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Multicast Capacity for VANETs With Directional Antenna and Delay Constraint Under Random Walk Mobility Model

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ABSTRACT In this paper, we investigate the multicast capacity for vehicular ad hoc networks with directional antennas and the end-to-end delay constraint. We consider a torus of unit area with n vehicles (nodes), there are n_s multicast sessions and each session contains one source vehicle which is associated with p destinations. We study the 2D and 1D random walk mobility models with two different time scales, i.e., fast and slow mobility. Given a delay constraint D and assuming that each vehicle is equipped with a directional antenna, we obtain the multicast capacity of the two mobility models with two different time scales in the order of magnitude, respectively. We then characterize the impact of the network parameters (i.e., the end-to-end delay constraint D, the beamwidth of directional antenna θ, and the number of destinations p in each session) on the multicast capacity. Moreover, we find that the unicast capacity can be considered as a special case of our multicast results when the beamwidth of directional antenna θ tends to 2π and the number of destinations p tends to 1 in the sense of probability.

INDEX TERMS Multicast capacity, VANETs, directional antenna, delay constraint, random walk.

I. INTRODUCTION

Emerging vehicular ad hoc networks (VANETs) have attracted lots of researchers' attention. A mass of work in the field of VANETs has made significant progress. With applying wireless communication technologies, more and more vehicles are equipped with on-board communication facilities, enabling them to communicate with surrounding vehicles or road infrastructures efficiently. The advantages of using vehicular communication networks not only provide information exchange service, but improve road safety by distributing incident warning signal. On account of these merits of VANETs, literatures are abound in investigating the vehicular communications. Jiang *et al.* [1] study the efficient multicast in vehicular networks based on vehicle trajectories. Kong *et al.* [2] provide a frequency-divided approach to analyze vehicle density information in Dedicated Short Range Communication (DSRC) vehicular networks. [3] investigates message deliveries with privacy preservation in VANETs by proposing a novel routing scheme BusCast. Kerrache *et al.* [4] investigate an adversary-oriented overview on the main trust models in VANETs. As large amounts of vehicular applications require vehicles to compete for data transmissions in a limited network area, it is desirable to know the fundamental throughput capacity of VANETs, which is greatly important and provides the guidance in designing a real network.

The asymptotic throughput capacity of static large-scale wireless networks was initiated by Gupta and Kumar in [5]. They prove that the throughput of each node under the protocol model in random wireless ad hoc network is $\Theta(\frac{W}{\sqrt{n\log n}})$[1] bits/sec, which means that while the number of nodes trends to infinity, the per-node throughput decreases to zero. Later, Grossglauser and Tse show that constant per-node capacity can be achieved by adopting a two-hop highly mobility model at the cost of tremendous delay [6]. Followed by their work, a plenty of studies have been done to

[1] Given two non-negative functions $f(n)$ and $g(n)$: $f(n) = O(g(n))$ means there exists a constant c such that $f(n) \leq cg(n)$ for n large enough; $f(n) = \Omega(g(n))$ if $g(n) = O(f(n))$; $f(n) = \Theta(g(n))$ means both $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$; $f(n) = o(g(n))$ means $\lim_{n\to\infty} f(n)/g(n) = 0$; and $f(n) = \omega(g(n))$ means $\lim_{n\to\infty} g(n)/f(n) = 0$.

investigate the capacity of mobile ad hoc networks (MANETs) under a variety of mobility models [7]–[10]. Other works focus on the delay-throughput tradeoffs for MANETs with variety scenarios [11]–[15]. All the above works focus on the investigation of unicast traffic flow, which appear low effective and little beneficial in both theory and application. For instance, multicast information flow can be utilized for delivering live multimedia contents in a variety of applications, e.g., weather prediction, wireless video conference etc. In [16], they study the multicast capacity of static wireless networks based on a comb routing scheme. Li *et al.* [17] prove the matching upper bound and lower bound for multicast capacity via a Euclidean minimum spanning tree method. Zhou and Ying [25] study the multicast capacity of large-scale MANETs under delay constraints by adopting raptor code technique.

