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# Event-Triggered Consensus Seeking of Heterogeneous First-Order Agents With Input Delay

MEI-MEI DUAN, CHENG-LIN LIU, AND FEI LIU

Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Institute of Automation, Jiangnan University, Wuxi 214122, China

Corresponding author: Cheng-Lin Liu (liucl@jiangnan.edu.cn)

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**ABSTRACT** Event-triggered consensus problem is studied for a class of heterogeneous first-order multi-agent system, which contains multiple single integrators with different nominal velocities, and an adaptive consensus algorithm in the leader-following structure is adopted. Under the event-triggered mechanism, the agents' inputs are only updated at discrete times determined by a centralized event-triggered function. By constructing proper Lyapunov function, sufficient consensus conditions are gained for the asymptotic consensus convergence. Furthermore, the consensus problem subject to identical input delay is considered, and consensus conditions in the form of linear matrix inequalities are obtained. Numerical simulations show the effectiveness of the event-triggered consensus control strategy.

**INDEX TERMS** Event-triggered consensus control, input delay, heterogeneous multi-agent system, leader-following control.

## I. INTRODUCTION

Recently, coordination control of spatially distributed multiple autonomous agents has been one of the key problems in the control theory field due to its various engineering applications [1]–[5], such as, formation control of unmanned aerial or ground vehicles [1], [2], consensus seeking control [3], [4], cooperative estimation of sensor networks [5], and so on.

Consensus problem is the most popular issue of coordination control of multi-agent systems, and has been extensively analyzed and synthesized in different research fields, e.g. mathematics, biology, and computer virtual simulation. Consensus problem means that several autonomous agents reach an agreement value on the state variable of interest [6]. As far as we know, the majority of research works on consensus problem were aimed at the homogeneous multi-agent systems composed of agents with identical dynamics, e.g. first-order agents [7]–[9], second-order agents [10]–[13] and high-order agents [14]. Meanwhile, heterogeneous multi-agent system composed of agents with distinct dynamics, has also attracted more and more researchers' interests in the past decade. For simplicity, mixed-order multi-agent systems composed of several type agents, e.g. first-order and second-order agents [15], first-order and second-order agents

together with nonlinear Euler-Lagrange agents [16], and so on, has been thoroughly studied, and consensus algorithms have been designed and analyzed based on different methods. To analyze the consensus seeking problem of general heterogeneous linear multi-agent systems, Lee and Spong [17] and Kim *et al.* [18] used spectral radius theorem and output regulation theory, respectively, to obtain consensus conditions for the agents with and without time delays. Liu and Liu [19] considered a multi-agent system of heterogeneous linear first-order agents with distinct nominal velocities, and designed new consensus algorithms by adding an adaptive variable into the usual consensus algorithm. Based on matrix theory and frequency-domain analysis, consensus conditions were provided for the multi-agent system without and with identical input delay, respectively, so as to guarantee the consensus convergence [19].

In real engineering application, the data for transmission and control input are usually time-triggered and sampled periodically and aperiodically [20], and the cost of information exchange is very high. In multi-agent network, the communication cost becomes critical, so event-triggered mechanism, where agents' states are sampled just at some triggered times determined by event functions, has been extensively proposed for traditional control [21] and coordination

control of multi-agent systems [22]–[24]. For first-order multi-agent systems, Dimarogonas and Johansson [22], [23] and Dimarogonas *et al.* [24] used centralized and decentralized event-triggered strategies, respectively, to deal with the consensus seeking problem. According to matrix theory and Lyapunov stability criterion, Dimarogonas and Johansson [22], [23] and Dimarogonas *et al.* [24] proved the consensus convergence conditions, and event-triggered functions and event intervals are obtained. For second-order multi-agent systems, the decentralized event-triggered consensus protocols were proposed to solve the dynamical consensus problem under fixed topology, and event-triggering function, consensus conditions and event-time intervals were presented by constructing proper Lyapunov functions [25]–[27]. For the second-order multi-agent systems without and with nonlinear dynamics, Li *et al.* [28] adopted a dynamical leader-following consensus protocol under the decentralized event-triggered mechanism, and obtained the consensus convergence conditions under fixed topology and switching topologies respectively. Besides, Wei and Xiao [29] investigated the synchronization problem of harmonic oscillators, which was regarded as special second-order multi-agent systems, and used the centralized and decentralized event-triggered control protocols respectively. Using Lyapunov stability criteria and non-smooth analysis, consensus conditions, event functions and event-time intervals were provided [29]. For general linear multi-agent systems under fixed and switching topologies, Zhang *et al.* [30] proposed asynchronous event-triggered consensus algorithms, and presented the consensus convergence criteria by using variable substitution method. In the light of event-triggered strategy, besides, Ding *et al.* [31] investigated the consensus problem for a class of discrete-time linear multi-agent systems with stochastic noises. By defining the consensus problem in probability, consensus seeking was proved based on the so-called input-to-state stability in probability, and control parameters and triggering threshold were gained in the form of LMIs [31].

In reality, information exchange among agents induces non-negligible time delay in the consensus control protocol [32]–[35]. In [32], event-triggered control was introduced to solve the consensus problem of multi-agent systems with time-varying communication delays, and two different decentralized event conditions and sufficient consensus conditions were obtained by using corresponding Lyapunov functions. Event-triggered consensus problem was also investigated for the multi-agent system with general directed topology and time delay [33], and an event-triggered control strategy was proposed together with a novel event-triggered function, where a time-varying offset was introduced so as to avoid the potential Zeno behavior and relax the conservative theoretical threshold. Besides, a sufficient condition was presented to guarantee the consensus convergence in [33]. Zhu and Jiang [34] considered an event-triggered leader-following consensus control for multi-agent systems with input delay, and necessary and sufficient

conditions were presented by using time-domain analysis method.

