Received February 28, 2017, accepted March 9, 2017, date of publication April 5, 2017, date of current version May 17, 2017.

Digital Object Identifier 10.1109/ACCESS.2017.2688518

Total-Amount Synchronous Control Based on Terminal Sliding-Mode Control

CHANGFAN ZHANG[1], ZHENZHEN LIN[1], SIMON X. YANG[2], (Senior Member, IEEE), AND JING HE[1]

[1]School of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou 412007, China
[2]Advanced Robotics and Intelligent Systems Laboratory, School of Engineering, University of Guelph, ON N1G 2W1, Canada

Corresponding author: Jing He (hejing@263.net)

This work was supported by the Natural Science Foundation of China under Grant 61473117.

ABSTRACT This paper presents a total-amount synchronous control (TASC) strategy for nonlinear systems with uncertainty based on finite-time control theory. In combination with a new type of terminal sliding-mode control strategy, finite-time convergence of TASC is realized. First, the specific mathematical expression of the system terminal sliding-mode surface is given. On the basis of this, according to the sliding-mode surface expression, the sliding-mode variable structure control laws of n regular nonlinear systems are derived, avoiding the singularity problem that can easily appear in ordinary terminal sliding-mode controllers. Meanwhile, the initial system is located on the sliding-mode surface. The approach process in sliding-mode control is eliminated, and the existence of the sliding phase is proved according to the Lyapunov stability theory. Finally, the effectiveness of the algorithm is verified by a numerical example.

INDEX TERMS Finite-time convergence, nonlinear system, synchronous control, terminal sliding-mode control.

I. INTRODUCTION

The synchronous behavior of biological groups attracts interest from researchers in biology, control, physics and other fields, research on synchronization has developed rapidly over the last two decades. Synchronization of dynamic systems is still a hot topic in the automatic control field. Synchronization means that all individual state variables of a system tend to the same value under the action of control laws [1]. Early literature focused mainly on synchronization of nominal systems without uncertainty; for example, Scardovi and Sepulchre [2] investigated the synchronization of a network of identical linear state-space models under a possibly time-varying and directed interconnection structure. In addition, In the absence of the double stochasticity assumption, Li *et al.* [3] investigated discrete-time single-integrator by means of the matrix transformation and the Lyapunov techniques. Fax and Murray [4] provided an approach that realizes a dynamical system that supplies each agent with a common reference to be used for cooperative.

However, the synchronization of systems in reality will inevitably be subjected to external disturbance and parameter perturbations. Researchers have therefore conducted much in-depth research on synchronization in the presence of uncertainty. Taking the effect of inherent unmodeled disturbances into account, X.Wang developed a reduced order observer based consensus protocol with a performance constraint, The anti-disturbance and output asymptotic synchronization of the closed-loop system are guaranteed [5]. Zhou *et al.* [6] investigated the locally and globally adaptive synchronization of an uncertain complex dynamical network, Several network synchronization criteria are deduced. By utilizing the concept of impulsive control and the stability results for impulsive systems, Liu *et al.* [7] studied robust impulsive synchronization of uncertain dynamical networks, several criteria for robust local and robust global impulsive synchronization are established for complex dynamical networks. Most existing studies have aimed for asymptotic synchronization, namely that individual state variable asymptotically converge to a given control state of the system. It also means that an individual trajectory will need an infinitely long time to coincide with a given reference trajectory. However, in practice, in many control systems (such as flexible spacecraft control, robot operation control, motor control, mechanical processing control, etc.), the required control process ends within a finite time. In addition, actual control systems require fast dynamic response, and their error can converge to zero in an adjustable finite time. Compared to asymptotically stable control law, the closed-loop system with finite time convergence usually demonstrates faster convergence rates and better disturbance rejection properties [8]–[10]. Therefore, the study of finite-time synchronization is more valuable for practical engineering applications.

Sliding-mode variable-structure control is a kind of discontinuous nonlinear control. When moving on a sliding-mode surface, the system has superior invariance and robustness [11], [12]. In addition, the algorithm is simple and robust in real time, facilitating engineering implementation. Yu and Long [13] investigated the distributed finite-time consensus problem of second-order multi-agent systems (MAS) in the presence of bounded disturbances. Mondal *et al.* [14] proposed a robust consensus controller for heterogeneous higher-order nonlinear multi-agent systems using sliding mode control, when the agent dynamics are involved with mismatched uncertainties. Du *et al.* [15] investigated the consensus tracking problem of multiple nonholonomic high-order chained-form systems using super-twisting algorithm.

These synchronization behaviors are all aimed at making individual state variables tend to be uniform. However, in some actual situations, synchronization of a single variable is not required, but the total amount of system output must be maintained constant. For example, in electric locomotive traction, under certain circumstances, it is required that the total traction be maintained constant. Whereas the sum of state variables of whole system is maintained constant, the total-amount synchronization problem has not yet been reported in the literature.

This paper represents a preliminary exploration of this problem, using the results of dynamic system synchronization efforts proposed in existing studies for reference. The research idea is to convert the total-amount synchronization problem into a system finite-time convergence problem. To realize finite-time convergence of the error system and at the same time avoid the singularity problem of the common terminal sliding-mode controller, as reported in the literature [16], this research has used a nonsingular terminal sliding-mode control strategy. Because the initial system state is located on a sliding-mode surface, the approach process of sliding-mode control is eliminated, guaranteeing the robustness of the whole process.

The rest of this paper is organized as follows. The second part presents a system description and the system design goal. The third part describes the design of the terminal sliding-mode surface. The fourth part discusses the design of the terminal sliding-mode controller, including a stability proof. The fifth part consists of a simulation example, and the sixth part concludes the paper.

II. SYSTEM DESCRIPTION

Because most electromechanical and mechanical systems in engineering practice have regular properties, this paper mainly studies the following n independent second-order regular nonlinear systems, as shown in Fig. 1. Fig. 1 shows the overall framework of Total-Amount Synchronous Control, the Total-Amount Synchronous controller clearly sums the outputs of all systems, views it as the total value, and then compares with the reference instruction. The controller generates the realtime control input for each system on the basis of the comparison; therefore, all systems are essentially coupled.

