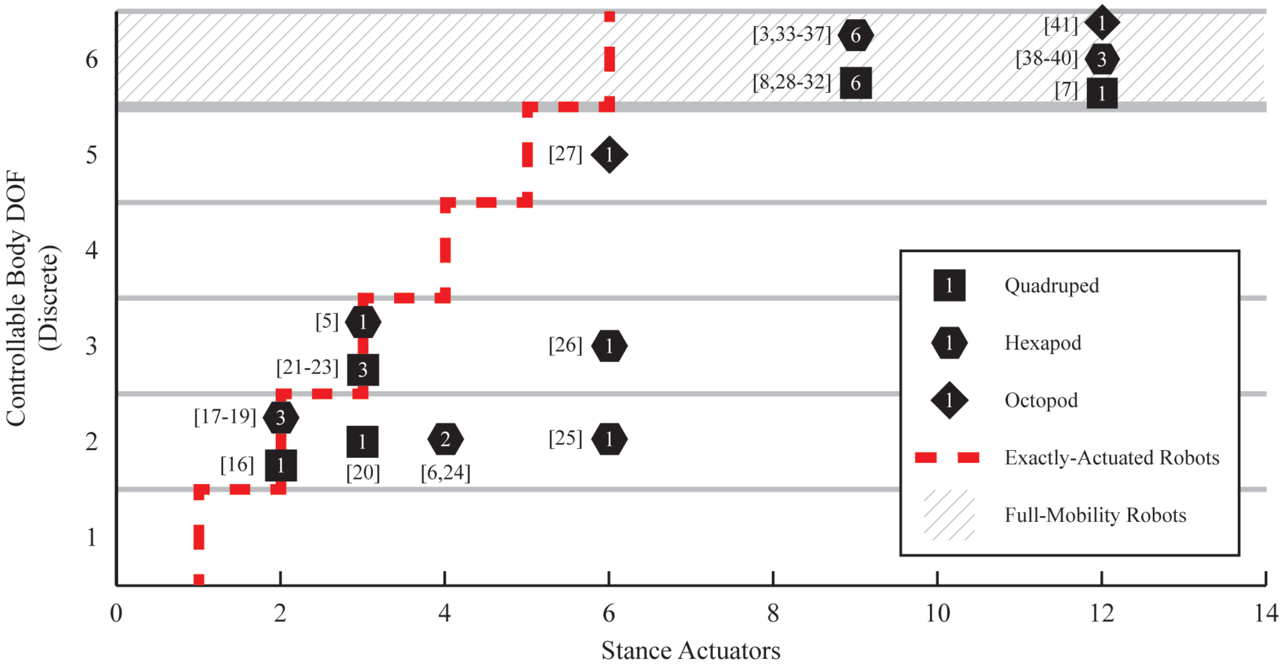

**FIGURE 1.** **Classification of legged robots by the number of stance actuators and the number of controllable body degrees of freedom. The vertical axis is discrete (i.e. 1, 2, 3, …, 6). The numbers inside of the markers represents the number of similar robots, whereas the shapes represents the number of legs the robots have (square: four, hexagon: six, diamond: eight). The dashed red line running up the left side represents exactly-actuated robots, or robots with as many actuators as controllable body DOF. Robots can be seen to fall generally into two categories: simpler robots with partial body mobility in stance, or more complex robots with full body mobility in stance.**

higher mobility may be necessary to achieve the required performance.

As seen at the top of Fig. 1, the robots that fully control their posture utilize redundant actuation. This approach provides several benefits – fast dynamic gaits, control over the ground contact forces [45], and robustness against component failure [46]. More generally, redundant actuation can also provide parallel mechanisms with singularity avoidance, controllable stiffness, and increased workspaces [47]. At the same time, redundancy results in more expensive and heavier robots with larger power requirements, resulting in shorter untethered operation and/or reduced carrying capacity. Additionally, the more complex the mechanical design, the more potential points of failure there are. Finally, controlling a robot with actuator redundancy can be difficult as the actuator space and task space have different dimensions, and resolving this can be computationally expensive [47]. Removing redundancy could strike a balance between the dynamic performance of existing full-mobility designs and the simplicity of lower-mobility designs while maintaining some terrain adaptability.

To the best of our knowledge, there have been no full-mobility walking robots that are exactly-actuated. These robots would be cheaper, mechanically simpler, and would require simpler control strategies while still allowing for full control over the robot's posture. The design process of this new category of legged robots involves answering several questions: which generalized leg topologies and specific kinematic design(s) should be used? How should the joints be actuated to exactly constrain the robot? How should the constraints between legs be distributed to achieve full control in stance? The following sections present a systematic approach to addressing those and other design issues for non-redundant walking robots.

