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# A Dimension Distance-Based SCMA Codebook Design

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**ABSTRACT** For sparse code multiple access (SCMA) with traditional codebooks, the initial information of message passing algorithm (MPA) receiver is easily susceptible to noise and multipath fading, and the convergence reliability of the first detected user in each decision process is unsatisfactory. Driven by these problems, an optimized codebook design for SCMA is presented in this paper. In the proposed SCMA codebook design, we first use turbo trellis coded modulation technology to design a basic complex multi-dimension constellation, which can increase the minimum Euclidean distance. Then, phase rotation and coordinate interleaving are added on the constellation to increase diversity and coordinate product distance between any constellation points. Based on these, we propose a novel criterion to select the most appropriate permutation set, which can capture as large as the sum of distance between dimensions of interfering codewords multiplexed on each resource node and maximize the diversity over the set of the sums of distance between dimensions of interfering codewords multiplexed on all resource nodes. Benefiting from the proposed codebook design, the quality of initial information of MPA receiver on each resource node and the convergence reliability of the first detected user in each decision process will be improved. Simulation results show that the bit error rate performance of SCMA with the proposed codebooks outperforms SCMA with traditional codebooks, low-density signature, and orthogonal frequency division multiple access under the same load.

**INDEX TERMS** Codebook, LDS, MPA, permutation set, SCMA.

## I. INTRODUCTION

Massive connectivity, better quality of service, higher throughput and lower latency are major requirements of future 5G wireless communication system [1]. Compared with 4G system, future 5G system improves spectral efficiency by 5~15 times. As a major technology in 4G system, OFDMA improves spectral efficiency by reasonably allocating two-dimension time-frequency resources [2]. However, limited by cyclic prefix and orthogonality between sub-carriers, OFDMA cannot meet the requirements of future 5G system. Driven by this problem, non-orthogonal multiple access, such as SCMA, becomes a hot research topic in recent years. SCMA is one multi-dimension codebook-based non-orthogonal spreading technology [3], which is similar to LDS. LDS is a special approach of CDMA sequence, and the codewords of LDS are built by the spreading of modulated QAM symbols using low-density spreading signatures [4].

Compared with LDS, SCMA improves spectral efficiency while it still provides the benefits of LDS in terms of overloading and moderate complexity of detection. In SCMA, incoming bits are first mapped to a complex  $N$ -dimension constellation. Then a complex  $N$ -dimension constellation point is mapped to a  $K$ -dimension codeword by a binary mapping matrix. The  $K$ -dimension codewords are sparse column vectors containing  $N < K$  nonzero elements. A codebook includes a plurality of codewords [5]. Each layer or user has its dedicated codebook (each codeword represents a transmission layer). In order to improve spectral efficiency, more than one layer is multiplexed on the same time-frequency resource nodes [6], [7]. All multiplexed layers have the same constellation size and length.

Although SCMA improves spectral efficiency, the inter-layer interference, noise and multipath fading are still problems that cannot be ignored. Traditional SCMA codebook

design emphasized on increasing power diversity over interfering codewords, which exploited Successive Interference Cancel (SIC) property to separate interfering users [5]. In traditional SCMA codebook design, the basic complex multi-dimension constellation was designed to increase power variation over constellation dimensions. Based on that, the traditional criterion of permutation set was to capture as much as power diversity over the interfering dimensions of codewords multiplexed on each resource node [5]. Traditional SCMA used a SIC-MPA receiver. The initial information of MPA receiver was the soft information output by SIC receiver which was mainly obtained by minimum mean square error (MMSE) estimation. As a basic rule, the quality of initial information is critical for the performance of MPA receiver. However, the MMSE estimation is easily susceptible to noise and multipath fading. In addition, traditional SCMA codebook design had increased the power diversity over the interfering dimensions of codewords multiplexed on each resource node. But the power diversity over interfering codewords did not reach its expectation, which would attenuate the convergence reliability of the first detected user in each decision process.

Driven by these problems, an optimized codebook design for SCMA is presented in this paper. The proposed SCMA codebook design includes basic complex multi-dimension constellation, operators on constellation (phase rotation and coordinate interleaving), mapping matrix and permutation. The basic complex multi-dimension constellation is designed by Turbo TCM technology, which can increase the minimum Euclidean distance [8]. Then phase rotation and coordinate interleaving are added on the constellation to increase the diversity (the number of distinct coordinates between any constellation points) and coordinate product distance between any constellation points [9], [10]. These in general imply an increase in the distance between dimensions of interfering codewords multiplexed on each resource node. SCMA with proposed codebooks uses a pure MPA receiver. For the pure MPA receiver, the initial information on each resource node is related to the distance between dimensions of interfering codewords multiplexed on corresponding resource node. So the proposed criterion first selects the permutation sets which can capture as large as the sum of distance between dimensions of interfering codewords multiplexed on each resource node. The proposed codebook design combines the constellation and permutation to increase the distance between dimensions of interfering codewords multiplexed on each resource node. The distance increased by the proposed codebook design is more than that increased by the codebook design which merely increases the minimum Euclidean distance, diversity and coordinate product distance between constellation points. It will result in a significant improvement on the quality of initial information of MPA receiver on each resource node. For the MPA receiver, the strongest user is first detected (its corresponding reliability is the most convergent) in each decision process and then it helps to detect the next strongest user. If the first detected user in

each decision process is an error, multiple access interference will increase, which will result in performance degradation. In order to improve the convergence reliability of the first detected user in each decision process, the most appropriate permutation set is selected from the selected permutation sets by the proposed criterion, which can maximize the diversity over the set of the sums of distance between dimensions of interfering codewords multiplexed on all resource nodes.