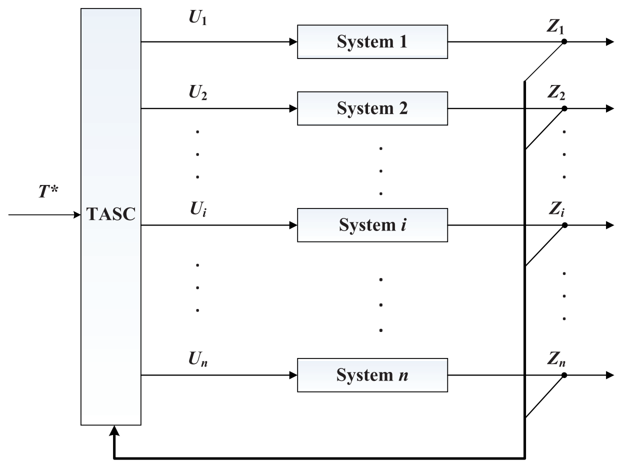


FIGURE 1. TASC block diagram.

In Fig. 1, the general expression for the $i-th$ differential equation of the system is:

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = f_i(x_i, t) + \Delta f_i(x_i, t) + a_i u_i + d_i(t) \\ z_i = x_{i1} \end{cases} \quad (i = 1, 2, \ldots, n) \qquad (1)$$

where $\boldsymbol{x}_i = [x_{i1}, x_{i2}]^T (i = 1, 2, \ldots, n)$ is the state vector; $\boldsymbol{u}_i$ is the control inputs; $f_i(x_i, t)$ is a known nonlinear function of system state; z_i is the $i-th$ system output,$\Delta f_i(x_i, t)$ and $d_i(t)$ are respectively the uncertainty and the external disturbance of an unknown object, which satisfy the inequality $|\Delta f_i(x_i, t)| \leq F_i$, $|d_i(x, t)| \leq D_i$, where F_i, D_i are known non-negative functions.

Previous studies have focused only on synchronization between individuals. In this paper, the design goal of n independent systems is as follows: under the existence of parameter perturbations and external disturbances, the controller is designed so that a linear combination of n regular nonlinear system outputs can track a given reference trajectory T^*within a finite time T, i.e., $\lim_{t \to T} (\sum_{i=1}^{n} z_i - T^*) = 0$.

III. DESIGN OF THE TERMINAL SLIDING-MODE SURFACE

In traditional sliding-mode control, a linear switching function is usually selected. After the controller forces the system trajectory onto the sliding-mode surface, the tracking error can gradually converge to zero, and the speed of gradual convergence can be adjusted arbitrarily by choosing the parameters of the sliding-mode surface. However, no matter how the parameter state is adjusted, the tracking error does not converge to zero within a finite time [17]. Therefore, some researchers have proposed a terminal sliding-mode control

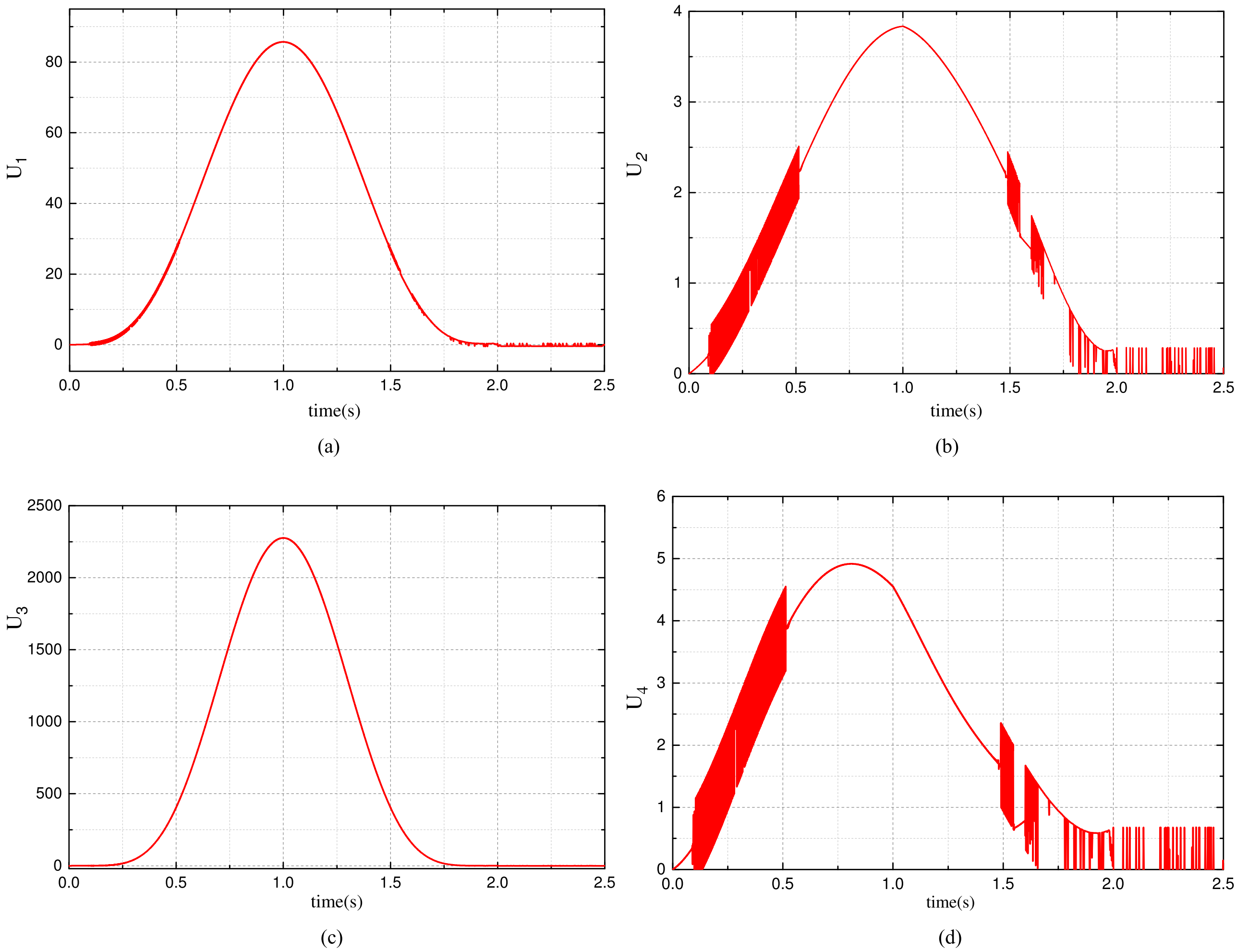


FIGURE 2. Sliding-mode controller output in case 1. (a) Control law U_1 of system 1. (b) Control law U_2 of system 2. (c) Control law U_3 of system 3. (d) Control law U_4 of system 4.

strategy by introducing a nonlinear function into the design of the sliding-mode surface, on the basis that guaranteeing the stability of sliding-mode control makes the system state converge to the desired state in a limited time.In traditional synchronous control, people focused the individual state, e.g., the speed and position tend to be identical. However, this work is the first to expand the individual synchronization to total synchronization. Based on the finite time convergence characteristics of the terminal sliding mode, this research applies terminal sliding mode theory to the design of the synchronous controller in order to realize the convergence of synchronous errors in finite time.