## III. TOPOLOGICAL SYNTHESIS

### *A. FINDING BODY MOBILITY*

In classical mechanism analysis, the mobility of a given kinematic structure can be determined using the CGK criterion, given as:

$$m_{CGK} = 6\,(N - j - 1) + \sum\nolimits_{i=1}^{j} f_i \qquad (1)$$

where $N$ is the number of rigid bodies (leg links + body + ground in the case of legged robots), $j$ is the number of joints, and $f_i$ is the number of DOFs of the $i^{\text{th}}$ joint. In general, the CGK criterion correctly predicts the mobility of architectures that can be described by just a list of the number of links and the type of joints between them [48], [49]; it gives the total number of independent parameters necessary to fully define the configuration of a robot. The degree of over-, under-, or exact constraint can then be determined by subtracting the total number of control inputs from the mobility. In order to properly use this criterion for our purposes, we assume that each leg of the robot consists only of a serial-link kinematic chain and that extra geometric constraints between legs are not necessary to describe the robot architecture.

Given a robot with $n$ legs, the $k^{\text{th}}$ of which has a total of $d_k$ links (and $d_k + 1$ joints, including ground contact), we can rewrite (1) as:

$$m_{CGK} = 6\,(1-n) + \sum\nolimits_{i=1}^{j} f_i \qquad (2)$$

If we define $f_k = \sum_{i=1}^{d_k} f_i$ where $f_k$ is the total DOFs of each leg excluding contact and $j_k$ is the number of joints on the $k^{\text{th}}$ leg, and defining $f_c$ is the number of freedoms provided by the contact constraint, our mobility expression becomes:

$$m_{CGK} = 6 + (f_c - 6)\,n + \sum\nolimits_{k=1}^{n} f_k \qquad (3)$$

This expression relates the body mobility in stance to the number of legs, the number of DOFs of each leg, and the contact assumption. Conversely, we can find the different combinations of leg freedoms that provide a desired body mobility given a specific number of legs. It does not describe the displacement of the platform relative to the ground, which can be fully evaluated using, for example, screw theory [50] once the kinematic design of the robot is known. However, from such analyses we know that any permissible motion of the body relative to the ground must be permitted by *all* of the legs. As such, if we desire $m$ DOFs of body mobility, each leg must have at least $m$ independent freedoms. This allows us to validate potential robot topologies to ensure that they are capable of providing a required body mobility.

Since we assume that our ground contact provides 3 DOFs per leg, we can include up to 3 one-degree-of-freedom joints per leg before introducing kinematic redundancy. Moreover, as this paper focuses on the design of full-mobility (i.e. 6 body DOFs) legged robots, only legs with 3 freedoms can be used. An unlimited number of full-mobility legs can be added to a robot without reducing its body mobility (but potentially changing its workspace). However, each additional leg makes constraint distribution more difficult and increases system complexity.

While all of the legs must have identical *mobility*, they do not have to be kinematically identical. The idea of designing a robot with multiple types of legs is notable; most designs to-date have had entirely symmetric topologies, i.e. all the legs have been identical. It may be possible to generate effective robot designs and gaits by specializing the roles of each leg. There is even support for this approach from the biological world – many spider species have different leg kinematics for the outer and middle legs and the two consequently play different roles in locomotion [51]. Additionally, the legs of cockroaches have different kinematics to improve their running performance– a concept explored in legged robots through systems such as Sprawl [6] and R-III [4].

### B. LEG DESIGN CHOICES

For a point contact with friction approximation, all of our legs must have three one-degree-of-freedom joints to allow for full body mobility. Following the selection of a set of leg freedoms, the kinematic structure of the legs themselves must be chosen. A set of potential 3-DOF designs is presented in Fig. 2. These leg designs utilize revolute and prismatic kinematic pairs with either parallel or orthogonal joint axes and represent all possible combinations of those pairs, independent of joint axis orientation. The two distal joints define the distance between the foot and hip within some plane and the proximal joint achieves motion normal to that plane. Tangentially, by locating the primary propulsive joints as close to body of the robot as possible we also reduce the inertial load of the leg during swing, but a more comprehensive dynamic analysis should be performed if necessary.

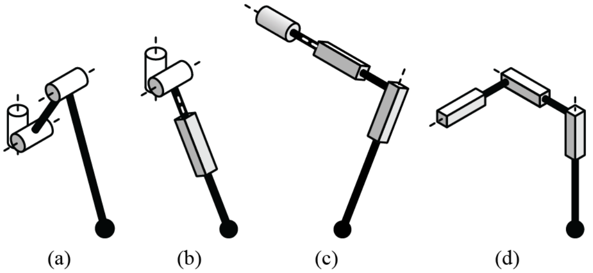

**FIGURE 2. Serial kinematic chains based on one-degree-of-freedom joints that may be used as legs in non-redundant walking robots of full mobility when ground contacts are modeled as point contacts with friction. Primary joint axes are indicated; the kinematic pairs used are R[evolute] and P[rismatic]: (a) RRR, (b) RRP, (c) RPP, and (d) PPP. Multiple configurations can be easily obtained by changing the angles between the joint axes.**

For the purposes of this paper we will take the four designs shown in Fig. 2 as examples, but our methodology is generalizable to other kinematic structures.