Referring to current works on event-triggered consensus problem, the investigation objects are mainly homogeneous multi-agent systems. Then, this paper considers the heterogeneous first-order multi-agent systems, of which each agent has different nominal velocities. To deal with the consensus problem, we adopt a leader-following adaptive consensus algorithm, and propose a centralized event-triggered mechanism. Based on Lyapunov stability criterion, sufficient conditions are presented for the agents converging to an asymptotic consensus. Moreover, we study the multi-agent systems with input delay, and sufficient conditions and event-triggered function are gained to guarantee the consensus convergence based on the Lyapunov-Krasovskii functional. Main contributions are summarized as follows.

- Event-triggered consensus strategy is applied into a heterogeneous first-order multi-agent systems.
- Consensus conditions are obtained for the heterogeneous multi-agent systems without and with identical input delay.

Throughout this paper,  $I_n$  denotes an  $n \times n$  identity matrix, and  $o_n$  is as a zero matrix. Denote  $1_n$  as an all-one vector with appropriate dimensions, and  $A^T$  is regarded as the transpose of matrix  $A$ .  $\|A\|$  and  $\lambda_{\min}(A)$  represent the Euclidean mode and the minimum eigenvalue of matrix  $A$ , respectively.

## II. PROBLEM FORMULATION

### A. GRAPH DESCRIPTION

A multi-agent network of  $n$  agents and a leader is described by an undirected interconnection graph  $\hat{G} = G \cup \{0\}$ . In  $\hat{G}$ , the leader corresponds to 0 and  $G = (V, E)$  denotes the topology of other  $n$  followers. In  $G$ ,  $V = \{1, 2, \dots, n\}$  is the set of agents and  $E \subseteq V \times V$  is the set of edges. In  $V$ ,  $i$  denotes the  $i$ th agent and the edge  $(i, j) \in E$  implies that agent  $i$  can reach the information of agent  $j$ .  $A = (a_{ij}) \in \mathbb{R}^{n \times n}$  is the weighted adjacency matrix of  $n$  agents where  $a_{ij} > 0$ , if  $(i, j) \in E$  and  $a_{ij} = 0$ , otherwise. To describe the leader-follower adjacency relationship, we define a diagonal matrix  $B = \text{diag}\{b_1, b_2, \dots, b_n\}$  where  $b_i > 0$  if the leader 0 is connected to the agent  $i$  and  $b_i = 0$ , otherwise. The set of agent  $i$ 's neighbors is defined as  $N_i = \{j \in V : (j, i) \in E\}$ .

In addition, the Laplacian matrix  $L$  is defined as  $L = D - A$ , where  $D = \text{diag}\{d_1, d_2, \dots, d_n\}$  with  $d_i = \sum_{j \in N_i} a_{ij}$ ,  $i \in V$ .  $G$  is called connected, if there exists a path between any two agents in  $G$ . From the definition of Laplacian matrix,  $L$  of an undirected graph is symmetric and positive semi-definite, and has a single zero eigenvalue with the corresponding eigenvector  $1_n$  [39]. In this paper, the eigenvalues of  $L$  are expressed as  $0 = \lambda_1(L) < \lambda_2(L) \leq \dots \leq \lambda_n(L)$ .

### B. SYSTEM MODEL

In this paper, we consider the following heterogeneous first-order agents

$$\dot{x}_i(t) = v_i + u_i(t), \quad i = 1, 2, \dots, n, \quad (1)$$

where  $x_i \in R$  and  $u_i \in R$  are the state and input of agent  $i$ , respectively, and  $w_i \in R$  denotes the agent  $i$ 's nominal velocity similar to that in [19]. To solve the consensus problem of agents (1), we take into account the leader-following coordination control structure, and the dynamical leader's model is described as

$$\dot{x}_0(t) = w_0, \quad (2)$$

where  $x_0(t) \in R$  and  $w_0 \in R$  are the leader's state and velocity. The first-order agents (1) converge to a *Leader-following Consensus* asymptotically, if the agents' states satisfy

$$\lim_{t \rightarrow \infty} (x_i(t) - x_0(t)) = 0, \quad i \in V. \quad (3)$$

### C. USEFUL LEMMAS

Two lemmas as follows will play important roles in the main results.

*Lemma 1:* [36] The symmetric linear matrix inequality

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0$$

is equivalent to either of the following conditions:

- 1)  $Q(x) > 0, R(x) - S^T(x)Q^{-1}(x)S(x) > 0;$
- 2)  $R(x) > 0, Q(x) - S(x)R^{-1}(x)S^T(x) > 0.$

*Lemma 2:* [27] For arbitrary  $x \in R, y \in R$  and  $\varepsilon > 0$ ,

$$xy \leq \frac{\varepsilon}{2}x^2 + \frac{1}{2\varepsilon}y^2$$

holds.

### III. EVENT-TRIGGERED CONSENSUS CONTROL

In this section, we adopt a leader-following adaptive consensus algorithm and propose an event-triggered mechanism to deal with the consensus problem of agents (1).

#### A. ADAPTIVE CONSENSUS ALGORITHM

The consensus algorithm of first-order agents (1) is usually designed as

$$u_i(t) = \kappa \left( \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t)) + b_i(x_0(t) - x_i(t)) \right), \quad i \in V, \quad (4)$$

where  $\kappa > 0$ , and  $a_{ij} > 0, j \in N_i$  and  $b_i > 0$ .

The closed-loop system of agents (1) with (4) is formulated as

$$\dot{x}_i(t) = w_i + \kappa \left( \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t)) + b_i(x_0(t) - x_i(t)) \right), \quad i \in V. \quad (5)$$

*Remark 1:* Specially, the system (5) can be considered as the linearized model of Kuramoto oscillators [37]. Different from the current works on Kuramoto oscillators, we considered the consensus seeking problem of  $x_i(t)$  but not that of the velocity  $\dot{x}_i(t)$ . Evidently, the followers in system (5) cannot reach the asymptotic leader-following consensus for the different velocities. For the heterogeneous first-order agents (1),

our former work [19] has proposed new adaptive consensus algorithms, which made the original heterogeneous multi-agent system become a homogeneous multi-agent system asymptotically.