Section II introduces the system model. The design of proposed SCMA codebook is presented in Section III. The performance analysis of the proposed codebook design is offered in Section IV. Finally, in Section V, the BER performance of SCMA with proposed codebooks is compared with that of OFDMA, LDS and SCMA with traditional codebooks according to simulations.

## II. SYSTEM MODEL

Consider a  $J$ -user SCMA system. For layer  $j$ ,  $\log_2 M_j$  bits are mapped to a complex  $N_j$ -dimension constellation  $C_j$  of size  $M_j$ .  $V_j$  is a binary mapping matrix of layer  $j$  which maps a complex  $N_j$ -dimension constellation point  $c$  to a  $K$ -dimension codeword  $x$ . Note that  $V_j$  contains  $K - N_j$  all-zero rows, and the rest can be represented by an identity matrix  $I_{N_j}$  after eliminating all-zero rows from  $V_j$ . It means that there are  $N_j$  nonzero elements and  $K - N_j$  zero elements in each codeword of layer  $j$ . Without loss of generality, we assume that all the layers have the same constellation size and length, i.e.  $M_j = M$ ,  $N_j = N$ ,  $C_j = C$ ,  $\forall j = 1, \dots, J$ . The received signal after the synchronous layer multiplexing can be expressed as:

$$y = \sum_{j=1}^J \text{diag}(h_j)x_j + n_0, \quad (1)$$

where  $x_j = (x_{1j}, x_{2j}, \dots, x_{Kj})^T$  is the codeword of layer  $j$ ,  $h_j = (h_{1j}, h_{2j}, \dots, h_{Kj})^T$  is the channel vector of layer  $j$ , and  $n_0$  is the white Gaussian noise vector. In the case that all the layers are transmitted from the same transmit point, all the channels to a target receiver are identical, i.e.  $h_j = h = (h_1, h_2, h_3, \dots, h_K)^T$ ,  $\forall j$ . By multiplexing  $J$  layers over  $K$  resources, the overloading factor is defined as  $\lambda = J/K$ .

The received signal at resource node  $k$  is presented as  $y_k$ . As the codewords are sparse, only a few of them collide over resource node  $k$ . The set of resources occupied by layer  $j$  is decided by the indices of nonzero elements of binary indicator vector  $f_j = \text{diag}(V_j V_j^T)$ . The structure of SCMA can be represented by a factor graph matrix defined as  $F = (f_1, \dots, f_J)$ . Layer node  $j$  and resource node  $k$  are connected if and only if  $(F)_{kj} = 1$ . Fig. 1 shows an example of a factor graph representation of  $F$  with  $J = 6$ ,  $K = 4$ .

## III. PROPOSED SCMA CODEBOOK DESIGN

As shown in Fig.2, the proposed codebook design of SCMA includes basic complex  $N$ -dimension constellation, operators on the constellation, mapping matrix and permutation. The operators on the constellation include phase rotation and coordinate interleaving.

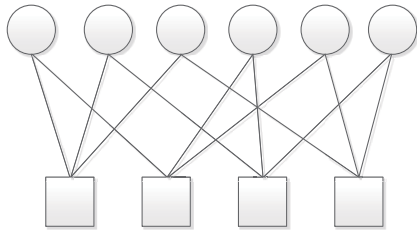


FIGURE 1. Factor graph of SCMA with  $J=6, K=4$ .

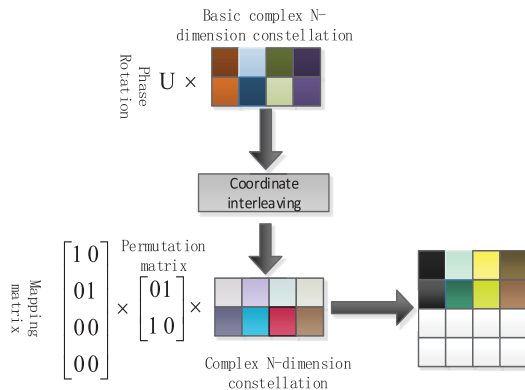


FIGURE 2. Proposed SCMA Codebook Design.

