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Statistical Analysis of Time-Varying Characteristics of Testability Index Based on NHPP

ZHIAO ZHAO, YONG ZHANG, GUANJUN LIU, AND JING QIU
Science and Technology on Integrated Logistics Support Laboratory, College of Mechatronic Engineering and Automation, National University of Defense Technology, Changsha 410073, China

Corresponding author: J. Qiu (qiujing16@sina.com)

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(Zhiao Zhao and Yong Zhang contributed equally to this work.)

ABSTRACT The failure detection rate (FDR) is the most common testability index used to evaluate the equipment testability level. According to the testability test theory of FDR, a widespread supposition is that the FDR value of a system is an unknown constant. However, there have been a few attempts to research the sample property of failure detection and the statistical characteristics of FDR to prove this fundamental premise. Considering the real maintenance effects on the failure occurrence process, the value of FDR catches time-varying characteristics, which can be depicted as a special statistical process. A failure occurrence model based on the non-homogeneous Poisson process (NHPP) is proposed to depict failure occurrence samples under the assumption of minimal maintenance policy. The binominal cumulative probability function (CDF) is used to depict the each failure detection action. Combining the NHPP based failure occurrence model and the failure detection model based on a binominal distribution, we can simulate the failure detection samples and statistical characteristics of FDR based on the Monte Carlo method. This paper mainly focuses on the expectation and variance of FDR, which are two key statistical characteristics. To validate the FDR time-varying characteristics, we perform a simulation using two Shop Replaceable Units in a level flight indicator of a helicopter to evaluate the FDR value. Based on theoretic and simulative methods, the FDR expectation of the level flight indicator has an increasing or decreasing tendency in the early stages and tends to be a constant in later stages, while the variation of FDR keeps monotonously decreasing. Under the assumptions made in this paper, the supposition that the FDR value of a system is a certain value is not suitable in all stages of the failure occurrence process.

INDEX TERMS Binominal distribution, failure detection rate, NHPP, statistical characteristics, testability index.

I. INTRODUCTION

High equipment testability levels can shorten the mean time to repair (MTTR) and give an immediate failure alarm, which has effects on maintenance actions, availability and safety [1]. The failure detection rate (FDR) is one of the fundamental testability indexes. It measures the ability to detect failures occurring in the system by prescriptive test means. FDR acts as the constraint and measurement of the testability level at two phases, including design for testability (DFT) and testability demonstration. At the design stage, FDR acts as a constraint of the product testability level. As a contract requirement, the product purchaser usually proposes the design value of FDR. At the demonstration stage, FDR is used as a measure to validate the product testability level. At this stage, the purchaser decides whether to accept the product by using statistical sampling methods. In order to validate the value of FDR, existing test theory considers that the FDR real value of a group of test subjects is a certain value. Therefore, quality inspection theory is used to validate the value FDR [2].

According to quality inspection theory, random sampling is the single source of statistical errors. Statistical errors can be divided into two sides, which are defined as the user's risk and manufacturer's risk. If the FDR test method based on the quality inspection theory is effective, it must accord with the assumption that the real value of FDR is a certain value.

Zhang *et al*. [3] proposed that the FDR has time-varying characteristics but they did not carry out further research. Based on the failure occurrence model under renewal theory, Zhao *et al*. [4] analysed the FDR expectation at different times. However, perfect repair assumptions are not suitable for most practical equipment and systems. Other failure-repair actions have received little attention with respect to the statistical characteristics of FDR and the influential factors. Accordingly, research on the characteristics of FDR is crucial, being the test theory basis of FDR demonstration. This paper mainly focuses on the statistical characteristics of FDR and aims to determine the validity of the assumptions we have made.

According to the definition of FDR, the value of FDR is affected by the failure occurrence process and each detection result [5]. Supposing that each failure detection result follows a binominal distribution, the FDR value can be defined as a probability value. Considering lifetime failure detection results, the FDR value is mainly influenced by the failure occurrence process. According to the repair policy when a failure occurs, systems can be categorized as non-repairable systems and repairable systems [6]. The renewal process can be used to model the failure occurrence process of non-repairable systems under the assumption of perfect repair. In reality, most systems, ranging from military equipment to civil products, are repairable [7]. Repairable systems can be restored to operating condition after repair actions such as adjustment, restoration, or lubrication etc.

Despite other differences between failures, the failure time interval is usually a common characteristics used to depict failure occurrence laws. Each failure time interval is a random time, which can be modelled by a probability density function (PDF). Existing FDR test theory normally assumed a constant failure rate, which means each failure time interval is identical and distributed [2]. However, some researchers questioned the assumptions of essentially unlimited life and a constant failure rate for electronics. From maintenance theory, different kinds of maintenance actions would change the PDF of failure time interval. The branching Poisson process (BPP), the superposed renewal process (SRP) and the non-homogeneous Poisson process (NHPP) are widely used to depict failure time intervals for non-renewal systems [8], [9]. In particular, the NHPP is able to describe the failure-repair process under minimal repairs. The minimal repair action means that the system is restored to the functioning state but is only as good as other equipment equal to its age at failure [10].

The statistical point process based NHPP is widely used in the field of reliability evaluation, maintenance cycle optimization and sample generation for virtual testability etc. Reliability engineering focuses on the reliability analyses and modelling of reparable systems [11]. Cui *et al.* [12] presented an analysis model for assessing the operational reliability of airborne equipment based on NHPP. Reference [13] proposed an optimum ramp accelerated life test (ALT) of m identical repairable systems using the non-homogeneous power law process (PLP) under a failure truncated case.