The investigation of capacity and delay for VANETs is of great importance in both theory and its potential applications. Pishro-Nik *et al.* [18], Nekoui and Pishro-Nik [19] initiate the study of scaling laws for VANETs. They show that the road geometry has great effect on the throughput capacity of VANETs. They propose a new concept of sparseness for analyzing the influence of road geometry on the capacity. Lu *et al.* [20] show the asymptotic capacity of social-proximity VANETs. They consider that vehicles move around a specific social spot in a restricted region. Wang *et al.* [21] analyze the throughput capacity by proposing a novel packet forwarding strategy. In the previous work [22], we investigate the multicast capacity of VANETs with directional antennas and end-to-end delay constraints under the i.i.d. mobility model.

In this paper, we study the multicast capacity of VANETs under the random walk mobility model. Specifically, we assume that every vehicle is equipped with a directional antenna. Data transmission is under the delay constraint, i.e., the source node transmits to its destinations directly within the transmission radius. Otherwise, data is transmitted by multihop fashion under a delay constraint D. The packets can be transmitted successfully if and only if each pair of transmitter and receiver is within their directional antenna beam coverage ranges.

Different from previous work, we adopt the random walk mobility model, which is a more general model with considerable applications. The random walk mobility model can be used to characterize the social characteristic of vehicle's mobility in a localized city region. For example, a vehicle often moves within a bounded area that is in the proximity of the driver's company or neighbourhood. In this mobility model, vehicles can move to adjacent regions or stay at the current region in the next time slot. Furthermore, in real life, a vehicle usually returns to its starting point after a long time travel. In [23], Polya have demonstrated that the node following random walk model on a d-dimensional ($d \leq 2$) surface returns to its origin with a probability of 1, which is similar with the mobile trajectory of vehicles.

In the analysis of the impact of mobility model on the multcast capacity of VANETs, we first study the two-dimensional random walk mobility model with fast mobility. Then, we derive the multicast capacity of VANETs under the two-dimensional random walk mobility model with slow mobility. Next, we analyze the one-dimensional random walk mobility model under conditions of fast and slow mobile vehicles. We find that the throughput capacity is improved in the one-dimensional mobility model, which is intuitively the same because the probability that the source chooses the location of its destinations becomes larger compared with the two-dimensional model. Finally, we obtain the upper bounds on the multicast capacity of VANETs, which show how the asymptotic results rely on the number of destinations p in a multicast session, the delay constraint D and the beamwidth of directional antenna θ. The main contributions of our paper are as follows:

- We present an asymptotic study of the multicast capacity for VANETs with directional antenna and delay constraint under random walk mobility model. We give the asymptomatic upper bounds on the muticast capacity.
- We adopt random walk mobility model with two time scales to characterize the vehicles mobility patterns. We analyze the impact of mobility models on the multicast capacity for two-dimensional and one-dimensional scenarios, which isn't considered in the state-of-art research.
- We investigate the impact of system parameters on the multicast capacity, i.e., the number of vehicles, the number of destinations of each session, the beamwidth of directional antenna, and the delay constraint. We also find that some of the previous work can be considered as a special case of our results. Compared with existing work, our results perform better in the capacity while other system parameters are fixed. We validate our results by providing extensive simulations.

The rest of the paper is organized as follows. In Section II, we introduce the definitions and notations. In Section III, we present the system models. In Section IV, we analyze the multicast capacity and give the main results for different scenarios. Finally, we conclude the paper in Section V.

II. DEFINITIONS AND NOTATIONS

In this section, we give the definitions and related notations for problem formulation and analysis.

A. FEASIBLE MULTICAST THROUGHPUT

For a VANET with n vehicles, we say that the multicast throughput, denoted by $\lambda(n)$, is feasible if there exists a spatial and temporal scheduling scheme that yields a throughput of $\lambda(n)$ bits/second.