We adopt an adaptive consensus algorithm similar to that in [19] as follows.

$$\begin{aligned} u_i(t) &= z_i(t) + \kappa \left( \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t)) \right. \\ &\quad \left. + b_i(x_0(t) - x_i(t)) \right), \\ \dot{z}_i(t) &= \gamma \left( \sum_{j \in N_i} a_{ij}(w_j + z_j(t) - w_i - z_i(t)) \right. \\ &\quad \left. + b_i(w_0 - w_i - z_i(t)) \right), \end{aligned} \quad (6)$$

where  $\gamma > 0$ , and  $z_i(t)$  is an added variable. In (6), the agents' velocities and the leader's velocity are supposed to be known and transmitted to its neighbors together with the adaptive variables.

With the algorithm (6), therefore, the agents (1) become

$$\begin{aligned} \dot{x}_i(t) &= w_i + z_i(t) + \kappa \left( \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t)) \right. \\ &\quad \left. + b_i(x_0(t) - x_i(t)) \right), \\ \dot{z}_i(t) &= \gamma \left( \sum_{j \in N_i} a_{ij}(w_j + z_j(t) - w_i - z_i(t)) \right. \\ &\quad \left. + b_i(w_0 - w_i - z_i(t)) \right). \end{aligned} \quad (7)$$

#### B. EVENT-TRIGGERED MECHANISM

In the centralized event-triggered structure, the agents update the controllers synchronously at a global triggered time and remain unchanged until the next trigger time. The triggered time is defined as the time when the triggered function reaches a threshold.

Event-triggered control law is given by

$$\begin{aligned} u_i(t) &= z(t_k) + \kappa \left( \sum_{j \in N_i} a_{ij}(x_j(t_k) - x_i(t_k)) \right. \\ &\quad \left. + b_i(x_0(t_k) - x_i(t_k)) \right), \\ \dot{z}_i(t) &= \gamma \left( \sum_{j \in N_i} a_{ij}(w_j + z_j(t_k) - w_i - z_i(t_k)) \right. \\ &\quad \left. + b_i(w_0 - w_i - z_i(t_k)) \right), \quad \forall t \in [t_k, t_{k+1}), \end{aligned} \quad (8)$$

where  $t_k > 0$  denotes the  $k$ th triggered time. The closed-loop form of agents (1) with (8) is formulated as

$$\begin{aligned} \dot{x}_i(t) &= z(t_k) + \kappa \left( \sum_{j \in N_i} a_{ij}(x_j(t_k) - x_i(t_k)) \right. \\ &\quad \left. + b_i(x_0(t_k) - x_i(t_k)) \right), \\ \dot{z}_i(t) &= \gamma \left( \sum_{j \in N_i} a_{ij}(w_j + z_j(t_k) - w_i - z_i(t_k)) \right. \\ &\quad \left. + b_i(w_0 - w_i - z_i(t_k)) \right), \quad \forall t \in [t_k, t_{k+1}). \end{aligned} \quad (9)$$

Let  $\tilde{x}_i(t) = x_i(t) - x_0(t)$  and  $\tilde{z}_i(t) = z_i(t) + w_i - w_0$ , and rewrite (9) as

$$\dot{\tilde{x}}_i(t) = \tilde{z}_i(t_k) + \kappa \left( \sum_{j \in N_i} a_{ij}(\tilde{x}_j(t_k) - \tilde{x}_i(t_k)) - b_i \tilde{x}_i(t_k) \right),$$

$$\dot{\tilde{z}}_i(t) = \gamma \left( \sum_{j \in N_i} a_{ij} (\tilde{z}_j(t_k) - \tilde{z}_i(t_k)) - b_i \tilde{z}_i(t_k) \right). \quad (10)$$

*Remark 2:* The convergence analysis of system (10) has been accomplished in our paper [38], but we also analyzed here in order to compare the following results with that of the next section on the event-triggered consensus seeking under input delay.

Set  $\tilde{x}(t) = [\tilde{x}_1(t), \dots, \tilde{x}_n(t)]^T$ ,  $\tilde{z}(t) = [\tilde{z}_1(t), \dots, \tilde{z}_n(t)]^T$ , and rewrite (10) as

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{z}(t_k) - \kappa(L+B)\tilde{x}(t_k), \\ \dot{\tilde{z}}(t) &= -\gamma(L+B)\tilde{z}(t_k), t \in [t_k, t_{k+1}). \end{aligned} \quad (11)$$

Let  $e(t) = [e_1^T(t), e_2^T(t)]^T$ , where  $e_1(t) = \tilde{x}(t_k) - \tilde{x}(t)$ ,  $e_2(t) = \tilde{z}(t_k) - \tilde{z}(t)$ ,  $[t_k, t_{k+1})$ . Then, the equation (11) turns to be

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{z}(t) + e_2(t) - \kappa(L+B)(e_1(t) + \tilde{x}(t)), \\ \dot{\tilde{z}}(t) &= -\gamma(L+B)(e_2(t) + \tilde{z}(t)). \end{aligned} \quad (12)$$

Furthermore, the equation (12) is expressed as

$$\dot{y}(t) = H(y(t) + e(t)), \quad (13)$$

where  $y(t) = [\tilde{x}^T(t), \tilde{z}^T(t)]^T$ ,  $e(t) = y(t_k) - y(t)$ , and  $H = \begin{bmatrix} -\kappa(L+B) & I_n \\ 0_n & -\gamma(L+B) \end{bmatrix}$ .

