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Energy-Efficient User Association and Power Control in the Heterogeneous Network

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ABSTRACT We consider a heterogeneous network (HetNet) containing primary users (PUs) and secondary users (SUs). Ordinary cellular users are characterized as PUs, while SUs are the unlicensed users, sensors, or some other Internet of Things equipments. The PUs occupy all the channels in the HetNet and the SUs try to reuse the channels of PUs. We consider two transmission modes for SUs, i.e., the SU can associate with the base station (BS) directly or through the help of its cooperative relay. The optimization of energy efficiency (EE) of SUs is considered. Particularly, we focus on user association (BS selection, channel allocation, and mode selection) and power control to optimize the uplink EE of the communication between the SU and the BS. The original problem is formulated as a non-convex and mixed-integer optimization problem. To get a tractable solution, we propose an iterative optimization algorithm. The alterative optimization method decomposes the original problem into three subproblems. In each iteration, the three subproblems are solved by using the sum-of-ratios programming algorithm, the parametric Dinkelbach algorithm, and convex optimization. Then, the proposed scheme repeats the iteration until convergence. Numerical results confirm that the proposed method can improve the uplink EE performance for SUs.

INDEX TERMS Heterogeneous networks, user association, power control, energy efficiency.

I. INTRODUCTION

Wireless networks tend to be more heterogeneous and more complex. Various technical targets, such as higher transmission rate, lower energy cost, lower delay, are pursued to satisfy the growing demand of users. To meet the ever-increasing wireless traffic requirement, heterogeneous networks (HetNets) have been proposed and have received great attention in recent years [1], [2]. However, multi-tier HetNets generate severe co-tier and cross-tier interference which may degrade the system performance observably. To reduce the impact caused by interference and improve the experience of users, it is necessary to investigate the resource allocation problem in HetNets [3], [4]. Besides, with the explosive growth of wireless traffic, the energy consumption for wireless communications is increasing dramatically. Since current battery technology cannot meet the ever-increasing demand of mobile users and other various energy-constrained applications, the energy consumption problem becomes more severe. Therefore, it is of great importance to improve the energy efficiency (EE) of users and other functional nodes for wireless communications, especially during the uplink transmission.

To improve EE in HetNets, several resource allocation and user association schemes have been proposed [5]–[8]. Authors of [5] investigated the energy-efficient congestion control and resource allocation scheme through traffic admission control, user association, resource block allocation and power allocation. By utilizing the matching theory and the Lagrangian dual decomposition method, authors in [6] proposed a hybrid user association scheme to maximize the energy efficiency, while considering two different types of user equipments. In [7], the authors studied the energy-efficient resource allocation problem with relay selection, power allocation and network selection. The complicated EE optimization problem in [7] was solved by Dinkelbach method and Lagrangian dual decomposition method. The EE maximization problem with resource block allocation and power control was investigated in [8] and the authors handled the resource allocation problem by resorting to several optimization methods. However, it is noted that the investigated scenarios in the abovementioned works are rather simplified. More specifically, in [5]–[8], it is assumed that the channels or resource blocks allocated to users are orthogonal or some interference coordination techniques are utilized to simply the interference model. In more complex scenarios where cross-tier interference exists, as more factors are taken into consideration, the EE optimization becomes increasingly complicated and hard to be solved.

For the purpose of further exploring the insight of energy-efficient resource optimization in HetNets, several researchers suggest to divide the users into two types, i.e., primary users (PUs) and secondary users (SUs) [9]–[12]. The ordinary licensed cellular users are classified as PUs. PUs already occupy all the channels in the HetNet. The unlicensed users, sensors and other Internet of Things (IoT) equipments are characterized as SUs. SUs try to reuse the channels of the PUs. For SUs like sensors and IoT equipments, the energy is quite limited, and hence, the EE optimization problem for SUs is highly considerable. Against this background, the resource allocation problems for SUs' EE optimization were investigated while guaranteeing the interference constraint of PUs satisfied [9]–[12]. In [9], the EE optimization problem for secondary small cells which served SUs was investigated and the authors solved the radio resource allocation problem by using the convex transformation and the Lagrangian dual decomposition method. Authors in [10] proposed a spectrum sharing scheme between the primary macrocell and secondary small cells and investigated the power allocation for the small cells to maximize their EE. The EE maximization problem with power and admission control was studied in [11] which admitted as many SUs as possible while guaranteeing SUs' rate requirements. Note that authors in [9]–[11] divide the users into two groups based on the served base stations (BSs), i.e., the PU is served by the primary BS which is typically the macro BS and the SU is served by the secondary BS which is often the small BS or other low power access node. However, under fixed BS selection, SUs at the edge of the small cell may suffer from the severe interference caused by PUs. To avoid this issue, authors in [12] investigated the scenario where SUs can connect to different BSs. Then the SUs' EE maximization problem in [12] was solved with the parametric Dinkelbach algorithm, the Lagrangian dual decomposition method and the Kuhn-Munkres algorithm. Nevertheless, authors in [12] assumed that each SU reused only one channel, which simplified the EE optimization problem and led to low utilization of channel resources.

To further improve the EE of SUs, different from the aforementioned works, we consider the scenario where each SU can connect to different BSs and can use multiple channels. Besides, when the power of the SU (e.g., sensors and IoT equipments) is limited, or the channel between the SU and the BS is of poor quality, the direct communication between the SU and the BS may not satisfy the quality of service (QoS) requirement. In this case, the SU may communicate with the BS through the help of the relay [13], [14]. Therefore, we consider two transmission modes for SUs, i.e., the SU can communicate with the BS directly or through the help of its cooperative relay. We aim to maximize the uplink EE of SUs through user association (BS selection, channel allocation, mode selection) and power control. To begin with, we investigate the BS selection and channel allocation problem which allocates channels from different BSs to different SUs. Next, the mode selection problem which decides whether the SU communicates with the BS directly or through the help of its cooperative relay, is considered. Then, we focus on the power control problem, in which we optimize the transmission power for the SU and its cooperative relay. Finally, we propose an iterative user association and power control scheme to maximize the EE of SUs. To sum up, the major contributions are summarized as follows:

1) We investigate the scenario where SUs can connect to different BSs and utilize multiple channels. Two transmission modes are considered for SUs, i.e., the SU can communicate with the BS directly or through the help of its cooperative relay. The considered EE maximization problem is quite complicated and we propose an effective and novel algorithm to solve the problem.

2) We propose a user association and power control scheme to optimize the uplink EE of SUs. The formulated problem is a non-convex and mixed-integer problem which is NP-hard. To get a tractable solution, we decompose the original problem into three subproblems. Then the three subproblems are solved with the sum-of-ratios programming [15], the parametric Dinkelbach algorithm [16] and convex optimization [17]. Numerical results verify that the proposed algorithm converges fast and the uplink EE of SUs can be improved with the proposed method.