The deviation between the linear combination of the output variables of systems and the given reference trajectory T^* is defined as $\boldsymbol{E}$, i.e., $\boldsymbol{E} = [e, \dot{e}, \ldots, e^{(n-1)}]^T$, where:

$$e = z_1 + z_2 + \ldots + z_n - T^* \tag{2}$$

The following form of interactive switching functions with global information is used:

$$s(x, t) = \boldsymbol{C}^* \boldsymbol{E} - \boldsymbol{W}(t) \tag{3}$$

where $C^* = [c_1, c_2, \ldots, c_n]$, $c_n = 1$,$c_i(i = 1, \ldots, n-1)$ is a constant to be designed. Define $W(t) = C^* P(t)$ and $\boldsymbol{P}(t) = [p(t)^T, \dot{p}\,(t)^T, \ldots, p^{(n-1)}(t)^T]$, select the nonlinear function $p(t)$ so that it satisfies the following Assumption 1.

Assumption 1 [16]: $p(t) : R_+ \rightarrow R, p(t) \in C^n[0, \infty), \dot{p}, \ldots, p^{(n)} \in L^\infty$*; for a certain constant* $T > 0$*,* $\boldsymbol{p}(t)$ *is bounded on time* $[0, T]$;$p(0) = e(0), \dot{p}(0) = \dot{e}(0), \ldots, p^{(n)} = e^{(n)}(0)$*, and when* $t \geq T$,$p = 0, \dot{p} = 0, \ldots, p^{(n)} = 0$,$C^n[0, \infty)$ *expresses the set of all nth* − *order derivable continuous functions defined on* $[0, \infty)$,*Where* $e(0)$ *is the initial error.*

The continuous function $p(t)$ is selected as shown in the Equation (4),

$$p(t) = \begin{cases} \sum_{k=0}^{n} \frac{1}{k!} e(0)^{(k)} t^k + \sum_{j=0}^{n} (\sum_{l=0}^{n} (\frac{a_{jl}}{T^{j-l+n+1}} e(0)^{(l)}) t^{j+n+1}, \\ \qquad t \leq T \\ 0, \quad t > T \end{cases} \tag{4}$$

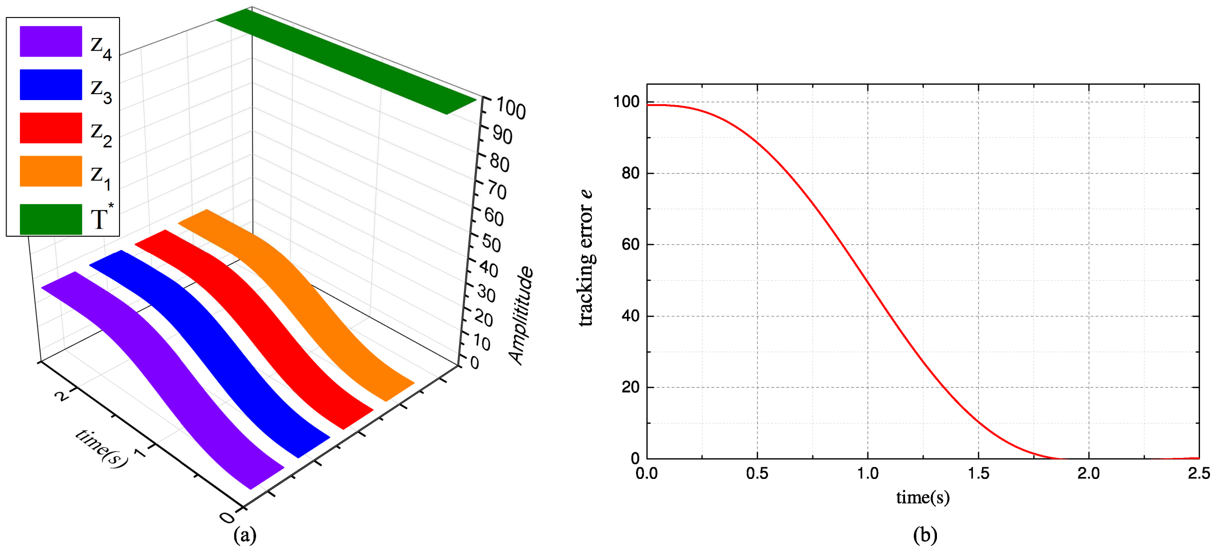


FIGURE 3. Output and tracking error in case 1. (a) Output and reference signal. (b) Tracking error of output state variables.

where the parameter a_{jl} can be determined by the conditions in Assumption 1. The following is the design process of the terminal function $p(t)$ which is determined by Equation (1) when $n = 2$. From Equation (4), function $p(t)$ can be described as shown in the Equation (5).

$$p(t) = \begin{cases} e_0 + \dot{e}_0 t + \frac{1}{2}\ddot{e}_0 t^2 + (\frac{a_{00}}{T^3}e_0 + \frac{a_{01}}{T^2}\dot{e}_0 + \frac{a_{02}}{T}\ddot{e}_0)t^3 \\ \quad +(\frac{a_{10}}{T^4}e_0 + \frac{a_{11}}{T^3}\dot{e}_0 + \frac{a_{12}}{T^2}\ddot{e}_0)t^4 + \\ (\frac{a_{20}}{T^5}e_0 + \frac{a_{21}}{T^4}\dot{e}_0 + \frac{a_{22}}{T^3}\ddot{e}_0)t^5, \quad t \le T \\ 0, \quad t > T \end{cases} \tag{5}$$