B. AGGREGATE MULTICAST THROUGHPUT CAPACITY

We say the aggregate multicast throughput capacity of a VANET is of order $O(f(n))$ bits/second if there is a

TABLE 1. Main notations.

n	The number of vehicles in a VANET
W	The maximum bandwidth of the VANET
θ	The beamwidth of a directional antenna
D	The maximum tolerant delay of the packets transmission
r_i	The transmission radius of vehicular node i
$X_i(t)$	Vehicular node i and its location at t time slot
L	The distance that the packet can hit its destination
$\lambda^d(n)$	The number of bits are successfully transmitted to destinations in [0,T] when the packets are directly transmitted from the source to its destinations
$\lambda^r(n)$	The number of bits are successfully transmitted to destinations in [0,T] when the packets transmitted from the relays to destinations
$\lambda(n)$	The total number of bits are successfully transmitted to destinations in [0,T]
P	The probability of an event

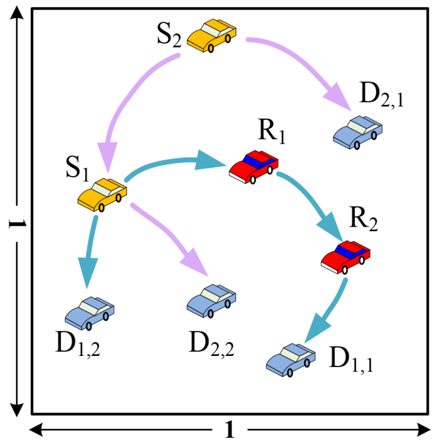

FIGURE 1. A VANET with two multicast sessions, where $D_{1,1}$ and $D_{1,2}$ are the destinations of source vehicle S_1, $D_{2,1}$ and $D_{2,2}$ are the destinations of source vehicle S_2. Any vehicle can be a relay node in the multicast sessions.

deterministic constant $c_1 < +\infty$ such that

$$\lim_{n\to\infty} \inf P(\lambda(n)) = c_1 f(n) \text{ is feasible}) = 0,$$

and is of order $\Theta(f(n))$ bits per second if there is a deterministic constant $0 < c_2 < c_3 < +\infty$ such that

$$\lim_{n\to\infty} \inf P(\lambda(n) = c_2 f(n) \text{ is feasible}) = 1,$$
$$\lim_{n\to\infty} \inf P(\lambda(n) = c_3 f(n) \text{ is feasible}) = 0.$$

C. DELAY CONSTRAINT

We say a successful transmission if the source node delivers packets to its p destinations within D consecutive time slots.

D. HITTING DISTANCE

At time slot t, the packet hits its destination if the distance between the packet and its destination is less than or equal to L [13].

To facilitate the understanding, some important notations used in this paper are listed in Table I.

III. SYSTEM MODEL

A. NETWORK MODEL

As shown in Fig. 1, we consider the vehicular ad hoc networks with n mobile vehicles (nodes) distributed in a unit square, which is assumed to be a torus. In the multicast transmission model, we assume that there are n_s multicast sessions and each session has one source and p destinations. Each vehicle is exactly the source of one session and the destination of another session. Moreover, every vehicle can serve as a relay node in any multicast session. Thus, we can obtain that $n = n_s(1 + p)$ vehicles in the VANET.

B. DIRECTIONAL ANTENNA MODEL

In order to improve the capacity of VANETs, we adopt the directional antenna model, which is directional transmission

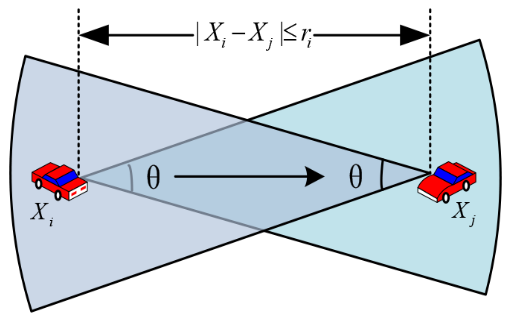

FIGURE 2. The packets are delivered from the source to its destination successfully under the directional antennal model when the distance between the source and the destination is no more than r_i and the antenna beam of two nodes will cover each other.

and directional reception (DTDR) in VANETs. We approximate the directional antenna as a circular sector with angle θ, the beam radius equals to the transmission/reception range r, as shown in Fig. 2. We further assume the angle of sector approximates the beamwidth of the antenna [26]. In reality, the directional antenna consists two portions: a mainlobe which is in the transmission direction and several smaller backlobes in nontransmission direction arising from low efficiency in antenna design. For the convenience of analysis, we ignore the impact of backlobes, i.e., the directional antenna gain is within specific angle θ and the backlobes gain is tend to zero.