Notations: $\mathbf{A}$ and $\mathbf{a}$ denote the matrix and the vector, respectively. $\mathbf{a} \succ 0$ indicates that all elements of $\mathbf{a}$ are positive and $\mathbf{a} \succeq 0$ means all elements of $\mathbf{a}$ are nonnegative. $\mathbf{0}$ denotes the vector whose elements are all 0. $\|\mathbf{A}\|$ is the Euclidean norm of $\mathbf{A}$.

The rest of this paper is organized as follows. Section II introduces the system model and Section III formulates the EE maximization problem. In section IV, a three-step iterative optimization algorithm is proposed. Numerical simulation results and analyses are presented in section V. Finally, section VI makes the conclusion of this paper.

II. SYSTEM MODEL

In this paper, we focus on a HetNet composed of a macro BS, several pico BSs and relays, as shown in Fig. 1. Denote the BS set as $\mathcal{B} = \{1, \ldots, B\}$, where the index 1 stands for the macro BS and others represent pico BSs. Consider a densely deployed area where each BS b has N_b orthogonal channels and channels between any two BSs are orthogonal, i.e., there is no interference between BSs in the HetNet. The total number of channels in the HetNet is $\sum_{b=1}^{B} N_b = M$ and each channel has the same spectrum bandwidth. U SUs coexist with C PUs in the HetNet. The PUs are served by different BSs according to BS coverage, load balancing, etc.

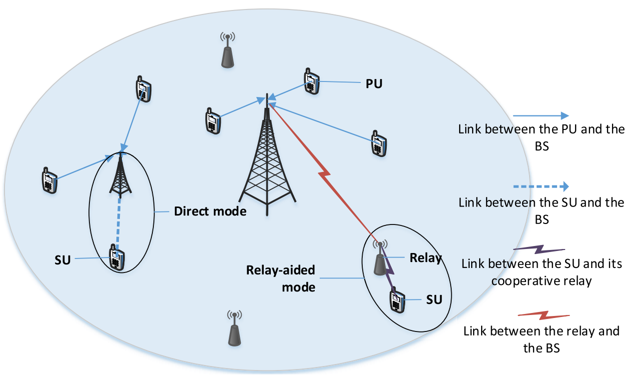

Fig. 1. **The HetNet with the macro BS and pico BSs.**

We consider a fully loaded network scenario where C PUs occupy M channels orthogonally ($C = M$). The SUs can reuse the channel of PUs. To avoid excessive interference to PUs, we assume that each channel can be reused by at most one SU and $U \le C$. For power-limited SUs like sensors or IoT equipments and SUs whose channels are of poor quality, the direct communication between the SU and the BS may not satisfy the QoS requirements. In this case, the SU can connect to the BS with the help of the relay. We assume that the cooperative relay is selected based on the coverage and each relay serves only one SU. In the following, we denote the index for the pair of the SU and its cooperative relay as $u \in \{1, 2, \ldots, U\}$. The SU and its cooperative relay are denoted as u_T and u_R, respectively. For simplicity, hereinafter we denote the channel index for the BS b as $n \in \{1, 2, \ldots, N\}$, where $N = \max\{N_1, N_2, \ldots, N_B\}$.[1] The PU who occupies the channel n of the BS b is denoted as c_{bn}.

We consider the uplink communication for SUs in the HetNet. The SU u_T can communicate with the BS b directly (the direct mode) or through the help of the its cooperative relay u_R (the relay-aided mode). In the latter case, the cooperative relay u_R operates in half-duplex mode using decode-and-forward (DF) protocol. If the SU u_T communicates with the BS b directly, the transmission rate for the SU u_T on the channel n is

$$R_{u_T bn} = W \log_2\left(1 + \frac{p_{u_T bn} g_{u_T bn}}{p_{bn} h_{bn} + W N_0}\right), \tag{1}$$

where $p_{u_T bn}$ is the transmission power of the SU u_T on the channel n of the BS b, $g_{u_T bn}$ is the channel power gain between the SU u_T and the BS b on the channel n, p_{bn} is the transmission power of the PU c_{bn} who occupies the channel n of the BS b, h_{bn} is the channel power gain between the PU c_{bn} and the BS b on the channel n, W is the bandwidth of each channel and N_0 is the thermal noise power density.

[1] For those BSs whose numbers of channels are smaller than N, we set the channel power gain of the nonexistent channel as zero to indicate that the BS does not own that channel. For instance, if the number of channels owned by the BS b is $N_b < N$, the channel power gains for BS b's channels from the $(N_b + 1)$-th one to the N-th one are all set as 0. Therefore there is no user occupying the nonexistent channel of BS b.

If the SU u_T associates with the BS b through the help of its cooperative relay u_R, the transmission rate between u_T and u_R on the channel n of the BS b is

$$r_{u_T u_R bn} = \frac{1}{2} W \log_2\left(1 + \frac{p_{u_T bn} g_{u_T u_R bn}}{p_{bn} h_{u_R bn} + W N_0}\right), \tag{2}$$

where $g_{u_T u_R bn}$ is the channel power gain between u_T and u_R on the channel n of the BS b, $h_{u_R bn}$ is the channel power gain between the PU c_{bn} and the cooperative relay u_R on the channel n of the BS b, $\frac{1}{2}$ denotes the fraction of time allocated to the communication between the SU and the cooperative relay on the channel n of the BS b, , i.e., we assume that the time is equally partitioned between the first and second hops. The transmission rate between u_R and the BS b on the channel n is

$$r_{u_R bn} = \frac{1}{2} W \log_2\left(1 + \frac{p_{u_R bn} g_{u_R bn}}{p_{bn} h_{bn} + W N_0}\right), \tag{3}$$

where $p_{u_R bn}$ is the transmission power of the cooperative relay u_R on the channel n of the BS b, $g_{u_R bn}$ is the channel power gain between the cooperative relay u_R and the BS b on the channel n. Then the transmission rate for the communication between the SU u_T and the BS b on the channel n in the relay-aided mode can be written as

$$R_{u_T u_R bn} = \min\left(r_{u_T u_R bn}, r_{u_R bn}\right). \tag{4}$$

Denote the mode selection index as $m \in \{0, 1\}$. $m = 1$ indicates that the SU u_T associates with the BS through the help of its cooperative relay u_R, and $m = 0$ corresponds to the direct communication between the SU u_T and the BS. Then the transmission rate of the uplink communication between the SU u_T and the BS b on the channel n can be rewritten as

$$R_{ubnm} = \begin{cases} R_{u_T bn} & m = 0 \\ R_{u_T u_R bn} & m = 1. \end{cases} \tag{5}$$

III. PROBLEM FORMULATION

In this paper, we focus on the uplink EE optimization problem for SUs with BS selection, channel allocation, mode selection and power control. More precisely, BS selection and channel allocation denote that we allocate channels from different BSs to different SUs, mode selection decides the transmission mode for SUs, i.e., the direct mode or the relay-aided mode, and power control corresponds to the transmission power optimization for the SU and its cooperative relay.

