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Direction Finding in MIMO Radar With Unknown Mutual Coupling

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ABSTRACT Multiple-input multiple-output (MIMO) radar has received much attention due to its potentials in offering improved performance for target detection and parameter estimation. It has been shown that the performance of direction finding techniques can be considerably enhanced in MIMO radar, since an extended virtual array can be employed. In general, the performance gain is achieved by assuming that both the transmitter and receiver are well calibrated without uncertainties. However, in practice, both the transmitter and receiver may suffer from various array imperfections such as mutual coupling. Hence, the problem of direction finding in MIMO radar with unknown mutual coupling is investigated in this paper. Computationally efficient algorithms are developed to estimate the directions-of-arrival by exploiting the structure of mutual coupling matrix of the transmitter and receiver equipped with uniform linear arrays. Moreover, it is shown that the proposed methods can be applied to scenarios in which the transmitter or receiver has other imperfections, for example, sensor position perturbations or gain/phase mismatches. Simulation results demonstrate that the proposed methods are able to effectively eliminate the negative influence of unknown mutual coupling on direction finding and offer improved performance over the existing methods.

INDEX TERMS Multiple-input multiple-output (MIMO) radar, direction finding, direction-of-arrival (DOA) estimation, mutual coupling.

I. INTRODUCTION

In the last decade, multiple-input multiple-output (MIMO) radar has been attracting significant attention. Compared with traditional phased array radar, MIMO radar possesses a number of advantages such as resolution enhancement, fading mitigation and barrage jamming [1]–[7]. MIMO radar utilizes multiple antennas to transmit independent waveforms as well as multiple antennas to receive the reflected signals such that the performance of parameter estimation can be improved with signal diversity. In general, MIMO radar can be classified into two categories according to the configuration of the transmit and receive antennas [8], [9]. More specifically, one is MIMO radar with widely separated antennas and the other is MIMO radar with colocated antennas. In this paper, we are concerned with the latter, and hence, the angle of target with respect to the transmit and receive antennas are the same.

It is known that angle estimation is an important aspect of parameter estimation in MIMO radar and a number of estimation algorithms have been presented [10]–[15]. For instance, the Capon method has been applied to estimate the direction of departure (DOD) and the direction of arrival (DOA) of targets in bistatic MIMO radar in [10]. However, this method needs two-dimensional (2-D) angular searching, which demands high computational complexity. In order to avoid the 2-D search, in [11], the ESPRIT method is employed by exploiting the shift invariant property of the transmit and receive arrays. Since an extra procedure of angle paring is required after the DOD and DOA have been obtained, another algorithm which also makes use of the shift invariant concept but is free of angle paring was developed in [12]. Compared with the approach in [11], this method can offer similar performance of angle estimation with less computational complexity.

The above methods provide good performance in the case of both transmit and receive array are well calibrated. However, in practical applications, due to the presence of mutual coupling, the performance of these algorithms will be degraded greatly [16]–[20]. Therefore, various methods have been developed to deal with this problem. For example, in [16], a MUSIC-like algorithm is introduced to tackle the unknown mutual coupling by taking advantage of the Toeplitz structure of the mutual coupling matrix (MCM). A decoupling-complex matrix (DCCM) method was proposed by using auxiliary sensors to eliminate the influence of mutual coupling in [17]. The property of shift invariance is extracted in the new data for the angle estimation. This method exploits the subarrays of the transmit and the receive arrays, which would lead to certain performance loss. In [18], a root-MUSIC based method is proposed by selecting output data of partial sensors.

To further investigate the problem of direction in MIMO radar with mutual coupling, a MUSIC-like algorithm based on rank reduction of a matrix related to the noise subspace is first introduced in this paper. To reduce the computational complexity, a new ESPRIT-like method is devised. The mutual coupling is tackled by a new parameterziation of the steering vectors of the transmitter and receiver. Unlike existing approaches, the proposed one is able to make a better use of the array aperture. Furthermore, the proposed ESPRIT-like method allows the transmitter or receiver has other uncertainties, such as gain and phase uncertainties [21]–[23] and sensor position perturbations, rather than mutual coupling, provided that either the receiver or transmitter (but not both) has unknown mutual coupling only.

The remainder of the paper is organized as follows. In Section II, the colocated MIMO radar signal model is introduced. The mutual coupling and associated data models are given in Section III. Direction finding methods with unknown mutual coupling are presented in Section IV and the consideration of other array imperfections is discussed in Section V. Simulation results are given in Section VI. Finally, conclusions are drawn in Section VII.

II. MIMO RADAR SYSTEM MODEL

Consider a MIMO radar system with the transmitter having M antennas and the receiver having N antennas. It is assumed that the transmitter and receiver are collocated and the transmitter transmit M orthogonal waveforms $s_m(t)$, $m = 1, 2, ..., M$, where t is the fast time index, i.e., the time index within one radar pulse. In other words, we have

$$\int_{T_p} s_i(t)s_j^*(t)dt = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where $*$ denotes the conjugate transpose and T_p represents the pulsewidth.

Assume that L targets with distinct DOAs $\{\theta_1, \theta_2, \cdots, \theta_L\}$ are present, then the radar return of the lth target is given by

$$r_l(t, \tau) = \alpha_l(\tau)\mathbf{a}_T^T(\theta_l)\mathbf{s}(t) \tag{2}$$

where τ is the index of the pulse and is also known as slow time index, $\alpha_l(\tau)$ denotes the reflection coefficient of the lth target, which is assumed to be a constant during each pulse but time-varying over the slow time index, $\mathbf{a}_T(\theta_l) \in \mathbb{C}^{M\times 1}$ denotes the steering vector of the transmitter corresponding to the lth target and $\mathbf{s}(t) \in \mathbb{C}^{M\times 1}$ is the waveform vector

$$\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_M(t)]^T. \tag{3}$$

Let $\mathbf{a}_R(\theta_l) \in \mathbb{C}^{N\times 1}$ be the steering vector of the receiver corresponding to the lth target, the observation vector $\mathbf{x}(t, \tau) \in \mathbb{C}^{N\times 1}$ at the receiver can be written as

$$\begin{aligned} \mathbf{x}(t, \tau) &= \sum_{l=1}^{L} \mathbf{a}_R(\theta_l) r_l(t, \tau) + \mathbf{v}(t, \tau) \\ &= \sum_{l=1}^{L} \alpha_l(\tau)\mathbf{a}_R(\theta_l)\mathbf{a}_T^T(\theta_l)\mathbf{s}(t) + \mathbf{v}(t, \tau) \end{aligned} \tag{4}$$

where $\mathbf{v}(t, \tau)$ is a vector of zero-mean additive white Gaussian noise (AWGN) which is independent of the signals. Matching the receiver observation with $s_m(t)$, we have $\mathbf{y}_m(\tau) \in \mathbb{C}^{N\times 1}$ as follows

$$\begin{aligned} \mathbf{y}_m(\tau) &= \int_{T_p} \mathbf{x}(t, \tau)s_m^*(t)dt \\ &= \sum_{l=1}^{L} \alpha_l(\tau)\mathbf{a}_R(\theta_l)\mathbf{a}_T^T(\theta_l) \int_{T_p} \mathbf{s}(t)s_m^*(t)dt + \tilde{\mathbf{v}}(t, \tau) \\ &= \sum_{l=1}^{L} \alpha_l(\tau)\mathbf{a}_R(\theta_l)a_{T,m}(\theta_l) + \tilde{\mathbf{v}}_m(t, \tau) \end{aligned} \tag{5}$$