**A. BASIC COMPLEX N-DIMENSION CONSTELLATION**

1) TRADITIONAL BASIC COMPLEX N-DIMENSION CONSTELLATION

A traditional basic complex  $N$ -dimension constellation was generated from a real  $2N$ -dimension constellation. The  $2N$  real dimensions could be paired in  $N_c = \prod_{n=1}^N (2N - 2n + 1)$  different ways. Among the available pairing options, the one with maximum power variance across the complex dimensions was selected [5]:

$$C^+ = \arg \max_{c \in N_c} v(c), \tag{2}$$

where

$$v(c) = \sum_{n=1}^N \sum_{m=1}^M (|c_{nm}|^2 - \frac{1}{N} \|c_m\|^2)^2. \tag{3}$$

2) PROPOSED BASIC COMPLEX N-DIMENSION CONSTELLATION

Compared with traditional basic complex  $N$ -dimension constellation emphasizing on increasing power variance across the complex dimensions, the proposed basic complex  $N$ -dimension constellation concentrates on increasing the minimum Euclidean distance. As Turbo code is applied as the channel coding in this paper [11], [12], Turbo TCM technology is used to design the basic complex  $N$ -dimension constellation. TCM, which combines coding with modulation, can obtain a coding gain in bandwidth and power limited system, but the coding gain of TCM is limited [13]. Benefiting from multi-dimension constellation, multi-dimension TCM increases not only the minimum Euclidean distance but

also the coding gain [14], [15]. As anti-interference ability is very important for modern wireless communication systems, a lot of researches turn to combining channel coding with multi-dimension TCM to increase the anti-interference ability in recent years, such as LDPC TCM [16] and Turbo TCM [17], [18]. A proposed basic complex  $N$ -dimension signal can be constructed by  $N$  cartesian products of two-dimension signals. In other words, the transmitter sends a complex  $N$ -dimension signal, which is equivalent to sending  $N$  two-dimension signals, i.e.  $S^N = S \times S \times \dots \times S(N \text{ times})$ . Turbo TCM can be seen as two parallel concatenated Ungerboeck Trellis codes with code rate  $k/(k + 1)$ .

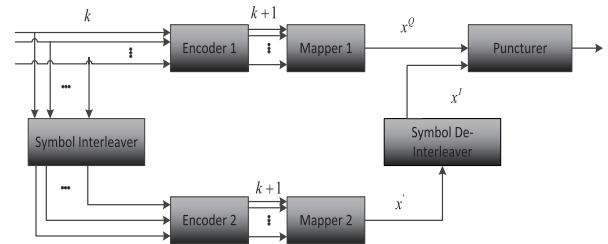


FIGURE 3. The structure of Turbo TCM.

As shown in Fig.3, Turbo TCM consists of component encoder, mapper, interleaver, de-interleaver and puncturer. The input data stream and its counterpart after interleaving are input into encoder 1 and encoder 2, respectively. Encoder 1 and encoder 2 use the same recursive systematic convolutional code. As encoder 1 and encoder 2 encode two input bits at a time, the interleaver must also interleave each pair of information bits instead of individual bits. Besides this constraint, the interleaver must also map even positions to even positions and odd positions to odd positions [17]. The coded bits are mapped to a constellation by mapper. We deinterleave the output symbols from mapper 2 to ensure that the ordering of the two information bits partly defining each symbol corresponds to that of encoder 1 [18]. Finally, the output symbols from these two component encoders are punctured and combined. In the combined output, all even symbols come from the output of encoder 1 and all odd symbols come from the output of encoder 2.

**B. PHASE ROTATION**

As a basic rule, the minimum Euclidean distance is the determinant of the performance of constellation in AWGN channel, while the performance of constellation in fading channel is related to the diversity and product distance between any constellation points. It can be achieved via phase rotation. The phase optimal algorithm can be expressed in two steps as follows:

*Step 1:* For a given constellation, choose a desired level of partitioning. For example, 16QAM can be partitioned such that there are eight, four, two, or one points in each subconstellation.

$$J(k1, k2, \theta_{k1,k2}) = \begin{bmatrix} 1 & \vdots & \vdots & \vdots & \vdots \\ \dots & \cos(\theta_{k1,k2}) & \dots & -\sin(\theta_{k1,k2}) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & \sin(\theta_{k1,k2}) & \dots & \cos(\theta_{k1,k2}) & \dots \\ \vdots & \vdots & \vdots & \vdots & 1 \end{bmatrix}. \quad (6)$$

Step 2: Find the optimal rotation parameter  $\theta^*$  such that

$$\theta^* = \arg \max_{\theta} \min(MCPD_{\theta}^{(1)}, \dots, MCPD_{\theta}^{(m_s)}, \dots, MCPD_{\theta}^{(M_s)}), \quad 1 \leq m_s \leq M_s, \quad (4)$$

$$CPD = \prod_{k=1}^{L_c} (c_k^i - c_k^j), \quad (5)$$

where  $M_s$  is the number of distinct subconstellations,  $MCPD_{\theta}^{(m_s)}$  is the minimum coordinate product distance (CPD) for the  $m_s$ -th subconstellation rotated by  $\theta$ ,  $L_c$  is the number of distinct coordinates between any constellation points, and  $c_k^i$  is the  $k$ -th dimension of constellation point  $i$ . In  $K$ -dimension constellation, a general rotation may be represented by the product of  $\frac{K(K-1)}{2}$  Givens rotation matrices, each parametrized by a single parameter  $\theta_{k1,k2}$  as shown in formula (6), at the top of this page.

Accordingly, the optimization must be carried out over the  $\frac{K(K-1)}{2}$ -dimension vector  $\theta = (\theta_{1,2}, \theta_{1,3}, \dots, \theta_{K-1,K})$ .