## IV. CONSTRAINT DISTRIBUTION

### A. STANCE OVER-CONSTRAINT AND ACTUATION REDUNDANCY

The simplest method of actuating a multi-legged robot is to actuate every joint in the legs. This allows for complete control over the motion of the foot in swing, but presents problems with the actuated mobility of the robot in stance. Adding an independent actuator to a robot exerts a single constraint on it, e.g. removes a single DOF from the system. An exactly-constrained robot would therefore have a mobility of zero once all of its actuators are considered, while an over-constrained robot would have negative mobility and an under-constrained robot positive mobility.

Returning to (3), if we have actuators at every joint and lock all of them, e.g. remove all of the leg freedoms from the mobility equation, we simply get:

$$m_{CGK} = 6 + (f_c - 6)\,n \qquad (4)$$

Substituting $f_c = 3$ for a spherical contact constraint yields $-3\,(n-2)$, meaning that a fully-actuated robot with more than two legs is always over-constrained. While full actuation may be desirable from a swing controllability and/or leg workspace perspective, it necessarily results in over-constraint in stance. It similarly means that fully-actuated stance legs lead to actuation redundancy.

Actuator redundancy has consequences beyond increased power requirements and complexity, particularly when we attempt to perform quasi-static force analyses on the system. For a robot whose configuration is known, we can express the relationship between joint torques and the wrench of the body as [52]:

$$-\boldsymbol{W} = \boldsymbol{J}^T \boldsymbol{\tau} \tag{5}$$

where $\boldsymbol{W}$ is the wrench acting on the body, $\boldsymbol{J}^T$ is a $6 \times j$ Jacobian matrix ($j$ being the number of joints in the robot) that can be constructed using the geometry of the robot, and $\boldsymbol{\tau}$ are the joint torques/forces (referred to simply as torques for simplicity). The joint torques are a combination of torques due to actuator inputs and torques due to compliance in the system, which we can express as:

$$\boldsymbol{\tau} = {}^a\boldsymbol{\tau} + {}^c\boldsymbol{\tau} \tag{6}$$

where ${}^a\boldsymbol{\tau}$ is a $j \times 1$ vector, each element representing the torque exerted about each joint by the actuators, and ${}^c\boldsymbol{\tau}$ is a $j \times 1$ vector, each element a function of the configuration / joint angles of the robot. Substituting (6) into (5) we get:

$$-\boldsymbol{W} = \boldsymbol{J}^{T\,a}\boldsymbol{\tau} + \boldsymbol{J}^{T\,c}\boldsymbol{\tau}. \tag{7}$$

If all joints are fully actuated, e.g. each element of ${}^a\boldsymbol{\tau}$ represents an independent and unknown actuator input, the system is underdetermined and there are multiple sets of joint actuator values that will balance a given wrench (and different corresponding ground reaction forces), whereas if a robot is exactly-actuated, e.g. some of the joints are unactuated so their elements in ${}^a\boldsymbol{\tau}$ are simply zero, the system is uniquely determined and only a single set of forces and input torques/forces will result in equilibrium. The presence of compliance at any of the joints does not change the order of the system, but merely adds constant offsets that modify the equilibrium solution. This will become relevant as we proceed.

Over-constraint can only be dealt with by increasing the number of unconstrained DOFs present across a robot's legs. The easiest way to increase the number of DOFs in the legs is to add additional passive joints; this however changes the kinematics of the robot, and, in the case of 3-DOF legs, introduce kinematic redundancy. It also has the consequence of increasing the complexity of the mechanical design of the robot and does nothing to address its actuation redundancy. Strategies to reduce the number of actuators used in controlling a robot's legs are required to *both* exactly-constrain and exactly-actuate the robot.

### B. CONSTRAINT DISTRIBUTION TECHNIQUES

The field of under-actuated robotics, particularly under-actuated manipulation has provided us with a wealth of design tools for reducing the number of actuators in a mechanism [53]. These tools have been used with great success, leading to robust grippers that can achieve many grasping and manipulation tasks while utilizing far fewer actuators than non-under-actuated alternatives.

There are several ways to actuate an underactuated kinematic chain. The most obvious method is to simply leave one or more joints passive but this prevents those joints from supporting any load. Another method is to couple two (or more) joints together adaptively, with fewer actuators than DOF, but with torque applied to all coupled joints. This has primarily been accomplished in one of two ways – either through an adaptive tendon coupling [9] or through linkages [54] – but can also be done via hydraulics or pneumatics [55]. Each adaptive coupling imposes a single constraint on the system; multiple adaptive constraints can be combined to control the robot, provided that they are independent.

There is an important distinction between *adaptive* and *rigid* joint couplings. For example, if two joints are connected by a gear train then the motion of one joint is entirely determined by the motion of the other. The key to adaptive joint couplings is that, while all joints are affected by the coupling input, they are not entirely constrained by it. The remaining freedoms are precisely what prevent the robot from being over-constrained in stance.

