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On the Capacity Scaling of Large Multipair Relay Networks With Successive Relaying Protocol

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ABSTRACT In this paper, we consider large multi-pair relay networks with K fixed source-destination pairs and M relays randomly distributed in a given area, where each node is equipped with a single antenna and works on half duplex. Each source communicates with its corresponding destination under the aid of the relays. With the conventional two-slot relaying protocol, the sum capacity was found to scale as $(K/2)\log(M) + \mathcal{O}(1)$, where K is fixed and $M \rightarrow \infty$. This paper proves that the capacity scaling law can be further improved to $K \log(M) + \mathcal{O}(1)$ with successive relaying protocol, as if the relays became "full duplex." To prove the scaling law, a distributed amplify-and-forward scheme is proposed, which only requires local channel state information (CSI) at each relay and statistical global CSI at the sources and destinations. Furthermore, we prove that imperfect CSI at the relays would not affect the scaling law.

INDEX TERMS Large relay networks, capacity scaling law, successive relaying protocol, amplify-and-forward.

I. INTRODUCTION

In recent few years, relaying communications systems have attracted continuous research interests due to the potential to extend the network coverage, improve the network capacity and enhance the link reliability. Especially, large relay networks receive considerable attentions, since the next generation network, e.g., Internet of Things (IoT) and Internet of Vehicles (IoV), typically supports a large number of devices which can serve as relays to improve the system performance.

The aim of this paper is to investigate the information-theoretic performance limits for large relay networks. It is known that in general the capacity of relay networks is still open. Nevertheless, there have been a plethora of good results on the asymptotic capacity (or capacity scaling law) of large relay networks.¹ As the pioneering work, Gupta and Kumar [1] introduced the framework to investigate the asymptotic capacity for wireless networks, and derived the capacity scaling law for ad hoc networks with a large number of nodes under AWGN channel. Then the

results of [1] were improved in [2] by using more sophisticated receivers and in [3] by enabling node mobility. Further, the capacity scaling law have been reported for large extended AWGN network in [4] and [5], for the case of multi-hop routing in [6], and for the case where the nodes have multiple antennas in [7], etc. As for large relay networks, literature [8] first derived the capacity scaling law over the relay number under AWGN channel. The power scaling law was investigated in [9] under the fading channel. The authors in [7], [10]–[12], and the references therein analyzed the capacity scaling law of large relay networks under the cases where nodes are equipped with multiple antennas. Besides, the scaling law of two-way relay networks was studied in the works [13], [14].

As the most related literature to this paper, [11] considered the one-way two-hop large relay network with single source and destination both equipped with K antennas and M relays. With the half-duplex restriction, a two-slot relaying protocol is applied where the source transmits the data frame in the first time slot and the relays forward in the second. It was proved that the capacity scales as $\frac{K}{2}\log(M) + \mathcal{O}(1)$ when $M \rightarrow \infty$ but K is fixed. Furthermore, as shown in [15], imperfect CSI and asynchronization

¹To avoid ambiguity, note that the capacity discussed here and in the mentioned literature is the one induced by a specific protocol, not the one of the network itself.

at relays would not affect the capacity scaling behavior. Morgenshtern and Bölcskei [16] extended the results in [11] to large multi-pair relay networks where both K and M approach infinity under some constraints on the growth rate. Note that all the above works applied a two-slot relaying protocol into the one-way relaying network, resulting in the $\frac{1}{2}$ rate-loss.

Instead of the aforementioned two-slot relaying protocol, this paper resorts to successive relaying protocol which can achieve a better capacity scaling law. This protocol is originally proposed for the one-way two-relay network to overcome the multiplexing loss brought by two-slot relaying protocol [17]–[20]. In this paper, we consider a large one-way multi-pair relay network with K pre-fixed source-destination pairs and M relays randomly distributed in a given area with a certain spatial distribution. Each source communicates to its corresponding destination and all the nodes are equipped with a single antenna each. Part of relays are selected and then divided into two groups to successively help the sources convey information to the destinations, which is named ‘cluster’ successive relaying [21]. With this protocol, each data frame of the source which contains $L - 1$ subframes is transmitted to the destination in L time slots.

The main contributions of this paper are summarized as follows:

- 1) We derive an upper bound on the sum capacity of the considered large multi-pair relay network with successive relaying protocol. We prove that the upper bound scales as $K \log(M) + \mathcal{O}(1)$ with probability 1 (w.p.1) when K is arbitrary but fixed, $M \rightarrow \infty$ and then $L \rightarrow \infty$.
- 2) We propose a distributed amplify-and-forward (AF) relaying scheme with successive relaying protocol, and derive the achievable sum rate, which can serve as a lower bound on the sum capacity. Note that the proposed scheme only requires perfect local CSI at each relay and statistical global CSI at each source and destination. We prove that, the capacity lower bound scales as $K \log(M) + \mathcal{O}(1)$ w.p.1, for an arbitrary but fixed K , $M \rightarrow \infty$ and then $L \rightarrow \infty$.
- 3) We further investigate the case of imperfect local CSI at the relays, i.e., each relay is assumed to suffer channel estimation error satisfying Gaussian distribution with zero-mean and bounded variance. We derive the achievable sum rate for the proposed distributed AF scheme under this case, and prove that it still scales as $K \log(M) + \mathcal{O}(1)$.
- 4) With the above results, we establish the capacity scaling law for the large multi-pair relay network with successive relaying protocol, which is $K \log(M) + \mathcal{O}(1)$ w.p.1, when K is arbitrary but fixed, $M \rightarrow \infty$ and then $L \rightarrow \infty$, in both cases where the relays have perfect or imperfect local CSI.

We note that this paper is an extension of our conference paper [23] which only considered the case of perfect local CSI at the relays. Moreover, in [23] we assumed that the distance

between each transmit and receiver pair is lower bounded by some fixed value without any precondition. The assumption is not valid for the studied network in this paper. As the number of relays in a fixed area increases, the distances between some relays will be arbitrarily small. In this paper, the assumption is removed and the rigorous proof for the capacity scaling law is provided.

The remaining of this paper is organized as follows. In Section II, both the system model and successive relaying protocol are introduced. A capacity upper bound and the corresponding scaling law are derived in Section III. Section IV presents a distributed AF relaying scheme, and then its achievable sum rate together with the corresponding scaling law are obtained when the relays have perfect or imperfect local CSI with channel estimation error, respectively. Performance simulation and comparison are conducted in Section V. Section VI concludes the whole paper.

II. SYSTEM MODEL AND TRANSMISSION PROTOCOL

A. NOTATION

Throughout the paper, vectors and matrices are denoted by boldface characters. For a matrix \mathbf{A} , \mathbf{A}^T and \mathbf{A}^H denote its transpose and conjugate transpose. $\text{Tr}(\mathbf{A})$ and $\|\mathbf{A}\|$ represent the trace and Frobenius norm of the matrix. \mathbf{I}_m denotes the m -by- m identity matrix. For a vector \mathbf{g} , $\text{diag}(\mathbf{g})$ represents the diagonal matrix whose diagonal entries are those of \mathbf{g} . We use $u(x) = \mathcal{O}(v(x))$ if $\left| \frac{u(x)}{v(x)} \right|$ remains bounded as $x \rightarrow \infty$. Note that we consider a wireless network where the relays are randomly distributed in a fixed area, and the wireless channel between each transmitter and receiver in the network experiences random fading. When the relay location is fixed, the expectation over the channel fading is denoted by $E_f(\cdot)$. On the other hand, $E_{p\{\cdot\}}(\cdot)$ is used to represent the expectation (or spatial average) conditioned on the location of some relay nodes. For instance, consider the channel coefficient $f_{k,i}$ between the relays R_k and R_i , the spatial average of the expectation (over channel fading) of $|f_{k,i}|^2$ conditioned on the location of R_i can be represented as $E_{p\{R_i\}}(E(|f_{k,i}|^2))$. At last, we use $E_p(\cdot)$ to denote the spatial average over the positions of all relays.

B. SYSTEM MODEL

We consider a wireless network with M relays (denoted by R_m , $m = 1, \dots, M$) and K designated pairs of sources and destinations (denoted by S_k and D_k , $k = 1, \dots, K$, respectively), as depicted in Fig. 1. The M relays are independently and randomly distributed according to a certain spatial distribution \mathcal{F} within a domain \mathcal{A} of fixed area. The sources and destinations are located outside \mathcal{A} . All the terminals are equipped with a single antenna each and on half-duplex. We assume no direct links between all the sources and the destinations due to the large distances or the obstacles between them. Part of the relays are selected to form two disjoint groups, while the unchosen ones always

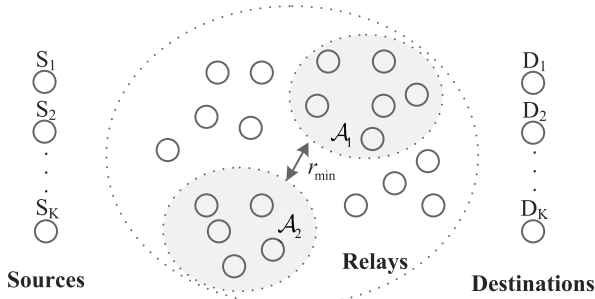


FIGURE 1. Large relay networks with K designated pairs of sources and destinations, and M relays randomly distributed in a given area.

keep idle. Each source S_k sends independent data frames to its paired destination D_k (for $k = 1, \dots, K$) with the help of the two relay groups using successive relaying protocol. The details of the protocol will be discussed in the next subsection.

We assume that there is no line-of-sight between each transmitter and receiver, and the channel experiences independent frequency-flat block fading which is modelled by circularly symmetric complex Gaussian distribution with zero mean. For each transmitter and receiver, the variance of channel coefficient is assumed to be bounded² if the distance between them is larger than a ‘minimal distance’ [8], [11] of some fixed value r_{min} . We also assume that the duration of one time slot equals the channel coherent time. Consequently, the channel coefficient for each transmitter-receiver pair is assumed identically and independently distributed (i.i.d.) across each time slot. Channel reciprocity is assumed, i.e., the channel between each transmitter and receiver has the same coefficient on both directions.