*Theorem 1:* For the heterogeneous first-order multi-agent system (9) with an undirected and connected topology, the leader-following consensus is reached asymptotically, if

$$\gamma > \frac{1}{\lambda_{\min}^2(L+B)} \quad (14)$$

holds, and the triggered function is given by

$$f(e(t)) = \|e(t)\| - \sigma \frac{(\gamma \lambda_{\min}^2(L+B) - 1) \|y(t)\|}{\|Q\|}, \quad (15)$$

where  $Q = \begin{bmatrix} -\kappa(\kappa + \gamma)(L+B)^2 & \kappa(L+B) \\ -\kappa(L+B) & I_n - \gamma(L+B)^2 \end{bmatrix}$ ,  $0 < \sigma < 1$ ,  $\kappa = \frac{\sqrt{\gamma^2 + 4\gamma - \gamma}}{2} > 0$ . When  $f(e(t))$  approaches zero, all the agents' controllers will be triggered, i.e., the control input of each agent will be updated, and the measurement error will be set to zero.

*Proof:* In order to analyze the effectiveness of the proposed strategy, we construct the following Lyapunov function,

$$V(t) = \frac{1}{2} y^T(t) P y(t), \quad (16)$$

where  $P = \begin{bmatrix} (k + \gamma)(L+B) & I_n \\ I_n & L+B \end{bmatrix}$ . Since (14) is satisfied, the matrix  $P$  is strictly positive according to Lemma 1. For simplicity, we set  $V = V(t)$ ,  $y = y(t)$ ,  $e = e(t)$ . Calculating time derivation of  $V$  yields

$$\dot{V} = y^T \begin{bmatrix} -k(k + \gamma)(L+B)^2 & k(L+B) \\ -k(L+B) & I_n - \gamma(L+B)^T \end{bmatrix} y$$

$$\begin{aligned} &+ y^T \begin{bmatrix} -k(k + \gamma)(L+B)^2 & k(L+B) \\ -k(L+B) & I_n - \gamma(L+B)^2 \end{bmatrix} e \\ &= -k(k + \gamma) \tilde{x}^T (L+B)^2 \tilde{x} + \tilde{z}^T \tilde{z} - \gamma \tilde{z}^T (L+B)^2 \tilde{z} \\ &+ y^T \begin{bmatrix} -k(k + \gamma)(L+B)^2 & k(L+B) \\ -k(L+B) & I_n - \gamma(L+B)^2 \end{bmatrix} e \\ &\leq -\gamma \lambda_{\min}^2(L+B) \|y\|^2 + \|y\|^2 \\ &+ y^T \begin{bmatrix} -k(k + \gamma)(L+B)^2 & k(L+B) \\ -k(L+B) & I_n - \gamma(L+B)^2 \end{bmatrix} e \\ &= \|y\|^2 (1 - \gamma \lambda_{\min}^2(L+B)) \\ &+ y^T \begin{bmatrix} -k(k + \gamma)(L+B)^2 & k(L+B) \\ -k(L+B) & I_n - \gamma(L+B)^2 \end{bmatrix} e \\ &\leq \|y\|^2 (1 - \gamma \lambda_{\min}^2(L+B)) + \|y\| \|Q\| \|e\|, \end{aligned} \quad (17)$$

where  $Q = \begin{bmatrix} -k(k + \gamma)(L+B)^2 & k(L+B) \\ -k(L+B) & I_n - \gamma(L+B)^2 \end{bmatrix}$ . Enforce  $\|e\|$  to satisfy

$$\|e\| \leq \sigma \frac{(\gamma \lambda_{\min}^2(L+B) - 1) \|y\|}{\|Q\|} \quad (18)$$

with  $0 < \sigma < 1$ , and we get

$$\dot{V} \leq (\sigma - 1) \frac{(\gamma \lambda_{\min}^2(L+B) - 1) \|y\|^2}{\|Q\|} < 0. \quad (19)$$

Thus, the centralized event-triggered strategy is effective for the agents in system (9) converging to the leader-following consensus asymptotically. Theorem 1 is proved.  $\square$

Moreover, the event time  $t_k$  is computed from the following equation,

$$\Delta(t) = \|e\| - \sigma \frac{(\gamma \lambda_{\min}^2(L+B) - 1) \|y\|}{\|Q\|} = 0. \quad (20)$$

Besides, the time interval between the adjacent events  $t_k$  and  $t_{k+1}$  is larger than a strictly positive constant given in the following theorem.

*Theorem 2:* For the heterogeneous first-order multi-agent systems (9) with an undirected and connected topology, if the condition (14) holds and the triggered function (15) is adopted, we get

$$\{t_{k+1} - t_k\} \geq \theta > 0, \quad (21)$$

and

$$\theta = \frac{\sigma(\gamma \lambda_{\min}^2(L+B) - 1)}{(\|H\| + 1)(\|Q\| + \sigma(\gamma \lambda_{\min}^2(L+B) - 1))}. \quad (22)$$

*Proof:* Similarly to [21], the time derivative of  $\frac{\|e\|}{\|y\|}$  is given by

$$\begin{aligned} \frac{d}{dt} \frac{\|e\|}{\|y\|} &= \frac{-e^T \dot{y}}{\|e\| \|y\|} - \frac{y^T \dot{y} \|e\|}{\|y\|^2 \|y\|} \\ &\leq \frac{\|\dot{y}\|}{\|y\|} + \frac{\|\dot{y}\| \|e\|}{\|y\|^2} \\ &= \left(1 + \frac{\|e\|}{\|y\|}\right) \frac{\|\dot{y}\|}{\|y\|}. \end{aligned} \quad (23)$$

From (13), we obtain

$$\|\dot{y}\| \leq \|y\| + \|H\| (\|y\| + \|e\|). \quad (24)$$

Substituting (24) into (23) yields

$$\begin{aligned} \frac{d}{dt} \frac{\|e\|}{\|y\|} &\leq \left(1 + \frac{\|e\|}{\|y\|}\right) \frac{\|\dot{y}\|}{\|y\|} \\ &\leq \left(1 + \frac{\|e\|}{\|y\|}\right) \frac{\|y\| + \|H\| (\|y\| + \|e\|)}{\|y\|} \\ &\leq (\|H\| + 1) \left(1 + \frac{\|e\|}{\|y\|}\right)^2. \end{aligned} \quad (25)$$