### C. COORDINATE INTERLEAVING

The coordinate interleaving uses both a I interleaver and a Q interleaver. I interleaver and Q interleaver are  $n \times n$  matrices. Data is written into I interleaver row-wise starting from the (1, 1) position and read out column-wise, starting from the same position. Without any loss of generality, we assume that data is written into Q interleaving row-wise starting with the (1, 1) position and read out column-wise starting from some position (not (1, 1) as in I interleaver) [9]. The suitable position, where the data of Q interleaver read out column-wise starts, is related to channel state information.

### D. MAPPING MATRIX

The sparse characteristic of SCMA is decided by the set of mapping matrices  $V = \{V_1, V_2, \dots, V_J\}$ . The mapping matrix design rules are as follows: i)  $V_j \in B^{K \times N}$  where  $B$  represents a binary matrix; ii)  $V_i \neq V_j, \forall i \neq j$ ; iii)  $V_j^{[\ominus]} = I_N$ , where  $V_j^{[\ominus]}$  is  $V_j$  after removing its all-zero rows. The mapping properties of  $V$  are as follows:

$$i) J = \binom{K}{N};$$

ii) The number of interfering layers per resource node is the same, i.e.  $d_{fj} = d_f = \binom{K-1}{N-1} = \frac{JN}{K}, \forall j$ ;

iii)  $\max(0, 2N - K) \leq l \leq N - 1$ , where  $l$  is the number of the overlapping elements of any distinct  $f_j$  vector.

Example: In order to obtain a reasonable level of sparsity,  $N$  should be small enough compared with  $K$ . If  $N = 2$ , the codewords of SCMA will be multiplexed over  $K > 2$  resource nodes to obtain the minimum number of interfering layer nodes. If  $K = 4$ , the factor graph matrix can be

$$\text{expressed as } F = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

### E. PERMUTATION SET

Let  ${}^n C = \{c_{nm} = (c_m)_n | \forall c_m \in C, m = 1, \dots, M\}, \forall n \in 1, \dots, N$  denotes the  $n$ -th dimension of constellation  $C$ . Assuming that  $z_n \in {}^n C$ , an arbitrary alphabet of constellation  $C$  can be represented by  $z(z, z^1, \dots, z^N)$ . Assume that the operator on constellation of layer  $j$  is limited to permutation matrix  $\pi_j$ . Under these assumptions, the codeword of layer  $j$  can be defined as  $x_j = q_j = V_j \pi_j z$  [5]. In AWGN channel, the aggregate received signal can be expressed as:

$$p(z) = \sum_{j=1}^J q_j(z) = \sum_{j=1}^J V_j \pi_j z, \quad (7)$$

where  $p(z) = (p_1(z), \dots, p_K(z))^T$  is a  $K \times 1$  vector in which  $p_k(z)$  denotes the interfering polynomial of resource node  $k$ . The interfering polynomial of resource node  $k$  can be modeled as  $p_k(z) = d_{k1}z + d_{k2}z^2 + \dots + d_{kN}z^N$ . As the number of interfering layers per resource node is  $d_f$ , we can conclude that  $\sum_{n=1}^N d_{kn} = d_f, \forall k$ .

An example: If  $N = 2, d_f = 3$ , the interfering polynomial of resource node 1 can be  $p_1(z) = 2z + z^2$ , which means that the resource node 1 takes three interfering layers. In the three interfering layers, two of them are from the first dimension of the mother constellation and the rest is from the second dimension of the mother constellation. There is a one-to-one mapping between  $p(z)$  and permutation set  $\prod = [\pi_j]_{j=1}^J$ . The total number of the constellation permutation choices is  $(N!)^J$ , while the total number of the distinct interfering polynomials is just limited to  $\binom{d_f + N - 1}{d_f}$ .

### 1) TRADITIONAL CRITERION OF PERMUTATION SET

Traditional criterion of permutation set can be expressed as:  $\{\prod^1, \prod^2, \dots\} = \arg \max_{\prod} \min_k w^2(p(z))$ . More precisely, the power variation of  $p(z) = d_1z + d_2z^2 + \dots + d_Nz^N$  (here  $k$

$$E_r^k = |x_{j1,n1}^{k,r} - x_{j2,n2}^{k,r}|^2 + \dots + |x_{j1,n1}^{k,r} - x_{jm_f, nm_f}^{k,r}|^2 + \dots + |x_{j(m_f-2), n(m_f-2)}^{k,r} - x_{jm_f, nm_f}^{k,r}|^2 + |x_{j(m_f-1), n(m_f-1)}^{k,r} - x_{jm_f, nm_f}^{k,r}|^2, \quad (11)$$

$$E_{im}^k = |x_{j1,n1}^{k,im} - x_{j2,n2}^{k,im}|^2 + \dots + |x_{j1,n1}^{k,im} - x_{jm_f, nm_f}^{k,im}|^2 + \dots + |x_{j(m_f-2), n(m_f-2)}^{k,im} - x_{jm_f, nm_f}^{k,im}|^2 + |x_{j(m_f-1), n(m_f-1)}^{k,im} - x_{jm_f, nm_f}^{k,im}|^2, \quad (12)$$

is dropped for the sake of simplicity) can be quantified as follows:

$$w(p(z)) = \frac{\sqrt{\sum_{n=1}^N d_n^2 (E(nC) - \bar{E})^2}}{\bar{E}}, \quad (8)$$

where

$$E(nC) = \frac{1}{M} \sum_{m=1}^M |c_{nm}|^2, \quad (9)$$

$$\bar{E} = \frac{\sum_{n=1}^N d_n E(nC)}{\sum_{n=1}^N d_n} = \frac{\sum_{n=1}^N d_n E(nC)}{d_f}. \quad (10)$$