In the field of preventive maintenance, the main goal focuses on designing reasonable maintenance intervals or inspection times. Therefore, the analysis of failure data is an important facet in the development of maintenance strategy for equipment. Cheng *et al*. supposed that transporter failure numbers obey NHPP and computed the equal and unequal maintenance interval [14]. Aiming at finding the optimal inspection interval of the k-out-of-n load-sharing system, Taghipour *et al*. developed a model to find the optimal inspection times based on the NHPP model, which minimizes the total expected cost incurred over the system life cycle [15].

In a physical test of testability demonstration, the failure sample size is solved by a hypothesis test. The widely used test sample plan determination methods include single sample plan, multi sample plan and SPRT(Sequential Probability Ratio Test) [2]. During the virtual testability demonstration test, Zhang etc. proposed a failure sample generation method of repairable systems under assumptions of minimal repair and considered two maintenance policies, scheduled repair and corrective maintenance [3], [5].

Obtaining the failure data, estimating the parameters of NHPP is paramount. Reference [16] compared the parameter and non-parameter estimation methods to establish the model of the failure intensity function by using the bootstrap sampling method. A Bayesian statistical approach is presented to yield posterior distributions of the parameters of the Power Law and the Log-Linear intensity functions, which are used to model the trend in the data observed [17], [18]. An overwhelming majority of publications on the NHPP considers just two monotonic forms of the NHPP's rate of occurrence of failures (ROCOF) [19]. Other aspects of the power-law NHPP and extensions of the model have been studied. These include the well-known failure intensity function "bathtub" curve, which is not easily depicted by a mathematical model. Reference [20] used the superposed PLP (S-PLP) and superposed log-linear process (LLP) to model earlier failures and deterioration failures. Considering that the system design or the operation environment experiences major changes, a single model is not appropriate to describe the failure behaviour of the entire timeline. A piecewise NHPP model is proposed for repairable systems with multiple stages [21].

FDR is an important parameter to scale the equipment testability level. Whether the FDR value of a system meets the constant assumption needs further investigation. As a statistical and time-varying parameter, its statistical characteristics are the main points of this paper. This paper proceeds as follows. Section II analyzes FDR test theory based on quality inspection and describes the statistical model of FDR. Under the minimal repair assumption, Section III models the failure occurrence process using the NHPP, and three basic kinds of ROCOF functions are analysed. A simulation flow is illustrated to calculate the statistical characteristics of FDR in Section IV. As two key stochastic characteristics, the FDR expectation and variance are validated using

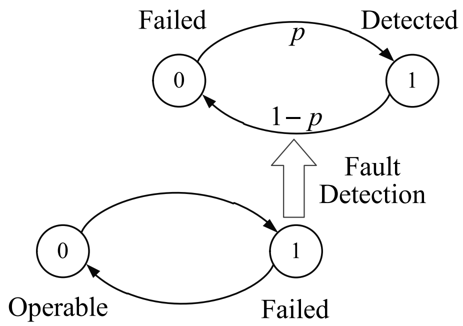

FIGURE 1. Failure occurrence and detection model.

a simulation example in Section V. Conclusions are given in Section VI.

II. PROBLEM DESCRIPTION

A. STATISTICAL MODEL OF FDR

In general, each failure detection process can be divided into two steps. First, a failure means that a system changes its state from operable to a failed state. This process is called the failure occurrence process. When the system changes its state from failed to operable, this process is called a repair action. Assuming that the maintenance time is negligible, each failure occurrence can be modeled by a statistical process. Second, the failure detection method detects the failure. When an exited failure is correctly detected, the test state is one; when the test fails, the test state is zero. The state transferring probability model can be defined as a 0-1 distribution.

Each failure detection process is an automatic indication of an existing failure. FDR is defined as a rate whereby the number of successfully detected failures is divided by the total number of failures during the specified time interval, which can be defined as [3], [22]

$$q_{FDR} = N^D/N = (N - N^F)/N \tag{1}$$

where N^D denotes the number of successfully detected failures, N^F denotes the number of failed detections, and N denotes the number of total failures. Normally, $N^D \leq N$ and $q_{FDR} \in [0, 1]$.

According to quality inspection theory, the detection results of the total occurred failures compose a sample population, which has size N. Restricted by time and money expenses, random sampling theory is widely used in the quality inspection process. Random sampling theory confirms that samples from the whole population can be used to estimate the statistical characteristics of the population. Randomly select n samples to estimate the value of FDR, which can be mathematically modeled by

$$P\{X = d\} = \frac{C_{N-N_D}^{n-d} C_{N_D}^{d}}{C_N^n} \tag{2}$$

where the random variable X denotes the number of failed detections in the n samples, and d denotes the number of detected failures in the samples.

Equation (2) indicates that $\hat{q}_{FDR}$ follows a Hypergeometric distribution. Actually, it may be difficult to obtain the whole population of occurred and detected failures as the time releases. Normally, researchers suppose that the number of total occurred failures and the selected sample size n have the relation $n/N < 0.1$. Under this assumption, the Hypergeometric distribution can be approximately calculated as a binominal distribution, i.e., $\hat{q}_{FDR}$ follows the distribution $B(n, q)$ and (2) can be calculated as

$$\begin{cases} P\{X = d\} = C_n^d (1-q)^{n-d} q^d \\ \hat{q}_{FDR} = d/n \end{cases} \tag{3}$$

where $P\{X = d\}$ denotes the event that random variable X is smaller or equal to the number d, and the random variable X denotes the successfully detected failures in the set of n samples.