Denoting  $\vartheta = \frac{\|e\|}{\|y\|}$ , we have  $\dot{\vartheta} \leq (\|H\| + 1)(1 + \vartheta)^2$  so that  $\vartheta$  satisfies the bound  $\vartheta \leq \mu(t, \mu_0)$ , where  $\mu(t, \mu_0)$  is the solution of  $\dot{\mu} \leq (\|H\| + 1)(1 + \mu)^2$ ,  $\mu(t, \mu_0) = \mu_0$ . Then, the solution of equation (25) is defined as

$$\mu(\theta, 0) = \frac{(1 + \|H\|)\theta}{1 - (1 + \|H\|)\theta}. \quad (26)$$

It follows from (18) that

$$\mu(\theta, 0) = \sigma \frac{(\gamma \lambda_{\min}^2(L + B) - 1)}{\|Q\|}. \quad (27)$$

With (26) and (27), we gain

$$\theta = \frac{\sigma(\gamma \lambda_{\min}^2(L + B) - 1)}{(\|H\| + 1)(\|Q\| + \sigma(\gamma \lambda_{\min}^2(L + B) - 1))}. \quad (28)$$

Hence, Theorem 2 is proved.  $\square$

#### IV. CONSENSUS SEEKING WITH INPUT DELAY

Now, we investigate the leader-following consensus problem of heterogeneous first-order multi-agent system (7) with identical input delay.

$$\begin{aligned} \dot{x}_i(t) &= w_i + z_i(t - \tau) \\ &\quad + \kappa \left( \sum_{j \in N_i} a_{ij}(x_j(t - \tau) - x_i(t - \tau)) \right. \\ &\quad \left. + b_i(x_0(t - \tau) - x_i(t - \tau)) \right), \\ \dot{z}_i(t) &= \gamma \left( \sum_{j \in N_i} a_{ij}(w_j + z_j(t) - w_i - z_i(t)) \right. \\ &\quad \left. + b_i(w_0 - w_i - z_i(t)) \right), \end{aligned} \quad (29)$$

where  $\tau > 0$  is the input delay.

By utilizing the event-triggered mechanism, the system (29) becomes

$$\begin{aligned} \dot{x}_i(t) &= w_i + z_i(t_k - \tau) \\ &\quad + \kappa \left( \sum_{j \in N_i} a_{ij}(x_j(t_k - \tau) - x_i(t_k - \tau)) \right. \\ &\quad \left. + b_i(x_0(t_k - \tau) - x_i(t_k - \tau)) \right), \\ \dot{z}_i(t) &= \gamma \left( \sum_{j \in N_i} a_{ij}(w_j + z_j(t_k) - w_i - z_i(t_k)) \right. \\ &\quad \left. + b_i(w_0 - w_i - z_i(t_k)) \right). \end{aligned} \quad (30)$$

Let  $\tilde{x}_i(t) = x_i(t) - x_0(t)$ ,  $\tilde{z}_i(t) = z_i(t) + w_i - w_0$ , and reformulate (30) as

$$\dot{\tilde{x}}_i(t) = \tilde{z}_i(t_k - \tau)$$

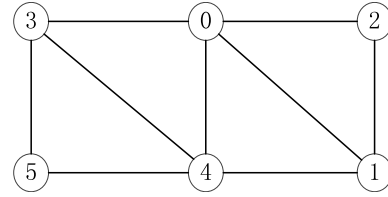


FIGURE 1. The topology graph.

$$\begin{aligned} &+ \kappa \left( \sum_{j \in N_i} a_{ij}(\tilde{x}_j(t_k - \tau) - \tilde{x}_i(t_k - \tau)) \right. \\ &\quad \left. - b_i \tilde{x}_i(t_k - \tau) \right), \\ \dot{\tilde{z}}_i(t) &= \gamma \left( \sum_{j \in N_i} a_{ij}(\tilde{z}_j(t_k) - \tilde{z}_i(t_k)) - b_i \tilde{z}_i(t_k) \right), \end{aligned} \quad (31)$$

where  $t_k > 0$  denotes the  $k$ th triggered time.

Denote  $\tilde{x}(t) = [\tilde{x}_1(t), \dots, \tilde{x}_n(t)]^T$  and  $\tilde{z}(t) = [\tilde{z}_1(t), \dots, \tilde{z}_n(t)]^T$ , and we obtain

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{z}(t - \tau) + e_2(t - \tau) - \kappa(L + B)(e_1(t - \tau) \\ &\quad + \tilde{x}(t - \tau)), \\ \dot{\tilde{z}}(t) &= -\gamma(L + B)(e_2(t) + \tilde{z}(t)), \end{aligned} \quad (32)$$

where  $e(t) = [e_1^T(t), e_2^T(t)]^T$  with  $e_1(t) = \tilde{x}(t_k) - \tilde{x}(t)$  and  $e_2(t) = \tilde{z}(t_k) - \tilde{z}(t)$ ,  $\forall t \in [t_k, t_{k+1})$ .