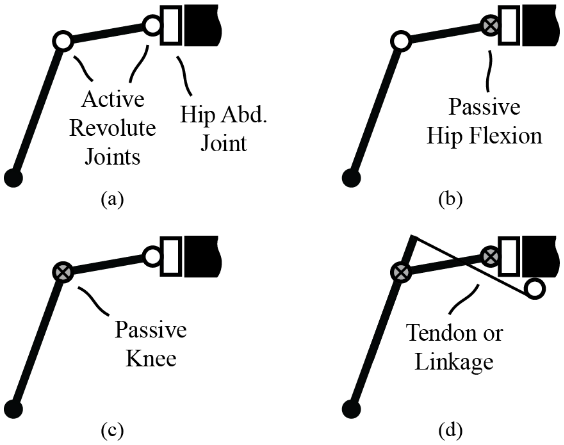

**FIGURE 3.** A number of options for constraint distribution in an RRR leg: (a) the fully-actuated leg, (b) the leg with a passive hip flexion joint, (c) a passive knee flexion joint, and (d) an adaptive coupling between the knee and hip. Crossed joints in (b), (c), and (d) are passive.

Fig. 3 illustrates an example of constraint distribution using the above methods. The fully-actuated mechanism, an RRR leg as in Fig. 2a, is shown in Fig. 3a. This is a common leg architecture, often identified as the Universal-Revolute-Spherical (URS) leg. If we simply want to remove an actuator and leave one joint passive, it makes sense to remove either the hip flexion joint (Fig. 3b) or the knee flexion joint (Fig. 3c). The hip abduction joint provides completely independent motion from the other two joints and is also closest to the body. Removing a constraint from those joints will likely reduce the controllable workspace of the foot but will not reduce the dimensionality of the space of controllable foot motions. Fig. 3d shows an example of adaptively coupling the hip and knee flexion joints; in this case the length of the tendon or angle of the link imposes a single constraint on the two passive joints. Even with that actuator locked the leg retains a single unconstrained DOF. The specific design determines the actual trajectory of the foot relative to the hip with the actuator locked, i.e. the location of the instant center of rotation of the leg. This will become particularly relevant in the following section.

One important consequence of introducing unconstrained DOF's to the legs is that they allow them to passively adapt to rough terrain. During swing, the actuation mechanism is under-constrained since we rely on stable contact with all feet for exact-constraint. This means that, depending on the scheme implemented, some or all of the legs will continue to lower until *all* of the legs make contact, while those that make contact first will impart minimal disturbance forces to the robot. This passive adaptability can minimize postural disturbances of the body from the swing legs regardless of variations in ground height and with no active sensing.

It is important to note that the introduction of unconstrained and adaptively-coupled joints introduce additional singular configurations where the robot gains some uncontrollable DOF. In certain singular or ill-conditioned configurations some constraints may become redundant, e.g. if two passive joints become aligned, resulting in a robot that either falls down or is susceptible to certain disturbances. It is possible to utilize Jacobian analysis [56] to find these configurations and avoid them through properly designed gaits.

### C. ELASTIC ELEMENTS

The introduction of unconstrained freedoms such as passive joints is not without consequence. While these freedoms can help improve stance performance or simplify control, they allow for uncontrollable motion of the legs in swing that will ultimately be determined by the energetics of the system. By adding elastic elements, e.g. springs, in parallel with our passive joints, we can impose some nominal behavior on the under-constrained swing legs. These elements do not impose hard kinematic constraints on the robot, but merely shape its energetics based on their stiffnesses.

Elastic elements, through their energetic shaping, can impact the behavior of the robot in stance as well. Depending on the relative stiffnesses of the elements involved, springs can help reduce the effort required from the actuators to support the weight of the robot. This will increase the effort required to swing the legs, but may allow for smaller actuators to be used. Elastic elements can also be used to mitigate the effects of a loss of the ground contact constraint. Properly tuned, springs can ensure that any slipping of the feet results in more stable stance configurations.

The stability or robustness of the ground contacts for most robots is dependent on the frictional interaction between the foot and the ground, which is almost entirely defined by the relationship between the direction of the contact force relative to the surface normal and the material properties of all the elements involved. In over-actuated legged robots, it may be possible to independently control the direction of the reaction forces to ensure stable contact. In exactly-actuated robots, the equilibrium reaction forces are uniquely determined by the joint torques and body wrench, as described above. Additionally, the presence of unactuated freedoms within the system, even those that have some compliance, means that any external forces must pass through the relevant instant centers of rotation in the equilibrium configuration.

For legs with a single unactuated freedom, it is fairly straightforward to find the instant center of rotation of the foot relative to the body. Looking at the examples in Fig. 3, the center of rotation in the cases where an actuator is removed from a single joint (Fig. 3b and Fig. 3c) is just at the center of those joints. In the cases where an adaptive coupling is used (Fig. 3d), the instant center for each configuration can be found using classic mechanism theory, but again, the reaction forces at the foot must pass through that point. The only difference in that case is that the location of the instant center will not be fixed on the robot but will be configuration-dependent [53], [57]. Finally, as noted above, the presence of compliance in the system does not change the fact that the equilibrium ground reaction force is unique given a robot configuration and body wrench; if we assume that the spring torques in (6) are small relative to the actuator torques, we can estimate the reaction force direction based on the instant center of the leg.