According to (2) and (3), let an event $\{X \leq c\}$, denote the failed detection number c in n samples. Let $P\{X \leq c\}$ denote the probability of event $\{X \leq c\}$

$$P\{X \leq c\} = \sum_{d=0}^{c} C_n^d (1-q)^{n-d} q^d \tag{4}$$

The probability value $P\{X \leq c\}$ is also defined as the receiving probability. When the failure sample size n and the failed detection number c is determined, the probability $P\{X \leq c\}$ relates to q, and the equation can be written as

$$L(q) = P\{X \leq c\} = \sum_{d=0}^{c} P(r) \tag{5}$$

Under the determined test sample (n, c), $L(q)$ is a function relating to the parameter q, and $L(q)$ is often defined as the operating characteristic curve (OC curve).

Single sample plan calculation based on hypothesis test theory is widely used under double sided risk [3], [23]. The user's risk and manufacturer's risk can be mathematically modeled by

$$\begin{cases} 1 - L(q_0) \leq \alpha \\ L(q_1) \leq \beta \end{cases} \tag{6}$$

where α and β denote the limited user's risk and manufacturer's risk respectively. q_0 is the allowable minimum value of FDR and q_1 is the manufacturer's design value and it must satisfy $q_0 < q_1$.

Considering that the failure occurrence is a statistical point process, let $N(t)$ denote the number of total occurred failures and $N^D(t)$ denote the total number of correctly detected failures in the time interval $(0, t)$. As the failure occurrence can be modeled according to its definition, FDR is time varying. Here we use the notation $q_{FDR}(t)$ to represent the failure detection rate at time t. Following its definition, $q_{FDR}(t)$ can be calculated as follows,

$$q_{FDR}(t) = \frac{N^D(t)}{N(t)} \tag{7}$$

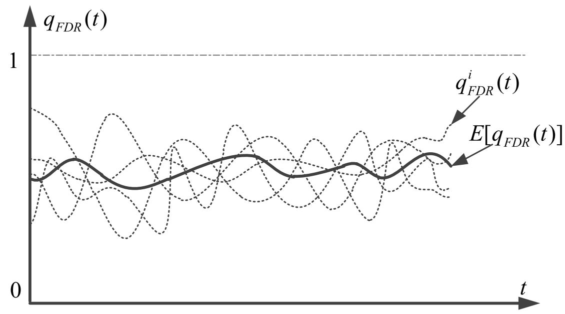

FIGURE 2. Time varying characteristic of FDR.

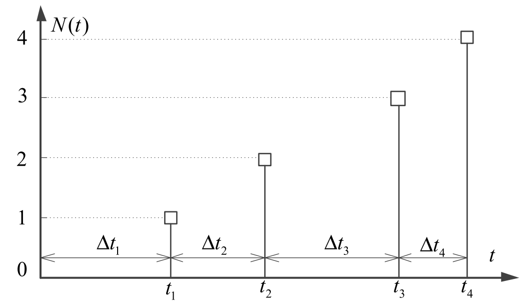

FIGURE 3. Counting process of failure occurrence

Generally, the relationship of $N(t)$ and $N^D(t)$ must satisfy $N(t) \geq N^D(t)$.

From the definition of (7), $q_{FDR}(t)$ always lies in the interval [0, 1]. As random variables $N(t)$ and $N^D(t)$ are time varying, $q_{FDR}(t)$ can be called a statistical process. Considering n components, let $q^i_{FDR}(t)$ denote the FDR value of the ith component at time t. Here, the term 'same component' denotes that the failure causes, such as design, hardware, software, installation, location, maintenance or operations people (and conditions), are the same or similar [24]. Under these design and operation conditions, each $q^i_{FDR}(t)$ can be assumed independent and identically distributed. As shown in Fig. 2, the dotted lines $q^i_{FDR}(t)|i = 1, 2, \cdots, n$ are the samples of $q_{FDR}(t)$, which fluctuate between 0 and 1. The heavy line is the expectation $E[q_{FDR}(t)]$ of the samples $q^i_{FDR}(t)|i = 1, 2, \cdots, n$. From the view point of statistics, we must pay a great deal of attention to the statistical characteristics of $q_{FDR}(t)$, such as expectation $E[q_{FDR}(t)]$. Therefore, the trend and the statistical characteristics of $q_{FDR}(t)$ are the key points in the following section.

We assume a system with M subsystems whose failure occurrence processes remain independent. Let q^S_{FDR} denote the FDR value of the system. Let q^i_{FDR} denote the FDR value of the ith subsystem. According to (1), the value q^S_{FDR} can be modeled by

$$q^S_{FDR} = \sum_{i=1}^{M} \frac{N^i_D(t)}{\sum\limits_{i=1}^{M} N^i(t)} \tag{8}$$

where $N^i_D(t)$ denotes the number of detected failures of ith subsystem in the time interval $(0, t]$ and $N^i(t)$ denotes the total occurred failure in the time interval $(0, t]$.

According to the binominal distribution, the expected number of detected failures of ith subsystem can be calculated as $E[N^i_D(t)] = q^i_{FDR} \cdot N^i(t)$. Approximately, the value q^S_{FDR} can be calculated as

$$q^S_{FDR} \approx \sum_{i=1}^{M} \frac{q^i_{FDR} \bullet N^i(t)}{\sum\limits_{i=1}^{M} N^i(t)} = \frac{1}{\sum\limits_{i=1}^{M} N^i(t)} \sum_{i=1}^{M} q^i_{FDR} \bullet N^i(t) \tag{9}$$

From (9), it clearly illustrated the truth that the FDR value q^S_{FDR} would be affected by the failure occurrence processes of the M subsystems. In other words, although all the FDR values q^i_{FDR} $(i = 1, 2, \cdots, k)$ remain constant, the system FDR value q^S_{FDR} would not be confirmed as satisfying the assumption that q^S_{FDR} is a constant.