In order to simplify the derivation, we rewrite the equation (32) as

$$\begin{aligned} \dot{\tilde{x}}(t) &= \tilde{z}(t - \tau) + e_2(t - \tau) - k(L + B)(e_1(t - \tau) \\ &\quad + \tilde{x}(t - \tau)), \\ \dot{\tilde{z}}(t - \tau) &= -\gamma(L + B)(e_2(t - \tau) + \tilde{z}(t - \tau)). \end{aligned} \quad (33)$$

Moreover, let  $y(t) = [\tilde{x}^T(t), \tilde{z}^T(t - \tau)]^T$ , and we get

$$\dot{y}(t) = Cy(t) + Fy(t - \tau) + He(t - \tau), \quad (34)$$

where

$$\begin{aligned} C &= \begin{bmatrix} 0 & I_N \\ 0 & \gamma(-L - B) \end{bmatrix}, \\ F &= \begin{bmatrix} k(-L - B) & 0 \\ 0 & 0 \end{bmatrix}, \\ H &= \begin{bmatrix} k(-L - B) & I_N \\ 0 & \gamma(-L - B) \end{bmatrix}. \end{aligned}$$

**Theorem 3:** For the heterogeneous first-order multi-agent systems (30) under an undirected and connected topology, the agents' states converge to the leader-following consensus asymptotically, if the following triggered function

$$\begin{aligned} f(e(t)) &= e^T(t)Qe(t) - \kappa_1 * y^T(t)Dy(t) \\ &\quad - \kappa_2 * y^T(t - \tau)Gy(t - \tau) \\ &\quad - \kappa_3 * e^T(t - \tau)Se(t - \tau) = 0 \end{aligned} \quad (35)$$

is adopted, where  $\kappa_1 > 0$ ,  $\kappa_2 > 0$ ,  $\kappa_3 > 0$ , and there exists symmetry matrices  $P = P^T > 0$ ,  $R = R^T > 0$ ,  $Q = Q^T > 0$ ,

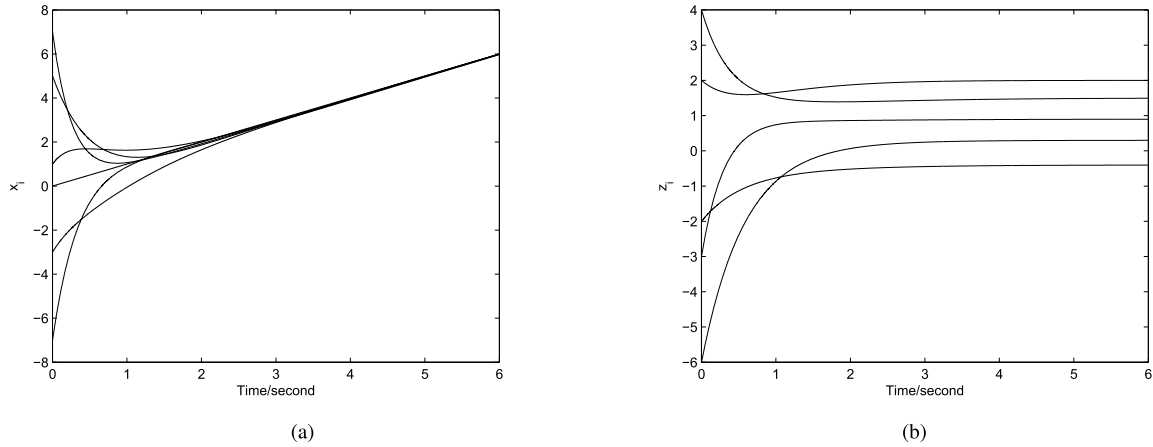


FIGURE 2. Consensus seeking without input delay. (a) Evolution of agents' states. (b) Trajectories of adaptive variable  $z_j$ .

$W = W^T > 0$ , and  $D > 0, G > 0, S > 0$  such that the following Linear Matrix Inequality

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ * & M_{22} & M_{23} \\ * & * & M_{33} \end{bmatrix} < 0, \quad (36)$$

holds, where

$$\begin{aligned} M_{11} &= PC + C^T + R + \tau C^T WC - \tau^{-1}W + \kappa_1 * D, \\ M_{12} &= PF + \tau C^T WF + \tau^{-1}W, \\ M_{13} &= PH + \tau C^T WH, \\ M_{22} &= -R + \tau F^T WF - \tau^{-1}W + \kappa_2 * G, \\ M_{23} &= \tau F^T WH, \\ M_{33} &= -Q + \tau H^T WH + \kappa_3 * S. \end{aligned}$$

When  $f(e(t))$  approaches zero, all the agents' controllers will be triggered, i.e., the control input of each agent will be updated, and the measurement error will be set to zero.

*Proof:* For the system (34), we construct the following Lyapunov-Krasovskii functional

$$\begin{aligned} V(t) &= y^T(t)Py(t) + \int_{t-\tau}^t y^T(s)Ry(s)ds \\ &\quad + \int_{t-\tau}^t e^T(s)Qe(s)ds \\ &\quad + \int_{-\tau}^0 \int_{t+\theta}^t \dot{y}^T(s)W\dot{y}(s)dsd\theta. \end{aligned}$$

Calculating the time derivation of  $V$  yields

$$\begin{aligned} \dot{V}(t) &= 2y^T(t)P\dot{y}(t) + y^T(t)Ry(t) \\ &\quad - y^T(t-\tau)Ry(t-\tau) \\ &\quad + e^T(t)Qe(t) - e^T(t-\tau)Qe(t-\tau) \\ &\quad + \tau \dot{y}^T(t)W\dot{y}(t) - \int_{t-\tau}^t \dot{y}^T(s)W\dot{y}(s)ds. \end{aligned}$$

According to Jensen Inequality [36], we have

$$- \int_{t-\tau}^t \dot{y}^T(s)W\dot{y}(s)ds$$

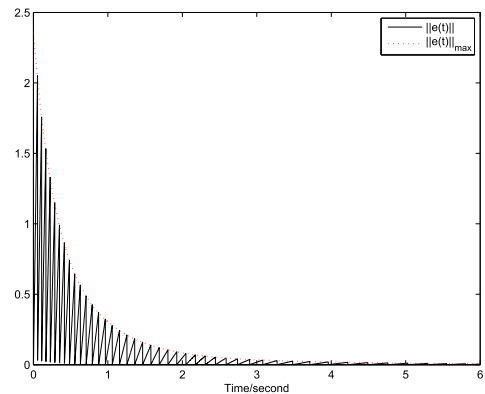


FIGURE 3. Evolution of measurement error norm  $\|e(t)\|$  without input delay.