At last, we have an assumption on the spatial distribution \mathcal{F} of the relays:

Assumption 1: Within the domain \mathcal{A} , there exists two disjoint domains \mathcal{A}_1 and \mathcal{A}_2 of non-zero areas which simultaneously satisfy the following two conditions:

- 1) For arbitrary two points $x_1 \in \mathcal{A}_1$ and $x_2 \in \mathcal{A}_2$, the distance between x_i (for $i = 1, 2$) and each source or destination, or between x_1 and x_2 is no smaller than r_{min} .
- 2) For an arbitrary relay R_m , $m = 1, \dots, M$, the probabilities that it is located in \mathcal{A}_1 or \mathcal{A}_2 are both non-zero, i.e.,

$$\Pr(R_m \in \mathcal{A}_1) \triangleq \lambda_1 > 0, \quad \Pr(R_m \in \mathcal{A}_2) \triangleq \lambda_2 > 0, \quad (1)$$

Remark: Note that Assumption 1 is weak in general, e.g. it is satisfied when the relays are uniformly distributed. Assumption 1 is required when we prove the lower bound of the capacity scaling law.

²Note that the variance of the channel coefficient depends on the locations of the corresponding transmitter and receiver. Hence the channel coefficient variance is a random variable related to the relay spatial distribution.

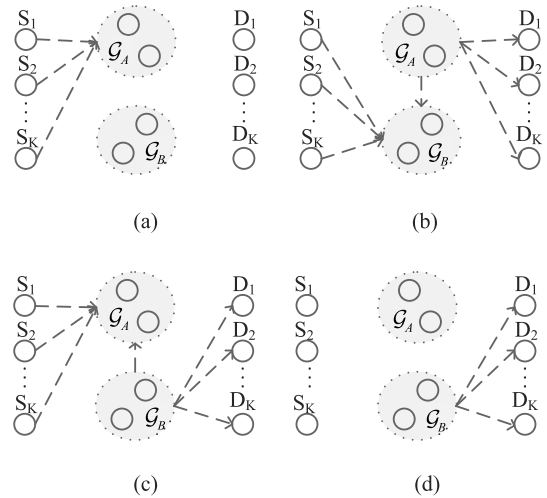


FIGURE 2. Successive relaying protocol in multipair relay networks. (a) the 1st slot. (b) the even slot. (c) the odd slot. (d) the last (odd) slot.

C. SUCCESSIVE RELAYING PROTOCOL

Before transmission, part of relays are selected to form two groups \mathcal{G}_A and \mathcal{G}_B , respectively.³ The sizes of \mathcal{G}_A and \mathcal{G}_B are denoted by M_A and M_B , respectively.

In each transmission round, each source transmits one data frame containing $L - 1$ subframes within L time slots. (Without loss of generality, an odd L is assumed throughout the paper.) Explicitly, each source transmits its l th subframe in the l th time slot for $l = 1, \dots, L - 1$. Meanwhile, relay groups \mathcal{G}_A and \mathcal{G}_B take turns to listen or forward the received subframes to the destinations from the second to the L th slots. The details are given in the following.

As shown in Fig.2 (a), in the first slot all the sources simultaneously transmit their first subframes in the network, while all the relays keep idle. The received signal of each relay in group \mathcal{G}_A is :

$$r_m^{(1)} = \mathbf{h}_m^{(1)} \mathbf{x}^{(1)} + w_m^{(1)}, \quad R_m \in \mathcal{G}_A, \quad (2)$$

where $\mathbf{x}^{(l)} = [x_1^{(l)}, \dots, x_K^{(l)}]^T$ and $x_k^{(l)}$ denotes the transmitted signal from source S_k (for $k = 1, \dots, K$) in the l th slot. $\mathbf{h}_m^{(l)} = [h_{m,1}^{(l)}, \dots, h_{m,K}^{(l)}]$ are the complex Gaussian distributed channel coefficients from source S_k (for $k = 1, \dots, K$) to relay R_m (for $m = 1, \dots, M$), where $h_{m,k}^{(l)} \sim \mathcal{CN}(0, \alpha_{m,k})$. $w_m^{(l)}$ is the additive white Gaussian noise (AWGN) sample at relay R_m with unit variance.

As depicted in Fig.2 (b), in the second slot and the following even slots l (for $l = 2, 4, \dots, L - 1$), all the sources and the relays belonging to group \mathcal{G}_A transmit in the network at the same time, and relay group \mathcal{G}_B is silent. The received signal at relay $R_n \in \mathcal{G}_B$ is given by:

$$r_n^{(l)} = \mathbf{h}_n^{(l)} \mathbf{x}^{(l)} + \sum_{R_m \in \mathcal{G}_A} f_{m,n}^{(l)} r_m^{(l)} + w_n^{(l)}, \quad R_n \in \mathcal{G}_B, \quad (3)$$

³In the derivation of the capacity upper bound, we do not restrict to any specific relay selection method. To derive the capacity lower bound, a relay selection method is given in Section IV.A.

where $f_{m,n}^{(l)}$ is the channel coefficient between relays R_m and R_n (for $m = 1, \dots, M, n = 1, \dots, M$ and $m \neq n$) and $f_{m,n}^{(l)} \sim \mathcal{CN}(0, \eta_{m,n})$. $t_m^{(l)}$ represents the transmitted signal by R_m in the l th slot. Meanwhile, destination D_k (for $k = 1, \dots, K$) receives the signals from relay group \mathcal{G}_A :

$$y_k^{(l)} = \sum_{R_m \in \mathcal{G}_A} g_{m,k}^{(l)} t_m^{(l)} + w_{D,k}^{(l)}, \quad k = 1, \dots, K, \quad (4)$$

where $g_{m,k}^{(l)}$ denotes the channel coefficient from relay R_m (for $m = 1, \dots, M$) to destination D_k (for $k = 1, \dots, K$) in the l th slot, and $g_{m,k}^{(l)} \sim \mathcal{CN}(0, \beta_{m,k})$.

In the third and the following odd slots as shown in Fig. 2 (c), the sources together with relay group \mathcal{G}_B transmit in the network. The relays in group \mathcal{G}_A listen. The received signals at the relays in \mathcal{G}_A and the destinations are given by (for $l = 3, 5, \dots, L$):

$$r_m^{(l)} = \mathbf{h}_m^{(l)} \mathbf{x}^{(l)} + \sum_{R_n \in \mathcal{G}_B} f_{n,m}^{(l)} t_n^{(l)} + w_m^{(l)}, R_m \in \mathcal{G}_A, \quad (5)$$

$$y_k^{(l)} = \sum_{R_n \in \mathcal{G}_B} g_{n,k}^{(l)} t_n^{(l)} + w_{D,k}^{(l)}, k = 1, \dots, K, \quad (6)$$

respectively.

In the last (L th) slot as shown in Fig 2 (d), only the relays belonging to group \mathcal{G}_B transmit to the destinations. The received signals at the destinations are similar to (6).

Finally, each source is subject to a long-term power constraint.⁴ Note that the sources do not transmit in the last slot and we have (for $k = 1, \dots, K$):

$$\frac{1}{L} \sum_{l=1}^{L-1} E_f \left(|x_k^{(l)}|^2 \right) \leq P_S. \quad (7)$$

All the relays have a long-term sum power constraint, i.e.:

$$\frac{1}{L} \sum_{l=2}^L \sum_{m=1}^M E_f \left(|t_m^{(l)}|^2 \right) \leq P_R. \quad (8)$$

Note that the relays not belonging to groups \mathcal{G}_A or \mathcal{G}_B keep idle all the time and hence have null inputs $t_m^{(l)}$.

III. UPPER BOUND ON THE SUM CAPACITY AND ITS SCALING LAW

In this section, the upper bound on the sum capacity of the multi-pair relay network with successive relaying protocol is derived using cut-set bound [22]. Explicitly, we consider the first hop from the sources to the relays which forms a distributed multiple-input-multiple-output (MIMO) system. The upper bound on the achievable rate through the first hop can serve as the capacity upper bound. Based on this, the scaling law on the capacity upper bound is given.

⁴Note that a long-term sum power constraint for all the sources will not affect the main results.

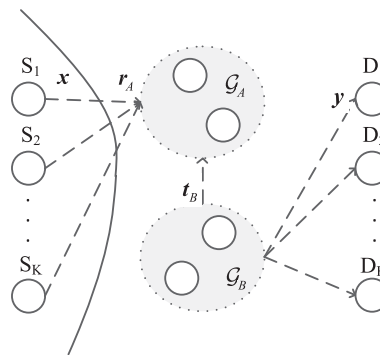


FIGURE 3. The cut across the relay network to derive the capacity upper bound. An odd slot is taken as an example.

Theorem 1: For the large multipair relay network with successive relaying protocol, the capacity is upper bounded by $K \log(M) + \mathcal{O}(1)$ w.p.1, with an arbitrary but fixed $K, M \rightarrow \infty$ and then $L \rightarrow \infty$.