In the following sections, a failure occurrence model based on NHPP is used to depict subsystem failures. We also work on the supposition that the subsystem FDR values are constant and can be modeled by the binomial distribution.

III. FAILURE OCCURRENCE MODEL BASED ON NHPP

As previously described, the failure occurrence process is not only affected by its reliability level and maintenance activities. Especially for repairable weapons and equipment, they will take a long time to work and experience many repair actions. During the whole life cycle, their failure occurrence situations should consider the effects of maintenance effects. In this section, we consider the minimal maintenance policy and mathematically model the failure occurrence process [25].

For a repairable system, let $\{N(t); t \geq 0\}$ denote the number of failures that occurred during time interval $[0, t]$, which can be simplified as $N(t)$. This is a counting process, which has the following properties:

(1) The value of $\{N(t); t \geq 0\}$ is an integer and increases monotonically as illustrated in Fig.3.

(2) $\{N(t); t \geq 0\}$ has independent increments;

(3) There are no failures to begin with $t = 0$, i.e. $N(0) = 0$;

(4) The probability that more than one failure will occur during $(t, t + \Delta t)$ is $o(\Delta t)$, and mathematically modeled as $P\{[N(t+\Delta t)-N(t)] \geq 2\} = o(\Delta t)$, where $o(\Delta t)$ is negligible when the time increment Δt is small;

(5) The probability that a failure will occur during $(t, t + \Delta t)$ is $\lambda(t)\Delta t + o(\Delta t)$, which can be modeled by $P\{[N(t + \Delta t) - N(t)] = 1\} = \lambda(t)\Delta t + o(\Delta t)$;

Determined by these five conditions, this stochastic point process $\{N(t); t \geq 0\}$ can be defined as an NHPP with the failure intensity function $\lambda(t)$. If $\lambda(t)$ is a constant, $\{N(t); t \geq 0\}$ is a homogeneous Poisson process (HPP). Therefore, the HPP is a special case of NHPP. The failure intensity function $\lambda(t)$ is also defined by the rates of occurrence of failures (ROCOF) and can be modeled as the time derivative of the number of failures in the assigned time interval:

$$\lambda(t)=\lim_{\Delta t\to 0}\frac{E[N(t+\Delta t)-N(t)]}{\Delta t}=\frac{d}{dt}E[N(t,t+\Delta t)] \quad (10)$$

where $E[N(t)] = E[N(0,t)]$ refers to the expectation of the occurred failure number in $(0,t)$. Naturally, an estimator of $\lambda(t)$ can be modeled by

$$\hat{\lambda}(t)=\frac{N[t+\Delta t,t]}{\Delta t} \quad (11)$$

Equally, let $\Lambda(t) = E\{N(0,t]\}$ denote the expectation of a failure number in $(0,t]$ and it can be defined as the cumulative intensity function. Therefore, the expected number of failures $\Lambda(t,0) = \Lambda(t)$ in the interval (t_1,t_2) is given by

$$\Lambda(t)=E\left[N(t)\right]=\int_{t_1}^{t_2}\lambda\left(u\right)du \quad (12)$$

Normally, $\Lambda(t)$ is referred to the MCF (Mean Cumulative Function).

A sample path of the counting process of failure occurrence is shown in Fig.3. Suppose that a repairable system is put into operation at time $t = 0$. The failure occurrence times are $(t_1, t_2, t_3, \cdots)$. Ignoring the repair time, i.e., the system is repaired and put into operation immediately after a failure. Let t_i denote the occurring time of the ith failure. The interval $\Delta t_i = t_i - t_{i-1} (i = 1, 2, \cdots)$ denotes the time interval length between the $(i-1)$th failure and ith failure. In addition, $t_0 = 0$. We use the term interarrival time to depict the time interval between two successive failures.

The probability $P\{[N(\Delta t + t) - N(t)] = n\}$ that n failures occurred in the $[t, t + \Delta t]$ is modeled as

$$P\{[N(t+\Delta t)-N(t)]=n\} = \frac{[\Lambda(t+\Delta t)-\Lambda(t)]^n}{n!}e^{-[\Lambda(t+\Delta t)-\Lambda(t)]^n} \quad (13)$$

where the probability $P\{[N(t + \Delta t) - N(t)] = n\}$ not only relates to counting start time t but also is affected by the time interval Δt. The interarrival times between failures are neither independent nor identically distributed.

From (13), an NHPP is able to model a time dependent intensity function. The importance of the NHPP resides in the fact that it does not require the condition of stationary increments. Thus, there is the possibility that events may be more likely to occur during specific time intervals. The NHPP has the memory property. Then, it is an adequate tool to study events where there may be time action [26]. In general, the MCF $\Lambda(t)$ changes over time.

A. BASIC FORMS OF ROCOF OF NHPP

Under the failure occurrence hypothesis of an NHPP model, time intervals between successive failures are neither independent nor identically distributed, which makes this model the most important and widely used in the modeling of repairable systems data. Actually, whenever a trend is found to be present in time between failure data, a non-stationary model such as the NHPP is mandatory, where the ROCOF form is the core of an NHPP.