As illustrated above, the traditional criterion of permutation set increased the power diversity over the interfering dimensions of codewords multiplexed on each resource node. However, the traditional criterion of permutation set did not increase the diversity over  $w = \{w(p_1(z)), w(p_2(z)), \dots, w(p_K(z))\}$ . So the power diversity over interfering codewords was unsatisfactory, which would attenuate the convergence reliability of the first detected user in each decision process.

## 2) PROPOSED CRITERION OF PERMUTATION SET

For  $p_k(z)$ , there may be different dimensions of the mother constellation multiplexed over resource node  $k$ . The proposed criterion of permutation set can be expressed as in (11) and (12), as shown at the top of this page.

If  $d_f = m_f$ , the sum of distance between dimensions of interfering codewords multiplexed on resource node  $k$  can be expressed as:

$$n(p_k(z)) = \sqrt{E_r^k + E_{im}^k}, \quad (13)$$

where  $x_{j,n}^{k,r}$  is the real part of the  $n$ -th dimension of codeword of layer  $j$  connecting to resource node  $k$ ,  $x_{j,n}^{k,im}$  is the imaginary part of the  $n$ -th dimension of codeword of layer  $j$  connecting to resource node  $k$ , and  $n(p_k(z))$  is the sum of distance between dimensions of interfering codewords multiplexed on resource node  $k$ . As illustrated in pioneer section, there is a one-to-one mapping between  $p(z)$  and permutation set  $\prod = [\pi_j]_{j=1}^J$ . So there is a one-to-one mapping between  $n(p(z))$  and permutation set  $\prod = [\pi_j]_{j=1}^J$ . The novel criterion of permutation set is divided into two steps. First, formula (14) selects the permutation sets which can maximize the minimum in corresponding  $n(p(z)) = \{n(p_1(z)), n(p_2(z)), \dots, n(p_K(z))\}$ :

$$\{\Pi^{1*}, \Pi^{2*}, \dots\} = \arg \max_{\prod} \min_k n(p_k(z)). \quad (14)$$

According to formula(14), there may be more than one solution for the above optimization problem, i.e.  $\Pi^* = \{\Pi^{1*}, \Pi^{2*}, \dots\}$ . For the pure MPA receiver, the first detected user in each decision process is very important. If the first detected user in each decision process is an error, multiple access interference will increase, which will result in performance degradation. In order to improve the convergence reliability of the first detected user in each decision process, formula (15) utilizes the variance function to capture the most appropriate permutation set  $\Pi^{l*}$  from  $\Pi^* = \{\Pi^{1*}, \Pi^{2*}, \dots\}$  which can maximize the diversity over  $n(p(z)) = \{n(p_1(z)), n(p_2(z)), \dots, n(p_K(z))\}$ :

$$\Pi^{l*} = \arg \max_{\Pi^*} \text{var}(n(p_1(z)), n(p_2(z)), \dots, n(p_K(z))), \quad (15)$$

where var is the variance function.

## IV. PERFORMANCE ANALYSIS OF THE PROPOSED CODEBOOK

### A. SIC-MPA RECEIVER

The process that a SIC receiver separates interfering users is as follows: i) The SIC receiver makes judgements on a plurality of users one by one in the received signal, and the strongest user will be first detected; ii) The received signal subtracts the interference generated from the detected user; iii) The SIC receiver repeats the process until all the multiple access interferences are eliminated. The size of a user's power decides its order of operation, and the user with the largest power will be first detected in each decision process.

A SIC-MPA receiver is a combination of a SIC receiver and a MPA receiver [19]. The MPA receiver is the main body of the hybrid Interference Cancel (IC) scheme, and the soft information output by the SIC receiver is the initial information of MPA receiver. For SIC-MPA receiver, the quality of initial information of MPA receiver and the first detected user in each decision process are very important. The soft information output by the SIC receiver is obtained by MMSE estimation. However, MMSE estimation is easily susceptible to noise and multipath fading, which can degrade the quality of initial information of MPA receiver. Moreover, if the first detected user in each decision process is an error, multiple access interference will increase. It will result in performance degradation.