We define the BS association and channel allocation variable as x_{ubn}, which is equal to 1 when the SU u_T selects the channel n of the BS b, otherwise it is equal to 0. Then the EE of the SU u_T in the direct mode can be expressed as

$$EE_{u_T} = \frac{\sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn} R_{u_T bn}}{\sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn} p_{u_T bn} + p_c}, \tag{6}$$

where p_c is the circuit power consumption for the SU u_T. In the relay-aided mode, the EE for the uplink communication of the SU u_T can be written as,

$$EE_{u_T u_R} = \frac{\sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn} R_{u_T u_R bn}}{\frac{1}{2}\sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn}\left(p_{u_T bn} + p_{u_R bn}\right) + p_c}. \quad (7)$$

For notation brevity, we rewrite the EE of the SU u_T as

$$EE_{um} = \begin{cases} EE_{u_T} & m = 0 \\ EE_{u_T u_R} & m = 1. \end{cases} \quad (8)$$

In this paper, we investigate the EE optimization problem for SUs during the uplink communication through BS selection, channel allocation, mode selection and power control, while considering the rate requirement of SUs, the interference threshold for PUs and the power constraint. particularly, we aim to maximize the weighted sum EE of SUs [18], [19]. The sum EE is chosen due to the fact that each SU competes for resources and SUs' power sources are independent. The weight coefficients are utilized to achieve fairness to some degree among all SUs and can be adjusted by the network operator. Denote the mode selection variable as y_{um}. $y_{um} = 1$ indicates that the SU u_T associates with the BS in the mode m, otherwise $y_{um} = 0$. Then the sum EE optimization problem can be formulated as follows

$$\max_{\mathbf{X},\mathbf{Y},\mathbf{P}_T,\mathbf{P}_R} \sum_{u=1}^{U} \lambda_u \sum_{m=0}^{1} y_{um} EE_{um} \quad (9a)$$
$$\text{s.t. } x_{ubn}, \quad y_{um} \in \{0,1\}, \quad \forall u, \forall b, \forall n, \forall m, \quad (9b)$$
$$\sum_{u=1}^{U} x_{ubn} \leq 1, \quad \forall b, \forall n, \quad (9c)$$
$$\sum_{m=0}^{1} y_{um} = 1, \quad \forall u, \quad (9d)$$
$$0 \leq p_{u_T}, \quad p_{u_R} \leq p_{max}, \quad \forall u, \quad (9e)$$
$$R_u \geq R_{min}, \quad \forall u, \quad (9f)$$
$$\delta_{u_T bn}, \quad \delta_{u_R bn} \leq \delta_{bn}, \quad \forall u, \forall b, \forall n, \quad (9g)$$

where $\mathbf{X}$ is the $U \times B \times N$ matrix variable whose element is x_{ubn}, $\mathbf{Y}$ is the $U \times 2$ matrix variable whose element is y_{um}, $\mathbf{P}_T$ and $\mathbf{P}_R$ are $U \times B \times N$ matrix variable corresponding to $p_{u_T bn}$ and $p_{u_R bn}$ respectively, p_{max} is the maximum transmission power for SUs and cooperative relays, $p_{u_T} = \sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn} p_{u_T bn}$ is the transmission power of the SU u_T, $p_{u_R} = \sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn} y_{u1} p_{u_R bn}$ is the transmission power of the cooperative relay u_R, $R_u = \sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn} \sum_{m=0}^{1} y_{um} R_{ubnm}$ is the transmission rate of the SU u_T, R_{min} is the minimum transmission rate required by u_T, $\delta_{u_T bn} = x_{ubn} p_{u_T bn} g_{u_T bn}$ is the interference from the SU u_T to the PU c_{bn} who occupies the channel n of the BS b, $\delta_{u_R bn} = x_{ubn} y_{u1} p_{u_R bn} g_{u_R bn}$ is the interference from the cooperative relay to the PU c_{bn}, δ_{bn} is the interference threshold for the PU c_{bn}. Note that we do not sum $\delta_{u_T bn}$ and $\delta_{u_R bn}$ since they are generated in different time. λ_u is the constant weight factor associated with the EE which satisfies $\sum_{u=1}^{U} \lambda_u = 1$ and is used to achieve fairness to some extent among all SUs [18], [19]. Constraint (9b) is the integer constraint for x_{ubn} and y_{um}. Constraint (9c) indicates that each channel can be reused by at most one SU. (9d) is mode selection constraint for the SU. (9e) is the power constraint for the SU and its cooperative relay. (9f) is the rate requirement of the SU and constraint (9g) is the interference limit for the PU. It should be noted that, the SU can choose different BSs and utilize multiple channels in the considered scenario, and each SU can choose the direct mode or the relay-aided mode, which makes the user association and power control problem rather complicated.

IV. USER ASSOCIATION AND POWER CONTROL

The problem (9) is a sum-of-ratios fractional programming problem, which is NP-hard in essence [20]. Besides, the integer constraint (9b) makes the problem further complicated. Since it is difficult to derive the optimal solution, we propose an iterative algorithm to get a tractable solution. The proposed algorithm divides the original problem into three subproblems, including BS selection and channel allocation problem, mode selection problem and power control problem. In each iteration, we solve these three subproblems separately. More specifically, the joint BS selection and channel selection problem is solved under fixed mode selection and power control. The other two subproblems are solved based on the same principle, i.e., solving certain variables with other variables fixed. At last, we repeat the iteration till convergence. In the following subsections, the solution of each subproblem will be derived.

A. BS SELECTION AND CHANNEL ALLOCATION

In this subsection, we consider the BS selection and channel allocation problem, given fixed mode selection and power control. Note that mode selection and power control should be solved under corresponding constraints in the problem (9). Denote the mode selection and power control as $\widetilde{y}_{um}$, $\widetilde{p}_{u_T bn}$ and $\widetilde{p}_{u_R bn}$. Then the EE of the SU u_T can be rewritten as,

$$\widetilde{y}_{u0}\widetilde{EE}_{u0} + \widetilde{y}_{u1}\widetilde{EE}_{u1} = \frac{\sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn} k_{ubn}}{\sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn} q_{ubn} + p_c}, \quad (10)$$

where

$$\widetilde{EE}_{u0} = \frac{\sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn} \widetilde{R}_{u_T bn}}{\sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn} \widetilde{p}_{u_T bn} + p_c},$$
$$\widetilde{EE}_{u1} = \frac{\sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn} \widetilde{R}_{u_T u_R bn}}{\frac{1}{2}\sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn}\left(\widetilde{p}_{u_T bn} + \widetilde{p}_{u_R bn}\right) + p_c},$$