*Proof:*⁵ Consider an arbitrary multipair relay network which is a realization of the relay distribution \mathcal{F} . For this network, consider an arbitrary relay selection method. In the odd time slot l (excluding the last slot), both the sources and relay group \mathcal{G}_B transmit in the network. The achievable sum rate of the network in the l th slot is upper bounded by the maximal mutual information across the cut as depicted in Fig.3:

$$R_{sum}^{(l)} \leq E_f \left(\max_{p(\mathbf{x}^{(l)})} I \left(\mathbf{x}^{(l)}; \mathbf{r}_A^{(l)}, \mathbf{y}^{(l)} | \mathbf{t}_B^{(l)} \right) \right), \quad (9)$$

where, (recalling that $E_f(\cdot)$ denote the expectation over channel fading), $\mathbf{x}^{(l)} = (x_1^{(l)}, x_2^{(l)}, \dots, x_K^{(l)})$ and $\mathbf{t}_B^{(l)} = (t_{m_1}^{(l)}, t_{m_2}^{(l)}, \dots, t_{m_{M_B}}^{(l)})$ represent the transmitted signal vectors from the sources and relay group \mathcal{G}_B , respectively, $\mathbf{y}^{(l)} = (y_1^{(l)}, y_2^{(l)}, \dots, y_K^{(l)})$ and $\mathbf{r}_A^{(l)} = (r_{n_1}^{(l)}, r_{n_2}^{(l)}, \dots, r_{n_{M_A}}^{(l)})$ represent the received signal vectors at the destinations and the relays of group \mathcal{G}_A in the l th slot, respectively. Assuming all the nodes have the knowledge of global CSI (since we are deriving the upper bound), we have [23]:

$$I(\mathbf{x}; \mathbf{r}_A, \mathbf{y} | \mathbf{t}_B) \leq I(\mathbf{x}; \tilde{\mathbf{r}}_A), \quad (10)$$

where the slot number l is omitted and each element \tilde{r}_m in $\tilde{\mathbf{r}}_A$ (i.e., $R_m \in \mathcal{G}_A$) is defined by: $\tilde{r}_m = r_m - \sum_{R_n \in \mathcal{G}_B} f_{n,m} t_n = \mathbf{h}_m \mathbf{x} + w_m$. Note that the right hand side of (10) is the sum rate with which all the sources can communicate to the relays. Then, using Gaussian channel inputs,

⁵This Theorem is proved in [23] under a per-slot power constraint for each source, i.e., $E_f \left(|x_k^{(l)}|^2 \right) \leq P_S$ for $l = 1, \dots, L$. Note that in this paper a long-term power constraint (7) is assumed, which introduce some difference in the proof.

we have [23]:

$$\begin{aligned}
 & E_f \left(\max_{p(\mathbf{x}^{(l)})} I(\mathbf{x}^{(l)}; \tilde{\mathbf{r}}_A^{(l)}) \right) \\
 & \leq K \log(M) + \sum_{k=1}^K \log \left(\frac{1}{M} + E_f \left(\frac{P_{S_k}^{(l)}}{M} \sum_{m=1}^M h_{k,m}^{(l)2} \right) \right) \\
 & \stackrel{(a)w.p.1}{\rightarrow} K \log(M) + \sum_{k=1}^K \log \left(E_f \left(\frac{P_{S_k}^{(l)}}{M} \sum_{m=1}^M \alpha_{k,m} \right) \right) \\
 & \stackrel{(b)}{\leq} K \log(M) + \sum_{k=1}^K \log \left(\bar{P}_{S_k}^{(l)} \alpha^{upper} \right) \tag{11}
 \end{aligned}$$

where $P_{S_k}^{(l)}$ ($k = 1, \dots, K$) is the transmit power of source S_k in the l th slot. Note that since a long-term power constraint is assumed, each source may have a different transmit power in different slots and for different realizations of channel fading. (a) is due to the law of large numbers [24] when $M \rightarrow \infty$. In (b), α^{upper} is an upper bound for all $\alpha_{m,k}$ which is a fixed value only depending on the locations of the sources and the given domain \mathcal{A} where relays are located, and $\bar{P}_{S_k}^{(l)}$ represents the average transmit power of S_k in the l th slot in each transmission round, i.e., $\bar{P}_{S_k}^{(l)} = E_f \left(P_{S_k}^{(l)} \right)$.

For the slot of an even number l , a similar upper bound can be derived as in (11). Note that in the last slot, the sources are silent. With (11), the achievable sum rate of the network is upper bounded by:

$$\begin{aligned}
 R_{sum} &= \frac{1}{L} \sum_{l=1}^{L-1} R_{sum}^{(l)} \\
 &\leq \frac{L-1}{L} K \log(M) + \frac{1}{L} \sum_{l=1}^{L-1} \sum_{k=1}^K \log \left(\bar{P}_{S_k}^{(l)} \alpha^{upper} \right) \\
 &\stackrel{(a)}{\leq} \frac{L-1}{L} K \log(M) + \frac{L-1}{L} K \log \left(\frac{LP_S \alpha^{upper}}{L-1} \right) \\
 &\triangleq C^U. \tag{12}
 \end{aligned}$$

where (a) is due to Jensen's inequality and the long-term power constraint for each source: $\sum_{l=1}^{L-1} \bar{P}_{S_k}^{(l)} = LP_S$. Then it is straightforward that C^U approaches $K \log(M) + \mathcal{O}(1)$ when K is fixed, $M \rightarrow \infty$ and then $L \rightarrow \infty$. Note that for an arbitrary realization of the relay distribution \mathcal{F} with arbitrary valid relay selection method, any achievable sum rate of the network is upper bounded by (12). Therefore, we complete the proof. ■

IV. LOWER BOUND ON THE SUM CAPACITY AND ITS SCALING LAW

In this section, we present a distributed AF relaying scheme assuming the relays have perfect local CSI. The achievable sum rate is derived which serves as a lower bound on the sum capacity of the network. Then we prove that the lower bound achieves the same scaling law as that of the upper bound. Moreover, the case of imperfect local CSI at relays is

investigated, wherein channel estimation errors with bounded variances are assumed for each relay. We derive the achievable sum rate by the similar distributed AF scheme in this case and prove that it has the same scaling law.

A. DISTRIBUTED AMPLIFY-AND-FORWARD RELAYING SCHEME

Firstly, we propose a simple but effective method to select part of the M relays within domain \mathcal{A} to form two groups \mathcal{G}_A and \mathcal{G}_B . According to Assumption 1 (given in Section 2) for the relay distribution \mathcal{F} , we simply choose the relays within domain \mathcal{A}_1 for group \mathcal{G}_A and those within \mathcal{A}_2 for \mathcal{G}_B .⁶ As a result, the distance between each relay in \mathcal{G}_A and each in \mathcal{G}_B has a fixed lower bound r_{min} . Again the sizes of \mathcal{G}_A and \mathcal{G}_B are denoted by M_A and M_B , respectively.

Then we give a distributed AF relaying scheme with which only local CSI is needed by each relay, i.e., for the channel from each source/destination to the relay itself. Moreover, each destination can recover the information from the corresponding source solely with statistical global CSI. In practice, the relay can acquire local CSI through the training signals sent in sequence by each source and destination before each time slot. Since the number of sources or destinations is fixed and much smaller than that of the relays, the cost for CSI acquisition by the relays is affordable.

Each source independently encodes its information with a Gaussian codebook, and the transmit power in each slot is fixed to P_S .⁷ Then the transmitted signals of the sources in all the slots (except the last slot) satisfy: $x_k^{(l)} \sim \mathcal{CN}(0, P_S)$ for $k = 1, \dots, K$, and $l = 1, \dots, L-1$.

Each relay in groups \mathcal{G}_A and \mathcal{G}_B applies a distributed AF relaying scheme, (which has been applied for two-slot relaying protocol in [11] and [13]). The basic idea of the scheme is to make the intended signals from source S_k (for $k = 1, \dots, K$) coherently superimposed at destination D_k , while the interferences (including inter-relay interference (IRI) and inter-source interference) and noises are added non-coherently. Let us take the even slots as an example to illustrate the scheme. In the even slot l (for $l = 2, 4, \dots, L$), each relay $R_m \in \mathcal{G}_A$ linearly processes and then forwards the previously received signals. The transmitted signal from relay R_m is:

$$t_m^{(l)} = \gamma_m^{(l)} \mathbf{g}_m^{(l)H} \mathbf{h}_m^{(l-1)H} r_m^{(l-1)}, \tag{13}$$

where $\gamma_m^{(l)}$ is the amplifying coefficient of relay R_m to satisfy the long-term power restriction (8) and recalling that $\mathbf{h}_m^{(l)} = [h_{m,1}^{(l)}, \dots, h_{m,K}^{(l)}]$ and $\mathbf{g}_m^{(l)} = [g_{m,1}^{(l)}, \dots, g_{m,K}^{(l)}]$. Note that $\gamma_m^{(l)}$ is independent on instant channel coefficients. Substituting (2) and (5) into (13), we have (recalling that an odd L is assumed) (14) shown on the top of the next page.

⁶It is clear that there may be multiple pair of domains \mathcal{A}_1 and \mathcal{A}_2 satisfying Assumption 1. We choose arbitrary one pair among them.

⁷Actually, the transmit power can be a little larger, i.e., $\frac{LP_S}{L-1}$, which, however, does not affect the final result.

$$t_m^{(l)} = \begin{cases} \gamma_m^{(l)} \mathbf{g}_m^{(l)H} \mathbf{h}_m^{(l-1)H} \mathbf{h}_m^{(l-1)} \mathbf{x}^{(l-1)} + \gamma_m^{(l)} \mathbf{g}_m^{(l)H} \mathbf{h}_m^{(l-1)H} w_m^{(l-1)}, & l = 2 \\ \gamma_m^{(l)} \mathbf{g}_m^{(l)H} \mathbf{h}_m^{(l-1)H} \mathbf{h}_m^{(l-1)} \mathbf{x}^{(l-1)} + \gamma_m^{(l)} \mathbf{g}_m^{(l)H} \mathbf{h}_m^{(l-1)H} \left(\sum_{R_n \in \mathcal{G}_B} f_{n,m}^{(l-1)} t_n^{(l-1)} + w_m^{(l-1)} \right), & l = 4, 6, \dots, L-1 \end{cases} \quad (14)$$

$$P_{R_m}^{(2)} = E_f \left(|t_m^{(2)}|^2 \right) = (\gamma_m^{(2)})^2 \left(P_S \left(\sum_{k=1}^K \sum_{i=1}^K \alpha_{m,k} \alpha_{m,i} \beta_{m,k} + \sum_{k=1}^K \alpha_{m,k}^2 \beta_{m,k} \right) + \sum_{k=1}^K \alpha_{m,k} \beta_{m,k} \right). \quad (15)$$

$$P_{R_m}^{(l)} = E_f \left(|t_m^{(l)}|^2 \right) = (\gamma_m^{(l)})^2 \left(P_S \left(\sum_{k=1}^K \sum_{i=1}^K \alpha_{m,k} \alpha_{m,i} \beta_{m,k} + \sum_{k=1}^K \alpha_{m,k}^2 \beta_{m,k} \right) + \sum_{k=1}^K \alpha_{m,k} \beta_{m,k} + \sum_{k=1}^K \alpha_{m,k} \beta_{m,k} \sum_{R_n \in \mathcal{G}_B} \eta_{n,m} P_{R_n}^{(l-1)} \right) \quad (16)$$

$$\gamma_m^{(2)} = \frac{1}{\sqrt{M_A}} \sqrt{\frac{P_R}{\left(P_S \left(\sum_{k=1}^K \sum_{i=1}^K \alpha_{m,k} \alpha_{m,i} \beta_{m,k} + \sum_{k=1}^K \alpha_{m,k}^2 \beta_{m,k} \right) + \sum_{k=1}^K \alpha_{m,k} \beta_{m,k} \right)}} \quad (17)$$

$$\gamma_m^{(l)} = \frac{1}{\sqrt{M_A}} \sqrt{\frac{P_R}{\left(P_S \left(\sum_{k=1}^K \sum_{i=1}^K \alpha_{m,k} \alpha_{m,i} \beta_{m,k} + \sum_{k=1}^K \alpha_{m,k}^2 \beta_{m,k} \right) + \sum_{k=1}^K \alpha_{m,k} \beta_{m,k} + \sum_{k=1}^K \alpha_{m,k} \beta_{m,k} \sum_{R_n \in \mathcal{G}_B} \eta_{n,m} \frac{P_R}{M_B} \right)}} \quad (18)$$