$$\leq -\tau^{-1} [y(t) - y(t - \tau)]^T * W * [y(t) - y(t - \tau)]. \quad (37)$$

By combining the inequalities (36) and (37), we obtain

$$\dot{V}(t) \leq \eta^T(t)\hat{M}\eta(t) + e^T(t)Qe(t),$$

where

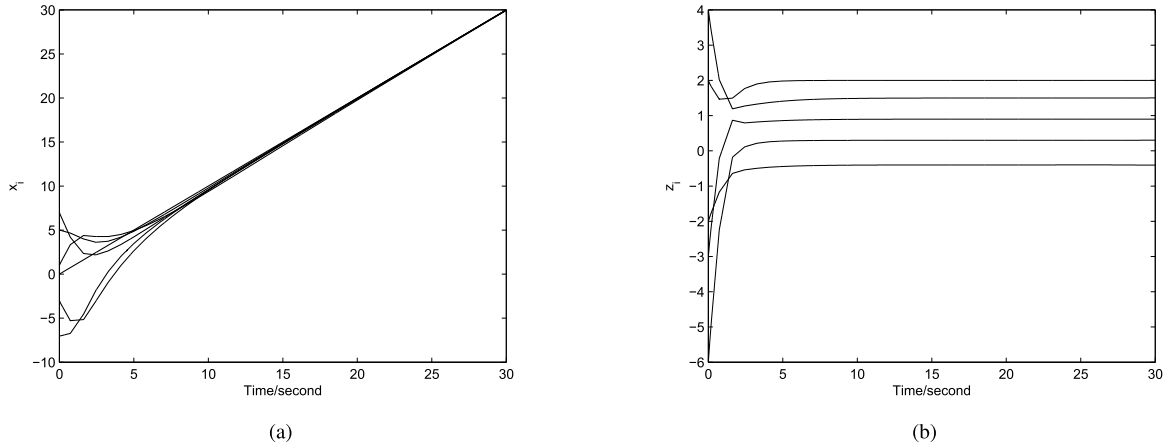
$$\begin{aligned} \eta(t) &= [y^T(t), y^T(t - \tau), e^T(t - \tau)]^T, \\ \hat{M} &= \begin{bmatrix} \hat{M}_{11} & M_{12} & M_{13} \\ * & \hat{M}_{22} & M_{23} \\ * & * & \hat{M}_{33} \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \hat{M}_{11} &= PC + C^T P + R + \tau C^T WC - \tau^{-1}W, \\ \hat{M}_{22} &= -R + \tau F^T WF - \tau^{-1}W, \\ \hat{M}_{33} &= -Q + \tau H^T WH. \end{aligned}$$

Define the triggered function as (35), and we get

$$\begin{aligned} e^T(t)Qe(t) &\leq \kappa_1 * y^T(t)Dy(t) + \kappa_2 * y^T(t-\tau)Gy(t-\tau) \\ &\quad + \kappa_3 * e^T(t-\tau)Se(t-\tau), \end{aligned}$$

which leads to  $\dot{V}(t) \leq \eta^T(t)M\eta(t) \leq 0$ , i.e., the consensus problem of (30) with input delay is solved. Theorem 3 is proved.  $\square$



**FIGURE 4.** Consensus seeking with input delay. (a) Evolution of agents' states. (b) Trajectories of adaptive variable  $z_i$ .

**Theorem 4:** For the heterogeneous first-order multi-agent systems (30) with an undirected and connected topology, the agents achieve the leader-following consensus asymptotically if the triggered function (35) is adopted and there exist symmetry matrices  $P = P^T > 0$ ,  $R = R^T > 0$ ,  $Q = Q^T > 0$ ,  $W = W^T > 0$ , and  $D > 0$ ,  $G > 0$ ,  $S > 0$  such that the inequality  $M < 0$  in (36) holds. Besides, the system (30) with the event triggered function (35) avoids the Zeno behavior.

*Proof:* Firstly, we set  $y_a(t) = [\tilde{x}^T(t) \tilde{z}^T(t)]^T$ . Then,

$$\dot{y}_a(t) = H_a(y_a(t) + e(t)) + H_b(y_a(t - \tau) + e(t - \tau)),$$

$$\text{where } H_a = \begin{bmatrix} 0_n & 0_n \\ 0_n & -\gamma(L + B) \end{bmatrix}, H_b = \begin{bmatrix} -k(L + B) & 0_n \\ 0_n & I_n \end{bmatrix}.$$

It follows from (35) that

$$\|e(t)\| \leq h_1 \|y(t)\| + h_2 \|y(t - \tau)\| + h_3 \|e(t - \tau)\|,$$

where  $h_1 = \sqrt{\lambda_{\max}(D/Q)}$ ,  $h_2 = \sqrt{\lambda_{\max}(G/Q)}$ ,  $h_3 = \sqrt{\lambda_{\max}(S/Q)}$ .

In order to derive a positive lower bound on the sampling interval, an upper bound on the measurement error  $e(t)$  for  $t > t_k$  is calculated. Since  $\dot{e}(t) = -\dot{y}_a(t)$ ,  $\|e(t)\| \leq \int_{t_k}^t \|y_a(s)\| ds$  holds, where

$$\|y_a(t)\| \leq \|H_a\| (\|y(t)\| + \|e(t)\|) + \|H_b\| (\|y(t - \tau)\| + \|e(t - \tau)\|).$$

Define  $\|y(t)\|_{\infty} = \sup_{t \geq 0} \|y(t)\|$ ,  $\|e(t)\|_{\infty} = \sup_{t \geq 0} \|e(t)\|$ ,  $\|y(t - \tau)\|_{\infty} = \sup_{t \geq 0} \|y(t - \tau)\|$  and  $\|e(t - \tau)\|_{\infty} = \sup_{t \geq 0} \|e(t - \tau)\|$ , then we get

$$\|e(t)\| \leq (t - t_k)(\|H_a\| (\|y(t)\|_{\infty} + \|e(t)\|_{\infty}) + \|H_b\| (\|y(t - \tau)\|_{\infty} + \|e(t - \tau)\|_{\infty})).$$