$\widetilde{R}_{u_T bn}$ and $\widetilde{R}_{u_T u_R bn}$ are calculated according to $\widetilde{p}_{u_T bn}$ and $\widetilde{p}_{u_R bn}$. $k_{ubn} = \widetilde{R}_{u_T bn}$ and $q_{ubn} = \widetilde{p}_{u_T bn}$ when $\widetilde{y}_{u0} = 1$, otherwise $k_{ubn} = \widetilde{R}_{u_T u_R bn}$ and $q_{ubn} = \frac{1}{2}\left(\widetilde{p}_{u_T bn} + \widetilde{p}_{u_R bn}\right)$. The above algebraic simplification in (10) is based on the fact that each SU only chooses one transmission mode, i.e., $\sum_{m=0}^{1} y_{um} = 1$. Then, we relax x_{ubn} to be a continuous variable between 0 and 1, in which case x_{ubn} can be interpreted as the fraction of time that the channel n of the BS

b is allocated to the SU u_T. With the formula (10) and the continuity relaxation of x_{ubn}, the problem (9) under fixed mode selection and power control can be formulated as,

$$\max_{\mathbf{X}} \sum_{u=1}^{U} \lambda_u \frac{\sum_{b=1}^{B}\sum_{n=1}^{N} x_{ubn}k_{ubn}}{\sum_{b=1}^{B}\sum_{n=1}^{N} x_{ubn}q_{ubn} + p_c} \tag{11a}$$
$$\text{s.t. } 0 \le x_{ubn} \le 1, \quad \forall u, \forall b, \forall n, \tag{11b}$$
$$\sum_{u=1}^{U} x_{ubn} \le 1, \quad \forall b, \forall n, \tag{11c}$$
$$0 \le \widetilde{p}_{u_T}, \quad \widetilde{p}_{u_R} \le p_{max}, \quad \forall u, \tag{11d}$$
$$\widetilde{R}_u \ge R_{min}, \quad \forall u, \tag{11e}$$
$$\widetilde{\delta}_{u_T bn}, \quad \widetilde{\delta}_{u_R bn} \le \delta_{bn}, \quad \forall u, \forall b, \forall n, \tag{11f}$$

where $\widetilde{p}_{u_T}$, $\widetilde{p}_{u_R}$, $\widetilde{R}_u$, $\widetilde{\delta}_{u_T bn}$ and $\widetilde{\delta}_{u_R bn}$ are corresponding algebraic expressions calculated according to $\widetilde{y}_{um}$, $\widetilde{p}_{u_T bn}$ and $\widetilde{p}_{u_R bn}$. To this end, we introduce Theorem 1 to transform the problem (11) into a tractable one. The proof of Theorem 1 is detailed in Appendix A.

Theorem 1: If $\mathbf{X}^*$ is optimal solution of the problem (11), then there exist $\boldsymbol{\beta}^* = (\beta_1^*, \beta_2^*, \ldots, \beta_U^*)$ and $\boldsymbol{\gamma}^* = (\gamma_1^*, \gamma_2^*, \ldots, \gamma_U^*)$ such that $\mathbf{X}^*$ is a solution of the following problem for $\boldsymbol{\beta} = \boldsymbol{\beta}^*$ and $\boldsymbol{\gamma} = \boldsymbol{\gamma}^*$.

$$\max_{\mathbf{X}} \sum_{u=1}^{U} \beta_u \left(f_u(\mathbf{X}) - \gamma_u h_u(\mathbf{X})\right) \tag{12}$$
$$\text{s.t. (11b), (11c), (11d), (11e), (11f),}$$

where $f_u(\mathbf{X}) = \sum_{b=1}^{B}\sum_{n=1}^{N} x_{ubn}\lambda_u k_{ubn}$ and $h_u(\mathbf{X}) = \sum_{b=1}^{B}\sum_{n=1}^{N} x_{ubn}q_{ubn} + p_c$. And $\mathbf{X}^*$ also satisfies the following system of equations for $\boldsymbol{\beta} = \boldsymbol{\beta}^*$ and $\boldsymbol{\gamma} = \boldsymbol{\gamma}^*$,

$$\beta_u = \frac{1}{h_u(\mathbf{X})}, \quad u = 1, 2, \ldots, U \tag{13}$$
$$f_u(\mathbf{X}) - \gamma_u h_u(\mathbf{X}) = 0, \quad u = 1, 2, \ldots, U. \tag{14}$$

Since $\boldsymbol{\beta}^*$ and $\boldsymbol{\gamma}^*$ are unknown, we propose a BS selection and channel allocation (BSCA) algorithm which gets close to the solution of the problem (11) iteratively. The detailed process is illustrated in Algorithm 1.

It can be derived that $\left\|\boldsymbol{\psi}^{(t)}\right\|$ is a monotonically decreasing function with respect to t and converges to zero [15]. Therefore the convergence of Algorithm 1 is guaranteed. The detailed convergence proof of the BSCA algorithm can be referred in [15].

After the problem transformation, we focus on the optimization problem (12) given $\boldsymbol{\beta}^{(t)}$ and $\boldsymbol{\gamma}^{(t)}$. It can be observed that the problem (12) is a linear programming problem, hence it can be solved with the Lagrangian dual method or other convex optimization methods [17] efficiently.

Algorithm 1 BS Selection and Channel Allocation (BSCA) Algorithm

Initialize:
$\beta_u^{(0)} = \frac{1}{h_u(\mathbf{X}^{(0)})}, \quad \gamma_u^{(0)} = \frac{f_u(\mathbf{X}^{(0)})}{h_u(\mathbf{X}^{(0)})}, \quad u = 1, 2, \ldots, U.$
Denote $\tau_u^{(0)} = -f_u(\mathbf{X}^{(0)}) + \gamma_u^{(0)} h_u(\mathbf{X}^{(0)})$, $\varphi_u^{(0)} = -1 + \beta_u^{(0)} h_u(\mathbf{X}^{(0)})$ and $\boldsymbol{\psi}^{(0)} = \left(\tau_1^{(0)}, \ldots, \tau_U^{(0)}, \varphi_1^{(0)}, \ldots, \varphi_U^{(0)}\right)$.
Set the iteration count $t = 0$, the maximum number of iterations T and the iteration threshold $\varepsilon > 0$ which is used for terminating the algorithm.
Repeat:
Solve the problem (12) given $\boldsymbol{\beta}^{(t)}$ and $\boldsymbol{\gamma}^{(t)}$ and obtain $\mathbf{X}^{(t)}$.
Update $\boldsymbol{\psi}^{(t)}$ with $\boldsymbol{\beta}^{(t)}$, $\boldsymbol{\gamma}^{(t)}$ and $\mathbf{X}^{(t)}$.
Update $\boldsymbol{\beta}^{(t)}$ and $\boldsymbol{\gamma}^{(t)}$ with the modified Newton method [15].
$t = t + 1$.
Until:
$\left\|\boldsymbol{\psi}^{(t)}\right\| < \varepsilon$ or $t > T$.