We consider that every vehicle is equipped with one directional antenna and each antenna can be steerable. That is to say, the antenna beam can be placed towards any direction at any time slot. Thus, the probability that the beam covers a direction is $\theta/2\pi \in (0, 1)$.

C. COMMUNICATION MODEL

In real VANETs, the radio signal propagation of a vehicle can be interfered by many factors (obstacles, road geometry, etc).

Martinez et al. [24] propose a new propagation model, which is called Real Attenuation and Visibility (RAV). However, for the convenience of analysis, in this paper we adopt the protocol model introduced in [5] to analyze the impact of interference on capacity in VANETs. For simplicity, denote r_i as the transmission radius of node i. We assume that all vehicles have the same transmission radius and common transmission power P. If node i transmits to node j successfully, then two following conditions need to be satisfied:

(1) The position of receiving node is within the transmission range of the transmitter, i.e.,

$$|X_i(t) - X_j(t)| \leq r_i.$$

(2) Other transmitters X_k delivering packets at the same time slot does not interfere the receiving node j, i.e.,

$$|X_k(t) - X_j(t)| \geq (1 + \Delta)r_i.$$

Here $\Delta > 0$ denotes the guard zone, which is a constant that doesn't depend on n. $X_i(t)$ not only denotes the location of a node but refers to the node itself at time slot t. According to the above two conditions, when node i transmits to node j successfully if and only if the antenna beams of i and j cover each other. We further assume that each transmitter-receiver pair can deliver W bits/s in a successful transmission.

D. MOBILITY MODEL

In this paper, we focus on the following mobility models.

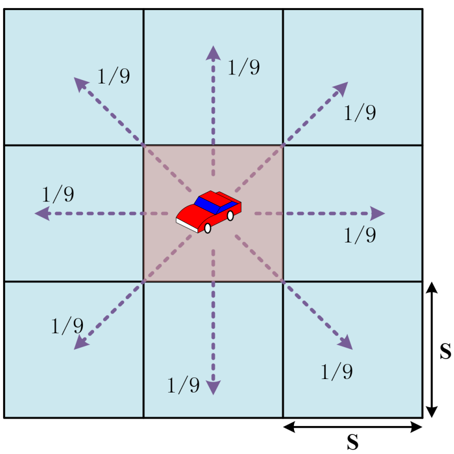

FIGURE 3. Two-dimensional random walk model.

(1) **Two-dimensional random walk model:** Consider that all vehicles are in a unit square network, which is further divided into $1/S^2$ smaller squares of the same size. We assume that S is an integer. Each smaller square will be called a cell. Two cells are said to be adjacent if they share a common point, as shown in Fig. 3. At initial time slot, a vehicular node will independently and uniformly select one of nine adjacent cells (including itself cell) and stay there in the next time slot. So the probability for each cell to be selected as the destination cell is $1/9$.

(2) **One-dimensional random walk model:** First, we assume that the total number of mobile vehicles n and the multicast sessions n_s are both even numbers. Then, let $n/2$ nodes, named H-nodes, move on the horizontal lines; and the other $n/2$ nodes, named V-nodes, move on the vertical lines. We further assume that sources and destinations are the same type of nodes. Next, each vehicle's horizontal (or vertical) orbit will be divided into $1/S$ intervals. At the beginning of time slot, a node will randomly and uniformly move to one of two adjacent intervals or stay at the current interval, as shown in Fig. 4. Finally, the probability for each interval to be selected as the destination interval is $1/3$.