As shown in Appendix A, the long-term average transmit power (over the channel fading) of each relay $R_m \in \mathcal{G}_A$ for each slot is given as follows:

For the second slot, the transmit power is given by (15), as shown at the top of this page. And for the other even slot l , see (16), as shown at the top of this page, where $P_{R_m}^{(l)}$ denotes the long-term average transmit power of relay R_m in the l th slot. Let $P_{R_m}^{(l)}$ be $\frac{P_R}{M_A}$ or $\frac{P_R}{M_B}$ for $R_m \in \mathcal{G}_A$ or $R_m \in \mathcal{G}_B$, respectively. Then according to (15), the amplifying coefficient of each relay $R_m \in \mathcal{G}_A$ in the second slot can be calculated by (17),⁸ as shown at the top of this page.

Similarly, according to (16), each $R_m \in \mathcal{G}_A$ has an invariant amplifying coefficient for all the other even slots, which is given by (for $l = 4, 6, \dots, L-1$) (18) on the above of this page.

With the distributed AF relaying scheme, the received signal at each destination D_k (for $k = 1, \dots, K$) in the even time slot l (for $l = 4, 6, \dots, L-1$, except the second one) is given by:

$$y_k^{(l)} = \sum_{R_m \in \mathcal{G}_A} u_{m,k}^{(l-1)} x_k^{(l-1)} + \sum_{i \neq k} \sum_{R_m \in \mathcal{G}_A} u_{m,k,i}^{(l-1)} x_i^{(l-1)} + \sum_{R_m \in \mathcal{G}_A} v_{m,k}^{(l-1)} \sum_{R_n \in \mathcal{G}_B} f_{n,m}^{(l-1)} r_n^{(l-1)} + \sum_{R_m \in \mathcal{G}_A} v_{m,k}^{(l-1)} w_m^{(l-1)} + w_{D,k}^{(l)}, \quad (19)$$

⁸It is assumed that each relay knows the statistical channel information for the links from itself to the sources, destinations and the relays of the other group. Based on this knowledge, each relay can derive its own amplifying coefficient.

where the following notations are used:

$$\begin{aligned} u_{m,k,k}^{(l)} &\triangleq \gamma_m^{(l+1)} g_{m,k}^{(l+1)} \left(\mathbf{g}_m^{(l+1)H} \mathbf{h}_m^{(l)H} \right) h_{m,k}^{(l)}, \\ u_{m,k,i}^{(l)} &\triangleq \gamma_m^{(l+1)} g_{m,k}^{(l+1)} \left(\mathbf{g}_m^{(l+1)H} \mathbf{h}_m^{(l)H} \right) h_{m,i}^{(l)}, \\ v_{m,k}^{(l)} &\triangleq \gamma_m^{(l+1)} g_{m,k}^{(l+1)} \left(\mathbf{g}_m^{(l+1)H} \mathbf{h}_m^{(l)H} \right). \end{aligned} \quad (20)$$

In (19), the first term represents the desired signal by destination D_k which is from source S_k , the second and third terms are the interferences from the other sources and relay group \mathcal{G}_B , respectively, and the fourth and last terms are the forwarded noises from the relays and the Gaussian noise at the destination. For the second slot, the received signal at the destination is slightly different from (19). Explicitly, the third term does not exist since relay group \mathcal{G}_B is silent in the first slot.

As for the recovery of the desired signal, each destination applies coherent detection with the mean effective channel gain, i.e., $E_f \left(\sum_{R_m \in \mathcal{G}_A} u_{m,k,k}^{(l-1)} \right)$ for destination D_k in the even slot l ($l \neq 2$), instead of the instantaneous CSI. Since the considered network consists of a very large number of relays, it is very beneficial and necessary to avoid the acquisition of the global CSI.

B. LOWER BOUND ACHIEVED BY DISTRIBUTED AF RELAYING SCHEME

Firstly, we calculate the achievable sum rate by the proposed distributed AF relaying scheme. Since each destination applies the coherent detection solely with the statistical CSI, additional interference is involved in, which would cause performance degradation. In the following we first focus on the even time slot except the second one.

We rewrite the received signals at destination D_k (for $k = 1, \dots, K$) in the even slot l (for $l = 4, 6, \dots, L-1$),

$$y_k^{(l)} = \underbrace{E_f \left(\sum_{R_m \in \mathcal{G}_A} u_{m,k,k}^{(l-1)} \right)}_{L_{1,k}} x_k^{(l-1)} + \underbrace{\left(\sum_{R_m \in \mathcal{G}_A} u_{m,k,k}^{(l-1)} - E_f \left(\sum_{R_m \in \mathcal{G}_A} u_{m,k,k}^{(l-1)} \right) \right)}_{L_{2,k}} x_k^{(l-1)} + \underbrace{\sum_{i \neq k} \sum_{R_m \in \mathcal{G}_A} u_{m,k,i}^{(l-1)} x_i^{(l-1)}}_{L_{3,k}} + \underbrace{\sum_{R_m \in \mathcal{G}_A} v_{m,k}^{(l-1)} \sum_{R_n \in \mathcal{G}_B} f_{n,m}^{(l-1)} r_n^{(l-1)}}_{L_{4,k}} + \underbrace{\sum_{R_m \in \mathcal{G}_A} v_{m,k}^{(l-1)} w_m^{(l-1)} + w_{D,k}^{(l-1)}}_{L_{5,k}} \quad (21)$$

$$E_f(|L_{1,k}|^2) = P_S \left(\sum_{R_m \in \mathcal{G}_A} \gamma_m \alpha_{m,k} \beta_{m,k} \right)^2,$$

$$E_f(|L_{2,k}|^2) = P_S \sum_{R_m \in \mathcal{G}_A} \gamma_m^2 \left(2\alpha_{m,k}^2 \beta_{m,k}^2 + \alpha_{m,k} \beta_{m,k} \sum_{j=1}^K \alpha_{m,j} \beta_{m,j} \right),$$

$$E_f(|L_{3,k}|^2) = P_S \sum_{i \neq k} \sum_{R_m \in \mathcal{G}_A} \gamma_m^2 \left(\alpha_{m,i} \alpha_{m,k} \beta_{m,k}^2 + \alpha_{m,i}^2 \beta_{m,i} \beta_{m,k} + \alpha_{m,i} \beta_{m,k} \sum_{j=1}^K \alpha_{m,j} \beta_{m,j} \right),$$

$$E_f(|L_{4,k}|^2) = \sum_{R_m \in \mathcal{G}_A} \gamma_m^2 \left(\alpha_{m,k} \beta_{m,k}^2 + \sum_{j=1}^K \alpha_{m,j} \beta_{m,j} \beta_{m,k} \right) \sum_{R_n \in \mathcal{G}_B} \eta_{n,m} \frac{P_R}{M_B},$$

$$E_f(|L_{5,k}|^2) = \sum_{R_m \in \mathcal{G}_A} \gamma_m^2 \left(\alpha_{m,k} \beta_{m,k}^2 + \sum_{j=1}^K \alpha_{m,j} \beta_{m,j} \beta_{m,k} \right). \quad (22)$$

i.e., (19), as (21) on top of this page, where destination D_k recovers the signal $x_k^{(l-1)}$ using the mean of the effective channel gain $E_f \left(\sum_{R_m \in \mathcal{G}_A} u_{m,k,k}^{(l-1)} \right)$ in the term $L_{1,k}$, the term $L_{2,k}$ is the residual interference caused by using the statistical channel information.

Then the achievable rate for each pair of source and destination in the even slot excluding the second one is given by the following proposition.

Proposition 1: For the multipair relay network with successive relaying protocol, the achievable rate of each communication pair (S_k, D_k) (for $k = 1, \dots, K$) for all the even slots l (for $l = 4, 6, \dots, L - 1$) is given by:

$$R_k^{(even)} = \log \left(1 + \frac{E_f(|L_{1,k}|^2)}{\sum_{n=2}^5 E_f(|L_{n,k}|^2) + 1} \right), \quad (23)$$

where each term $E_f(L_n^2)$ (for $n = 1, \dots, 5$) is calculated as (22) and γ_m is given by (18) where the superscript is omitted.

Proof: For each source S_k ($k = 1, \dots, K$), define a ‘super frame’ which consists of all the even subframes (excluding the second one) of all frames. Let its codeword span the whole ‘super frame’ which goes through all the even time slots (except the second one) of each transmission round. Then according to (19), it can be regarded that the codeword experiences an inter-symbol-interference channel with

inaccurate CSI at the receiver. Note that the channel coefficient of each link is independent on each other and independent from one time slot to the other. Therefore, the terms $L_{i,k}$ in (21) (for $i = 2, \dots, 5$) are uncorrelated to each other, and their means are zero. According to [25], the achievable rate can be derived by treating the terms $L_{i,k}$ (for $i = 2, \dots, 5$) as Gaussian noise with the variances $E_f(|L_{i,k}|^2)$. Then the achievable rate is given by (23). The detailed calculation of $E_f(|L_{i,k}|^2)$ (for $i = 1, \dots, 5$) is provided in Appendix B. ■

Following the similar line as in the proof of Proposition 1, the achievable rate of each pair S_k and D_k in the odd time slots can be derived accordingly. We denote the achievable rate by $R_k^{(odd)}$ and its detailed expression omitted here. Finally, as for the second slot, note that the received signal does not include the term $L_{4,k}$ in (21) (since relay group \mathcal{G}_B does not transmit in the first slot). Then the achievable rate of the second slot for the pair S_k and D_k :

$$R_k^{(2)} = \log \left(1 + \frac{E_f(|L_{1,k}|^2)}{\sum_{n=2, n \neq 4}^5 E_f(|L_{n,k}|^2) + 1} \right), \quad (24)$$

where $E_f(|L_{i,k}|^2)$ (for $i = 1, \dots, 5, i \neq 4$) are also given by (22), except that the amplifying factor γ_m should be $\gamma_m^{(2)}$ in (17) instead of $\gamma_m^{(l)}$ given in (18). With this, we derive the achievable sum rate of the multipair relay network with successive relaying protocol:

Proposition 2: For the multipair relay network with successive relaying protocol, the distributed AF scheme can achieve the following sum rate:

$$R_{sum} = \frac{1}{L} \sum_{k=1}^K \left(\frac{L-1}{2} R_k^{(odd)} + \frac{L-3}{2} R_k^{(even)} + R_k^{(2)} \right). \quad (25)$$

Now we can prove the scaling law of the achievable sum rate with the distributed AF scheme:

Theorem 2: For the large multipair relay network with successive relaying protocol, the achievable sum rate by the distributed AF scheme scales as $K \log(M) + \mathcal{O}(1)$ w.p.1, with an arbitrary but fixed $K, M \rightarrow \infty$ and then $L \rightarrow \infty$.