1) POWER LAW PROCESS (PLP)

The power law non-homogeneous Poisson process (PL-NHPP) model is usually called a Weibull process. The failure intensity function of PL-NHPP is modeled by

$$\lambda(t)=\alpha\beta t^{\beta-1} \quad t\geq 0 \quad (14)$$

where α is the scale parameter and β is the shape parameter, and $\alpha, \beta > 0$. The PL-NHPP intensity function has different shapes depending on the range of β. One of the reasons for the popularity of the PL-NHPP stems from the fact that the form of $\lambda(t)$ is flexible. More precisely, the PLP setup can accommodate both increasing $\beta > 1$ and decreasing $\beta < 1$ intensities. Moreover, in the special case that $\beta = 1$, the failure intensity function $\lambda(t)$ is constant and the PL-NHPP becomes an Homogeneous Poisson Process.

2) LOG-LINEAR PROCESS (LLP)

The log-linear model is often used to depict the failure occurrence process of electronic systems [5]. The log-linear intensity function is defined as

$$\lambda(t)=\alpha\exp\left(\beta t\right) \quad t\geq 0, \alpha>0 \quad (15)$$

The parameter β in the log-linear model affects the number of failures. If $\beta > 0$, a repairable system would be in a deterioration state. The failure occurrence rate would be in a decrease state when $\beta < 0$. When $\beta = 0$, the log-linear model would be reduced to an HPP with the parameter α, denoted by HPP(α).

3) SUPERPOSED PLP AND LLP

Although the PL-NHPP model and the log-linear model are widely used to depict failure occurrence, they both have a strong drawback that only the NHPP can model monotonic (increasing/decreasing) failure intensity functions. To describe the bathtub ROCOF curve, Pulcini proposed the superposed power law process, which can be modeled as [27]

$$\lambda(t)=\frac{b_1}{a_1}\left(\frac{t}{a_1}\right)^{b_1-1}+\frac{b_2}{a_2}\left(\frac{t}{a_2}\right)^{b_2-1} \quad t\geq 0$$
$$a_1,b_1,a_2,b_2>0 \quad (16)$$

where the parameters a_1, b_1, a_2, b_2 meet $(b_1 - 1)(b_2 - 1) < 0$, the superposed PLP is suitable to describe the bathtub ROCOF curve.

The superposed PLP further enlarges the application range of the NHPP model. However, it would lead mutation at time $t = 0$. Therefore, [20] proposed the superposed LLP, and its failure intensity function can be described as

$$\lambda(t)=\alpha_1 e^{-\beta_1 t}+\alpha_2 e^{\beta_2 t} \quad t\geq 0$$
$$\alpha_1,\beta_1,\alpha_2,\beta_2>0 \quad (17)$$

The superposed LLP can also be used to model the bathtub curve and avoids the mutation at time $t = 0$.

IV. SIMULATION CALCULATION OF FDR STATISTICAL CHARACTERISTICS

This section aims at simulating the failure occurrence and detection process based on the Monte Carlo method. The Monte Carlo simulation method is a computerized technique that provides an approximate solution for a wide variety of problems [28]. The core of the Monte Carlo method is the process of generating random numbers.

The simulation of a failure occurrence model based on NHPP mainly focuses on simulating the failure time interval between two failures [29]. The NHPP failure occurrence series can be simulated as follows:

(1) Statistical sequence $\{u_i, i = 1, 2, \cdots, n\}$ numbers are independent and identically distributed (i.i.d.), which follow a uniform distribution $u_i \sim U(0, 1)$. Suppose that $\{\eta_i, i = 1, 2, \cdots, n\}$ is independent and identically exponentially distributed, which can be calculated by

$$\eta_i = -\ln u_i \tag{18}$$

(2) Let y_j denote the sum of $\{\eta_1, \eta_2, \cdots, \eta_n\}$

$$y_j = \sum_{i=1}^{n} \eta_i, j = 1, 2, \cdots, n \tag{19}$$

The joint PDF of random variables $y_1, y_2, \cdots, y_n$ is

$$f(y_1, y_2, \cdots, y_n) = e^{-x_n}, 0 < x_1 < x_2 < \cdots < x_n \tag{20}$$

The statistical sequence $\{y_j, j = 1, 2, \cdots, n\}$ is the reaching time of an HPP(1).

(3) Suppose

$$\tau_j = \Lambda^{-1}(y_j), j = 1, 2, \cdots, n \tag{21}$$

where $\Lambda^{-1}(\cdot)$ denotes the inverse function of MCF, which is defined in (12).

The time series $\tau_1, \cdots, \tau_n$ is a sample of NHPP with MCF $\Lambda(t)$ [30]. For $\forall n \geq 1$, the joint PDF of $\tau_1, \cdots, \tau_n$ is

$$f(x_1, x_2, \cdots, x_n) = \begin{cases} \prod_{i=1}^{n} \lambda(x_i) \exp[-\Lambda(x_n)], 0 < x_1 < x_2 < \cdots < x_n \\ 0, \text{other} \end{cases} \tag{22}$$

Each failure detection result b can be simulated by the binominal distribution $b \sim B(1, q)$, where q is defined as the FDR prevalue of a subsystem. The simulating result of $B(1, q)$ would be $b = 1$ with probability q and $b = 0$ with probability $1 - q$, where $b = 1$ represents the successful detection and $b = 0$ denotes failed detection.

In general terms, the simulation calculation procedure of FDR statistical characteristics is depicted as follows:

Step 1: Determine the statistical time T_F, which determined the last failure occurrence time. Initialize the simulation times ST, the number of subsystems M and the FDR prevalue q_k of the kth subsystem.