According to Assumption 1, through design of a_{jl},$p(t)$, $\dot{p}(t)$, can be made equal to zero at $t = T$s, and thereby the following system of linear equations can be obtained:

$$\begin{cases} a_{00} + a_{10} + a_{20} = -1 \\ 3a_{00} + 4a_{10} + 5a_{20} = 0 \\ 6a_{00} + 12a_{10} + 20a_{20} = 0 \end{cases} \tag{6}$$

$$\begin{cases} a_{01} + a_{11} + a_{21} = -1 \\ 3a_{01} + 4a_{11} + 5a_{21} = -1 \\ 6a_{01} + 12a_{11} + 20a_{21} = 0 \end{cases} \tag{7}$$

$$\begin{cases} a_{02} + a_{12} + a_{22} = -\frac{1}{2} \\ 3a_{02} + 4a_{12} + 5a_{22} = -1 \\ 6a_{02} + 12a_{12} + 20a_{22} = -1 \end{cases} \tag{8}$$

According to this system of linear equations, the value of parameter $a_{jl}(j = 0, 1, 2; l = 0, 1, 2)$ can be determined:

$$\begin{cases} a_{00} = -10 \\ a_{10} = 15 \\ a_{20} = -6 \end{cases} \quad \begin{cases} a_{01} = -6 \\ a_{11} = 8 \\ a_{21} = -3 \end{cases} \quad \begin{cases} a_{02} = -1.5 \\ a_{12} = 1.5 \\ a_{22} = -0.5 \end{cases}$$

The terminal sliding-mode surface of the system can then be determined. The corresponding parameter a_{jl} of the *nth − order* system can also be determined with reference to the previous solution process, and its derivation is not repeated here.

After the terminal sliding-mode surface of the system has been determined, the next stage is to design a sliding-mode controller so as to ensure the existence of the sliding-mode phase.

IV. DESIGN OF TOTAL-AMOUNT SYNCHRONOUS CONTROLLER

In sliding-mode control, the control law $\boldsymbol{u} = [u_1, u_2, \ldots, u_n]^T$ should enable all state trajectories of the system to reach the sliding-mode surface $s(x, t) = 0$ in finite time. After reaching the sliding-mode surface, the system begins sliding-mode motion under the action of the sliding-mode control law. During the sliding phase, the system has invariance to parameter perturbations and external disturbances. From Equation (2), the tracking error e and its derivative can be obtained

$$e = z_1 + z_2 + \ldots + z_n - T^* \tag{9}$$

$$\begin{aligned} \dot{e} &= \dot{x}_{11} + \dot{x}_{21} + \ldots + \dot{x}_{n1} - \dot{T}^* \\ &= x_{12} + x_{22} + \ldots + x_{n2} - \dot{T}^* \end{aligned} \tag{10}$$

$$\ddot{e} = \dot{x}_{12} + \dot{x}_{22} + \ldots + \dot{x}_{n2} - \ddot{T}^* \tag{11}$$

From Equation (1)

$$\begin{aligned} \ddot{e} = & f_1(x_1, t) + \Delta f_1(x_1, t) + a_1 u_1 + d_1(t) \\ & + f_2(x_2, t) + \Delta f_2(x_2, t) + a_2 u_2 + d_2(t) \\ & + \vdots \\ & \vdots \\ & + f_n(x_n, t) + \Delta f_n(x_n, t) + a_n u_n + d_n(t) - \ddot{T}^* \end{aligned} \tag{12}$$

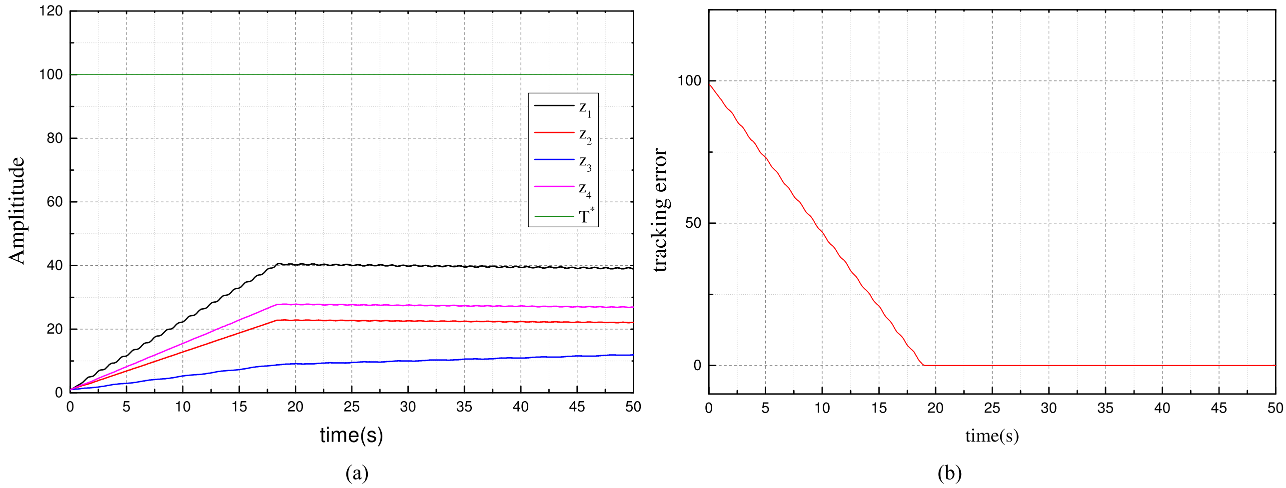


FIGURE 4. Output and tracking error in case 1 under linear switching function. (a) Output and reference signal. (b) Tracking error of output state variables.

Through the derivation of Equation (3), the following can be obtained:

$$\begin{aligned}\dot{s}(x,t) &= c_1(\dot{e}-\dot{p})+\ddot{e}-\ddot{p}\\ &= c_1(x_{12}+x_{22}+\ldots+x_{n2}-\dot{T}^*-\dot{p})\\ &\quad +f_1(x_1,t)+\Delta f_1(x_1,t)+a_1u_1+d_1(t)\\ &\quad +f_2(x_2,t)+\Delta f_2(x_2,t)+a_2u_2+d_2(t)\\ &\quad +\vdots\\ &\quad \vdots\\ &\quad +f_n(x_n,t)+\Delta f_n(x_n,t)+a_nu_n+d_n(t)\\ &\quad -\ddot{T}^*-\ddot{p} \end{aligned} \tag{13}$$