where $a_{T,m}(\theta_l)$ is the mth entry of $\mathbf{a}_T(\theta_l)$ and $\tilde{\mathbf{v}}(t, \tau) = \int_{T_p} \mathbf{v}(t, \tau)s_m^*(t)dt \in \mathbb{C}^{N\times 1}$. Stacking the outputs of all matched filters, one gets $\mathbf{y}(\tau) \in \mathbb{C}^{MN\times 1}$ as

$$\begin{aligned} \mathbf{y}(\tau) &= [\mathbf{y}_1^T(\tau), \mathbf{y}_2^T(\tau), \cdots, \mathbf{y}_M^T(\tau)]^T \\ &= \sum_{l=1}^{L} (\mathbf{a}_T(\theta_l) \otimes \mathbf{a}_R(\theta_l))\, \alpha_l(\tau) + \mathbf{v}(\tau) \end{aligned} \tag{6}$$

where $\otimes$ denotes the Kronecker product and $\mathbf{v}(\tau) = [\tilde{\mathbf{v}}_1^T(\tau), \tilde{\mathbf{v}}_2^T(\tau), \cdots, \tilde{\mathbf{v}}_M^T(\tau)]^T \in \mathbb{C}^{MN\times 1}$ is the noise term with covariance matrix of $\sigma_n^2 I$, where I denotes an identity matrix with appropriate dimension.

It is worth mentioning that the outputs of the matched filters can be also stacked as follows

$$\begin{aligned} \mathbf{z}(\tau) &= \text{vec}\{[\mathbf{y}_1(\tau), \mathbf{y}_2(\tau), \cdots, \mathbf{y}_M(\tau)]^T\} \\ &= \sum_{l=1}^{L} (\mathbf{a}_R(\theta_l) \otimes \mathbf{a}_T(\theta_l))\, \alpha_l(\tau) + \mathbf{n}(\tau) \end{aligned} \tag{7}$$

where $\mathbf{z}(\tau) \in \mathbb{C}^{MN\times 1}$, $\mathbf{n}(\tau) \in \mathbb{C}^{MN\times 1}$, and $\text{vec}\{\cdot\}$ denotes the vectorization operator. It has been well studied that when the steering vectors of the transmitter and receiver, i.e., $\mathbf{a}_T(\theta)$, $\mathbf{a}_R(\theta)$, are exactly known, conventional high-resolution algorithms such as MUSIC and ESPRIT can be used to determine the unknown DOAs. However, in this paper we are interested in the cases in which the transmitter and receivers suffer from imperfections. In particular, the unknown mutual coupling of the transmitter and receiver are considered.

III. ARRAY MUTUAL COUPLING

In practice, both the transmitter and receiver would suffer from mutual coupling. Typically, mutual coupling is undesirable because energy that should be radiated away is absorbed by a nearby antenna. Similarly, energy that could have been captured by one antenna is instead absorbed by a nearby antenna. Hence, mutual coupling reduces the antenna efficiency and performance of antennas. According to the principle of reciprocity (i.e., the receive and transmit properties of an antenna are identical), antennas have the same properties in the transmit and receive mode. Let us take the receiver for example, the mutual coupling matrix (MCM) $\mathbf{C} \in \mathbb{C}^{N\times N}$ can be expressed as [24], [25]

$$\mathbf{C} = (Z_A + Z_L)(\mathbf{Z} + Z_L\mathbf{I})^{-1} \tag{8}$$

where Z_A and Z_L are the antenna impedance and terminating load, respectively. $\mathbf{Z}$ denotes the mutual impedance matrix. In the absence of mutual coupling, the off-diagonal entries of $\mathbf{Z}$ are zero and hence, $\mathbf{C}$ is an identity matrix, i.e., $\mathbf{C} = I$.

In general, mutual coupling has no specific property except that it is inversely proportional to the distance between two antenna elements. However, in uniform linear arrays (ULAs), the MCM has a symmetric Toeplitz structure owing to the symmetric geometry of the array. Thus, a simplified parameterization of the MCM can be expressed as (see [26] and related references therein)

$$C_{ij} = c_{|i-j|}, \quad \text{with } c_0 = 1 \tag{9}$$

where C_{ij} denotes the (i, j)-th entry of C. Furthermore, it is known that the magnitude of the coupling parameter decreases quite fast, therefore, the mutual coupling can be sufficiently approximated as zero when two antennas are separated by few inter-element spacings. Thus, it is possible to further simplify the MCM as

$$C_{ij} = \begin{cases} 1, & i = j \\ c_k, \; k = |i-j| \le P-1, & i \ne j \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

which shows that the mutual coupling is ignorable when two elements are separated by at least P inter-element spacings. Thus, the mutual coupling coefficient vector composed of the nonzero entries as $\mathbf{c} = [1, c_1, c_2, \cdots, c_{P-1}]^T \in \mathbb{C}^{P\times 1}$. It can be seen that the MCM in ULAs is a banded complex symmetric Toeplitz matrix given by $\mathbf{C} = \text{Toeplitz}\{[\mathbf{c}^T, \mathbf{0}^{1\times(N-P)}]\}$, where Toeplitz$\{\cdot\}$ returns a symmetric Toeplitz matrix having the bracketed vector as its first row.

To avoid confusions, we now utilize $\mathbf{C}_T$ and $\mathbf{C}_R$ to denote the MCM of the transmitter and receiver, respectively. Accordingly, let $\mathbf{c}_T \in \mathbb{C}^{P_T\times 1}$ and $\mathbf{c}_R \in \mathbb{C}^{P_R\times 1}$ denote the mutual coupling coefficient vectors, P_T and P_R are the parameters of P defined in (10). Hence, in the presence of mutual coupling, the steering vectors of the transmitter and receiver should be respectively written as

$$\mathbf{a}_T = \mathbf{C}_T\breve{\mathbf{a}}_T(\theta) \text{ and } \mathbf{a}_R = \mathbf{C}_R\breve{\mathbf{a}}_R(\theta) \tag{11}$$

where $\breve{\mathbf{a}}_T(\theta)$ and $\breve{\mathbf{a}}_R(\theta)$ are the nominal steering vectors. As a result, the output vector of the matched filters should be written as

$$\mathbf{y}(\tau) = \sum_{l=1}^{L} \big((\mathbf{C}_T\breve{\mathbf{a}}_T(\theta_l)) \otimes (\mathbf{C}_R\breve{\mathbf{a}}_R(\theta_l))\big)\,\alpha_l(\tau) + \mathbf{v}(\tau) \tag{12}$$

or in the other form which stems from (7) as

$$\mathbf{z}(\tau) = \sum_{l=1}^{L} \big((\mathbf{C}_R\breve{\mathbf{a}}_T(\theta_l)) \otimes (\mathbf{C}_T\breve{\mathbf{a}}_R(\theta_l))\big)\,\alpha_l(\tau) + \mathbf{n}(\tau) \tag{13}$$

In the following section, the unknown mutual coupling is taken into account in the problem of direction finding in MIMO radar deployed ULAs in the transmitter and receiver.