B. MODE SELECTION

In this subsection, we focus on the mode selection problem. The BS selection, channel allocation and power control are given by $\check{x}_{ubn}$, $\check{p}_{u_T bn}$ and $\check{p}_{u_R bn}$. Then the problem (9) with fixed $\check{x}_{ubn}$, $\check{p}_{u_T bn}$ and $\check{p}_{u_R bn}$ can be formulated as follows,

$$\max_{\mathbf{Y}} \sum_{u=1}^{U} \lambda_u \sum_{m=0}^{1} y_{um}\breve{EE}_{um} \tag{15a}$$
$$\text{s.t. } y_{um} \in \{0, 1\}, \quad \forall u, \forall m, \tag{15b}$$
$$\sum_{m=0}^{1} y_{um} = 1, \quad \forall u, \tag{15c}$$
$$0 \le \breve{P}_{u_R} \le p_{max}, \quad \forall u, \tag{15d}$$
$$\breve{R}_u \ge R_{min}, \quad \forall u, \tag{15e}$$
$$\breve{\delta}_{u_R bn} \le \delta_{bn}, \quad \forall u, \forall b, \forall n, \tag{15f}$$

where $\breve{EE}_{um}$, $\breve{P}_{u_R}$, $\breve{R}_u$ and $\breve{\delta}_{u_R bn}$ are algebraic expressions calculated according to $\check{x}_{ubn}$, $\check{p}_{u_T bn}$ and $\check{p}_{u_R bn}$. The optimization problem (15) can be reformulated as follows,

$$\sum_{u=1}^{U} \lambda_u \max_{\mathbf{Y}} \sum_{m=0}^{1} y_{um}\breve{EE}_{um}$$
$$\text{s.t. (15b), (15c), (15d), (15e), (15f).} \tag{16}$$

Since different SUs' transmission modes are not correlated, the optimal mode selection for the SU u_T can be simply written as follows,

$$\begin{cases} y_{u0} = 1, & \breve{EE}_{u0} \ge \breve{EE}_{u1}. \\ y_{u1} = 1, & \breve{EE}_{u0} < \breve{EE}_{u1}. \end{cases} \tag{17}$$

The solution is quite intuitive, i.e., each SU u_T chooses the mode which has higher $\breve{EE}_{um}$. It should be noted that the chosen transmission mode should satisfy the constraints of the problem (15).

C. POWER CONTROL

Given the fixed BS selection, channel allocation and mode selection, in this subsection, we investigate the power control problem. We assume that BS selection, channel allocation and mode selection are denoted as $\hat{x}_{ubn}$ and $\hat{y}_{um}$. Then the problem (9) with fixed $\hat{x}_{ubn}$ and $\hat{y}_{um}$ can be expressed as,

$$\max_{\mathbf{P}_T,\mathbf{P}_R} \sum_{u=1}^{U} \lambda_u \sum_{m=0}^{1} \hat{y}_{um}\hat{EE}_{um} \tag{18a}$$
$$\text{s.t. } 0 \le \hat{p}_{u_T},\quad \hat{p}_{u_R} \le p_{max}, \forall u, \tag{18b}$$
$$\hat{R}_u \ge R_{min}, \quad \forall u, \tag{18c}$$
$$\hat{\delta}_{u_T bn},\quad \hat{\delta}_{u_R bn} \le \delta_{bn}, \quad \forall u, \forall b, \forall n, \tag{18d}$$

where $\hat{EE}_{um}$, $\hat{p}_{u_T}$, $\hat{p}_{u_R}$, $\hat{R}_u$, $\hat{\delta}_{u_T bn}$ and $\hat{\delta}_{u_R bn}$ are corresponding algebraic expressions calculated based on $\hat{x}_{ubn}$ and $\hat{y}_{um}$. Since each SU's transmission mode is fixed and channels are allocated to SUs, the power control problem (18) can be decoupled into U subproblems, i.e.,

$$\max_{\mathbf{P}_T,\mathbf{P}_R} \sum_{u=1}^{U} \lambda_u \sum_{m=0}^{1} \hat{y}_{um}\hat{EE}_{um} = \sum_{u=1}^{U} \lambda_u \max_{\mathbf{P}_{u_T},\mathbf{P}_{u_R}} \sum_{m=0}^{1} \hat{y}_{um}\hat{EE}_{um}, \tag{19}$$

where $\mathbf{P}_{u_T}$ and $\mathbf{P}_{u_R}$ are $B\times N$ matrix variables whose elements are $p_{u_T bn}$ and $p_{u_R bn}$ for the SU u_T, respectively. For each SU, the power control problem can be rewritten as follows,

$$\max_{\mathbf{P}_{u_T},\mathbf{P}_{u_R}} \sum_{m=0}^{1} \hat{y}_{um}\hat{EE}_{um}$$
$$\text{s.t. (18b), (18c), (18d).} \tag{20}$$

For the SU who chooses the direct mode, the EE maximization problem is denoted as,

$$\max_{\mathbf{P}_{u_T}} \frac{\sum_{b=1}^{B}\sum_{n=1}^{N} \hat{x}_{ubn} R_{u_T bn}}{\sum_{b=1}^{B}\sum_{n=1}^{N} \hat{x}_{ubn} p_{u_T bn} + p_c}$$
$$\text{s.t. (18b), (18c), (18d).} \tag{21}$$

It can be seen that the problem (21) is still a non-convex problem. However, the fractional optimization problem (21) can be transformed into an equivalent one in subtractive form which is formulated as

$$\max_{\mathbf{P}_{u_T}} \hat{R}_{u0} - \eta^* \hat{P}_{u0}$$
$$\text{s.t. (18b), (18c), (18d),} \tag{22}$$

where $\hat{R}_{u0} = \sum_{b=1}^{B}\sum_{n=1}^{N} \hat{x}_{ubn} R_{u_T bn}$, $\hat{P}_{u0} = \sum_{b=1}^{B}\sum_{n=1}^{N} \hat{x}_{ubn} p_{u_T bn} + p_c$ and η^* denotes the optimal value of the objective function (21) [16]. Since η^* cannot be obtained directly, the parametric Dinkelbach algorithm proposed in [16] is used to get the optimal solution. The detailed process is illustrated in Algorithm 2.

Since it can be derived that $\eta^{(t+1)} \ge \eta^{(t)}$ and $\left\|\eta^{(t+1)} - \eta^{(t)}\right\|$ is a monotonically nonincreasing function for $t = 1, 2, \ldots$, the convergence of Algorithm 2 is guaranteed. The detailed convergence proof is provided in [16].

Algorithm 2 Power Control for Direct Communication (PCDC)

Initialize:
Set the iteration count $t = 0$, the maximum number of iterations T and the iteration threshold $\varepsilon > 0$ which is used for terminating the algorithm.
Initialize $\mathbf{p}_{u_T}^{(0)}$ with an achievable value and the corresponding $\eta^{(0)}$.
Repeat:
Solve the problem (23) with a fixed $\eta^{(t)}$ and obtain $\mathbf{p}_{u_T}^{(t)}$.
Update $\eta^{(t+1)} = \frac{\hat{R}_{u0}^{(t)}}{\hat{P}_{u0}^{(t)}}$ with the obtained $\mathbf{p}_{u_T}^{(t)}$.
Until:
$\left\|\eta^{(t+1)} - \eta^{(t)}\right\| < \varepsilon$ or $t > T$.