Theorem 1: For the class of n independent regular nonlinear systems described by Equation (1), if the sliding-mode variable structure control law of Equation (14) is used, the tracking error of a closed-loop system can be guaranteed to converge to zero in finite time:

$$\begin{aligned} u_1 &= -(a_1)^{-1}[c_1x_{12}-\frac{1}{n}c_1(\dot{T}^*+\dot{p})+f_1(x_1,t)\\ &\quad +(F_1+D_1+\eta_1)\,sgn(s)-\frac{1}{n}(\ddot{T}^*+\ddot{p})]\\ u_2 &= -(a_2)^{-1}[c_1x_{22}-\frac{1}{n}c_1(\dot{T}^*+\dot{p})+f_2(x_2,t)\\ &\quad +(F_2+D_2+\eta_2)\,sgn(s)-\frac{1}{n}(\ddot{T}^*+\ddot{p})]\\ &\vdots\\ &\vdots\\ u_n &= -(a_n)^{-1}[c_1x_{n2}-\frac{1}{n}c_1(\dot{T}^*+\dot{p})+f_n(x_n,t)\\ &\quad +(F_n+D_n+\eta_n)\,sgn(s)-\frac{1}{n}(\ddot{T}^*+\ddot{p})] \end{aligned} \tag{14}$$

where $\eta_1, \eta_2, \ldots, \eta_n$ are arbitrary constants greater than zero and $sgn(\cdot)$ is a symbolic function.

Proof: To prove the stability of the proposed control law, the Lyapunov method is used. Consider a positive definite Lyapunov function:

$$V = \frac{1}{2}s^2$$

The time derivative of this Lyapunov function is given by:

$$\begin{aligned}\dot{V} &= s\dot{s}\\ &= s\,[c_1(\dot{e}-\dot{p})+\ddot{e}-\ddot{p}]\\ &= s[c_1(x_{12}+x_{22}+\ldots+x_{n2}-\dot{T}^*-\dot{p})\\ &\quad +f_1(x_1,t)+\Delta f_1(x_1,t)+a_1u_1+d_1(t)\\ &\quad +f_2(x_2,t)+\Delta f_2(x_2,t)+a_2u_2+d_2(t)\\ &\quad +\vdots\\ &\quad \vdots\\ &\quad +f_n(x_n,t)+\Delta f_n(x_n,t)+a_nu_n+d_n(t)-\ddot{T}^*-\ddot{p}] \end{aligned}$$

Substituting Equation (13) into the above yields:

$$\begin{aligned}\dot{V} &= s[\Delta f_1(x_1,\mathrm{t})+d_1(t)+\Delta f_2(x_2,\mathrm{t})+d_2(t)\\ &\quad +\ldots+\Delta f_n(x_n,\mathrm{t})+d_n(t)-(F_1+D_1+\eta_1)\,\mathrm{sgn}(s)\\ &\quad -(F_2+D_2+\eta_2)\,\mathrm{sgn}(s)-\ldots-(F_n+D_n+\eta_n)\,\mathrm{sgn}(s)]\\ &= s[-\eta_1\,\mathrm{sgn}(s)-\eta_2\,\mathrm{sgn}(s)-\ldots-\eta_n\,\mathrm{sgn}(s)]\\ &= -(\eta_1+\eta_2+\ldots+\eta_n)\,|s| \le 0 \end{aligned}$$

Let $\boldsymbol{\xi}(t) = \boldsymbol{E}(t) - \boldsymbol{P}(t)$;according to the definition of the sliding-mode surface $s(x,t)$, $s(x,t) = \boldsymbol{C}(\boldsymbol{E}-\boldsymbol{P}) = \boldsymbol{C}\boldsymbol{\xi}$ is obtained. According to Assumption 1,$\xi(0) = 0$,and a closed-loop system goes into the sliding stage in its initial moments. According to the sliding-mode equivalence principle, $s(x,t) = 0$ and the system is on the sliding surface at any time $t \ge 0$. Select $p(t)$ according to Equation (4), so that