IV. DIRECTION FINDING WITH MUTUAL COUPLING

A. MUSIC-LIKE METHOD

In order to determine the DOAs in the presence of unknown mutual coupling, a MUSIC-like approach has been developed based on the rank reduction of a matrix related to the subspace. Basically, the following transformation is exploited [16]

$$\mathbf{C}\breve{a}(\theta) = \mathbf{T}(\theta)\mathbf{c} \tag{14}$$

where $\mathbf{T}(\theta) = \mathbf{T}_1(\theta) + \mathbf{T}_2(\theta) \in \mathbb{C}^{N\times P}$, the (i, j)-th entry of $\mathbf{T}_1(\theta)$ and $\mathbf{T}_2(\theta)$ are respectively given by

$$[\mathbf{T}_1(\theta)]_{ij} = \begin{cases} \breve{a}_{i+j-1}(\theta), & i + j \le N + 1 \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

and

$$[\mathbf{T}_2(\theta)]_{ij} = \begin{cases} \breve{a}_{i-j+1}(\theta), & i \ge j \ge 2 \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

Using the above identities, one gets

$$\begin{aligned} \mathbf{b}(\theta_l) &\triangleq (\mathbf{C}_T\breve{\mathbf{a}}_T(\theta_l)) \otimes (\mathbf{C}_R\breve{\mathbf{a}}_R(\theta_l)) \\ &= (\mathbf{T}_T(\theta)\mathbf{c}_T) \otimes (\mathbf{T}_R(\theta)\mathbf{c}_R) \\ &= (\mathbf{T}_T(\theta) \otimes \mathbf{T}_R(\theta))\,(\mathbf{c}_T \otimes \mathbf{c}_R) \\ &= \mathbf{T}_{TR}(\theta)\mathbf{c}_{TR} \end{aligned} \tag{17}$$

where $\mathbf{T}_{TR}(\theta) \triangleq \mathbf{T}_T(\theta) \otimes \mathbf{T}_R(\theta) \in \mathbb{C}^{MN\times P_T P_R}$ and $\mathbf{c}_{TR} \triangleq \mathbf{c}_T \otimes \mathbf{c}_R \in \mathbb{C}^{P_T P_R\times 1}$. Recalling the subspace principle, it is known that the matrix $\mathbf{B} = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \cdots, \mathbf{b}(\theta_L)] \in \mathbb{C}^{MN\times L}$ spans the same space as the matrix $\mathbf{U}_S \in \mathbb{C}^{MN\times L}$, which is composed of the L principal eigenvectors of the output covariance matrix $\mathbf{R}_\mathbf{y} = E\{\mathbf{y}(t)\mathbf{y}^H(t)\}$. This implies that we have span$\{\mathbf{B}\}$ = span$\{\mathbf{U}_S\}$ and hence

$$\mathbf{B} = \mathbf{U}_S\mathbf{W} \tag{18}$$

where $\mathbf{W} \in \mathbb{C}^{L\times L}$ is a nonsingular matrix. Moreover, the matrix B (and vectors $\mathbf{b}(\theta_l)$, $l = 1, 2, \cdots, L$) is orthogonal to the matrix $\mathbf{U}_N \in \mathbb{C}^{MN\times(MN-L)}$, which is composed of the eigenvectors associated with the $MN - L$ smallest eigenvalues. As a result, we have

$$\mathbf{b}^H(\theta_l)\mathbf{U}_N\mathbf{U}_N^H\mathbf{b}(\theta_l) = (\mathbf{c}_T \otimes \mathbf{c}_R)^H\mathbf{Q}(\theta_l)(\mathbf{c}_T \otimes \mathbf{c}_R)$$
$$= \mathbf{c}_{TR}^H\mathbf{Q}(\theta_l)\mathbf{c}_{TR} = 0 \tag{19}$$

where $\mathbf{c}_{TR} \triangleq \mathbf{c}_T \otimes \mathbf{c}_R$ and $\mathbf{Q}(\theta_l)$ is given by

$$\mathbf{Q}(\theta_l) = \mathbf{T}_{TR}^H(\theta_l)^H\mathbf{U}_N\mathbf{U}_N^H\mathbf{T}_{TR}^H(\theta_l). \tag{20}$$

In general, $\mathbf{Q}(\theta)$ has full rank, however, it has reduced rank if θ coincides with the true DOAs $\{\theta_1, \theta_2, \cdots, \theta_L\}$. Therefore, the following MUSIC-like spectrum can be utilized to find the DOAs

$$G(\theta) = \frac{1}{\det\{\hat{\mathbf{Q}}(\theta)\}} \tag{21}$$

where $\det\{\cdot\}$ denotes the determinant of the bracketed matrix, $\hat{\mathbf{Q}}(\theta_l) = \mathbf{T}_{TR}^H(\theta_l)^H\hat{\mathbf{U}}_N\hat{\mathbf{U}}_N^H\mathbf{T}_{TR}^H(\theta_l)$, and $\hat{\mathbf{U}}_N$ is typically obtained from the eigendecomposition of the covariance matrix estimate

$$\hat{\mathbf{R}} = \frac{1}{J}\sum_{\tau=1}^{J}\mathbf{y}(\tau)\mathbf{y}^H(\tau) \tag{22}$$

where J denotes the number of pulse. The positions of the L highest peaks indicate the DOAs. However, the MUSIC-like method is computationally expensive due to the exhaustive spectral grid search. Moreover, it is sensitive to other array imperfections (other than mutual coupling) as discussed later in the next section. To this end, a computationally more efficient algorithm is presented below.

B. ESPRIT-LIKE METHOD

For notational simplicity, let us denote the nominal steering vector of ULA as $\breve{\mathbf{a}}(\theta) = [1, \beta(\theta), \cdots, \beta(\theta)^{N-1}]^T \in \mathbb{C}^{N\times 1}$, where $\beta(\theta) = \exp(j2\pi\lambda^{-1}d\sin\theta)$, λ represents the signal carrier wavelength, and d is the inter-element spacing. Then, as shown in [26], the true steering vector can be reparameterized as follows

$$\mathbf{C}\breve{a}(\theta) = \gamma(\theta)\mathbf{\Gamma}(\theta)\breve{\mathbf{a}}(\theta) \tag{23}$$

where $\gamma(\theta)$ is given by

$$\gamma(\theta) = 1 + \sum_{i=1}^{P-1} c_i\left(\beta(\theta)^i + \beta(\theta)^{-i}\right). \tag{24}$$

It should be mentioned that $\gamma(\theta)$ is assumed to be nonzero, otherwise, the transmitter/receiver cannot transmit/receive signal at the angle θ. In (23), $\mathbf{\Gamma}(\theta) \in \mathbb{C}^{N\times N}$ is a diagonal matrix defined as

$$\mathbf{\Gamma}(\theta) = \mathrm{diag}\{[\mu_1, \cdots, \mu_{P-1}, 1, \cdots, 1, \alpha_1, \cdots, \alpha_{P-1}]\} \tag{25}$$

where $\mathrm{diag}\{\cdot\}$ constructs a diagonal matrix from the bracketed vector, μ_k and α_k, $k = 1, 2, \cdots, P$, are respectively given by

$$\mu_k = \frac{1 + \sum_{i=1}^{P-1} c_i\beta(\theta)^i + \sum_{i=1}^{k-1} c_i\beta(\theta)^{-i}}{\gamma(\theta)} \tag{26a}$$
$$\alpha_k = \frac{1 + \sum_{i=1}^{P-1} c_i\beta(\theta)^{-i} + \sum_{i=1}^{P-1-k} c_i\beta(\theta)^i}{\gamma(\theta)} \tag{26b}$$