It can be seen that the most important step in Algorithm 2 is to solve the power control problem with a fixed η. The problem is written as follows,

$$\max_{\mathbf{P}_{u_T}} \hat{R}_{u0} - \eta \hat{P}_{u0}$$
$$\text{s.t. (18b), (18c), (18d).} \tag{23}$$

We can observe that the objective function (23) is concave [17] and the inequality constraints are convex. Hence the problem (23) is a convex optimization problem which can be solved effectively with the Lagrangian dual method or other convex optimization methods [17].

Consider the EE maximization problem when a SU u_T chooses the relay-aided mode. The corresponding power control problem is formulated as,

$$\max_{\mathbf{P}_{u_T},\mathbf{P}_{u_R}} \frac{\sum_{b=1}^{B}\sum_{n=1}^{N} \hat{x}_{ubn} R_{u_T u_R bn}}{\frac{1}{2}\sum_{b=1}^{B}\sum_{n=1}^{N} \hat{x}_{ubn_b}\left(p_{u_T bn} + p_{u_R bn}\right) + p_c}$$
$$\text{s.t. (18b), (18c), (18d).} \tag{24}$$

Note that in the situation that u_T communicates with the BS b on the channel n through the help of u_R, the optimal power allocation on the channel n must be subject to the following condition,

$$r_{u_T u_R bn} = r_{u_R bn} = R_{u_T u_R bn} = \overline{R}_{ubn}. \tag{25}$$

With the formula (25), the original problem (24) can be simplified as,

$$\max_{\overline{\mathbf{R}}_u} \frac{\sum_{b=1}^{B}\sum_{n=1}^{N} \hat{x}_{ubn} \overline{R}_{ubn}}{\frac{1}{2}\sum_{b=1}^{B}\sum_{n=1}^{N} \hat{x}_{ubn}\left(\overline{p}_{u_T bn} + \overline{p}_{u_R bn}\right) + p_c} \tag{26a}$$
$$\text{s.t. } 0 \le \overline{p}_{u_T},\quad \overline{p}_{u_R} \le p_{max}, \tag{26b}$$
$$\overline{R}_u \ge R_{min}, \tag{26c}$$
$$\overline{\delta}_{u_T bn},\ \overline{\delta}_{u_R bn} \le \delta_{bn}, \quad \forall b, \forall n, \tag{26d}$$

where $\overline{p}_{u_T bn} = \frac{\left(2^{\frac{2\overline{R}_{ubn}}{W}} - 1\right)(p_{bn} h_{u_R bn} + W N_0)}{g_{u_T u_R bn}}$ and $\overline{p}_{u_R bn} = \frac{\left(2^{\frac{2\overline{R}_{ubn}}{W}} - 1\right)(p_{bn} h_{bn} + W N_0)}{g_{u_R bn}}$ denote the power corresponding to the transmission rate $\overline{R}_{ubn}$ on the channel n of

Algorithm 3 Power Control for Relay-Aided Communication (PCRC)

Initialize:

Set the iteration count $t = 0$, the maximum number of iterations T and the iteration threshold $\varepsilon > 0$ which is used for terminating the algorithm.

Initialize $\overline{\mathbf{R}}_u^{(0)}$ and $\eta^{(0)}$.

Repeat:

Solve the problem (27) with a fixed $\eta^{(t)}$ and obtain $\overline{\mathbf{R}}_u^{(t)}$.

Update $\eta^{(t+1)} = \frac{\overline{R}_{u1}}{\overline{P}_{u1}}$ with the obtained $\overline{\mathbf{R}}_u^{(t)}$.

Until:

$\left\|\eta^{(t+1)} - \eta^{(t)}\right\| < \varepsilon$ or $t > T$.

the BS b, $\overline{\mathbf{R}}_u$ is $B \times N$ matrix variable whose element is $\overline{R}_{ubn}$ for the SU $u_T, \overline{p}_{u_T}, \overline{p}_{u_R}, \overline{R}_u, \overline{\delta}_{u_T bn}$ and $\overline{\delta}_{u_R bn}$ are corresponding algebraic expressions generated based on the recalculation of $\hat{p}_{u_T}, \hat{p}_{u_R}, \hat{R}_u, \hat{\delta}_{u_T bn}$ and $\hat{\delta}_{u_R bn}$ with $\overline{R}_{ubn}, \overline{p}_{u_T bn}$ and $\overline{p}_{u_R bn}$. Note that it is still a non-convex optimization problem. Like before, we apply the parametric Dinkelbach algorithm to solve the problem (26). The whole process of the algorithm is shown in Algorithm 3. For notation brevity, we denote the numerator and the denominator of the objective function (26a) as $\overline{R}_{u1}$ and $\overline{P}_{u1}$ afterwards.

The most critical part in Algorithm 3 is to solve the power control problem written as follows,

$$\max_{\overline{\mathbf{R}}_u} \ \overline{R}_{u1} - \eta \overline{P}_{u1}, \quad \text{s.t. (26b), (26c), (26d).} \tag{27}$$

It can be derived that the objective function (27) is concave with respect to $\overline{\mathbf{R}}_u$ and the inequality constraints are convex, thus the Lagrangian dual method or other convex optimization methods [17] can be utilized to solve the problem (27) efficiently.

Recall that in each iteration, we solve the above three subproblems separately. And the whole algorithm repeat the iteration until convergence. The detailed user association and power control (UAPC) algorithm is summarized in Algorithm 4.

It should be noted that, the objective function to be optimized is the same in each iteration and the convergence of Algorithm 1, 2 and 3 is guaranteed. Besides, in each optimization subproblem, we solve the corresponding EE maximization problem optimally. This means that $\eta_{sum}^{(t)}$ is nondecreasing with respect to t, and hence, the convergence of Algorithm 4 is guaranteed.

Since the convex optimization problem can be solved efficiently with the Lagrangian dual method or other convex optimization methods [17], the computational complexity of the BSCA algorithm mainly depends on the number of iterations in Algorithm 1. Similarly, The complexity of the power control algorithm also relies on the number of iterations in Algorithm 2 and 3. Note that the three subproblems are solved repeatedly in the UAPC algorithm, so the whole computational complexity of the proposed method highly depends on the number of iterations in Algorithm 1, 2, 3 and 4 which will be illustrated in simulation results.

Algorithm 4 User Association and Power Control (UAPC) Algorithm

Initialize:

Initialize $\mathbf{X}^{(0)}, \mathbf{Y}^{(0)}, \mathbf{P}_T^{(0)}, \mathbf{P}_R^{(0)}$ and the corresponding sum EE $\eta_{sum}^{(0)}$.

Set the iteration count $t = 0$, the maximum number of iterations T and the iteration threshold $\varepsilon > 0$ which is used for terminating the algorithm.