### B. MPA RECEIVER

In this paper, the pure MPA receiver uses a min-sum algorithm [20]. Moreover, the whole structure of SCMA can be represented by a factor graph F with  $J$  check nodes and  $K$  variable nodes. Resource nodes can be seen as variable nodes, and layers can be seen as check nodes [21]. For the

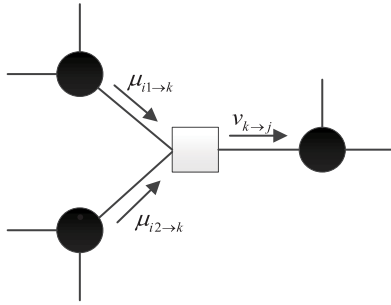


FIGURE 4. The process that message is exchanged from variable node  $k$  to check node  $j$ .

pure MPA receiver, the process that messages are exchanged between variable nodes and check nodes is as follows:

The message exchanged from the variable node  $k$  to check node  $j$  is given by

$$v_{k \rightarrow j}(x_j) = \gamma_k(x_j) + \sum_{i \in \Psi(k) \setminus j} \mu_{i \rightarrow k}(x_i), \quad (16)$$

$$\gamma_k(x_j) = -\ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\|y_k - \sum_{i \in \Psi(k)} x_{i,k} h_k\|^2}{2\sigma^2}\right)\right), \quad (17)$$

where  $v_{k \rightarrow j}(x_j)$  is the cost function that message is exchanged from variable node  $k$  to check node  $j$  when the value of check node  $j$  is  $x_j$ ,  $\gamma_k(x_j)$  is the initial information function on variable node  $k$  when the value of check node  $j$  is  $x_j$ ,  $\sigma^2$  is noise power,  $\mu_{i \rightarrow k}(x_i)$  is the cost function that message is exchanged from check node  $i$  to variable node  $k$  when the value of check node  $i$  is  $x_i$  and  $\Psi(k) \setminus j$  represents all check nodes connecting to the variable node  $k$  except check node  $j$ . Fig.4 shows the process that message is exchanged from variable node  $k$  to check node  $j$ .

The message exchanged from the check node  $j$  to variable node  $k$  is given by

$$\mu_{j \rightarrow k}(x_j) = \min\left(\sum_{l \in \Phi(j) \setminus k} v_{l \rightarrow j}(x_j)\right), \quad (18)$$

where  $\Phi(j) \setminus k$  represents all variable nodes connecting to check node  $j$  except variable node  $k$ . Fig.5 shows the process that message is exchanged from check node  $j$  to variable node  $k$ .

After several iterations, the final cost function of check node  $j$ , when the value of check node  $j$  is  $x_j$ , is

$$\mu(x_j) = \sum_{l \in \Phi(j)} v_{l \rightarrow j}(x_j). \quad (19)$$

### C. PERFORMANCE ANALYSIS

As illustrated in [22]–[24], the larger the Euclidean distance between adjacent constellation points is, the larger the decision region of constellation point. So increasing the Euclidean distance between adjacent constellation points can be efficiently against noise and multipath fading. According to formula (17),  $\gamma_k(x_j)$  is related to  $\hat{y}_k$  ( $\hat{y}_k$  is the expected symbol

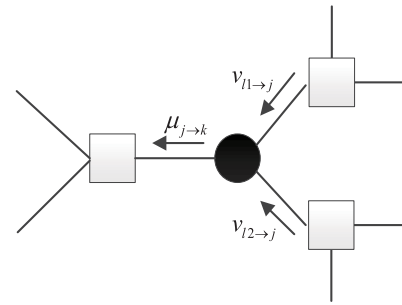


FIGURE 5. The process that message is exchanged from check node  $j$  to variable node  $k$ .

on resource node  $k$ ), noise and channel state information. Increasing  $n(p_k(z))$  will enlarge the decision region of  $\hat{y}_k$ , which can be efficiently against noise and multipath fading. So the pure MPA receiver can improve the quality of initial information on each variable node by increasing  $n(p(z))$ . As illustrated in pioneer section, the proposed basic complex multi-dimension constellation is designed to increase the minimum Euclidean distance. Then phase rotation and coordinate interleaving are added on the constellation to increase the diversity and coordinate product distance of constellation. These in general imply an increase in  $n(p(z))$ . Moreover, formula (14) selects the permutation sets  $\Pi^*$  which can capture as large as  $n(p_k(z))$ . So the proposed codebook design will obtain a significant improvement on the quality of initial information of MPA receiver on each resource node. If  $K = 4$ ,  $d_f = 3$ , the process that messages are exchanged between check nodes and variable nodes can be expressed as:

$$\begin{aligned} \mu_{1 \rightarrow 1}(x_1) &= \min(v_{2 \rightarrow 1}(x_1)) \\ \mu_{1 \rightarrow 2}(x_1) &= \min(v_{1 \rightarrow 1}(x_1)) \\ &\vdots \\ \mu_{6 \rightarrow 3}(x_6) &= \min(v_{4 \rightarrow 6}(x_6)) \\ \mu_{6 \rightarrow 4}(x_6) &= \min(v_{3 \rightarrow 6}(x_6)). \quad (20) \\ v_{1 \rightarrow 1}(x_1) &= \gamma_1(x_1) + \mu_{2 \rightarrow 1}(x_2) + \mu_{3 \rightarrow 1}(x_3) \\ v_{2 \rightarrow 1}(x_1) &= \gamma_2(x_1) + \mu_{4 \rightarrow 2}(x_4) + \mu_{5 \rightarrow 2}(x_5) \\ &\vdots \\ v_{3 \rightarrow 6}(x_6) &= \gamma_3(x_6) + \mu_{2 \rightarrow 3}(x_2) + \mu_{4 \rightarrow 3}(x_4) \\ v_{4 \rightarrow 6}(x_6) &= \gamma_4(x_6) + \mu_{3 \rightarrow 4}(x_3) + \mu_{5 \rightarrow 4}(x_5). \quad (21) \end{aligned}$$