Note that there are $\tilde{N} = N - 2P + 2$ ones between μ_{P-1} and α_1 in (25). To avoid confusions, in the sequel, we shall use $\tilde{N} = N - 2P_R + 2$, and $\tilde{M} = M - 2P_T + 2$ for the receiver and transmitter of the MIMO radar, respectively. According to (23), we have

$$\begin{aligned}\mathbf{b}(\theta) &= \mathbf{a}_T(\theta) \otimes \mathbf{a}_R(\theta)\\ &= \mathbf{a}_T(\theta) \otimes (\gamma_R(\theta)\mathbf{\Gamma}_R(\theta)\breve{\mathbf{a}}_R(\theta))\\ &= \gamma_R(\theta)\mathbf{a}_T(\theta) \otimes (\mathbf{\Gamma}_R(\theta)\breve{\mathbf{a}}_R(\theta)).\end{aligned} \tag{27}$$

Let us define a matrix $\bar{\mathbf{J}}_1 = \mathbf{I}^{M\times M} \otimes \mathbf{J}_1 \in \mathbb{C}^{M(\tilde{N}-1)\times MN}$, where $\mathbf{J}_1 = [\mathbf{0}^{(\tilde{N}-1)\times(P_R-1)}, \mathbf{I}^{(\tilde{N}-1)\times(\tilde{N}-1)}, \mathbf{0}^{(\tilde{N}-1)\times P_R}] \in \mathbb{C}^{(\tilde{N}-1)\times N}$. Similarly, let $\bar{\mathbf{J}}_2 = \mathbf{I}^{M\times M} \otimes \mathbf{J}_2 \in \mathbb{C}^{M(\tilde{N}-1)\times MN}$ with $\mathbf{J}_2 = [\mathbf{0}^{(\tilde{N}-1)\times P_R}, \mathbf{I}^{(\tilde{N}-1)\times(\tilde{N}-1)}, \mathbf{0}^{(\tilde{N}-1)\times(P_R-1)}] \in \mathbb{C}^{(\tilde{N}-1)\times N}$. Therefore, we have

$$\begin{aligned}\mathbf{b}_1(\theta) &= \bar{\mathbf{J}}_1\mathbf{b}(\theta)\\ &= \gamma_R(\theta)(\mathbf{I}^{M\times M} \otimes \mathbf{J}_1)(\mathbf{a}_T(\theta) \otimes (\mathbf{\Gamma}_R(\theta)\breve{\mathbf{a}}_R(\theta)))\\ &= \gamma_R(\theta)\mathbf{a}_T(\theta) \otimes (\mathbf{J}_1\mathbf{\Gamma}_R(\theta)\breve{\mathbf{a}}_R(\theta))\\ &= \gamma_R(\theta)\mathbf{a}_T(\theta) \otimes (\mathbf{J}_1\breve{\mathbf{a}}_R(\theta))\\ &= \gamma_R(\theta)\mathbf{a}_T(\theta) \otimes \breve{\mathbf{a}}_{R1}(\theta)\end{aligned} \tag{28}$$

where $\breve{\mathbf{a}}_{R1}(\theta) = [\breve{\mathbf{a}}_R(\theta)]_{P:(N-P)}$ is a subvector composed of the P-th to $(N-P)$-th entries of $\breve{\mathbf{a}}_R(\theta)$. Similarly, one gets

$$\mathbf{b}_2(\theta) = \bar{\mathbf{J}}_2\mathbf{b}(\theta) = \gamma_R(\theta)\mathbf{a}_T(\theta) \otimes \breve{\mathbf{a}}_{R2}(\theta) \tag{29}$$

where $\breve{\mathbf{a}}_{R2}(\theta) = [\breve{\mathbf{a}}_R(\theta)]_{(P+1):(N-P+1)}$. Careful examination shows that $\breve{\mathbf{a}}_{R2}(\theta) = \beta(\theta)\breve{\mathbf{a}}_{R1}(\theta)$, which yields

$$\mathbf{b}_2(\theta) = \beta(\theta)\gamma_R(\theta)\mathbf{a}_T(\theta) \otimes \breve{\mathbf{a}}_{R1}(\theta) = \beta(\theta)\mathbf{b}_1(\theta) \tag{30}$$

and hence, $\mathbf{B}_2 = \bar{\mathbf{J}}_2\mathbf{B} = \mathbf{B}_1\mathbf{\Phi} \in \mathbb{C}^{M(\tilde{N}-1)\times L}$, where $\mathbf{\Phi}$ is an $L \times L$ diagonal matrix as

$$\mathbf{\Phi} = \mathrm{diag}[\beta(\theta_1), \beta(\theta_2), \cdots, \beta(\theta_L)]. \tag{31}$$

Following the above procedure, it can be obtained that

$$\mathbf{U}_{S1} = \bar{\mathbf{J}}_1\mathbf{U}_S = \bar{\mathbf{J}}_1\mathbf{BW} = \mathbf{B}_1\mathbf{W} \in \mathbb{C}^{M(\tilde{N}-1)\times L} \tag{32a}$$
$$\mathbf{U}_{S2} = \bar{\mathbf{J}}_2\mathbf{U}_S = \bar{\mathbf{J}}_2\mathbf{BW} = \mathbf{B}_2\mathbf{W} \in \mathbb{C}^{M(\tilde{N}-1)\times L}. \tag{32b}$$

Using the identity $\mathbf{B}_2 = \mathbf{B}_1\mathbf{\Phi}$ and after some manipulations, we can further get

$$\mathbf{U}_{S2} = \mathbf{U}_{S1}\mathbf{W}^{-1}\Phi W = \mathbf{U}_{S1}\mathbf{\Omega} \tag{33}$$

where $\mathbf{\Omega} = \mathbf{W}^{-1}\mathbf{\Phi}W \in \mathbb{C}^{L\times L}$. It is readily known that $\mathbf{\Phi}$ corresponds to the eigenvalues of $\mathbf{\Omega}$, i.e.,

$$\beta(\theta_l) = \omega_l, \; l = 1, 2, \cdots, L \tag{34}$$

where ω_l denote the l-th eigenvalue of $\mathbf{\Omega}$.

The above analysis shows that the DOAs can be estimated from the estimate of $\mathbf{\Omega}$. According to (33), it is known that, once the estimates of $\mathbf{U}_{S1}$ and $\mathbf{U}_{S2}$ are available, $\mathbf{\Omega}$ can be estimated from the following total least squares (TLS) problem

$$\begin{aligned}&\underset{\mathbf{\Delta}_1,\mathbf{\Delta}_2,\Omega}{\text{minimize}} \ \|[\mathbf{\Delta}_1, \mathbf{\Delta}_2]\|_F \\ &\text{subject to } (\hat{\mathbf{U}}_{S1} + \mathbf{\Delta}_1)\mathbf{\Omega} = \hat{\mathbf{U}}_{S2} + \mathbf{\Delta}_2 \end{aligned} \tag{35}$$

where $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$ are the estimation errors of $\mathbf{U}_{S1}$ and $\mathbf{U}_{S2}$, respectively, and $\|\cdot\|_F$ denotes the Frobenius norm. In order to solve the above problem, let the singular value decomposition of $[\mathbf{U}_{S1}\ \mathbf{U}_{S2}]$ as

$$[\hat{\mathbf{U}}_{S1}\ \ \hat{\mathbf{U}}_{S2}] = [\hat{\mathbf{U}}_1\ \ \hat{\mathbf{U}}_2]\begin{bmatrix}\hat{\mathbf{\Sigma}}_1 & \mathbf{0}\\ \mathbf{0} & \hat{\mathbf{\Sigma}}_2\end{bmatrix}\begin{bmatrix}\hat{\mathbf{V}}_{11} & \hat{\mathbf{V}}_{12}\\ \hat{\mathbf{V}}_{21} & \hat{\mathbf{V}}_{22}\end{bmatrix}^H \tag{36}$$

where $\hat{\mathbf{U}}_i \in \mathbb{C}^{M(\tilde{N}-1)\times L}$, $\hat{\mathbf{\Sigma}}_i \in \mathbb{C}^{L\times L}$, $\hat{\mathbf{V}}_{ij} \in \mathbb{C}^{L\times L}$, and $i, j \in \{1, 2\}$. Therefore, the solution to the TLS problem (35) is given by

$$\hat{\mathbf{\Omega}} = -\hat{\mathbf{V}}_{12}\hat{\mathbf{V}}_{22}^{-1}. \tag{37}$$