Notice that the sampling will not be triggered before  $\|e(t)\| \leq h_1 \|y(t)\| + h_2 \|y(t - \tau)\| + h_3 \|e(t - \tau)\|$ . Thus, we obtain

$$t_{k+1} - t_k \geq \bar{\theta},$$

and

$$\bar{\theta} = (h_1 \|y(t)\| + h_2 \|y(t - \tau)\| + h_3 \|e(t - \tau)\|)(\|H_a\| + \|H_b\| (\|y(t - \tau)\|_{\infty} + \|e(t - \tau)\|_{\infty})).$$

$$+ \|e(t - \tau)\|_{\infty})^{-1}.$$

If the system (30) achieves the consensus, i.e.,  $\|y(t)\| = \|y(t - \tau)\| = \|e(t - \tau)\| = 0$ ,  $\bar{\theta}$  must be zero. In other words, there is no Zeno behavior before the consensus is reached. Theorem 4 is proved.  $\square$

## V. NUMERICAL EXAMPLE

Take into account a multi-agent network of five agents and a leader, and the undirected and connected topology is plotted in Fig. 1. In addition, the Laplacian matrix  $L$  and  $B$  are chosen as

$$L = \begin{bmatrix} 5 & -2 & 0 & -3 & 0 \\ -2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 6 & -2 & -4 \\ -3 & 0 & -2 & 9 & -4 \\ 0 & 0 & -4 & -6 & 10 \end{bmatrix},$$

and

$$B = \begin{bmatrix} 8 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 15 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For the leader (2) and the first-order agents (1), the nominal velocities are given by  $w_0 = 1.0$ ,  $w_1 = 0.7$ ,  $w_2 = -1.0$ ,  $w_3 = 0.1$ ,  $w_4 = 1.4$ ,  $w_5 = -0.5$ . For the system (7) without input delay, we choose  $\sigma = 0.99$  and get  $1/\lambda_{\min}^2(L + B) = 0.0338$ . Then, we choose  $\gamma = 0.1338$  satisfying the inequality (14) in Theorem 1. The agents (7) converge to a dynamical consensus asymptotically (see Fig. 2a), and the trajectories of the adaptive variables and the evolution of the errors are shown in Fig. 2b and Fig. 3, respectively.

For the multi-agent system (30) with input delay, we take  $\gamma = 0.1338$ ,  $k = 0.02$ ,  $\kappa_1 = 0.2$ ,  $\kappa_2 = 0.7$ ,  $\kappa_3 = 5.8$ , and set input delay  $\tau = 0.1s$ , which make the condition in Theorem 3 hold. Then, the agents (30) converge to the dynamical consensus asymptotically (see Fig. 4a), the convergence properties of the adaptive variables are shown in Fig. 4b, and

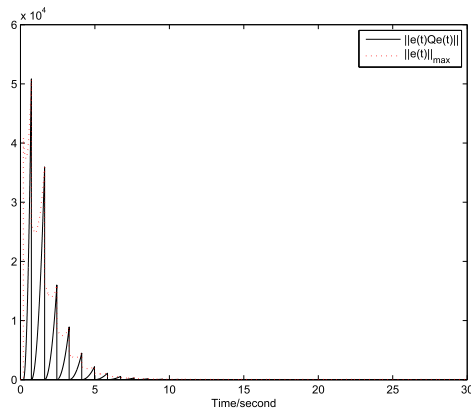


FIGURE 5. Evolution of the measurement error norm  $\|e(t)\|$  with input delay.

the evolution of the errors are illustrated in Fig. 5. Based on the simulation results above, it is easy to conclude that the update periods which are aperiodic and all the followers without or with input delay follow the leader's trajectory asymptotically.

## VI. CONCLUSIONS

In this paper, a centralized event-triggered consensus control is proposed for heterogeneous first-order multi-agent systems with different nominal velocities. We adopt an adaptive leader-following consensus algorithm, and examined the consentability of the closed-loop system without and with input delay, respectively, by using the Lyapunov stability criterion. Besides, it is also proved that there is no Zeno behavior when the proposed event-triggered control strategy is applied. However, event-triggered strategy in the centralized way that all the agents are triggered at the same event-times is really idealized, because the centralized information gathering is difficult for a large network. We will focus on designing a decentralized event-triggered consensus control for the heterogeneous first-order multi-agent systems in our future work. In our future work, in addition, some good results on switched systems [41], [42], stochastic systems [20], [41] and intelligent control [35], [40], [43], will provide good ideas on the analysis and synthesis of event-triggered consensus problem under switching topologies, stochastic packet loss, etc.

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**MEI-MEI DUAN** received the B.Sc. degree in automation from Jiangnan University, China, in 2014, where she is currently pursuing the M.Sc. degree. Her research interest includes the event-triggered control of multi-agent systems.



**CHENG-LIN LIU** received the bachelor's degree in electrical engineering and automation from the Nanjing University of Science and Technology, Nanjing, China, in 2003, and the Ph.D. degree in control theory and control engineering from Southeast University, China. Since 2008, he has been with Jiangnan University, Wuxi, China, where he is currently an Associate Professor with the Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), School of Internet of Things Engineering. His current research interests include cyber-physical systems, sensor networks, and coordination control of multi-agent systems.



**FEI LIU** received the Ph.D. degree in control science and control engineering from Zhejiang University, China. He is currently a Professor with the Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Institute of Automation, Jiangnan University. His research interests include advanced control theory and applications, batch process control engineering, statistical monitoring and diagnosis in industrial process, and intelligent technique with emphasis on fuzzy and neural systems.

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