Assuming that the values of  $x_1, x_2, \dots, x_6$  are expectations and the initial values of  $v_{k \rightarrow j}(x_j)$  and  $\mu_{j \rightarrow k}(x_j)$  are 0. After several iterations, the difference between  $\mu(x_1)$  and  $\mu(x_2)$  can be expressed as:

$$\mu(x_1) - \mu(x_2) = \gamma_3(x_2) - \gamma_2(x_1). \quad (22)$$

In the decision region of  $\hat{y}_k$ , the larger  $n(p_k(z))$  is, the smaller the value of  $\gamma_k(x_j)$  ( $\gamma_k(x_j)$  is decreasing function). If  $n(p_3(z))$  is less than  $n(p_2(z))$ , the value of  $\gamma_3(x_2)$  is more than that of  $\gamma_2(x_1)$ . Under this case, the cost when the value of check node 1 is judged as  $x_1$  is more than

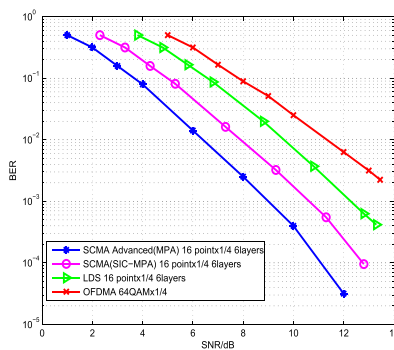
that when the value of check node 2 is judged as  $x_2$ . So check node 2 will be detected before check node 1. If  $K = 4, d_f = 3$ , the factor graph matrix can be

$$\text{expressed as } F = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}. \text{ For fac-}$$

tor graph matrix  $F$ , check node 1 occupies variable node 1 and variable node 2, and check node 2 occupies variable node 1 and variable node 3. The difference between  $\mu(x_1)$  and  $\mu(x_2)$  is mainly generated from variable node 2 and variable node 3. It will do the same between other check nodes. According to the characteristic of factor graph matrix  $F$ , we can conclude that increasing the diversity over  $n(p(z)) = \{n(p_1(z)), n(p_2(z)), \dots, n(p_K(z))\}$  will improve the convergence reliability of the first detected layer in each decision process (a layer can be seen as a check node). So the most appropriate permutation set is selected from the selected permutation sets by formula (15), which can maximize the variance of  $n(p(z))$ .

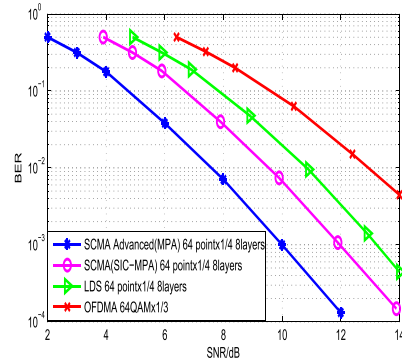
**V. SIMULATION**

In this section, simulations are based on long term evolution (LTE) system. For simplicity, ideal channel estimation is assumed. Turbo code is applied as channel code. Here SCMA Advanced is short for SCMA with proposed codebooks, SCMA is short for SCMA with traditional codebooks, and MPA receiver is short for the pure MPA receiver.



**FIGURE 6.** BER performance of SCMA Advanced with MPA receiver, SCMA with SIC-MPA receiver, LDS and OFDMA system over AWGN channel with the spectral efficiency 1.5 bits/tonne.

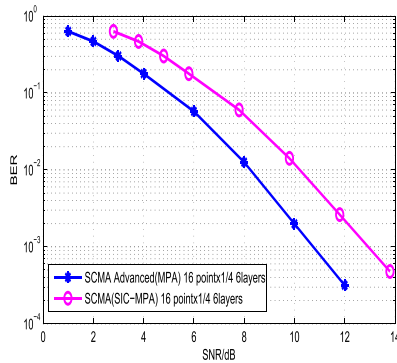
As the spectral efficiency in Fig. 6 is 1.5 bits/tonne, we set  $J = 6, K = 4$  for SCMA Advanced and SCMA. For SCMA Advanced, SCMA, LDS and OFDMA, the rate of Turbo code is 1/4. Fig. 6 is the BER performance of SCMA Advanced with MPA receiver, SCMA with SIC-MPA receiver, LDS and OFDMA system over AWGN channel. As can be observed in Fig.6, the BER performance of SCMA Advanced with MPA receiver outperforms that of SCMA with SIC-MPA receiver, while the BER performance of SCMA with SIC-MPA receiver outperforms that of LDS and OFDMA. The gain of SCMA Advanced with MPA receiver is over 1.3 dB compared to SCMA with



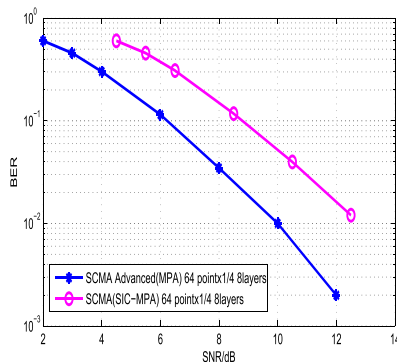
**FIGURE 7.** BER performance of SCMA Advanced with MPA receiver, SCMA with SIC-MPA receiver, LDS and OFDMA system over AWGN channel with the spectral efficiency 2 bits/tonne.