Finally, assume $\hat{\omega}_l$ is the lth eigenvalue of $\hat{\mathbf{\Omega}}$ and recalling that $\omega_l = \beta(\theta_l) = \exp(j2\pi\lambda^{-1}d\sin\theta_l)$, then the lth DOA can be estimated as follows

$$\hat{\theta}_l = \arcsin\left(\frac{\lambda\angle\hat{\omega}_l}{2\pi d}\right) \tag{38}$$

where $\angle$ returns the angle of the complex argument.

Remark: It is worth mentioning that the above derivations are based on the data model in (6), and accordingly, the reparameterization (23) is applied to the receiver steering vector only. In fact, it is straightforward to apply the proposed approach to the data model in (7) by reparameterizing the transmitter steering vector in the same manner.

Complexities: In order to obtain the estimates of the signal and noise subspaces, i.e., $\hat{\mathbf{U}}_S$ and $\hat{\mathbf{U}}_N$, both the MUSIC-like and ESPRIT-like algorithms require an eigendecomposition of $\hat{\mathbf{R}}$ with a complexity of $\mathcal{O}(M^3N^3)$. In addition, the MUSIC-like algorithm requires a spectral search with a complexity of $\mathcal{O}(\kappa P_T^3 P_R^3)$, where κ is the number of sampling points in the search domain, and the ESPRIT-like algorithm needs to carry out the singular value decomposition (36) with a complexity of $\mathcal{O}(M^3(\tilde{N}-1)^3)$. Since $N > \tilde{N} - 1$ and κ is usually large, in summary the complexity of the MUSIC-like algorithm is $\mathcal{O}(\kappa P_T^3 P_R^3)$ and the complexity of the MUSIC-like algorithm is $\mathcal{O}(M^3N^3)$.

V. CONSIDERATION OF OTHER ARRAY IMPERFECTIONS

It is well known that apart from mutual coupling, antenna arrays may also suffer from other imperfections such as gain/phase uncertainties and sensor position perturbations [27]–[29]. Therefore, in a general form, the steering vector should be written as

$$\mathbf{a}(\theta) = \mathbf{P}(\theta)\breve{\mathbf{a}}(\theta) \tag{39}$$

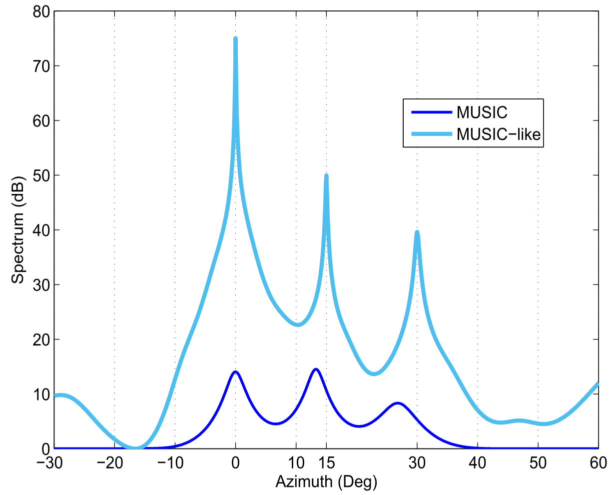


FIGURE 1. Comparison of the spatial spectra of MUSIC algorithm and MUSIC-like method (Case-I).

where $\mathbf{P}(\theta)$ denotes the uncertainty matrix and can be modeled according to different types of uncertainties as follows:

- In the case of mutual coupling, we have $\mathbf{P}(\theta) = \mathbf{C}$.
- In the case of angularly independent gain/phase, $\mathbf{P}(\theta)$ is a diagonal matrix with its diagonal entries being unknown angularly independent sensor gains/phases.
- In the case of sensor position perturbations, $\mathbf{P}(\theta)$ is also a diagonal matrix, but its diagonal entries are angularly dependent.
- In the case of mixed uncertainties, $\mathbf{P}(\theta)$ is general matrix without specific structure.

It should be noted that mutual coupling can be also regarded as angularly dependent gain/phase. As shown in (23), we have $\mathbf{P}(\theta) = \gamma(\theta)\mathbf{\Gamma}(\theta)$.

An interesting question may be raised is how to estimate the DOA when other array imperfections exist or when there is no specific structure of $\mathbf{P}(\theta)$? From the analysis in Section IV, it is noticed that the MUSIC-like method can only handle the case where both the transmitter and receiver have mutual coupling only. On the contrary, the ESPRIT-like method has no specific requirement of the steering vector of the transmitter if we use the data model (6). This implies that the transmitter can theoretically have any kinds of uncertainties. On the other hand, when the model (7) is employed, the ESPRIT-like method is applicable to cases where the transmitter has mutual coupling only whereas the receiver has any other uncertainties. Unfortunately, if both the transmitter and reviver have arbitrary uncertainties other than mutual coupling, the proposed method is not applicable. Since this direction is out the scope of this work, it is not further considered herein.

VI. SIMULATION RESULTS

In this section, various simulations are performed to demonstrate the performance of the proposed methods. It is assumed that both transmitter and receiver are equipped with ULAs with half-wavelength inter-element spacing. Three targets are

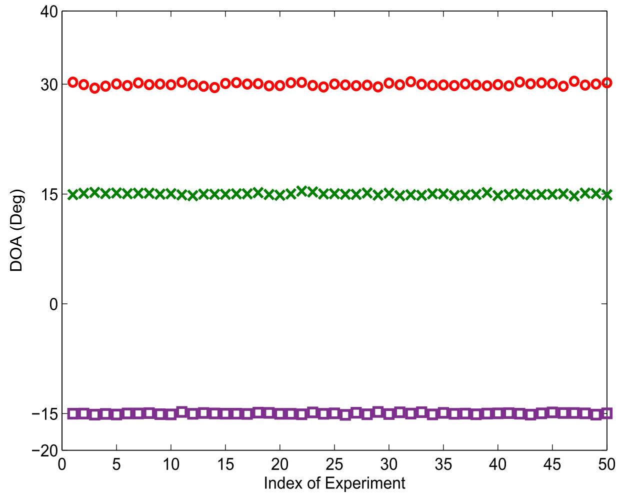


FIGURE 2. DOA estimation results of 50 experiments of the proposed ESPRIT-like algorithm, SNR = 0 dB (Case-I).