Step 2: Initialize the simulation indices i, j, and k, where i represents the ith simulation, j represents the jth failure, and k denotes the subsystem label number .

Step 3: Determine the MCF $\Lambda_k(t)$ and its parameters of the kth subsystem.

Step 4: Calculate the inverse function $\Lambda_k^{-1}(t)$ of $\Lambda_k(t)$.

Step 5: Generate a random number $u_j = U(0, 1)$, which obeys a uniform distribution, and calculate $\eta_j = -\ln u_j$.

Step 6: Calculate the sum $y_j = \sum_{m=1}^{j} \eta_m$.

Step 7: Calculate the failure time $\tau_i = \Lambda^{-1}(y_i)$.

Step 8: Generate a random number $b_j \sim B(1, q)$, which represents the detection result of a current simulative failure.

Step 9: Judge whether the current failure time meets $\tau_j \geq T_F$. If it satisfies the condition $\tau_j \geq T_F$, we obtain a complete sample of the failure occurrence process up to time T_F. If not, repeat steps 5 to 8.

Step 10: In the time interval $[0, T_F]$, at different time t_n, calculate the total number of occurred failures $N_k^i(t_n)$ and the number of successfully detected failures $N_{k,D}^i(t_n)$. Calculating the $q_{k,FDR}^i(t_n) = N_{k,D}^i(t_n)/N_k^i(t_n)$ and we can obtain an FDR curve in the interval $[0, T_F]$.

Step 11: Repeat the simulation flow ST times and obtain ST FDR sample curves of the kth subsystem.

Step 12: Simulate other subsystems' failure occurrence and detection processes. The FDR samples of whole system can be calculated as $q_{FDR}^i = \sum_{m=1}^{k} N_{k,D}^i(t_n) / \sum_{m=1}^{k} N_k^i(t_n)$, where $\sum_{m=1}^{k} N_{k,D}^i(t_n)$ and $\sum_{m=1}^{k} N_k^i(t_n)$ denote the number of detected failures and the total occurrences of failures in the system.

At different time $t_n \in [0, T_F]$, calculate the expectation and variance of the ST FDR sample curves using (23).

$$E[q_{FDR}(t_n)] = \frac{1}{ST} \sum_{i=1}^{ST} q_{FDR}^i(t_n)$$
$$V[q_{FDR}(t_n)] = \left[\frac{1}{ST-1} \sum_{i=1}^{ST} \left\{ [q_{FDR}^i(t_n) - E[q_{FDR}(t_n)]] \right\}^2 \right]^{\frac{1}{2}} \tag{23}$$

The simulation flowchart of FDR statistical characteristics is illustrated in Fig. 4.

V. SIMULATION CASE

Verifying the theory and simulation method proposed in this paper, a level flight indicator is applied as a study case that is used to simulate failure detection process and calculate the values of $E[q_{FDR}(t)]$ and $V[q_{FDR}(t)]$. The level flight indicator is an LRU (Line Replaced Unit) product that is composed of four SRUs (Shop Replaceable Units), including a static converter, gyroscope and upending mechanism and synchronizer. The structure of the level flight indicator is shown in Fig. 8.

Reference [23] collected failure occurrence time data to model the failure intensity functions of static converter (SRU1) and the righting mechanism (SRU2). Using the maximum likelihood estimation (MLE) method, the