SIC-MPA receiver. As the spectral efficiency in Fig. 7 is 2 bits/tonne, we set  $J = 8, K = 6$  for SCMA Advanced and SCMA. For SCMA Advanced, SCMA and LDS, the rate of Turbo code is 1/4. For OFDMA, the rate of Turbo code is 1/3. Fig. 7 is the BER performance of SCMA Advanced with MPA receiver, SCMA with SIC-MPA receiver, LDS and OFDMA system over AWGN channel. As can be observed in Fig.7, the BER performance of SCMA Advanced with MPA receiver outperforms that of SCMA with SIC-MPA receiver, while the BER performance of SCMA with SIC-MPA receiver outperforms that of LDS and OFDMA [19]. However, the initial information of SIC-MPA receiver is easily susceptible to noise. Moreover, the power diversity over interfering codewords of SCMA does not reach its expectation, which attenuates the convergence reliability of the first detected user in each decision process. Driven by these problems, SCMA Advanced increases  $n(p_k(z))$  and maximizes the diversity over  $n(p(z)) = \{n(p_1(z)), n(p_2(z)), \dots, n(p_K(z))\}$  to improve the quality of initial information of MPA receiver on each resource node and the convergence reliability of the first detected layer in each decision process. Simulation results show that the proposed codebook design will enable SCMA to obtain better BER performance compared to SCMA with traditional codebooks over AWGN channel.

In Fig.8 and Fig.9, the fading channel model is pedestrian B with speed of 3 km/h, and the rate of Turbo code is 1/4. The fading channel has 6 paths. The carrier frequency is 2 GHz and the frequency spacing is 15 KHz. A data payload occupies 6 LTE resource blocks (RBs). The antenna array exploits the SIMO model. Fig. 8 is the BER performance of SCMA Advanced with MPA receiver and SCMA with SIC-MPA receiver over fading channel with spectral efficiency 1.5 bits/tonne. As the spectral efficiency is 1.5 bits/tonne, we set  $J = 6, K = 4$  for SCMA Advanced



**FIGURE 8.** BER performance of SCMA Advanced with MPA receiver and SCMA with SIC-MPA receiver over fading channel with spectral efficiency 1.5 bits/tonne.



**FIGURE 9.** BER performance of SCMA Advanced with MPA receiver and SCMA with SIC-MPA receiver over fading channel with spectral efficiency 2 bits/tonne.

and SCMA. As can be observed in Fig.8, the BER performance of SCMA Advanced with MPA receiver outperforms that of SCMA with SIC-MPA receiver. The gain of SCMA Advanced with MPA receiver is over 1.8 dB compared to SCMA with SIC-MPA receiver. Fig. 9 is the BER performance of SCMA Advanced with MPA receiver and SCMA with SIC-MPA receiver over fading channel with spectral efficiency 2 bits/tonne. As the spectral efficiency is 2 bits/tonne, we set  $J = 8, K = 6$  for SCMA Advanced and SCMA. As can be observed in Fig.9, the BER performance of SCMA Advanced with MPA receiver outperforms that of SCMA with SIC-MPA receiver. The gain of SCMA Advanced with MPA receiver is over 2.5 dB compared to SCMA with SIC-MPA receiver. In addition, the intervals between the curve of SCMA Advanced and the curve of SCMA over fading channel in Fig.8 and Fig.9 are more than those over AWGN channel. In other words, for traditional SCMA, the quality of initial information of MPA receiver over fading channel is worse than that over AWGN channel, and the proposed SCMA codebook design can be efficiently against multipath fading.

## VI. CONCLUSIONS

In order to improve the quality of initial information of MPA receiver on each resource node and convergence reliability of the first detected layer in each decision process,

an optimized codebook design for SCMA is presented in this paper. The proposed SCMA codebook design includes basic complex multi-dimension constellation, operators on constellation (phase rotation and coordinate interleaving), mapping matrix and permutation. The basic complex multi-dimension constellation is designed to increase the minimum Euclidean distance. Then phase rotation and coordinate interleaving are added on the constellation to increase the diversity and coordinate product distance between any constellation points. Based on these, we propose a novel criterion to select the most appropriate permutation set which captures as large as the sum of distance between dimensions of interfering codewords multiplexed on each resource node and maximizes the diversity over the set of the sums of distance between dimensions of interfering codewords multiplexed on all resource nodes. Simulation results show that the BER performance of SCMA with proposed codebooks outperforms that of SCMA with traditional codebooks, LDS and OFDMA under the same load.

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