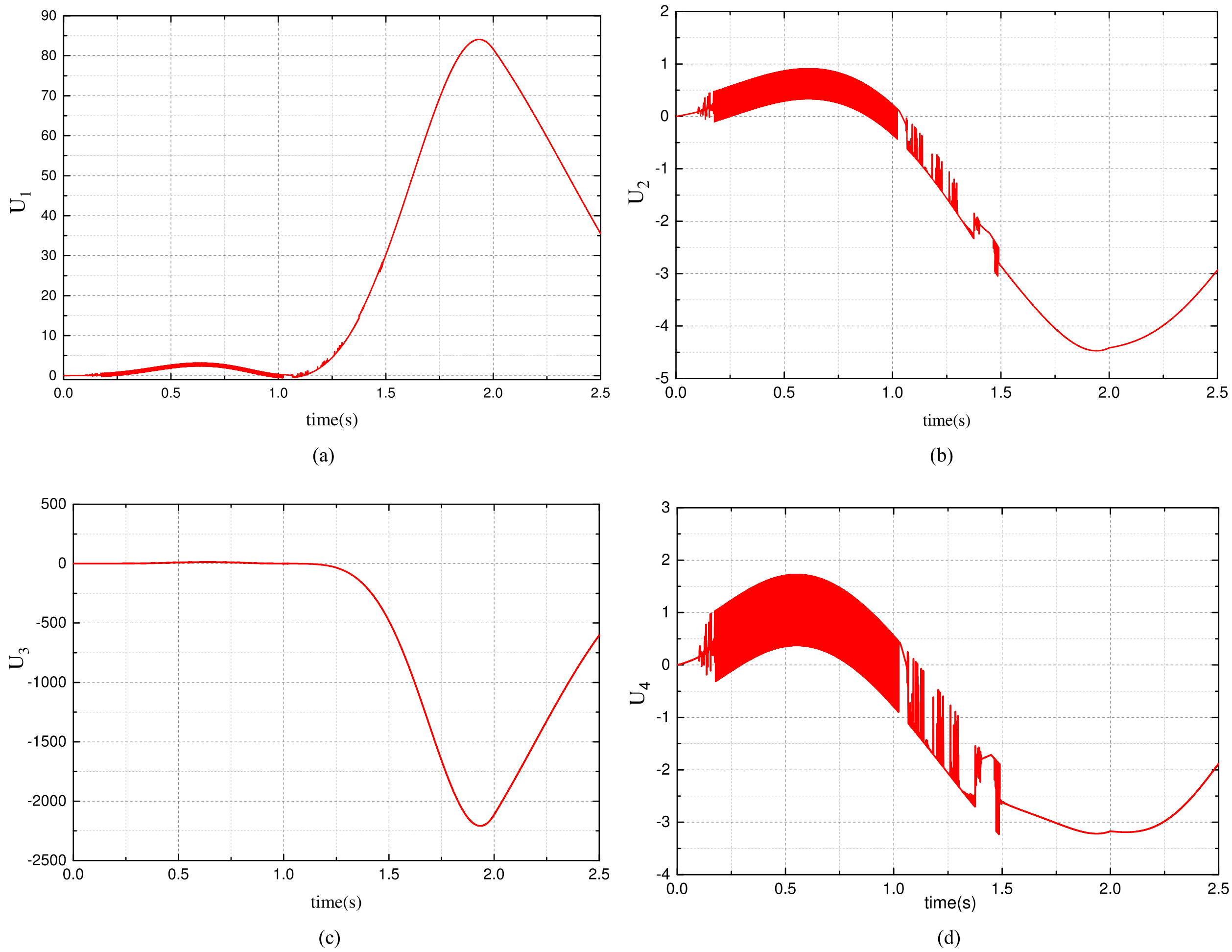


FIGURE 5. Sliding-mode controller output in case 2. (a) Control law U_1 of system 1. (b) Control law U_2 of system 2. (c) Control law U_3 of system 3. (d) Control law U_4 of system 4.

$\forall t \geq T$, all have $\boldsymbol{P}(t) = 0$ It can be guaranteed that the output tracking error converges to zero in a limited time.

This completes the proof. □

Remark 1: The sigmoid function $H(s) = 1/(1 + e^{-rs})$ *is used to replace the symbol function sgn(s). where r is a positive parameter to be designed, and r is used to adjust the slope of the sigmoid function, s represents the sliding-mode surface. For sliding-mode variable structure control, replacing the traditional switching function with a continuous sigmoid function can effectively reduce the chattering phenomenon arising from the controller.*

Remark 2: V is a positive definite function, but $\dot{V}$ *is a negative definite function, thus ensuring the existence of the sliding-mode surface. On the other hand, the following can be obtained from Assumption 1:* $s(x, 0) = c_1(e(0)-p(0))+\dot{e}(0)-\dot{p}(0) = 0$, $\dot{s}(x, 0) = c_1(\dot{e}(0)-\dot{p}(0))+\ddot{e}(0)-\ddot{p}(0) = 0$ *In other words, the initial system state is located on the sliding-mode surface, eliminating the arrival stage of sliding-mode control and making the whole process of sliding-mode control more robust.*

V. SIMULATION EXAMPLES

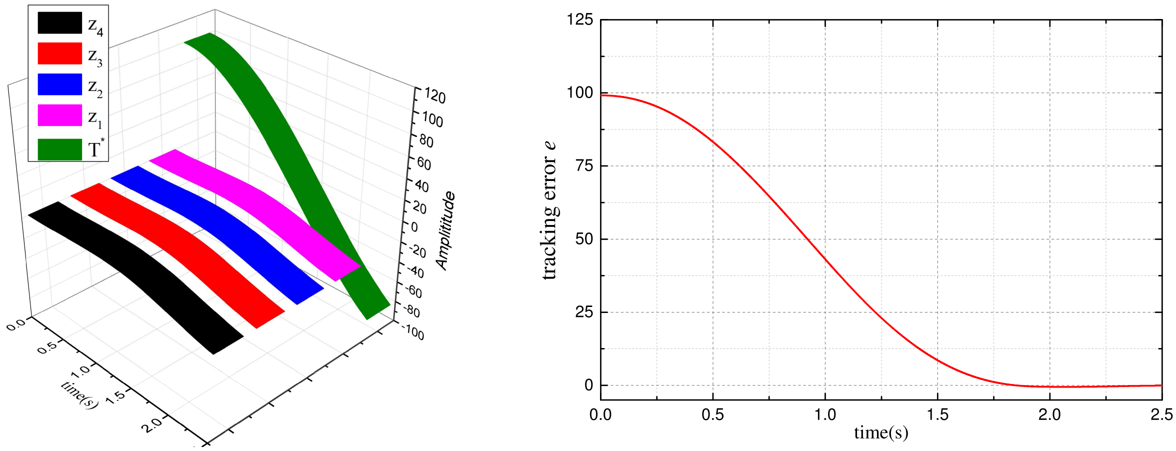
Considering four regular nonlinear systems, the system differential equations can be described as follows:

$$\begin{cases} \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = -11x_{12}^2 + 133u_1 + d_1(t) + \Delta f_1(x_{12}, t) \\ z_1 = x_{11} \end{cases} \quad (15)$$

$$\begin{cases} \dot{x}_{21} = x_{22} \\ \dot{x}_{22} = -\dfrac{25}{1 + e^{-t}} x_{22} + 111u_2 + d_2(t) + \Delta f_2(x_{22}, t) \\ z_2 = x_{21} \end{cases} \quad (16)$$

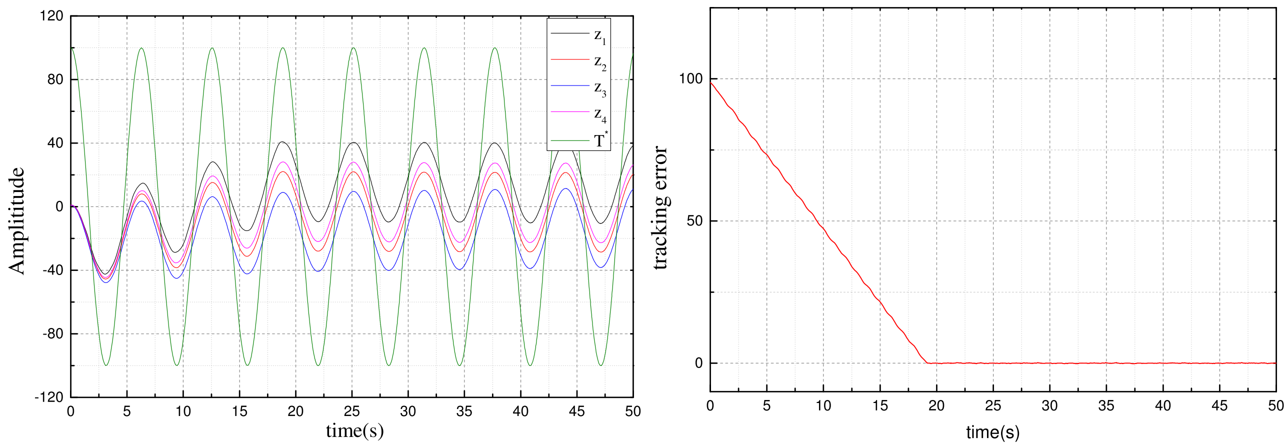
$$\begin{cases} \dot{x}_{31} = x_{32} \\ \dot{x}_{32} = -18x_{32}^3 + 100u_3 + d_3(t) + \Delta f_3(x_{32}, t) \\ z_3 = x_{31} \end{cases} \quad (17)$$

$$\begin{cases} \dot{x}_{41} = x_{42} \\ \dot{x}_{42} = -11(1 + e^{-t^2})x_{42} + 77u_4 + d_4(t) \\ \qquad + \Delta f_4(x_{42}, t) z_4 = x_{41} \end{cases} \quad (18)$$