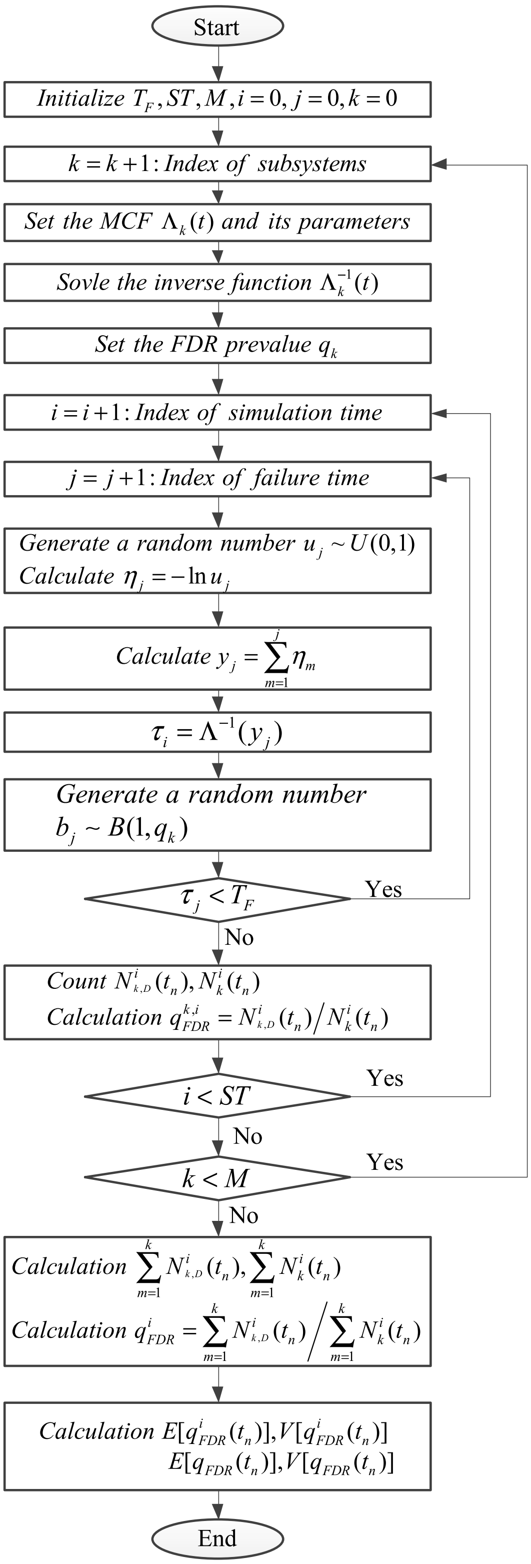

FIGURE 4. Simulation flowchart of FDR statistical characteristics

upending mechanism is modeled by a PLP model; its failure intensity function is

$$\lambda(t) = 0.002t^{0.11} \tag{24}$$

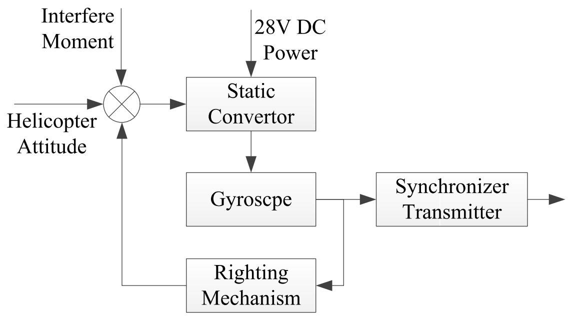

FIGURE 5. Structure of the level flight indicator

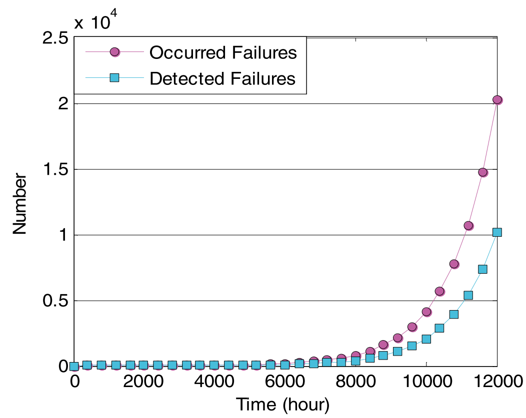

FIGURE 6. Expectation numbers of occurred and detected failures of SRU2.

Similarly, the failure intensity function of the static converter is modeled by LLP, which is denoted as

$$\lambda(t) = 0.001\exp(0.0008t) \tag{25}$$

The inverse functions of MCF of these two SRUs are as follows:

$$\begin{aligned} \Lambda_{SRU1}^{-1}(\tau) &= 555.556 \times \tau^{0.9009} \\ \Lambda_{SRU2}^{-1}(\tau) &= 1250 \times \ln(1 + 0.7273 \times \tau) \end{aligned} \tag{26}$$

where $\Lambda_{SRU1}^{-1}(\tau)$ denotes the inverse function of MCF of SRU1 and $\Lambda_{SRU1}^{-1}(\tau)$ denotes the inverse function of MCF of SRU2.

Set the simulation time $ST = 1000$ and the maximum failure time $T_F = 12000(hour)$. Set the prevalues of FDR of SRU1 and SRU2 0.95 and 0.5 respectively.

Fig. 6 illustrates the simulation results of the expected numbers of occurred and detected failures of SRU2. It can be clearly found that the two curves keep the same shape. At different times, the number of detected failures is half of the total number occurred failures, which accords with the FDR prevalue setting of SRU2.

To check the simulation precision in the number of occurred failures, we compare the simulative expectation number of occurred failures to the MCF $\Lambda_{SRU2}(t)$ of SRU2. The absolute error is calculated as shown in Fig. 7.

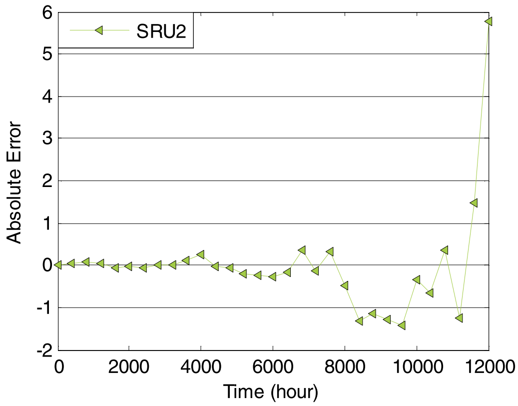

FIGURE 7. Absolute error expectation number of occurred failures

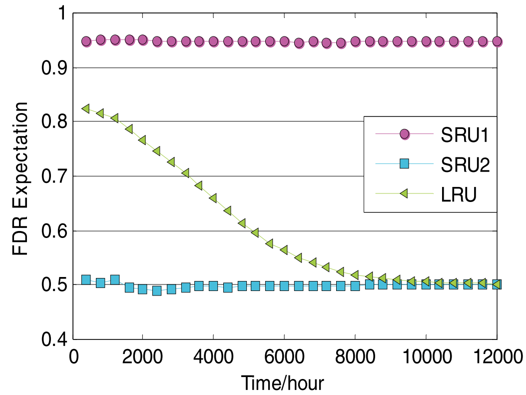

FIGURE 9. FDR expectation of two SRUs and LRU.