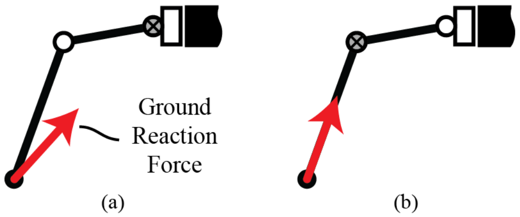

**FIGURE 4. The equilibrium ground reaction force directions for (a) the passive hip design and (b) the passive knee design. Note that the angle of the force in (b) is simply the angle of the distal link.**

If we compare the cases of the passive knee and the passive hip (Fig. 4), it is clear that while the angle of the reaction force in both cases is dependent on the vertical distance between the foot and the hip (i.e. the ground clearance), it is also strongly related to the lateral distance between the foot and hip in the passive hip case (Fig. 4a). This lateral distance is essentially the classical static stance stability margin [58], so there is a trade-off between contact robustness (e.g. a more vertical reaction force) and stance stability. In the case of the passive knee (Fig. 4b), the reaction force must be aligned with the distal link. While the distal link angle cannot be controlled directly, it is possible to use elastic elements to try and ensure contact robustness.

Elastic elements do not only have to take the form of simple parallel springs. The use of elastic linkages or cross-coupled springs, e.g. springs whose deflection is defined by multiple joints, allows for more complex trajectory tuning.

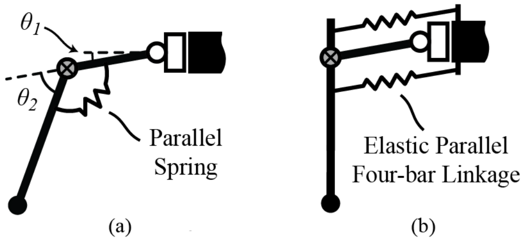

**FIGURE 5.** **Two options for the addition of elastic elements to a leg with uncontrolled DOF: (a) simple parallel springs and (b) elastic linkages / cross-coupled springs. Note the joint angle definitions. The foot in (a) will follow a circular path centered around the hip joint in free swing, whereas the foot in (b) will nominally follow the trajectory of the knee joint in swing since the distal link will be kept vertical relative to the body. In both cases the leg retains its unconstrained DOF.**

Fig. 5 illustrates two basic options for adding elastic elements to a leg with passive DOF. Starting with the leg in Fig. 3c (passive knee joint), we can simply add a spring parallel to the joint (Fig. 5a). This will set the nominal angle of the distal leg link relative to the proximal link, but independently of any other joint angles. This energy term takes the form $\frac{1}{2}k\Delta\theta_2^2$, where $k$ is the spring stiffness and $\Delta\theta_2$ is the difference between the actual and nominal distal joint angles, causing the system to incur an energetic cost for deviations from the nominal knee joint value.

Alternatively, if we wanted to specify a nominal distal link angle relative to the robot *body*, not the proximal link, we can add more complex compliance through an elastic parallel four-bar linkage (Fig. 5b). This will ensure that the angle of the distal link is nominally aligned with the body $z$-axis and that the foot trajectory in the plane of the hip/knee joints during swing will be more linear than the leg in Fig. 5a. In this case, the energy term takes the form $\frac{1}{2}k\Delta\theta_{12}^2$, where $\Delta\theta_{12}$ is the difference between the actual sum of the proximal and distal joint angles (e.g. $\theta_1 + \theta_2$) and the nominal sum (e.g. $\pi/2$). In this case, the system will incur energetic costs for deviations from the nominal knee joint value *as a function of* the hip joint angle. This is only one option for such cross-coupling / elastic linkages, but it illustrates that such compliance allows for nearly arbitrary tuning of the passive leg behavior.

Elastic elements have other effects on system behavior, e.g. the system's dynamic characteristics and resonant behavior. These aspects are less important for quasi-static walking but can play a role in robots designed for both statically- and dynamically-stable gaits, and would require further analysis.

### *D. BETWEEN-LEG COUPLING*

For some robotic systems, it is necessary to distribute constraints between legs to maintain the ability to both achieve full control in stance as well as undergo meaningful swing-phase leg motion without over-constraint. We noted earlier that we need $|-3\,(n-2)|$ unactuated DOF for a robot with $n$ legs for exact constraint; for three legs, the minimum number for static walking, this equals one DOF per leg. However, for larger stance sets the distribution of actuators cannot be even when the legs are all actuated independently.

Under-actuated mechanisms have been using between-finger adaptive couplings for quite some time to great success [9]. The idea of coupling the legs of walking or running robots is also not new; it has been used successfully in robots such as Whegs and iSprawl [18], [24]. The specific coupling between actuators and joints will determine the actuator to task space mapping used for control and can also be used to model the behavior of the robot. So long as the number of independent constraints is equal to the robot mobility, any set of constraints will do.