(a) (b)

FIGURE 6. **Output and tracking error in case 2. (a) Output and reference signal. (b) Tracking error of output state variables.**



(a) (b)

FIGURE 7. **Output and tracking error in case 2 under linear switching function. (a) Output and reference signal. (b) Tracking error of output state variables.**

where the perturbation portion is:

$$d_1(t) = \begin{cases} 0, & t < 1 \\ 30\sin(2*pi*t), & t \geq 1 \end{cases}$$

$$d_2(t) = \begin{cases} 1, & t < 1 \\ 15\sin(-t^2)e^{-t}, & t \geq 1 \end{cases}$$

$$d_3(t) = \begin{cases} 0, & t < 1 \\ 3\cos(2t), & t \geq 1 \end{cases}$$

$$d_4(t) = \begin{cases} 0, & t < 1 \\ 20*\sin(2*pi*t), & t \geq 1 \end{cases}$$

and the parameter perturbation portion is:

$$\Delta f_1(x_{12}, t) = e^{-2t}\sin t, \quad \Delta f_2(x_{22}, t) = \sin x_{21}.$$
$$\Delta f_4(x_{42}, t) = \arcsin(2t), \quad \Delta f_3(x_{32}, t) = \cos(3t).$$

Select $c_1 = 15$, then $s = 15e + \dot{e} - 15p(t) - \dot{p}(t)$

$$p(t) = \begin{cases} e_0 + \dot{e}_0 + \frac{1}{2}\ddot{e}_0 t^2 - (\frac{10}{T^3}e_0 + \frac{6}{T^2}\dot{e}_0 + \frac{3}{2T}\ddot{e}_0)t^3 \\ \quad + (\frac{15}{T^4}e_0 + \frac{8}{T^3}\dot{e}_0 + \frac{3}{2T^2}\ddot{e}_0)t^4 \\ \quad - (\frac{6}{T^5}e_0 + \frac{3}{T^4}\dot{e}_0 + \frac{1}{2T^3}\ddot{e}_0)t^5, & 0 \leq t \leq T \\ 0, \quad t > T \end{cases}$$

According to Equation (14), the control laws are obtained:

$$u_1 = -\frac{1}{133}[15x_{12} - \frac{15}{4}(\dot{T}^* + \dot{P}) - 11x_{12}^2 + (F_1 + D_1 + \eta_1)\,\mathrm{sgn}(s) - \frac{1}{4}(\ddot{T}^* + \ddot{p})]$$

$$u_2 = -\frac{1}{111}[15x_{22} - \frac{15}{4}(\dot{T}^* + \dot{P}) - \frac{25}{1+e^{-t}}x_{22} + (F_2 + D_2 + \eta_2)\,\mathrm{sgn}(s) - \frac{1}{4}(\ddot{T}^* + \ddot{p})]$$

$$u_3 = -\frac{1}{100}[15x_{32} - \frac{15}{4}(\dot{T}^* + \dot{P}) - 18x_{32}^3 + (F_3 + D_3 + \eta_3)\operatorname{sgn}(s) - \frac{1}{4}(\ddot{T}^* + \ddot{p})]$$

$$u_4 = -\frac{1}{77}[15x_{22} - \frac{15}{4}(\dot{T}^* + \dot{P}) - 11(1 + e^{-t^2})x_{22} + (F_4 + D_4 + \eta_4)\operatorname{sgn}(s) - \frac{1}{4}(\ddot{T}^* + \ddot{p})]$$

Let the external perturbation upper bounds be $D_1 = 50, D_2 = 30, D_3 = 40, D_4 = 50$ and the parameter perturbation upper bounds be $F_i = 1 (i = 1, 2, 3, 4)$.Using the saturation function instead of the actual switching function, let the boundary layer thickness be $\delta = 0.02$, the initial system conditions be $x_1 = [0.15, 0], x_2 = [0.05, 0], x_3 = [0.4, 0], x_4 = [0.25, 0]$ and the terminal time be T=2.0.

Case 1: the reference trajectory is a constant value, $T_1^* = 100$

Figure 2 shows the output waveform of the control law for the four systems. The figure shows that after replacing the symbol function with the saturation function, the control signal is softened to some extent, and the chattering amplitude is very small. Figure 3(a) shows the waveform of the output state variables for every system. Figure 3(b) shows the waveform of the output tracking error. It is clear from Fig 3(b) that the terminal sliding-mode control technique can make the output tracking error converge to zero within a finite time T. Fig 4 shows that the sum of each individual state converges to given value is 18s under linear switching function.

Case 2: The reference trajectory is a time-varying value, $T_2^* = 100\sin(t + pi/2)$

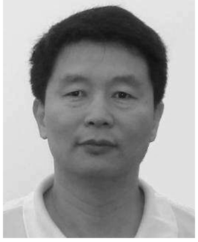
When the output waveform of the TASC controller shown in Figure 5 is compared with that of the tracking error in case 1, when the reference trajectory is a time-varying value $T_2^* = 100\sin(t + pi/2)$,the controller also shows good tracking performance. For t=1.75 s, the tracking error is in the range of engineering error, and the tracking error converges to zero at t=2 s, as shown in Figure 6. Figure 7 shows that the convergence time of the linear switch function is 18 s. Moreover, the terminal sliding-mode control technique suppresses the influence of uncertainties such as external disturbances and parameter perturbations on the system.

VI. CONCLUSION

This paper presents a method that extends the synchronous problem of individual states such as position and velocity to the synchronous consistent problem of state total amount. The problem of maintaining constant the sum of the output state variables of n regular nonlinear systems with parameter perturbations and external disturbances is discussed. The research idea is to convert the total-amount synchronization problem into a system finite-time convergence problem. In combination with a terminal sliding-mode control strategy, a simple TASC controller is derived. The controller can guarantee that the system state is on the sliding-mode surface at any time and that the output tracking error can converge to zero within a finite time T. In addition, the convergence time can be selected as any expectation value during the design process.