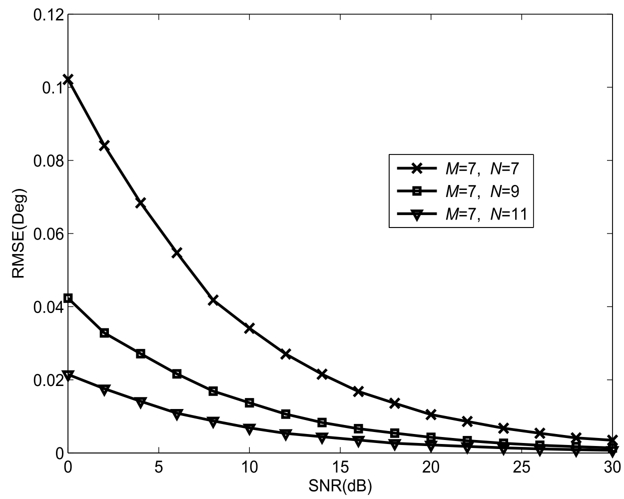


FIGURE 3. RMSE of DOA estimation of the proposed ESPRIT-like algorithm versus SNR for different N (Case-I).

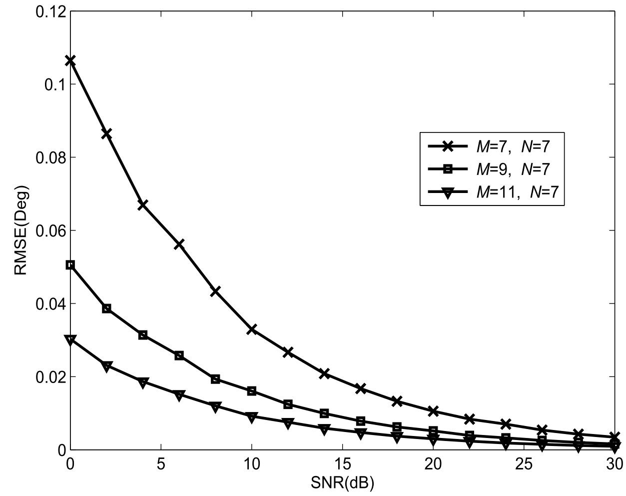


FIGURE 4. RMSE of DOA estimation of the proposed ESPRIT-like algorithm versus SNR for different M (Case-I).

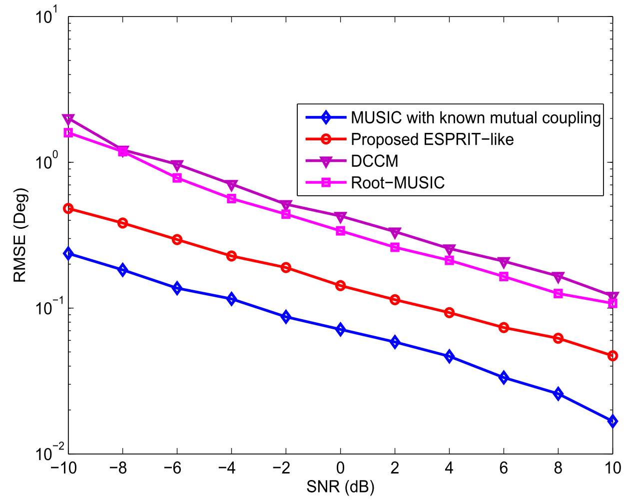


FIGURE 5. Comparison of the RMSEs of DOA estimation of different methods versus SNR (Case-I).

located at $-15°$, $15°$ and $30°$. In our examples, it is assumed that $M = N = 7$, unless otherwise specified in Example 2. The performance is measure in terms of the root mean square error (RMSE) calculated from 1000 Monte Carlo trials.

A. CASE I-MUTUAL COUPLING ONLY

First, we assume that the transmitter and receiver are not calibrated with unknown mutual coupling. Moreover, it is assumed that $P_T = P_R = 3$ and the mutual coupling coefficient vectors for the transmitter and receiver arrays are $\mathbf{c}_T = \mathbf{c}_R = [1, 0.1174 + 0.0577j, -0.0121 - 0.1029j]^T$. The snapshot number is 500 and SNR=0dB.

Fig. 1 shows the resultant normalized spectra of the MUSIC and MUSIC-like methods. It can be noticed that if we perform MUSIC algorithm without properly taking the unknown mutual coupling into account, the accuracy as well as the resolution of this method are significantly degraded. However, all DOAs can be correctly estimated by the MUSIC-like method.

Fig. 2 depicts the DOA estimation results of 50 independent experiments by using the ESPRIT-like algorithm. Obviously, it is observed that this method performs stably and the DOAs can be accurately estimated. Comparing with the MUSIC-like algorithm, the ESPRIT-like method is computationally more efficient.

To further examine the performance of ESPRIT-like algorithm in the cases of different numbers of antennas deployed. Fig. 3 (Fig. 4) displays the curves of RMSE versus SNR in the case of same number of antennas in the transmitter (receiver) but different numbers of antennas in the receiver (transmitter). As expected, the DOA estimation performance improves along with the increase of SNR as well as number of antennas either in the transmitter or receiver.

We now compare the ESPRIT-like algorithm with existing methods including the DCCM [17], root-MUSIC [18], and the MUSIC with exactly known mutual coupling. The SNR is varied from -10 dB to 10 dB. Fig. 5 shows the resulting

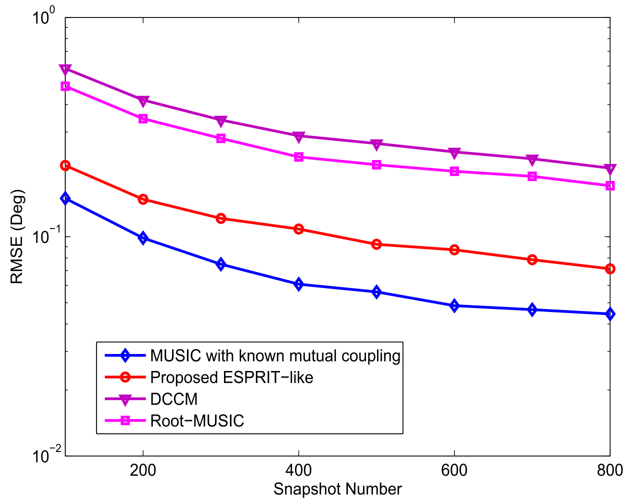


FIGURE 6. RMSE of DOA estimation of different methods versus the number of snapshots (Case-I).

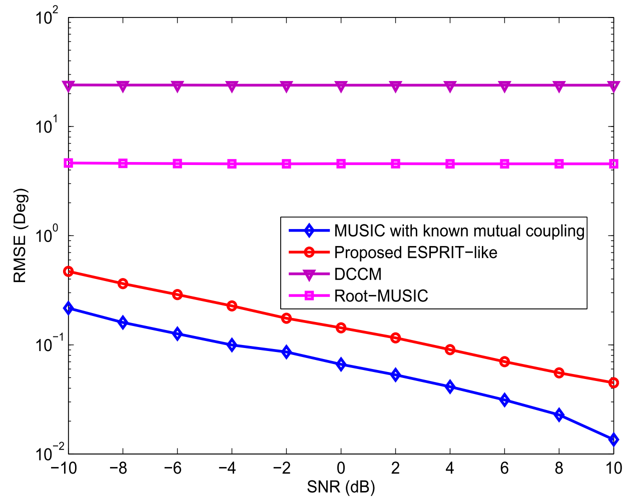


FIGURE 8. RMSE of DOA estimation of proposed ESPRIT-like algorithm versus SNR (Case-II).