Proof: Here we just give the sketch of the proof, the details of which can be found in Appendix VI. For an arbitral relay network which is a realization of the relay distribution \mathcal{F} , relay groups \mathcal{G}_A and \mathcal{G}_B are formed by the relays within domains \mathcal{A}_1 and \mathcal{A}_2 , respectively. Since \mathcal{F} satisfies Assumption 1, when $M \rightarrow \infty$, the ratio of the sizes of \mathcal{G}_A or \mathcal{G}_B to M , i.e., $\frac{M_A}{M}$ or $\frac{M_B}{M}$, will tend to the constant λ_1 or λ_2 w.p.1, respectively. Since the ‘minimal distance’ restriction is satisfied, the variance of the channel coefficient for the link between each transmitter-receiver pair is bounded. Then using the results in Proposition 1 and 2 and the law of large numbers [24], we can derive the scaling law of the achievable sum rate. ■

Finally, Theorems 1 and 2 show that the gap between the scaling laws for the upper and lower bounds on the sum capacity is an $\mathcal{O}(1)$ term. Therefore, the capacity scaling law is established:

Theorem 3: For the large multipair relay network with successive relaying protocol, the sum capacity scales as $K \log(M) + \mathcal{O}(1)$, w.p.1, with an arbitrary but fixed K , when $M \rightarrow \infty$ and then $L \rightarrow \infty$.

Compared with two-slot relaying protocol, with which the capacity of the network scales as $\frac{K}{2} \log(M) + \mathcal{O}(1)$, the successive relaying protocol completely recovers the $\frac{1}{2}$ multiplexing loss in the sense of capacity scaling law.

C. EFFECT OF CHANNEL ESTIMATION ERRORS AT THE RELAYS

In this subsection, we further investigate the case when each relay has imperfect local CSI.

The estimated channel coefficients at each relay R_m (for $m = 1, \dots, M$) are given by:

$$\hat{h}_{m,k}^{(l)} = h_{m,k}^{(l)} + \varepsilon_{m,k}^{(l)}, \quad \text{and} \quad \hat{g}_{m,k}^{(l)} = g_{m,k}^{(l)} + \xi_{m,k}^{(l)}, \quad (28)$$

where (for $k = 1, \dots, K, l = 1, \dots, L$) $\varepsilon_{m,k}^{(l)}$ and $\xi_{m,k}^{(l)}$ denote the zero-mean channel estimation error at each relay for the channel from source S_k to relay R_m and from R_m to destination D_k , respectively. $\varepsilon_{m,k}^{(l)}$ and $\xi_{m,k}^{(l)}$ are independent variables for each m, k and l , and also independent on the actual channel coefficients. $\varepsilon_{m,k}^{(l)}$ and $\xi_{m,k}^{(l)}$ are assumed to satisfy i.i.d. complex Gaussian distribution with the same variance σ . Note that the extension to the case of different but bounded variances for $\varepsilon_{m,k}^{(l)}$ and $\xi_{m,k}^{(l)}$ is straightforward.

We take the even slot (except the 2nd slot) as an example to illustrate how the proposed AF scheme works. Each relay uses the estimated channel coefficients $\hat{h}_{m,k}^{(l)}$ and $\hat{g}_{m,k}^{(l)}$ to do AF. The transmitted signals from relay $R_m \in \mathcal{G}_A$ in (13) is now given by:

$$\hat{t}_m^{(l)} = \hat{\gamma}_m^{(l)} \hat{g}_m^{(l)H} \hat{\mathbf{h}}_m^{(l-1)H} \hat{r}_m^{(l-1)}, \quad l = 2, 4, \dots, L-1, \quad (29)$$

where $\hat{\mathbf{g}}_m^{(l)} = (\hat{g}_{m,1}^{(l)}, \dots, \hat{g}_{m,K}^{(l)})^T$, $\hat{\mathbf{h}}_m^{(l)} = (\hat{h}_{m,1}^{(l)}, \dots, \hat{h}_{m,K}^{(l)})$, and $\hat{r}_m^{(l-1)}$ is the received signal in the $(l-1)$ th slot. $\hat{\gamma}_m^{(l)}$ is again the amplifying coefficient. For the relay belonging to \mathcal{G}_A and $l = 4, \dots, L-1$, $\hat{\gamma}_m^{(l)} = \sqrt{\frac{P_R}{M_A} \hat{P}_{R_m}^{(l)}}$, where the transmit power of the relay is calculated by (26), as shown at the bottom of this page.

The received signals at the destinations in the even slots (excluding the second one) are written as (for $l = 4, 6, \dots, L-1$ and $k = 1, 2, \dots, K$) (27) on the bottom of this page, where the notations are slightly different from those in (20):

$$\begin{aligned} \hat{u}_{m,k,k}^{(l)} &\triangleq \hat{\gamma}_m^{(l+1)} g_{m,k}^{(l+1)} \left(\hat{\mathbf{g}}_m^{(l+1)H} \hat{\mathbf{h}}_m^{(l)H} \right) h_{m,k}^{(l)}, \\ \hat{u}_{m,k,i}^{(l)} &\triangleq \hat{\gamma}_m^{(l+1)} g_{m,k}^{(l+1)} \left(\hat{\mathbf{g}}_m^{(l+1)H} \hat{\mathbf{h}}_m^{(l)H} \right) h_{m,i}^{(l)}, \\ \hat{v}_{m,k}^{(l)} &\triangleq \hat{\gamma}_m^{(l+1)} g_{m,k}^{(l+1)} \left(\hat{\mathbf{g}}_m^{(l+1)H} \hat{\mathbf{h}}_m^{(l)H} \right). \end{aligned} \quad (30)$$

Similarly, each destination uses the statistical CSI, i.e., $E_f \left(\sum_{R_m \in \mathcal{G}_A} \hat{u}_{m,k,k}^{(l-1)} \right)$, to do coherent detection.

Following the same line as in Appendix B, the achievable rate for each source-destination pair can be calculated:

Proposition 3: When each relay suffers complex Gaussian distributed channel estimation error with zero mean

$$\hat{P}_{R_m}^{(l)} = E_f \left(\left| \hat{t}_m^{(l)} \right|^2 \right) = (\hat{\gamma}_m^{(l)})^2 \left(P_S \left(\sum_{k=1}^K \sum_{i=1}^K \hat{\alpha}_{m,k} \alpha_{m,i} \hat{\beta}_{m,k} + \sum_{k=1}^K \alpha_{m,k}^2 \hat{\beta}_{m,k} \right) + \sum_{k=1}^K \hat{\alpha}_{m,k} \hat{\beta}_{m,k} + \sum_{k=1}^K \hat{\alpha}_{m,k} \hat{\beta}_{m,k} \sum_{R_n \in \mathcal{G}_B} \eta_{n,m} \hat{P}_{R_n}^{(l-1)} \right) \quad (26)$$

$$\hat{y}_k^{(l)} = \sum_{R_m \in \mathcal{G}_A} \hat{u}_{m,k,k}^{(l-1)} x_k^{(l-1)} + \sum_{i \neq k} \sum_{R_m \in \mathcal{G}_A} \hat{u}_{m,k,i}^{(l-1)} x_i^{(l-1)} + \sum_{R_m \in \mathcal{G}_A} \hat{v}_{m,k}^{(l-1)} \sum_{R_n \in \mathcal{G}_B} f_{n,m}^{(l-1)} \hat{r}_n^{(l-1)} + \sum_{R_m \in \mathcal{G}_A} \hat{v}_{m,k}^{(l-1)} w_m^{(l-1)} + w_{D,k}^{(l)} \quad (27)$$

and variance of σ , the achievable rate by the distributed AF scheme for each communication pair (S_k, D_k) (for $k = 1, \dots, K$) in the even slots (excluding the second one) is given by:

$$\hat{R}_k^{(even)} = \log \left(1 + \frac{E_f(|\hat{L}_{1,k}|^2)}{\sum_{n=2}^5 E_f(|\hat{L}_{n,k}|^2) + 1} \right), \quad (31)$$

where each term in the above expression is calculated as (32), as shown at the bottom of this page.

The achievable rate in the odd slot and in the second slot of each transmission round, i.e., $\hat{R}_k^{(odd)}$ and $\hat{R}_k^{(2)}$, can be derived accordingly. Compared with (23) in Proposition 1, it can be seen from (31) that the channel estimation error at the relays brings a loss on the achievable rate. Similar to Proposition 2, the achievable sum rate of the relay network is $\hat{R}_{sum} = \frac{1}{L} \sum_{k=1}^K \left(\frac{L-3}{2} \hat{R}_k^{(even)} + \frac{L-1}{2} \hat{R}_k^{(odd)} + \hat{R}_k^{(2)} \right)$. Following the same line as in Appendix C, it is easy to show that the scaling law of the achievable rate will still be $K \log(M) + \mathcal{O}(1)$. The intuition is that, with the distributed AF scheme, the additional interference caused by channel estimation error at each relay is superimposed non-coherently at the desired destination, which is the same as the inter-source interference and inter-relay interference. Note that although the result is derived under the case where the channel estimation error at each relay has a same variance, the extension to the case of a different but bounded variance for each relay is straightforward.