REFERENCES

[1] H. Su, M. Z. Q. Chen, X. Wang, and J. Lam, "Semiglobal observer-based leader-following consensus with input saturation," *IEEE Trans. Ind. Electron.*, vol. 61, no. 6, pp. 2842–2850, Jun. 2014.
[2] L. Scardovi and R. Sepulchre, "Synchronization in networks of identical linear systems," *Automatica*, vol. 45, no. 11, pp. 2557–2562, Nov. 2009.
[3] D. Li, Q. Liu, and X. Wang, "Distributed quantized consensus for agents on directed networks," *J. Syst. Sci. Complex.*, vol. 26, no. 4, pp. 489–511, Aug. 2013.
[4] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1465–1476, Sep. 2004.
[5] X. Wang, D. Xu, and H. Ji, "Robust almost output consensus in networks of nonlinear agents with external disturbances," *Automatica*, vol. 70, pp. 303–311, Aug. 2016.
[6] J. Zhou, J. Lu, and J. Lu, "Adaptive synchronization of an uncertain complex dynamical network," *IEEE Trans. Autom. Control*, vol. 51, no. 4, pp. 652–656, Apr. 2006.
[7] B. Liu, X. Liu, and G. Chen, "Robust impulsive synchronization of uncertain dynamical networks," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 52, no. 7, pp. 1431–1441, Jul. 2005.
[8] S. P. Bhat and D. S. Bernstein,"Finite-time stability of continuous autonomous systems," *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751–766, Feb. 2000.
[9] H. Du, Y. He, and Y. Cheng, "Finite-time synchronization of a class of second-order nonlinear multi-agent systems using output feedback control," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 61, no. 6, pp. 1778–1788, Jun. 2014.
[10] Z. Zuo and L. Tie, "Distributed robust finite-time nonlinear consensus protocols for multi-agent systems," *Int. J. Syst. Sci.*, vol. 47, no. 6, pp. 1366–1375, Jul. 2016.
[11] Y. Shtessel, C. Edwards, L. Fridman, and A. Levant, *Sliding Mode Control and Observation*. Basel, Switzerland: Springer, 2014.
[12] H. Du, X. Yu, M. Z. Q. Chen, and S. Li, "Chattering-free discrete-time sliding mode control," *Automatica*, vol. 68, pp. 87–91, Jun. 2016.
[13] S. Yu and X. Long, "Finite-time consensus for second-order multi-agent systems with disturbances by integral sliding mode," *Automatica*, vol. 54, pp. 158–165, Apr. 2015.
[14] S. Mondal, R. Su, and L. Xie, "Heterogeneous consensus of higher-order multi-agent systems with mismatched uncertainties using sliding mode control," *Int. J. Robust Nonlinear Control*, to be published, doi: 10.1002/rnc.3684.
[15] H. Du, G. Wen, and X. Yu, "Finite-time consensus of multiple nonholonomic chained-form systems based on recursive distributed observer," *Automatica*, vol. 62, pp. 236–242, Dec. 2015.
[16] K.-Y. Zhuang, H.-Y. Su, K.-Q. Zhang, and J. Chu, "Adaptive terminal sliding mode control for high-order nonlinear dynamic systems," *J. Zhejiang Univ.*, vol. 4, no. 1, pp. 58–63, Jun. 2003.
[17] M. Rahmani, A. Ghanbari, and M. M. Ettefagh, "Hybrid neural network fraction integral terminal sliding mode control of an Inchworm robot manipulator," *Mech. Syst. Signal Process.*, vol. 80, no. 1, pp. 117–136, Dec. 2016.



CHANGFAN ZHANG received the M.S. degree in electronics engineering from Southwest Jiaotong University, Chengdu, China, in 1989, and the Ph.D. degree in control theory and engineering from Hunan University, Changsha, China, in 2001. He was a Post-Doctoral Fellow with Central South University, Changsha, from 2001 to 2003, and a Visiting Research Fellow with the University of Waterloo, Waterloo, Canada, from 2007 to 2008. Since 1982, he has been with the College of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou, where he has been a Professor, since 2001.His research interests include fault diagnosis on electric machines and industrial process control.

ZHENZHEN LIN received the B.S degree in physics from Liaoning University, Shenyang, China, in 2014. He is currently pursuing the M.S. degree with the College of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou, China. His research interests include fault diagnosis on electric machines and industrial process control.

SIMON X. YANG (S'97–M'99–SM'08) received the B.Sc. degree in engineering physics from Beijing University, Beijing, China, in 1987, the M.Sc. degree in biophysics from the Chinese Academy of Sciences, Beijing, in 1990, the M.Sc. degree in electrical engineering from the University of Houston, Houstan, TX, USA, in 1996, and the Ph.D. degree in electrical and computer engineering from the University of Alberta, Edmonton, AB, Canada, in 1999. He joined the School of Engineering, University of Guelph, Guelph, ON, Canada, in 1999, where he is currently a Professor and the Head of the Advanced Robotics and Intelligent Systems Laboratory. He has authored over 400 refereed papers, including over 200 journal papers. His current research interests include intelligent systems, robotics, sensors and multisensor fusion, wireless sensor networks, control systems, soft computing, and computational neuroscience. He was a recipient of the Distinguished Professor Award at the University of Guelph. He was the General Chair of the 2011 IEEE International Conference on Automation and Logistics. He serves as an Editor-in-Chief of the *International Journal of Robotics and Automation* and the *Journal of Robotics and Artificial Intelligence*, and an Associate Editor of the IEEE Transactions on Cybernetics, Sensors, and several other journals. He has been involved in the organization of many conferences.

JING HE received the M.S. degree in computer engineering from the Central South University of Forestry and Technology, Changsha, China, in 2002, and the Ph.D. degree in mechatronics engineering from the National University of Defense Technology, Changsha, China, in 2009. She was a Visiting Research Fellow with the Helsinki University of Technology, Helsinki, Finland, from 2004 to 2005, and with the University of Waterloo, Waterloo, Canada, from 2007 to 2008. Since 1992, she has been with the College of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou, where she has been a Professor, since 2009. Since 2011, she has been a Post-Doctoral Fellow with the National University of Defense Technology. Her research interests include fault diagnosis on mechatronics and industrial process control.