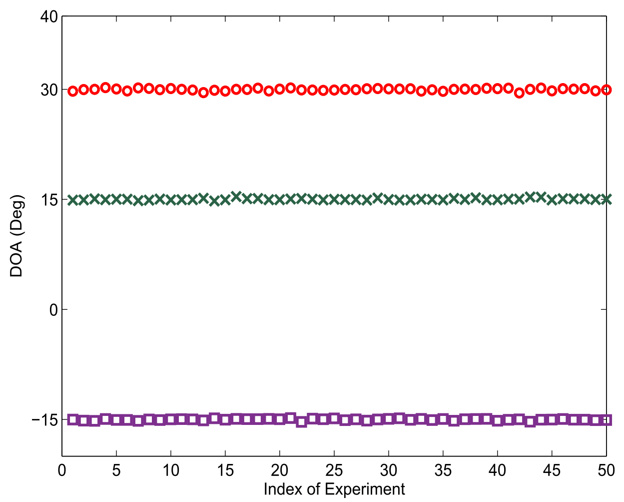


FIGURE 7. DOA estimation results of 50 experiments of the proposed ESPRIT-like algorithm, $\mathrm{SNR} = 0$ dB (Case-II).

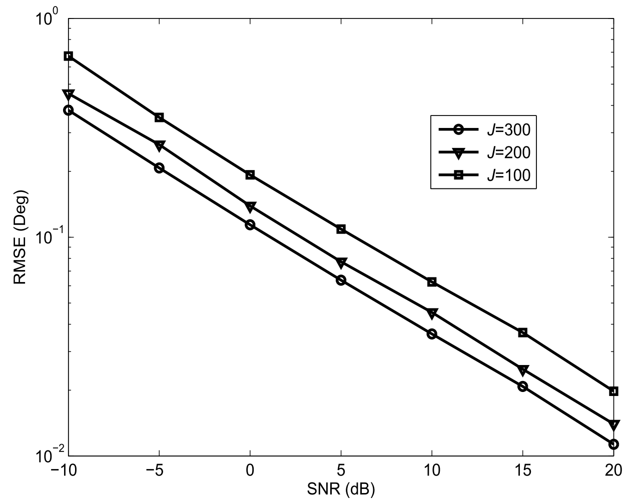


FIGURE 9. RMSE of DOA estimation of the proposed ESPRIT-like algorithm versus SNR for different numbers of snapshots J (Case-II).

RMSEs versus the SNR. It is seen that root-MUSIC method perform slightly better than the DCCM algorithm. However, both of these two algorithms are outperformed by the proposed ESPRIT-like method. The main reason is that both the DCCM and root-MUSIC only exploit partial array elements of the transmitter and receiver, whereas the proposed one is able to make better use of all array elements.

Following the above setting, we further evaluate the the proposed ESPRIT-like method by varying the SNR from −10dB to 10dB when $M = N = 7$. The RMSE of DOA estimation at each tested SNR is calculated, In this simulation, RMSEs are obtain 1000 independent realization. Fig. 5 shows the resultant RMSEs versus the SNR. The proposed method is compared with the DCCM [17], root-MUSIC [18], and MUSIC with exactly known mutual coupling. It can be noticed that the root-MUSIC method [18] is better than the ESPRIT-like method [17], but the angle estimation performance of our method is better than both of them. Because the DCCM and root-MUSIC only exploit partial array elements of the transmit and receive. So our method can achieve the better angle estimation performance.

We vary the number of snapshots from 100 to 800. The RMSE of DOA estimation is calculated for each tested number of snapshots. The resultant RMSEs versus the number of snapshots are shown in Fig. 6. It can be observed that the proposed method performs a good performance which is better than ESPRIT-like algorithm and root-MUSIC algorithm.

B. CASE II-WITH GAIN/PHASE ERRORS

Now, the second case of transmit or receive with gain/phase errors is considered. The gains and phases are generated from uniform distributions $U[1-\rho, 1+\rho]$ and $U[-\phi, \phi]$, and assume that $\rho = 0.5$, $\phi = \pi$, the snapshot number is 500, Fig. 7 shows the results of 50 Monte Carlo tests with $\mathrm{SNR} = 0dB$ for all three targets. From the result, it can be concluded that the proposed ESPRIT-like method have good

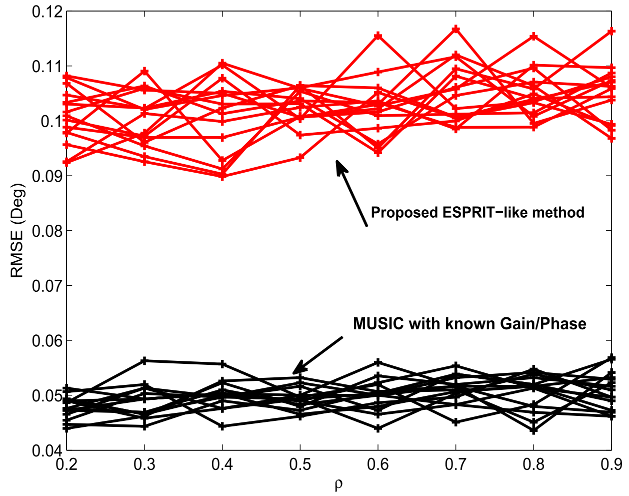


FIGURE 10. Comparison of the RMSEs of DOA estimation versus ρ with $\phi \in [\pi/10, \pi]$ (Case-II).

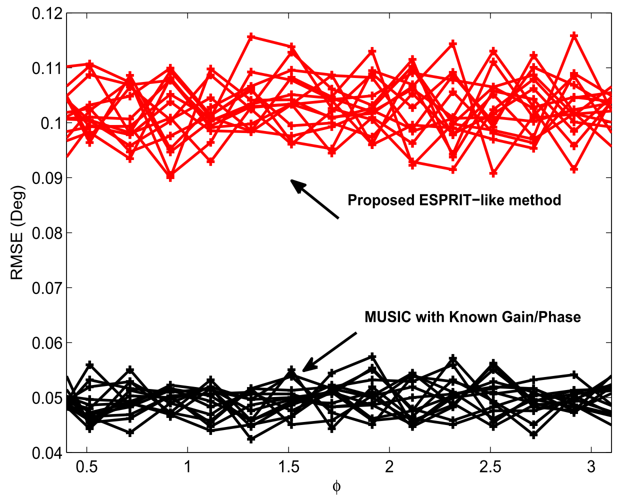


FIGURE 11. Comparison of the RMSEs of DOA estimation versus ϕ with $\rho \in [0.2, 0.9]$ (Case-II).

performance when the transmit has gain/phase and receive has mutual coupling. Furthermore, the proposed method can not only suffer from the mutual coupling but also deal with the other imperfections of the transmit or receive.

We vary the SNR from -10dB to 10dB. The resultant RMSEs versus the SNR are shown in Fig. 8. As the simulation results are described, the DCCM and the root-MUSIC algorithms are completely ineffective when the transmit or receive have gain/phase error. However, the proposed method still has good performance.

Fig. 9 presents the angle estimation performance of our scheme with $M = N = 7$, and different values of J. The SNR from -10dB to 20dB. It has been shown that the angle estimation error will be reduced when J increases. Fig. 10 and Fig. 11 shows the RMSEs of DOA estimation verse ρ and ϕ when the transmitter has mutual coupling and receiver has gain/phase, respectively. Note that, we vary ϕ from $\pi/10$ to π with a step size of 0.2 in Fig. 10 and vary ρ from 0.2 to 0.9 with a step size of 0.05. From the results, it can be concluded that the proposed methods are insensitive to the values of the gains and phases, though the performance may slightly perturb due to different values of the gains and phases.