Repeat:

Solve the BS selection and channel allocation problem with Algorithm 1 under fixed $\mathbf{Y}^{(t)}, \mathbf{P}_T^{(t)}$ and $\mathbf{P}_R^{(t)}$ and obtain $\mathbf{X}^{(t+1)}$.

Solve the mode selection problem under fixed $\mathbf{X}^{(t+1)}$, $\mathbf{P}_T^{(t)}$ and $\mathbf{P}_R^{(t)}$ and obtain $\mathbf{Y}^{(t+1)}$.

Solve the power control problem with Algorithm 2 and 3 under fixed $\mathbf{X}^{(t+1)}$ and $\mathbf{Y}^{(t+1)}$ and obtain $\mathbf{P}_T^{(t+1)}$ and $\mathbf{P}_R^{(t+1)}$.

Calculate $\eta_{sum}^{(t+1)}$ with $\mathbf{X}^{(t+1)}, \mathbf{Y}^{(t+1)}, \mathbf{P}_T^{(t+1)}$ and $\mathbf{P}_R^{(t+1)}$.

$t = t + 1$.

Until:

$\left\|\eta_{sum}^{(t+1)} - \eta_{sum}^{(t)}\right\| < \varepsilon$ or $t > T$.

TABLE 1. Simulation parameters.

Parameters	Values
Number of macro BSs	1
Number of pico BSs	8
Number of channels for one macro BS	40
Number of channels for one pico BS	5
Number of PUs	80
Number of SUs	50
Number of relays	60
The transmission power of PU	0.2W
The circuit power	0.1W
Thermal noise power density	-174dBm/Hz
The frequency bandwidth of each channel	100kHz
Path loss	$128.1+37.6\log_{10}(d)$
The Standard deviation of lognormal shadowing	8dB

To evaluate the performance of the UAPC algorithm, a compared algorithm which is modified from the algorithm in [12] is proposed. The detailed process of the BASE algorithm is illustrated in Algorithm 5 and the convergence the parametric Dinkelbach algorithm and the Kuhn-Munkres (KM) algorithm which are used in the BASE algorithm is shown in [12] and [21].

V. SIMULATIONS

In this section, the EE performance of the proposed algorithm is confirmed with simulations. Simulation parameters are detailed in Table 1. Pico BSs, users and relays are uniformly and randomly distributed in the macrocell. The weight coefficient is assumed to be equal for all SUs. The channel power gain contains the path loss, shadow fading and frequency-selective fading [22]. The frequency-selective fading is modeled as Rayleigh random process with unit variances.

Algorithm 5 The BASE Algorithm

Reorder the channels from all BSs as $m \in \{1, 2, \ldots, M\}$, M is the total number of channels in the HetNet.

For each SU u_T, calculate its maximum achievable EE on each channel m in the direct mode with the parametric Dinkelbach algorithm and the Lagrangian dual method. The calculation process is similar to Algorithm 2, except that the EE is optimized with only one channel.

Formulate the sum EE maximization problem for all SUs as follows,

$$\max_{\mathbf{Z}} \sum_{u=1}^{U} \sum_{m=1}^{M} z_{um} w_{um} \tag{28a}$$

$$\text{s.t. } z_{um} \in \{0, 1\}, \quad \forall u, \forall m, \tag{28b}$$

$$\sum_{u=1}^{U} z_{um} \leq 1, \quad \forall m, \tag{28c}$$

$$\sum_{m=1}^{M} z_{um} = 1, \quad \forall u, \tag{28d}$$

where w_{um} denotes the maximum achievable EE when the SU u_T uses the channel m. Solve the problem (28) with the KM algorithm [21]. After the convergence of the KM algorithm, each SU is allocated with one and only one orthogonal channel.

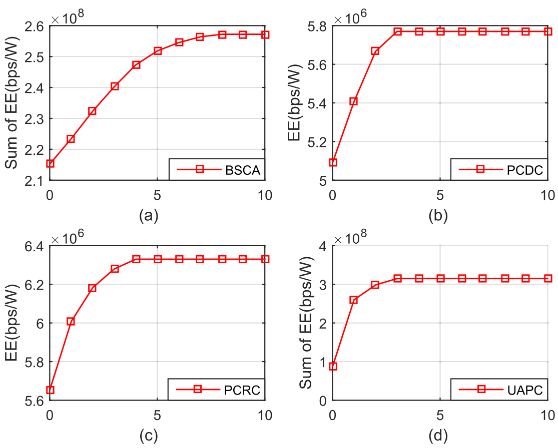

Fig. 2. **Convergence of the proposed algorithm. (a) Inner iterations–BSCA. (b) Inner iterations–PCDC. (c) Inner iterations–PCRC. (d) Outer iterations–UAPC.**

Fig. 2 shows the convergence behavior of the proposed algorithm. The rate constraint is 100kbps, the interference threshold is −100dB and the maximum transmission power for each SU is 0.2W. Fig. 2 (a) illustrates the number of iterations for the BS selection and channel allocation algorithm. The convergence of the power control algorithm for the direct mode and the relay-aided mode is shown in Fig. 2 (b) and Fig. 2 (c). Fig. 2 (d) shows the number of outer iterations in the UAPC algorithm. It can be observed that, for the inner loop, the BSCA algorithm converges to the stable value within 10 iterations. The proposed power control algorithms, i.e., PCDC and PCRC methods, converge to the stable value with only 5 iterations, as shown in Fig. 2 (b) and Fig. 2 (c). For the outer loop, the convergence of the UAPC algorithm takes about 3 iterations.

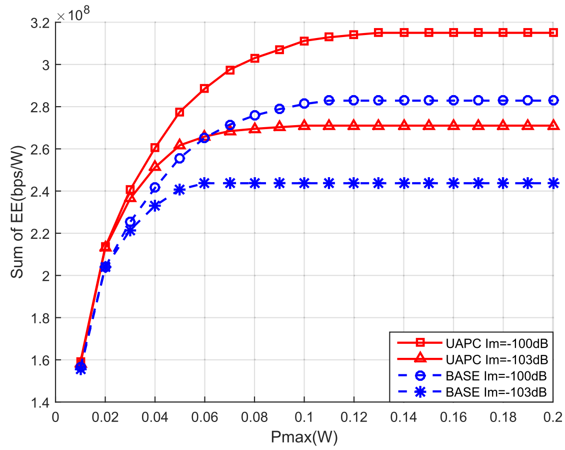

Fig. 3. **EE performance under different power constraint.**

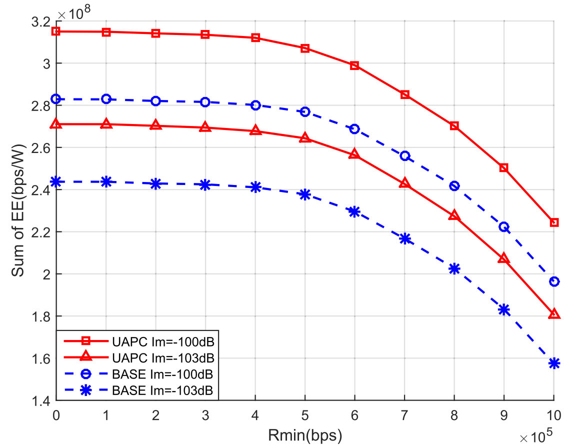

Fig. 4. **EE performance under different rate constraint.**

Fig. 3 compares the EE performance of the proposed UAPC algorithm with that of the compared method under different power constraint. The rate constraint is 100kbps. I_m denotes the maximum interference threshold for each PU. It can be observed that the proposed method outperforms the compared scheme obviously. When the power constraint is low, the EE of the UAPC method and the EE of the BASE method are both small, because the low power constraint restricts the feasible region of the problem. With the growth of the maximum transmission power, the feasible region expands and the UAPC method can choose proper transmission mode and allocate power to different channels. Compared with the BASE method which only selects the direct mode and allocates only one channel to each SU, the UAPC algorithm achieves higher EE.