As seen in (7), the number of independent actuators used to control a legged robot in stance determines the order of the system of static force balance equations. Given a robot where the number of joints is greater than the number of independent body DOF, the simplest way to make the statics system uniquely determined is to assume that some of them are passive (with or without springs), i.e. ${}^a\tau_i = 0$. As mentioned previously, there are benefits to having some degree of control over each joint. If some joints are adaptively coupled together such that the torque at some joint $i$ is a linear combination of some set of $m$ independent actuator constraints, e.g. ${}^a\boldsymbol{\tau} = \boldsymbol{J}_T\,{}^a\boldsymbol{T}$ where $\boldsymbol{J}_T$ is the $j \times m$ linear mapping between joint torques and actuator inputs ${}^a\boldsymbol{T}$. We can then express (7) as:

$$-\boldsymbol{W} = \boldsymbol{J}_a^T\,{}^a\boldsymbol{T} + \boldsymbol{J}^T\,{}^c\boldsymbol{\tau} \tag{8}$$

where $\boldsymbol{J}_a^T$ is a $6 \times m$ matrix obtained by multiplying the Jacobian with the actuator mapping. For fully-mobile robots, if there are six independent actuator inputs, (8) represents a square linear system for any given configuration, confirming that any given set of actuator inputs results in a unique body wrench (as applied by the ground reaction forces, for example). This can also be extended to robots with less than 6-DOF body mobility. If a robot is under-actuated (i.e. fewer actuators than controllable body DOFs), (8) is technically overdetermined but still only allows for at most a single solution of joint torques.

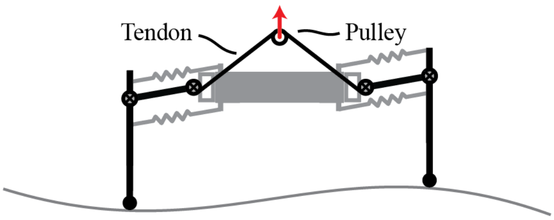

**FIGURE 6.** **Schematic representation of between-leg adaptive coupling. Note the tendon rigidly coupled to both hip flexion joints and the pulley through which the sum of the joint excursions is controlled.**

Fig. 6 shows a schematic representation of what between-leg tendon coupling might look like for a robot with legs from Fig. 5b. A single tendon is rigidly coupled to the two hip flexion joints and is routed over a free-spinning pulley. That pulley is moved to change the overall tendon excursion between the two joints, but they are free to vary so long as the

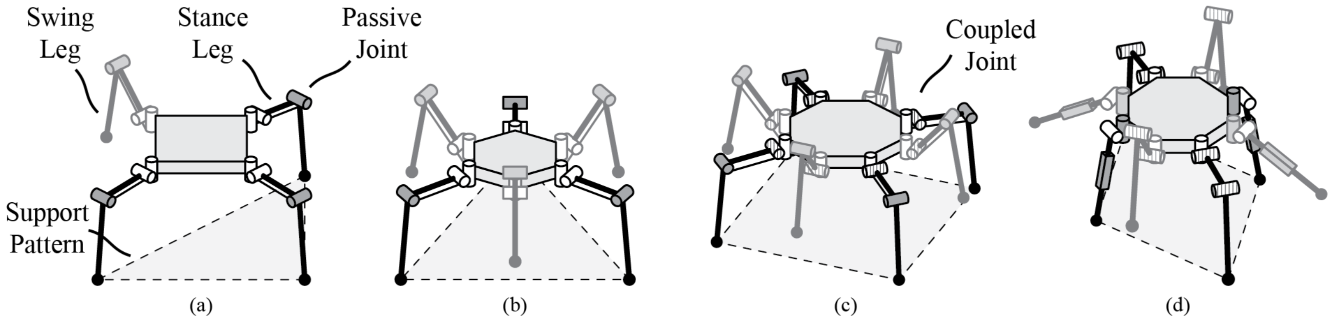

**FIGURE 7. Examples of non-redundant walking robot designs: (a) 4-RRR quadruped and (b) 6-RRR hexapod (the two hip joints of each leg are actuated and the knee joint is passive, with an elastic coupling between the hip and distal link to ensure a nominally vertical distal link), (c) 8-RRR octopod (the hip flexion joints of opposite legs are adaptively coupled to avoid over-constraint), and (d) 4-RRR + 4-RRP octopod (the hip and knee flexion joints of the RRR legs are adaptively coupled within each leg to avoid over-constraint). White joints are active, gray joints are passive, and striped joints are adaptively coupled.**

sum of the joint excursions matches the tendon constraint, e.g. $\theta_1 r_1 + \theta_2 r_2 = C$.

Another consideration, particularly with robots that use two identical stance sets and alternate between them, is that it may be possible to further reduce the number of actuators used in a robot by rigidly coupling the control inputs for each stance set and adding some phase offset. This essentially sacrifices some gait control for additional simplicity.