Theorem 4: For the large multipair relay network with successive relaying protocol, consider the case where each relay suffer channel estimation error on the local CSI which is complex Gaussian distributed with zero mean and bounded variances. The achievable sum rate by the distributed AF scheme scales as $K \log(M) + \mathcal{O}(1)$ w.p.1, with an arbitrary but fixed $K, M \rightarrow \infty$ and then $L \rightarrow \infty$.

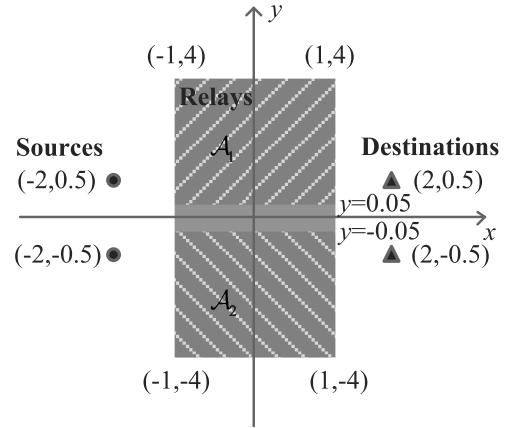


FIGURE 4. The simulation scenario. The sources and destinations are black circles and triangles, respectively. The relays are uniformly located in the grey rectangle area, which is divided into three areas by the lines $y = 0.05$ and $y = -0.05$.

V. NUMERICAL RESULTS

In this section, the results on the capacity scaling law for large multipair relay networks with successive relaying protocol are examined through numerical simulations. As for the simulation scenario, we consider the network depicted in Fig.4. Two sources reside on $(-2, 0.5)$ and $(-2, -0.5)$ and two destinations on $(2, 0.5)$ and $(2, -0.5)$, respectively. All the M relays are independently and uniformly distributed in the rectangle region whose vertices are $(1, 4)$, $(1, -4)$, $(-1, 4)$ and $(-1, -4)$. The ‘minimal distance’ r_{min} is set to 0.1. The channel coefficient between each transmitter and receiver satisfies zero-mean complex Gaussian distribution. The corresponding variance is calculated by $\frac{1}{d^3}$ where d is the distance between the transmitter and receiver. The long-term power constraint for each source and the long-term sum power constraint for all the selected relays (i.e., belonging to \mathcal{G}_A and \mathcal{G}_B) are set to 50 and 100, respectively. The variance of the noise at each receiver is set to 1.

$$\begin{aligned} E_f(|\hat{L}_{1,k}|^2) &= P_S \left(\sum_{R_m \in \mathcal{G}_A} \hat{\gamma}_m \alpha_{m,k} \beta_{m,k} \right)^2 \\ E_f(|\hat{L}_{2,k}|^2) &= P_S \sum_{R_m \in \mathcal{G}_A} \hat{\gamma}_m^2 \left(\alpha_{m,k} \hat{\alpha}_{m,k} \beta_{m,k}^2 + \alpha_{m,k}^2 \beta_{m,k} \hat{\beta}_{m,k} + \alpha_{m,k} \beta_{m,k} \sum_{j=1}^K \hat{\alpha}_{m,j} \hat{\beta}_{m,j} \right) \\ E_f(|\hat{L}_{3,k}|^2) &= P_S \sum_{i \neq k} \sum_{R_m \in \mathcal{G}_A} \gamma_m^2 \left(\alpha_{m,i} \hat{\alpha}_{m,k} \beta_{m,k}^2 + \alpha_{m,i}^2 \hat{\beta}_{m,i} \beta_{m,k} + \alpha_{m,i} \beta_{m,k} \sum_{j=1}^K \hat{\alpha}_{m,j} \hat{\beta}_{m,j} \right), \\ E_f(|\hat{L}_{4,k}|^2) &= \sum_{R_m \in \mathcal{G}_A} \hat{\gamma}_m^2 \left(\hat{\alpha}_{m,k} \beta_{m,k}^2 + \sum_{j=1}^K \hat{\alpha}_{m,j} \hat{\beta}_{m,j} \beta_{m,k} \right) \sum_{R_n \in \mathcal{G}_B} \eta_{n,m} \frac{P_R}{M_B} \\ E_f(|\hat{L}_{5,k}|^2) &= \sum_{R_m \in \mathcal{G}_A} \hat{\gamma}_m^2 \left(\hat{\alpha}_{m,k} \beta_{m,k}^2 + \sum_{j=1}^K \hat{\alpha}_{m,j} \hat{\beta}_{m,j} \beta_{m,k} \right) \end{aligned} \quad (32)$$

Firstly, we exam the scaling behavior of the capacity upper and lower bounds in the case when each relay has perfect local CSI. For the capacity upper bound, C^U in (12) is used where α^{upper} equals 1 which is derived according to the least possible distance between sources and relays. On the other hand, as $r_{min} = 0.1$, we separate the rectangle area (where the relays randomly reside) into three areas by two lines $y = 0.05$ and $y = -0.05$, as shown in Fig. 4. The relays within \mathcal{A}_1 and \mathcal{A}_2 are picked out to form relay groups \mathcal{G}_A and \mathcal{G}_B , respectively. It is straightforward that the least distance of the relays belonging to the two groups is no smaller than r_{min} . Based on this, R_{sum} in Proposition 2 is evaluated as the capacity lower bound.

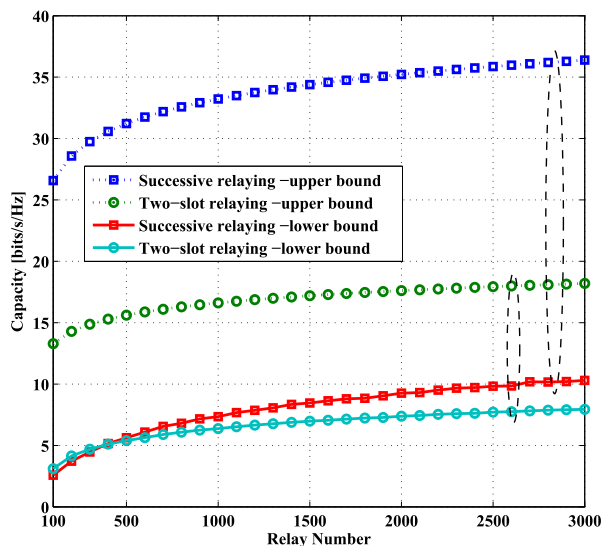


FIGURE 5. Capacity upper bound and lower bound versus relay number for relay networks with successive relaying and two-slot relaying, where the source number $K = 2$ and slot number $L = 100$.

Fig. 5 plots the capacity upper bound and lower bound of relay networks with successive relaying under various relay numbers. For comparison, the upper and lower capacity bounds with two-slot relaying protocol are also plotted, where all the relays are selected to help transmission. Explicitly, for the two-slot relaying, the lower bound is derived by calculating the achievable sum rate by adopting the proposed distributed AF relaying strategy and the upper bound is from [11]. It can be observed that, the scaling behavior of the upper bound with successive relaying protocol is similar to the corresponding lower bound, so is the upper and lower bounds with two-slot relaying protocol. Besides, both the upper and lower bounds with successive relaying increase much faster than that with two-slot relaying. Furthermore, note that the simulation results for each relay number are solely for one arbitrary realization of the relay distribution. This validates that the derived scaling laws for the capacity upper and lower bounds are achieved with a high probability, which agrees with Theorems 1, 2 and 3. We also note that the gap between the upper and lower bounds is large. This is due to the fact that both bounds are loose, especially for the

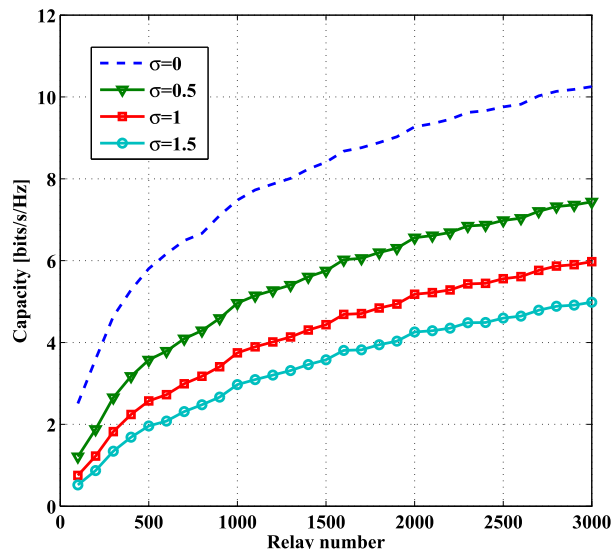


FIGURE 6. The achievable sum rate versus the relay number for relay networks with successive relaying in the case when each relay suffers a channel estimation error, where the source number $K = 2$ and slot number $L = 100$.

upper bound which is in fact the maximal rate over the source-relay cut. Nevertheless, from the figure one can see that it is sufficient to establish the scaling law with the proposed bounds.

Then we exam the capacity lower bound where each relay suffers channel estimation error, which is given by Proposition 3. As shown in Fig. 6, under the channel estimation error with variances of 0, 0.05, 0.1 and 0.15, the scaling behaviors of the achievable sum rate over the relay number are similar, which verifies Theorem 4. Note that the curves have fluctuations under small relay numbers since each point is the simulation result for only one relay distribution realization.