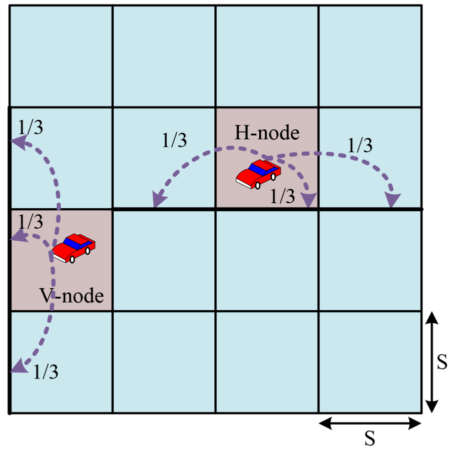

FIGURE 4. One-dimensional random walk model.

Two above mobility models can be very practical in real VANETs. For example, one vehicle is at two lane intersection in a single time slot. The vehicle will have two choices in the next time slot, one of which is moving across the intersection. The other is turning around and moving on the opposite lane. The trajectory of vehicle resembles the one-dimensional random walk model. However, the real trajectory of vehicles may have some differences with theoretical random walk mobility model, we find that these theoretical mathematical models can capture the essential mobility features of vehicles.

We consider two time scales of vehicular mobility in this paper. One is the fast mobility, i.e., the mobility of vehicles is at the same time scale as the packets transmission. So, the packets delivered to destinations can be finished only by one-hop in a single time slot and W is a constant independent of n. Fast mobility can simulate the scenario that the vehicles move in the uncrowded road condition. The other is slow mobility, i.e., the mobile speed of vehicles is much slower than the packets transmission. That is, the packets transmission can be achieved by multihop fashion within a single time slot and $W = \Omega(n)$. Thus, the packet size can be considered as $W/h(n)$ for $h(n) = O(n)$ to guarantee $h(n)$-hops transmission in a time slot. Slow mobility can simulate the scenario that the vehicles move in the traffic jam condition.

IV. THE UPPER BOUND CAPACITY ANALYSIS

A. TWO-DIMENSIONAL RANDOM WALK FAST MOBILITY MODEL

In this section, we investigate the upper bound on the multicast capacity of VANETs with the two-dimensional random walk fast mobility model under the conditions of the directional antenna and a delay constraint D. First, we introduce some notations, which will be used in the following proof.

- $\lambda^d(T)$: The number of bits that are successfully transmitted to destinations in [0,T], when the packets are directly transmitted to their destinations.
- $\lambda^r(T)$: The number of bits that are successfully transmitted to destinations in [0,T], when the packets are delivered to their destinations by relays.
- $\lambda(T)$: $\lambda(T) = \lambda^d(T) + \lambda^r(T)$, which denotes the total number of bits that are successfully transmitted to destinations in [0,T].
- B: Index of a bit of packet stored in the VANET.
- α_B: The transmission radius used to deliver bit B.

Then, before deriving the aggregate multicast throughput capacity of a VANET, we give some important lemmas, which are useful in the following analysis.

Lemma 1: Assume that all nodes are equipped with directional antennas under the protocol model, the following inequalities hold:

$$\lambda^d(T) \leq n_s p WT, \quad (1)$$

$$\lambda^r(T) \leq n_s(p+1)WT, \quad (2)$$

$$\sum_{B=1}^{B[T]} \frac{\Delta^2}{4}(\alpha_B)^2 \leq \frac{2WT}{\theta}. \quad (3)$$

Inequality (1) holds since the number of bits direct delivered in T time slots cannot exceed $n_s pWT$. Inequality (2) holds since the maximum number of bits transmitted by relays cannot exceed $n_s(p+1)WT$ in T time slots. This is because that the source vehicle can serve as a relay node. Inequality (3) holds since the probability that the directional antenna covers a direction is $\theta/2\pi$ and the disk of radius $\Delta\alpha_B/2$ times the length of transmission range centered at receivers should be disjoint under the protocol model.

Lemma 2: Consider the protocol model and the random walk mobility model with the directional antenna, there exists $k > 0$, the following inequality holds [22]:

$$E[\lambda(T)] \leq 5k\log(\theta n_s p)E[B[T]] + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_s p).$$

Now, we consider the scenario that the source vehicles transmit packets directly to the destinations with directional antennas without delay constraints.