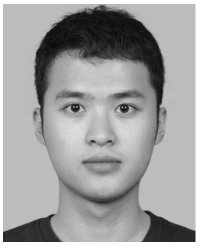
VII. CONCLUSION

Since the ignorance of unknown mutual coupling in MIMO radar may lead to significant performance of direction finding, a new ESPRIT-like algorithm is developed by exploiting the structure of the MCM of transmitter and receiver equipped with ULAs. Unlike the existing MUSIC-like algorithm, the proposed method is more computationally efficient. Moreover, compared the DCCM algorithm which also uses a shift invariance concept, the proposed method can make better use of the array aperture, and hence, better DOA estimation performance. Additionally, extensions of the proposed method can be made to handle other uncertainties such as gain/phase errors. Simulation results demonstrate the improved performance of the proposed method and its extended version.

REFERENCES

[1] J. Li and P. Stoica, *MIMO Radar Signal Processing*. Hoboken, NJ, USA: Wiley, 2008.
[2] J. Li and P. Stoica, "MIMO radar-diversity means superiority," in *Proc. 14th Adapt. Sensor Array Process. Workshop (ASAP)*, Lincoln Lab, MA, USA, 2006, pp. 1–6.
[3] E. Fishler, A. Haimovich, R. Blum, D. Chizhik, L. Cimini, and R. Valenzuela, "MIMO radar: An idea whose time has come," in *Proc. IEEE Radar Conf.*, Apr. 2004, pp. 71–78.
[4] R. Sharma, "Analysis of MIMO radar ambiguity functions and implications on clear region," in *Proc. IEEE Radar Conf. (RADAR)*, May 2010, pp. 544–548.
[5] J. Li and P. Stoica, "MIMO radar with colocated antennas," *IEEE Signal Process. Mag.*, vol. 24, no. 5, pp. 106–114, Sep. 2007.
[6] I. Bekkerman and J. Tabrikian, "Target detection and localization using MIMO radars and sonars," *IEEE Trans. Signal Process.*, vol. 54, no. 10, pp. 3873–3883, Oct. 2006.
[7] E. Fishler, A. Haimovich, R. S. Blum, L. J. Cimini, D. Chizhik, and R. A. Valenzuela, "Spatial diversity in radars-models and detection performance," *IEEE Trans. Signal Process.*, vol. 54, no. 3, pp. 823–838, Mar. 2006.
[8] A. M. Haimovich, R. S. Blum, and L. J. Cimini, "MIMO radar with widely separated antennas," *IEEE Signal Process. Mag.*, vol. 25, no. 1, pp. 116–129, Jan. 2008.
[9] J. Li and P. Stoica, "MIMO radar with colocated antennas," *IEEE Signal Process. Mag.*, vol. 24, no. 5, pp. 106–114, Sep. 2007.
[10] H. Yan, J. Li, and G. Liao, "Multitarget identification and localization using bistatic MIMO radar systems," *EURASIP J. Adv. Signal Process.*, vol. 2008, p. 283483, Dec. 2007, doi: 10.1155/2008/283483.
[11] C. Duofang, C. Baixiao, and Q. Guodong, "Angle estimation using ESPRIT in MIMO radar," *Electron. Lett.*, vol. 44, no. 12, pp. 770–771, Jun. 2008.
[12] C. Jinli, G. Hong, and S. Weimin, "Angle estimation using ESPRIT without pairing in MIMO radar," *Electron. Lett.*, vol. 44, no. 24, pp. 1422–1423, Nov. 2008.
[13] M. L. Bencheikh, Y. Wang, and H. He, "Polynomial root finding technique for joint DOA DOD estimation in bistatic MIMO radar," *Signal Process.*, vol. 90, no. 9, pp. 2723–2730, 2010.
[14] R. Xie, Z. Liu, and Z.-J. Zhang, "DOA estimation for monostatic MIMO radar using polynomial rooting," *Signal Process.*, vol. 90, no. 12, pp. 3284–3288, 2010.
[15] X. Zhang and D. Xu, "Low-complexity ESPRIT-based DOA estimation for colocated MIMO radar using reduced-dimension transformation," *Electron. Lett.*, vol. 47, no. 4, pp. 283–284, 2011.
[16] X. Liu and G. Liao, "Direction finding and mutual coupling estimation for bistatic MIMO radar," *Signal Process.*, vol. 92, no. 2, pp. 517–522, 2012.

[17] Z. Zheng, J. Zhang, and J. Zhang, "Joint DOD and DOA estimation of bistatic MIMO radar in the presence of unknown mutual coupling," *Signal Process.*, vol. 92, no. 12, pp. 3039–3048, 2012.
[18] J. Li, X. Zhang, and W. Chen, "Root-MUSIC based angle estimation for MIMO radar with unknown mutual coupling," *Int. J. Antennas Propag.*, vol. 2014, Jan. 2014, Art. no. 918964, doi: 10.1155/2014/918964.
[19] J. Li and X. Zhang, "Joint angles and mutual coupling estimation algorithm for bistatic MIMO radar," *Int. J. Antennas Propag.*, vol. 2012, Apr. 2012, Art. no. 921878, doi: 10.1155/2012/921878.
[20] J. Liu, X. Wang, and W. Zhou, "Covariance vector sparsity-aware DOA estimation for monostatic MIMO radar with unknown mutual coupling," *Signal Process.*, vol. 119, pp. 21–27, Feb. 2016.
[21] Y. D. Guo, Y. S. Zhang, and N. N. Tong, "ESPRIT-like angle estimation for bistatic MIMO radar with gain and phase uncertainties," *Electron. Lett.*, vol. 47, no. 17, pp. 996–997, Aug. 2011.
[22] B. Liao and S.-C. Chan, "Direction finding in MIMO radar with unknown transmitter and/or receiver gains and phases," *Multidimensional Syst. Signal Process.*, vol. 28, no. 2, pp. 691–707, 2015.
[23] J. Li, J. Ding, and D. Jiang, "Joint direction finding and array calibration method for MIMO radar with unknown gain phase errors," *IET Microw., Antennas Propag.*, vol. 10, no. 14, pp. 1563–1569, Nov. 2016.
[24] I. J. Gupta and A. A. Ksienski, "Effect of mutual coupling on the performance of adaptive arrays," *IEEE Trans. Antennas Propag.*, vol. 31, no. 5, pp. 785–791, Sep. 1983.
[25] T. Svantesson, "Modeling and estimation of mutual coupling in a uniform linear array of dipoles," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, Phoenix, AZ, USA, Mar. 1999, pp. 2961–2964.
[26] B. Liao, Z.-G. Zhang, and S.-C. Chan, "DOA estimation and tracking of ULAs with mutual coupling," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 1, pp. 891–905, Jan. 2012.
[27] B. C. Ng and C. M. S. See, "Sensor-array calibration using a maximum-likelihood approach," *IEEE Trans. Antennas Propag.*, vol. 44, no. 6, pp. 827–835, Jun. 1996.
[28] Z. Ma and K. C. Ho, "A study on the effects of sensor position error and the placement of calibration emitter for source localization," *IEEE Trans. Wireless Commun.*, vol. 13, no. 10, pp. 5440–5452, Oct. 2014.
[29] H. Liu, L. Zhao, Y. Li, X. Jing, and T.-K. Truong, "A sparse-based approach for DOA estimation and array calibration in uniform linear array," *IEEE Sensors J.*, vol. 16, no. 15, pp. 6018–6027, Aug. 2016.



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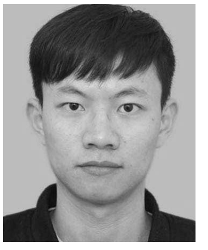

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