## V. EXAMPLES

### *A. 4/6-RRR (3-RRR STANCE PLATFORM)*

The smallest valid stance set for a kinetostatic walking robot requires three legs. Utilizing the legs shown in Fig. 5b, we can easily construct a full-mobility legged robot with no actuation redundancy, as described above. In terms of the complete system, this can be minimally built as a quadruped (Fig. 7a), putting only one leg in swing at a time, or as a hexapod (Fig. 7b), allowing for more effective gaits (e.g. alternating tripod) where the robot switches between two independent stance sets in order to move. This increases the overall system complexity and weight/power requirements, but allows for simpler and faster gaits, and, as described in the previous section, could allow for one set of actuators to be shared by both stance sets.

### *B. 8-RRR (4-RRR STANCE PLATFORM)*

If a larger support polygon is desired, the stance set could be expanded to four legs. This does make the constraint distribution problem trickier since there are 12 joints across four legs with only 6 independent constraints driving them. Using legs with a single passive joint, as in Fig. 5b, still leaves 8 joints that must be driven; one option is to drive all of the hip abduction joints and adaptively couple opposite pairs of hip flexion joints to raise and lower the legs. An illustration of this design is shown in Fig. 7c.

### *C. 4-RRR + 4-RRP (2-RRR + 2-RRP STANCE PLATFORM)*

A combination of different leg topologies could allow for an asymmetric stance workspace, as illustrated in Fig. 7d. The stance set is composed of two RRR legs at the front and back of the robot and two RRP legs in the middle, forming a tetrapod stance structure. The legs are assigned different ‘‘roles’’: the RRR legs primarily generate forward propulsion by pulling and pushing the body with the front and back legs, respectively. The RRP legs primarily provide lateral stability but still allow for the large forward motions generated by the other legs. The leg and body geometry can be varied to change the size and shape of the body workspace.

In terms of actuation, one could utilize an adaptive coupling between the hip and knee flexion joints of the front and back legs (see Fig. 3d), each of which would be independently driven to move the robot forward and control its pitch, along with the hip abduction for each leg to steer (four constraints). For the side legs, one could leave the hip abduction passive and lightly sprung as to not impede the robot’s motion, independently drive the hip flexion joints to raise and lower the legs, and leave the prismatic joints passive with parallel springs to ensure some nominal leg length. This design indicates the feasibility of using leg specialization in exactly-constrained walking robots.

### *D. PROOF-OF-CONCEPT PROTOTYPE*

In order to verify the feasibility of exactly constrained robot walking, a prototype robot was constructed based on the principles outlined above as a proof-of-concept, a rendering of which is shown in Fig. 8 [15] (referred to in that manuscript as a 4-URS robot). As mentioned above, the simplest robot with a 3-legged stance set would have a total of 4 legs, with one leg being transferred per gait step to shift the weight of the robot to a new stance set. A four-legged walker is not particularly efficient at walking, due primarily to the awkward gait that results from having only one ‘‘swing’’ leg per cycle and the need for alternating this leg, but it is capable of demonstrating the stability and mobility of a tripod stance, and the stability of support transitions.

The prototype was built with four RRR legs constructed using 3D-printed parts (Stratasys ABS*plus*). The universal hip joints of each leg were arranged in a square pattern 30 mm

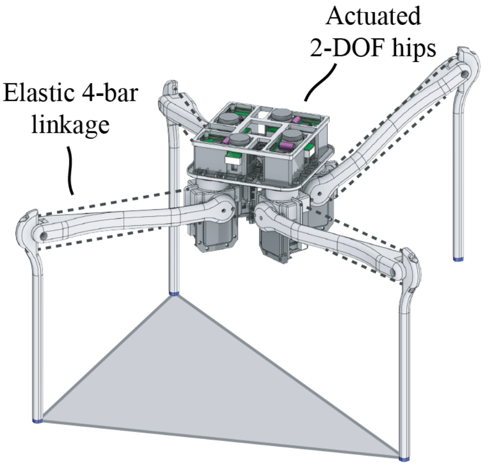

**FIGURE 8.** **Rendering of the prototype 4-RRR exactly-constrained quadruped whose topology is shown in Fig. 7a. Preloaded extension springs were used to create the elastic 4-bar linkage as indicated.**

on a side, with the yaw axis of each hip oriented vertically. Both hip joints were actuated with Robotis Dynamixel RX-28 servo motors. The knee joints are passive, and were connected to the body with a four-bar parallel elastic linkage made of preloaded extension springs, as shown in Fig. 7a, to ensure that the robot's legs stay parallel to the z axis of the robot body while in swing phase. The proximal leg links are 150 mm long, and the distal links are 160 mm long. At the end of each leg, a molded rubber foot improves the frictional contact with the ground.

Because it is easy to maneuver the walker into a configuration where the center-of-mass projection falls outside the support pattern and because only one leg of the walker could be repositioned at a time, the choice of gait for this robot was highly constrained. Figure 9 shows the tripod gait, with a step length of roughly 1/2 body length, that was used to control the robot. Three legs were held widely apart, and one of the two rear legs was used during each step to support the body while the front legs were repositioned. Body motion was achieved by using the tripod as a parallel platform to reposition the body in between stance changes. A robot with more legs would allow for more flexibility when synthesizing gaits, both in terms of simultaneously repositioning several legs as well as avoiding unstable configurations. The prototype was capable of repositioning the body with three translational and three rotational degrees of freedom. The distal leg links remained mostly vertical during these motions, keeping the ground reaction forces from leaving the friction cone of the contact with the floor. In experiments, little slippage was observed.