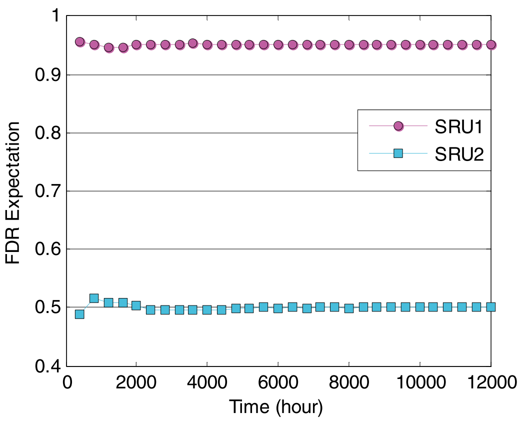

FIGURE 8. FDR Expectation curves of two SRUs.

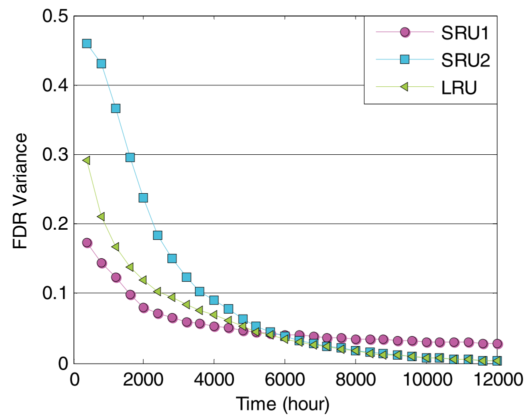

FIGURE 10. FDR variance of two SRUs and LRU

It illustrates that the absolute error is very small and the simulation procedure is reliable.

The FDR expectation curves of two SRUs are illustrated in Fig. 8. The two curves have the same characteristics, which tends to be constants. In early stages, the $E[q_{FDR}(t)]$ fluctuates slightly, which is attributed to the small number of failure occurrences; in later stages, the FDR expectation curves tend to be the FDR prevalues of the two SRUs. According to the simulation results, the FDR expectation values of the two SRUs accord with the constant supposition.

As mentioned above, considering the FDR value variation process of LRU, we suppose that the failure occurrence and detection process of SRU1 and SRU2 remain independent. Therefore, the number of occurred failures and detected failures of LRU is the sum of two SRUs. Fig. 9 illustrates the FDR expectation variation process of the LRU and two SRUs. Comparing the variation process of SRUs to LRU, the variation of the FDR expectation of LRU can be divided into two intervals. Obviously, the FDR expectation of LRU has a decreasing tendency before 8000 hours. The FDR expectation value of LRU decreases by about 0.3, which is much larger than the simulation error described in Fig. 8. After 8000 hours, the FDR expectation curve becomes gradually close to SRU2.

Fig. 10 illustrates the curves of the two SRUs and LRU. The variation characteristics of the FDR variance continuously tends toward zero at later stages.

VI. CONCLUSIONS

This paper focuses on the statistical characteristics of FDR, which is one of the most widely used testability indexes. The work of this paper can be concluded as follows.

(1) This paper discussed existing test plan design theory based on the random sampling technology and hypothesis test theory.

(2) Under the minimal maintenance assumption, we construct the failure occurrence process of the subsystems based on NHPP theory. The supposition that the FDR values remain constant is made in this paper and the failure detection process is modeled by the binominal distribution.

(3) Under the assumptions that the FDR values of all subsystems are constant and failure occurrence processes follows the NHPP model, we study the expectation and variance of $q_{FDR}(t)$ of the system constructed by several independent subsystems. From the simulation result, the expectation value of $q_{FDR}(t)$ does not always follow the constant assumption. As the number of occurred failures increases, the variance value of $q_{FDR}(t)$ continuously decreases and tends to be zero.

(4) When the failures occurrence processes obey the NHPP model, the failure detection samples collected at different times have diverse population characteristics and the existed test theory based on the hypothesis test theory is not be suitable to evaluate or validate the FDR value. Moreover, using the quality inspection method to evaluate the FDR value of a system, the validation step of the rates of occurrence of failures should be carried on at first.

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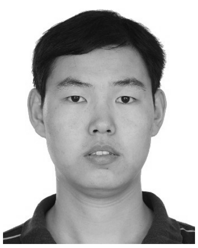
ZHIAO ZHAO received the B.S. degree from North China Electric Power University in 2010 and the M.S. degree from the National University of Defense Technology in 2012, where he is currently pursuing the Ph.D. degree with the Laboratory of Science and Technology on Integrated Logistics Support. His area of research includes the testability demonstration and validation of PHM.



YONG ZHANG received the Ph.D. degree from the National University of Defense Technology (NUDT) in 2012. He is currently an Assistant Professor with the Laboratory of Science and Technology on Integrated Logistics Support, NUDT. His area of research includes testability engineering and fault diagnosis.

GUANJUN LIU received the B.S. and Ph.D. degrees in mechanical engineering from the National University of Defense Technology (NUDT), in 1994 and 2000, respectively. Since 2000, he has been with NUDT, where he is currently a Professor and a Ph.D. Supervisor in the field of mechanical engineering. He has authored over 100 articles, primarily in condition monitoring, fault diagnosis techniques, and the technology of decreasing false alarms. His research interests include fault diagnosis, testability, and maintenance.

JING QIU received the B.S. degree from the Beijing University of Aeronautics and Astronautics in 1985, and the M.S. and Ph.D. degrees from the National University of Defense Technology (NUDT), Changsha, China, in 1988 and 1998, respectively. Since 1988, he has been teaching and conducting research at NUDT, where he became a Teaching Assistant in 1988, a Lecturer in 1991, an Associate Professor in 1994, and a Professor in 1998. He is currently a Professor and a Ph.D. Supervisor with the College of Mechatronic Engineering and Automation, NUDT. His research interests include design for testability, testability demonstration, fault diagnosis, and fault prognostics.