VI. CONCLUSION

In this work, we investigate the capacity scaling law of large relay networks with successive relaying protocol. The network is constituted of K pairs of sources and destinations with fixed positions and M relays randomly distributed in a bounded domain. Each transmission round of the protocol consists of L slots. Firstly we prove that the sum capacity of the network is upper bounded by $K \log(M) + \mathcal{O}(1)$ when K is fixed, $M \rightarrow \infty$ and then $L \rightarrow \infty$. Then by choosing a part of the M relays to form two groups and using the distributed AF scheme, we show that a sum rate of $K \log(M) + \mathcal{O}(1)$ can be achieved, where perfect local CSI and statistical CSI are assumed to be available at the relays and sources/destinations, respectively. Therefore, the capacity scaling of large relay networks with successive relaying protocol is established. Furthermore, we consider the case in which each relay suffers channel estimation error with a bounded variance on the local CSI and show that the achievable sum rate by the distributed AF is still $K \log(M) + \mathcal{O}(1)$. Hence the capacity scaling law is not affected in this case.

$$\begin{aligned}
 E_f \left(\left\| \mathbf{g}_m^H \mathbf{h}_m^H \mathbf{h}_m \right\|^2 \right) &= \text{Tr} \left(E_f \left(\mathbf{g}_m \mathbf{g}_m^H \right) E_f \left(\mathbf{h}_m^H \mathbf{h}_m \mathbf{h}_m^H \mathbf{h}_m \right) \right) = \text{Tr} \left(E_f \left(\mathbf{g}_m \mathbf{g}_m^H \right) E_f \left(\sum_{k=1}^K |h_{m,k}|^2 \mathbf{h}_m^H \mathbf{h}_m \right) \right) \\
 &= \text{Tr} \left(\text{diag} \left(\beta_{m,1}, \dots, \beta_{m,K} \right) \text{diag} \left(E_f \left(\sum_{k=1}^K |h_{m,k}|^2 |h_{m,1}|^2 \right), \dots, E_f \left(\sum_{k=1}^K |h_{m,k}|^2 |h_{m,K}|^2 \right) \right) \right) \\
 &\stackrel{(a)}{=} \text{Tr} \left(\text{diag} \left(\beta_{m,1}, \dots, \beta_{m,K} \right) \text{diag} \left(c_{m,1}, \dots, c_{m,K} \right) \right) = \sum_{k=1}^K \beta_{m,k} c_{m,k}
 \end{aligned} \tag{34}$$

Finally, although the results in this work are derived based on the assumption that the coefficients of the wireless channel satisfy complex Gaussian distribution, extensions to more general cases can be made. As an example, it is easy to show that the results derived in the present paper are still valid when assuming the channel coefficients are zero-mean and have bounded secondary and fourth-order moments. Another possible extension is the case where the number of source-destination pairs tends to infinity along with the number of relays. The capacity scaling law will behave differently under this case and we leave it as our future work.

**Appendix A
CALCULATION OF THE TRANSMIT POWER
OF EACH RELAY**

We calculate the long-term average transmit power for each relay in group \mathcal{G}_A , as given in (14). Firstly, consider the signal transmitted in the second slot. Recalling the fact that the amplifying factor $\gamma_m^{(l)}$ solely depends on m and l . Also note that the channel coefficient of an arbitrary transmitter-receiver pair is independent on that of the other pair and i.i.d. over each slot. Then the long-term average power of $t_m^{(2)}$ (which is the transmit signal from relay R_m in the second slot) can be calculated by:

$$E_f \left(|t_m|^2 \right) = \gamma_m P_S E_f \left(\left\| \mathbf{g}_m^H \mathbf{h}_m^H \mathbf{h}_m \right\|^2 \right) + \gamma_m E_f \left(\left| \mathbf{g}_m^H \mathbf{h}_m^H \right|^2 \right). \tag{33}$$

where the superscript ‘(2)’ is omitted for simplicity. Then for the first term we have (34), as shown at the top of this page,

where for (a), we have (recalling that $g_{m,k}$ and $h_{m,k}$ are independent complex Gaussian):

$$\begin{aligned}
 c_{m,k} &= E_f \left(\sum_{i=1}^K |h_{m,i}|^2 |h_{m,k}|^2 \right) \\
 &= E_f \left(|h_{m,k}|^4 + |h_{m,k}|^2 \sum_{i \neq k}^K |h_{m,i}|^2 \right) \\
 &= \alpha_{m,k}^2 + \sum_{k=1}^K \sum_{i=1}^K \alpha_{m,k} \alpha_{m,i}.
 \end{aligned} \tag{37}$$

Similarly, for the second term in (33),

$$E_f \left(\left| \mathbf{g}_m^H \mathbf{h}_m^H \right|^2 \right) = \sum_{k=1}^K \alpha_{m,k} \beta_{m,k}. \tag{38}$$

With this we derive the average transmit power (15) for the relay of group \mathcal{G}_A in the second slot.

Then consider the transmit signal of the relay in the other even slots (see (14)). Compared with that in the second slot, there exists an additional term $\gamma_m^{(l)} \mathbf{g}_m^{(l)H} \mathbf{h}_m^{(l-1)H} \sum_{R_n \in \mathcal{G}_A} f_{n,m}^{(l-1)} t_n^{(l-1)}$, which is uncorrelated to the other two terms. Therefore, the transmit power is calculated by (the superscript ‘(l)’ is also omitted):

$$\begin{aligned}
 E_f \left(|t_m|^2 \right) &= \gamma_m P_S E_f \left(\left\| \mathbf{g}_m^H \mathbf{h}_m^H \mathbf{h}_m \right\|^2 \right) + \gamma_m E_f \left(\left| \mathbf{g}_m^H \mathbf{h}_m^H \right|^2 \right) \\
 &\quad + \gamma_m E_f \left(\left| \mathbf{g}_m^H \mathbf{h}_m^H \right|^2 \right) \sum_{R_n \in \mathcal{G}_A} P_{R_n} E_f \left(|f_{n,m}|^2 \right),
 \end{aligned} \tag{39}$$

where P_{R_n} denotes the long-term average power of $t_n^{(l-1)}$, i.e., $P_{R_n} = E_f \left(|t_n^{(l-1)}|^2 \right)$. With (38) and $E_f \left(|f_{n,m}|^2 \right) = \eta_{n,m}$ for the above equation, the average relay transmit power is derived, as given by (16).

**Appendix B
CALCULATION OF $E_f \left(|L_{i,k}|^2 \right)$ FOR PROPOSITION 1**

We calculate the variances of $L_{i,k}$ for $i = 1, \dots, 5$ in (23). Recalling the fact that the channel coefficient of each link is independent on each other, it is straightforward that $E_f \left(|L_{i,k}|^2 \right)$ can be calculated as follows (note that the superscript for the slot number ‘(l)’ is omitted):

$$\begin{aligned}
 E_f \left(|L_{1,k}|^2 \right) &= P_S \left| \sum_{R_m \in \mathcal{G}_A} E_f \left(u_{m,k,k} \right) \right|^2, \\
 E_f \left(|L_{2,k}|^2 \right) &= P_S \sum_{R_m \in \mathcal{G}_A} E_f \left(|u_{m,k,k}|^2 \right) - |E_f \left(u_{m,k,k} \right)|^2, \\
 E_f \left(|L_{3,k}|^2 \right) &= P_S \sum_{i \neq k}^K \sum_{R_m \in \mathcal{G}_A} E_f \left(|u_{m,k,i}|^2 \right), \\
 E_f \left(|L_{4,k}|^2 \right) &= \frac{P_R}{M_B} \sum_{R_m \in \mathcal{G}_A} E_f \left(|v_{m,k}|^2 \right) \sum_{R_n \in \mathcal{G}_B} \eta_{n,m}, \\
 E_f \left(|L_{5,k}|^2 \right) &= \sum_{R_m \in \mathcal{G}_A} E_f \left(|v_{m,k}|^2 \right).
 \end{aligned} \tag{40}$$

$$\begin{aligned}
 E_f \left(|u_{m_B,k,i}|^2 \right) &= E_f \left(\gamma_m^2 g_{m,k} \left(\mathbf{g}_m^H \mathbf{h}_m^H \right) h_{m,i} h_{m,i}^* \left(\mathbf{h}_m \mathbf{g}_m \right) g_{m,k}^* \right) = \gamma_m^2 E_f \left(|g_{m,k}|^2 |h_{m,i}|^2 \left| \sum_{j=1}^K g_{m,j} h_{m,j} \right|^2 \right) \\
 &= \gamma_m^2 E_f \left(|g_{m,k}|^4 |h_{m,i}|^2 |h_{m,k}|^2 + |g_{m,i}|^2 |g_{m,k}|^2 |h_{m,i}|^4 + |g_{m,k}|^2 |h_{m,i}|^2 \sum_{j \neq i,k}^K |g_{m,j} h_{m,j}|^2 \right) \\
 &= \gamma_m^2 \alpha_{m,i} \beta_{m,k} \left(\alpha_{m,k} \beta_{m,k} + \alpha_{m,i} \beta_{m,i} + \sum_{j=1}^K \alpha_{m,j} \beta_{m,j} \right) \tag{35}
 \end{aligned}$$

$$\tilde{\gamma}_m^{(l)} = \frac{1}{\sqrt{M_A}} \sqrt{\frac{P_R}{\left(P_S \left(\sum_{k=1}^K \sum_{i=1}^K \alpha_{m,k} \alpha_{m,i} \beta_{m,k} + \sum_{k=1}^K \alpha_{m,k}^2 \beta_{m,k} \right) + \sum_{k=1}^K \alpha_{m,k} \beta_{m,k} + \sum_{k=1}^K \alpha_{m,k} \beta_{m,k} + P_R E_{p(R_m)} (\eta_{n,m}) \right)}} \tag{36}$$

As for $E_f \left(u_{m,k,k} \right)$, we have (for the definition of $u_{m,k,k}$, refer to (20)):

$$\begin{aligned}
 E_f \left(u_{m,k,k} \right) &= E_f \left(\gamma_m g_{m,k} \left(\mathbf{g}_m^H \mathbf{h}_m^H \right) \mathbf{h}_{m,k} \right) \\
 &= \gamma_m E_f \left(g_{m,k} \mathbf{g}_m^H \right) E \left(\mathbf{h}_m^H h_{m,k} \right) \\
 &= \gamma_m \alpha_{m,k} \beta_{m,k}. \tag{41}
 \end{aligned}$$

For the second equation in (40), $E_f \left(|u_{m,k,k}|^2 \right)$ is calculated by:

$$\begin{aligned}
 E_f \left(|u_{m,k,k}|^2 \right) &= E_f \left(\gamma_m^2 g_{m,k} \left(\mathbf{g}_m^H \mathbf{h}_m^H \right) h_{m,k} h_{m,k}^* \left(\mathbf{h}_m \mathbf{g}_m \right) g_{m,k}^* \right) \\
 &= \gamma_m^2 E_f \left(|g_{m,k}|^2 |h_{m,k}|^2 \left| \sum_{j=1}^K g_{m,j} h_{m,j} \right|^2 \right) \\
 &= \gamma_m^2 E_f \left(|g_{m,k}|^4 |h_{m,k}|^4 + |g_{m,k}|^2 |h_{m,k}|^2 \sum_{j \neq k}^K |g_{m,j}|^2 |h_{m,j}|^2 \right) \\
 &= \gamma_m^2 \left(3\alpha_{m,k}^2 \beta_{m,k}^2 + \alpha_{m,k} \beta_{m,k} \sum_{j=1}^K \alpha_{m,j} \beta_{m,j} \right). \tag{42}
 \end{aligned}$$

Then we calculate $E_f \left(|u_{m,k,i}|^2 \right)$ in the third equation in (40) by (35), as shown at the top of this page.