Fig. 4 shows the EE performance of the two schemes with respect to different rate constraint. The power constraint is 0.2W. Still, the EE performance of the UAPC method is better than that of the compared scheme. This is because the

UAPC method is able to allocate multiple channels to one SU and achieve better resource allocation. As the rate constraint grows, the EE performance decreases slightly first. When the minimum required rate is larger than 0.6Mbps, the EE of the two schemes both decline distinctly as a result of the narrowing of the feasible region.

VI. CONCLUSIONS

In this paper, we consider the uplink EE optimization problem for the communication between the SU and the BS with BS selection, channel allocation, mode selection and power control. The original problem is a non-convex and mixed-integer optimization problem which is difficult to be solved. To get a tractable solution, we propose an iterative algorithm which decomposes the problem into three subproblems. Then we solve these subproblems with the sum-of-ratios programming algorithm, the parametric Dinkelbach algorithm and convex optimization. Simulation results confirm the effectiveness of the proposed method.

Appendix

A. Proof of Theorem 1

To begin with, we transform the problem (11) into an equivalent form,

$$\max_{\mathbf{X}} \sum_{u=1}^{U} \gamma_u \tag{29a}$$

$$\text{s.t.}\ \frac{f_u(\mathbf{X})}{h_u(\mathbf{X})} \geq \gamma_u, \quad u = 1, \ldots, U, \tag{29b}$$

$$g_i(\mathbf{X}) \geq 0, \quad i = 1, ..., I, \tag{29c}$$

where $f_u(\mathbf{X}) = \sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn}\lambda_u k_{ubn}$ and $h_u(\mathbf{X}) = \sum_{b=1}^{B} \sum_{n=1}^{N} x_{ubn} q_{ubn} + p_c$. For simplicity, we denote $g_i(\mathbf{X})$ as the inequality constraints of the problem (11) and I as the total number of the inequality constraints in the problem (11). The constraint $\frac{f_u(\mathbf{X})}{h_u(\mathbf{X})} \geq \gamma_u$ is equivalent to $f_u(\mathbf{X}) - \gamma_u h_u(\mathbf{X}) \geq 0$. Define the following function for the problem (29),

$$L(\mathbf{X}, \boldsymbol{\gamma}, w, \boldsymbol{\beta}, \boldsymbol{\nu}) = w \sum_{u=1}^{U} \gamma_u + \sum_{u=1}^{U} \beta_u (f_u(\mathbf{X}) - \gamma_u h_u(\mathbf{X})) + \sum_{i=1}^{I} \nu_i g_i(\mathbf{X}), \tag{30}$$

where $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_U)$, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_U)$ and $\boldsymbol{\nu} = (\nu_1, \ldots, \nu_I)$. Denote $\mathbf{X}^*$ and $\boldsymbol{\gamma}^* = (\gamma_1^*, \gamma_2^*, \ldots, \gamma_U^*)$ as the optimal solution of the problem (29). By Fritz-John optimality condition [23], there exist w^*, $\boldsymbol{\beta}^* = (\beta_1^*, \beta_2^*, \ldots, \beta_U^*)$ and $\boldsymbol{\nu}^* = (\nu_1^*, \ldots, \nu_I^*)$ satisfying,

$$\frac{\partial L}{\partial \mathbf{X}} = \sum_{u=1}^{U} \beta_u^* (\nabla f_u(\mathbf{X}^*) - \gamma_u^* \nabla h_u(\mathbf{X}^*)) + \sum_{i=1}^{I} \nu_i^* \nabla g_i(\mathbf{X}^*) = 0 \tag{31}$$

$$\frac{\partial L}{\partial \gamma_u} = w^* - \beta_u^* h_u(\mathbf{X}^*) = 0, \quad u = 1, \ldots, U \tag{32}$$

$$\beta_u^* \frac{\partial L}{\partial \beta_u} = \beta_u^* (f_u(\mathbf{X}^*) - \gamma_u^* h_u(\mathbf{X}^*)) = 0, \quad u = 1, \ldots, U \tag{33}$$

$$\nu_i \frac{\partial L}{\partial \nu_i} = \nu_i^* g_i(\mathbf{X}^*) = 0, \quad i = 1, \ldots, I \tag{34}$$

$$g_i(\mathbf{X}^*) \geq 0, \quad \nu_i^* \geq 0, \quad i = 1, \ldots, I \tag{35}$$

$$f_u(\mathbf{X}^*) - \gamma_u^* h_u(\mathbf{X}^*) \geq 0, \quad \beta_u^* \geq 0,\ u = 1, \ldots, U \tag{36}$$

$$w^* \geq 0, (w^*, \boldsymbol{\beta}^*, \boldsymbol{\nu}^*) \neq (0, \mathbf{0}, \mathbf{0}) \tag{37}$$

where $\mathbf{0}$ denotes the vector whose elements are all 0. For notation brevity, we omit the dimensional indicator.

Following similar steps in [15], it can be derived that $w^* > 0$. Denote $\frac{\boldsymbol{\beta}^*}{w^*}$ and $\frac{\boldsymbol{\nu}^*}{w^*}$ by $\boldsymbol{\beta}^*$ and $\boldsymbol{\nu}^*$ again, we can see that (32) is equivalent to (13), then (33) is equivalent to (14) since $\beta_u^* > 0, u = 1, \ldots, U$ by (32).

With $\boldsymbol{\beta} = \boldsymbol{\beta}^*$ and $\boldsymbol{\gamma} = \boldsymbol{\gamma}^*$, it can be observed that (31), (34) and (35) are the KKT conditions for the problem (12). Since the problem (12) is convex programming for parameters $\boldsymbol{\beta} \succ 0$ and $\boldsymbol{\gamma} \succeq 0$, the KKT condition is also sufficient optimality condition and then $\mathbf{X}^*$ is the solution of the problem (12) for $\boldsymbol{\beta} = \boldsymbol{\beta}^*$ and $\boldsymbol{\gamma} = \boldsymbol{\gamma}^*$.

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