## VI. DISCUSSION AND FUTURE WORK

In this paper, we have outlined a design strategy for the development of fully-mobile kinetostatic walking robots with no actuation or kinematic redundancy that are, as a result, exactly-constrained in stance. A survey of past and current legged robots showed a lack of exactly-actuated, fully-mobile robots; such designs would be lighter, cheaper, and have lower power requirements while maintaining the ability to control the posture of the robot's body during locomotion. Additionally, such robots can achieve passive adaptability during swing, reconfiguring to find stable contact on unknown terrain profiles without active sensing or destabilizing contact forces. A set of robot topologies was generated using basic mobility theory and examples of basic leg kinematics were presented. We then discussed a number of strategies for distributing actuator effort across a robot's joints along with the importance of elastic elements in ensuring stability during the stance and swing phases. Finally, several full-mobility robot architectures were presented with non-redundant actuation schemes, and a working prototype was built to demonstrate an exactly-constrained platform with 6-DOF controllable mobility.

This new approach to legged robots has potential to achieve

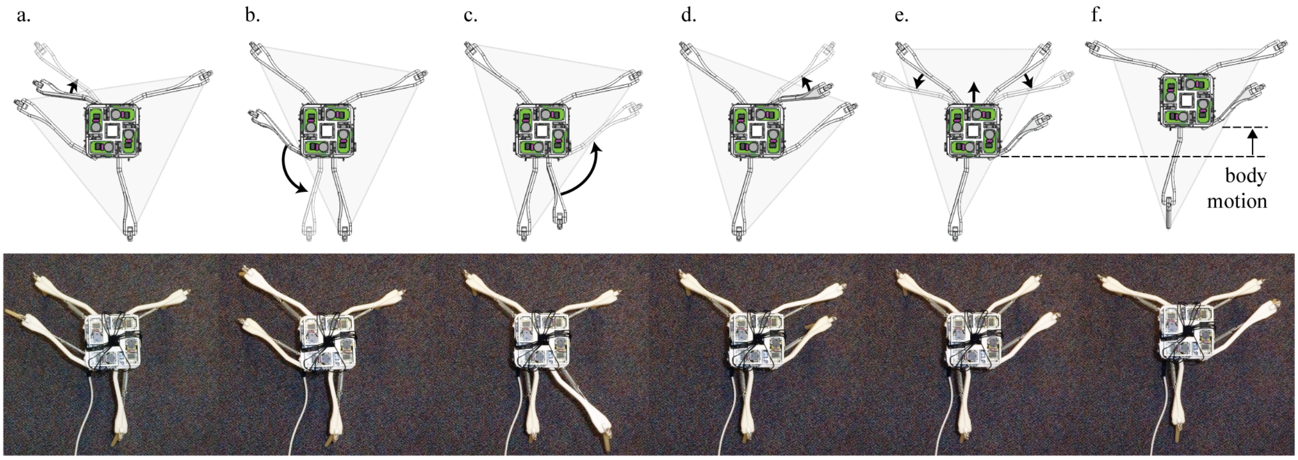

**FIGURE 9.** **Locomotive gait of prototype 4-RRR walker. The number of distinct motions was due to the fact that only one leg could be moved at a time as well as the fact that unstable configurations had to be avoided. The robot's configuration in (f) mirrors its configuration in (a).**

the posture control and ground clearance performance of highly-actuated dynamic running robots at much lower power and economic cost. By leveraging mechanical intelligence in the design and actuation of the legs and tuning the stance workspace of the body, we can design robots with kinetostatic walking gaits that can traverse rough terrain without requiring complex redundant control schemes and the ability to exert arbitrary ground reaction forces, often achieved through actuation redundancy. While these gaits may be slower than the dynamic running gaits of existing robots, we believe that simpler designs would be more suitable in situations where cost and weight are critical.

Looking forward, we would like to extend this strategy using mechanism synthesis tools to select appropriate leg architectures for specific body mobility, especially for applications where less than 6-DOF mobility is desired. This would also make the design approach more useful for applications other than legged robots, for example in the design of novel manipulators or parallel mechanisms. Using redundancy as a design constraint could yield simpler and cheaper mechanisms that would still be capable of controlling specific desired DOFs. We would also like to perform a more systematic analysis of the effect of elastic elements on robot stability and formulate a methodology for tuning those elements to achieve specific stance behavior (e.g. reduce actuator effort, increase stance stability when the feet slip). We also plan on systematically analyzing the design of actuation schemes, allowing for optimization of joint couplings for specific gait motions.

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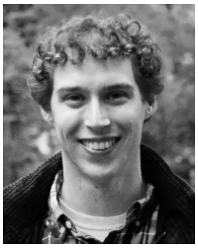

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