Finally, $E_f \left(|v_{m,k}|^2 \right)$ in the last two equations is calculated as below:

$$\begin{aligned}
 E_f \left(|v_{m,k}|^2 \right) &= E_f \left(\gamma_m^2 g_{m,k} \left(\mathbf{g}_m^H \mathbf{h}_m^H \right) \left(\mathbf{g}_m \mathbf{h}_m \right) g_{m,k}^* \right) \\
 &= \gamma_m^2 E_f \left(|g_{m,k}|^2 \left| \sum_{j=1}^K g_{m,j} h_{m,j} \right|^2 \right) \\
 &= \gamma_m^2 E_f \left(|g_{m,k}|^4 |h_{m,k}|^2 + |g_{m,k}|^2 \sum_{j \neq k}^K |g_{m,j} h_{m,j}|^2 \right) \\
 &= \gamma_m^2 \left(\alpha_{m,k} \beta_{m,k}^2 + \sum_{j=1}^K \alpha_{m,j} \beta_{m,j} \beta_{m,k} \right). \tag{43}
 \end{aligned}$$

Substituting the above terms into (40), we finish the proof for Proposition 1.

Appendix C PROOF OF THEOREM 2

Consider an arbitrary relay network which is a realization of the relay spatial distribution \mathcal{F} .

The achievable rate for the k th source-destination pair in the even slot (except the second one) of each transmission round is given in (23). We rewrite the term $E_f \left(|L_{1,k}|^2 \right)$ in (22) as follows (the superscript for the slot number ‘1’ is omitted):

$$\begin{aligned}
 E_f \left(|L_{1,k}|^2 \right) &= P_S \left(\sum_{R_m \in \mathcal{G}_A} \gamma_m \alpha_{m,k} \beta_{m,k} \right)^2 \\
 &= \left(\frac{1}{\sqrt{M_A}} \sum_{R_m \in \mathcal{G}_A} l_{1,m,k} \right)^2, \tag{44}
 \end{aligned}$$

where $l_{1,m,k} \triangleq \sqrt{P_S} \tilde{\gamma}_m \alpha_{m,k} \beta_{m,k}$ and $\tilde{\gamma}_m^{(l)} \triangleq \gamma_m^{(l)} \sqrt{M_A}$ for $R_m \in \mathcal{G}_A$. Similarly, the remaining terms $E_f \left(L_i^{(l)} \right)$

(for $i = 2, \dots, 5$) are rewritten as follows:

$$E_f \left(|L_{i,k}|^2 \right) = \frac{1}{M_A} \sum_{R_m \in \mathcal{G}_A} l_{i,m,k}, \quad (45)$$

where

$$\begin{aligned} l_{2,m,k} &\triangleq P_S \tilde{\gamma}_m^2 \left(2\alpha_{m,k}^2 \beta_{m,k}^2 + \alpha_{m,k} \beta_{m,k} \sum_{j=1}^K \alpha_{m,j} \beta_{m,j} \right), \\ l_{3,m,k} &\triangleq P_S \sum_{i \neq k} \tilde{\gamma}_m^2 \left(\alpha_{m,i} \alpha_{m,k} \beta_{m,k}^2 + \alpha_{m,i}^2 \beta_{m,i} \beta_{m,k} \right. \\ &\quad \left. + \alpha_{m,i} \beta_{m,k} \sum_{j=1}^K \alpha_{m,j} \beta_{m,j} \right), \\ l_{4,m,k} &\triangleq \tilde{\gamma}_m^2 \left(\alpha_{m,k} \beta_{m,k}^2 + \sum_{j=1}^K \alpha_{m,j} \beta_{m,j} \beta_{m,k} \right) \sum_{R_n \in \mathcal{G}_B} \eta_{n,m} \frac{P_R}{M_B}, \\ l_{5,m,k} &\triangleq \tilde{\gamma}_m^2 \left(\alpha_{m,k} \beta_{m,k}^2 + \sum_{j=1}^K \alpha_{m,j} \beta_{m,j} \beta_{m,k} \right). \end{aligned} \quad (46)$$

Note that the terms $l_{i,m,k}$ (for $i = 1, \dots, 5$) are random variables related to the relay spatial distribution. From (18) and the above definitions of $l_{i,m,k}$, it can be easily verified that $l_{i,m,k}$ are bounded under fixed K . One may notice the term $l_{4,m,k}$, which is special compared with the other terms as it contains the summation of M_B elements, i.e., $\sum_{R_n \in \mathcal{G}_B} \eta_{n,m} \frac{P_R}{M_B}$. Note that since $\eta_{n,m}$ is upper bounded, $\sum_{R_n \in \mathcal{G}_B} \eta_{n,m} \frac{P_R}{M_B}$ is also bounded, so is the term $l_{4,m,k}$. Now consider $E_f \left(|L_{1,k}|^2 \right)$ in (44), we have:

$$\begin{aligned} \frac{1}{\sqrt{M_A}} \sum_{R_m \in \mathcal{G}_A} l_{1,m,k} &= \frac{M}{\sqrt{M_A}} \frac{\sum_{R_m \in \mathcal{G}_A} l_{1,m,k}}{M} \\ &= \sqrt{M} \sqrt{\frac{M}{M_A}} \frac{\sum_{m=1}^M I(m) l_{1,m,k}}{M}, \end{aligned} \quad (47)$$

where $I(m)$ is the indicator function to indicate whether relay $R_m \in \mathcal{G}_A$. According to Assumption 1, $\frac{M}{M_A} \rightarrow \frac{1}{\lambda_1}$ w.p.1 as $M \rightarrow \infty$. Note that each $l_{1,m,k}$ will asymptotically approach $\tilde{l}_{1,m,k}$, where $\tilde{l}_{1,m,k} = \sqrt{P_S} \tilde{\gamma}_m \alpha_{m,k} \beta_{m,k}$ and $\tilde{\gamma}_m$ denotes variable $\tilde{\gamma}_m$ with the term $\sum_{R_n \in \mathcal{G}_B} \eta_{n,m} \frac{P_R}{M_B}$ in the denominator replaced by $P_R E_{p\{R_m\}}(\eta_{n,m})$. That is (for $R_m \in \mathcal{G}_A$) shown in (36), as shown at the top of the previous page, where $E_{p\{R_m\}}(\eta_{n,m})$ represents the spatial average of $\eta_{n,m}$ conditioned on the location of relay R_m . Since $E_{p\{R_m\}}(\eta_{n,m})$ only depends on the location of R_m , it is independent for different R_m . Next we define a new random variable $\tilde{l}_{1,m,k}$ which takes the value of $\tilde{l}_{1,m,k}$ when $R_m \in \mathcal{G}_A$ and otherwise randomly takes a value according to the distribution of $\tilde{l}_{1,m,k}$ as if $R_m \in \mathcal{G}_A$. Then $\tilde{l}_{1,m,k}$ and $I(m)$ is independent on

each other. Now back to the summation term of (47). According to the law of large numbers, we have (when $M \rightarrow \infty$):

$$\begin{aligned} \frac{1}{M} \sum_{m=1}^M I(m) l_{1,m,k} &\xrightarrow{w.p.1} \frac{1}{M} \sum_{m=1}^M I(m) \tilde{l}_{1,m,k} \\ &= \frac{1}{M} \sum_{m=1}^M I(m) \tilde{l}_{1,m,k} \xrightarrow{w.p.1} E_p \left(I(m) \tilde{l}_{1,m,k} \right) = \lambda_1 E_p \left(\tilde{l}_{1,m,k} \right) \end{aligned} \quad (48)$$

With this we have:

$$E_f \left(|L_{1,k}|^2 \right) \xrightarrow{w.p.1} M \lambda_1 \left(E_p \left(\tilde{l}_{1,m,k} \right) \right)^2 \quad (49)$$

Similarly, $E_f \left(|L_{i,k}|^2 \right)$ (for $i = 2, \dots, 5$) will tend to the spatial average of $\tilde{l}_{i,m,k}$, i.e., $E_f \left(L_{i,k} \right) \xrightarrow{w.p.1} E_p \left(\tilde{l}_{i,m,k} \right)$, where $\tilde{l}_{i,m,k}$ is defined analogously as $\tilde{l}_{1,m,k}$. Note that these terms are all bounded. Substituting them into (23), the achievable rate for the k th source-destination pair in the even slots will tend to the following term w.p.1:

$$\begin{aligned} R_k^{(even)} &\xrightarrow{w.p.1} \log \left(1 + \frac{M \lambda_1 E_p^2 \left(\tilde{l}_{1,m,k} \right)}{\sum_{z=2}^5 \left(E_p \left(\tilde{l}_{z,m,k} \right) \right) + 1} \right) \\ &= \log(M) + \log \left(\frac{1}{M} + \frac{\lambda_1 E_p^2 \left(\tilde{l}_{1,m,k} \right)}{\sum_{z=2}^5 \left(E_p \left(\tilde{l}_{z,m,k} \right) \right) + 1} \right) \\ &= \log(M) + \mathcal{O}(1). \end{aligned} \quad (50)$$

Similarly, it can be proved that the scaling law of $R_k^{(odd)}$ and $R_k^{(2)}$ is also $\log(M) + \mathcal{O}(1)$. Then according to Proposition 2, the achievable sum rate of the K source-destination pairs will approach $K \log(M) + \mathcal{O}(1)$ w.p.1, when K is fixed, $M \rightarrow \infty$ and then $L \rightarrow \infty$.

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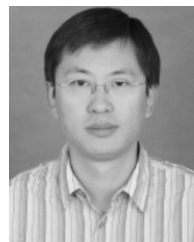
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