Lemma 3: Under the conditions of the protocol model and DTDR model, we consider the two-dimensional random walk fast mobility, when the packets are directly transmitted from the source vehicle to their destinations, then we have

$$E[\lambda^d(T)] \leq 5k\log(\theta n_s p)\left(\frac{4\sqrt{\pi}WT}{\Delta(1-\theta/2\pi)}\sqrt{n_s p}\right) + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_s p).$$

Proof: Let s_i denote the source of multicast session i, and let $d_{i,j}$ denote the j^{th} destination in the multicast session i, the minimum transmission distance between source node s_i and its nearest destination denoted by $D(s_i, t)$, i.e.,

$$D(s_i,t) = \min_{1\leq j\leq p} \text{dist}(s_i(t), d_{i,j}(t)).$$

Inspired by the definition of hitting distance, we know a successful transmission at a time slot can be achieved if and only if the transmission radius r_i of the source node s_i needs to be at least L. Hence, we assume all nodes utilize a common transmission radius L, i.e., $r_i = L$. The packet can hit its one of destinations with probability $\frac{\theta^2\pi L^2}{4\pi^2 S^2}$. Then, the probability that a packet can hit all of its destinations is $1-(1-\frac{\theta^2\pi L^2}{4\pi^2 S^2})^p$. Based on the properties of random walk model in [13], we obtain that

$$P(D(s_i,t)\leq L,\ \text{covered by } s_i) \leq 1-(1-\frac{\theta^2\pi L^2}{4\pi^2 S^2})^{\frac{99}{10}S^2 p} \leq \frac{99}{40\pi}\theta^2 L^2 p \leq \theta^2 L^2 p,$$

which implies

$$E\left[\sum_{t=1}^{T}\sum_{i=1}^{n_s} 1_{D(s_i,t)\leq L,\text{covered by } s_i}\right] \leq \theta^2 L^2 T n_s p.$$

A source node can send at most W bits in each successful transmission, then we have

$$\begin{aligned} E[B[T]] &= E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B\leq L}\right] + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B> L}\right] \\ &\leq WE\left[\sum_{t=1}^{T}\sum_{i=1}^{n_s} 1_{D(s_i,t)\leq L,\text{covered by } s_i}\right] \\ &\quad + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B> L}\right] \\ &\leq \theta^2 L^2 T n_s p W + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B> L}\right]. \end{aligned}$$

Using Cauchy-Schwarz inequality and inequality (3), we have

$$\left(\sum_{B=1}^{B[T]}\alpha_B\right)^2 \leq \left(\sum_{B=1}^{B[T]} 1\right)\left(\sum_{B=1}^{B[T]}(\alpha_B)^2\right) \leq B[T]\frac{8WT}{\theta\Delta^2}.$$

which implies

$$E\left[\sum_{B=1}^{B[T]}\alpha_B\right] \leq \left(\sqrt{\frac{8WT}{\theta\Delta^2}}\right)E\left[\sqrt{B[T]}\right]. \quad (4)$$

This inequality shows the upper bound on the expected distance travelled of all bits. Furthermore, according to Jensen's inequality and inequality (4), we obtain that

$$\sqrt{\frac{8WT}{\theta\Delta^2}}\sqrt{E[B[T]]} \geq E\left[\sqrt{\frac{8WT}{\theta\Delta^2}B[T]}\right] \geq E\left[\sum_{B=1}^{B[T]}\alpha_B\right] \geq LE\left[\sum_{B=1}^{B[T]}1_{\alpha_B>L}\right] \geq L(E[B[T]] - \theta^2L^2Tn_spW),$$

then, let $L = \sqrt{\frac{E[B[T]]}{2\pi\theta WTn_sp}}$, and substituting L into above inequality, we have

$$\frac{4\sqrt{\pi}WT}{\Delta(1-\theta/2\pi)}\sqrt{n_sp} \geq E[B[T]].$$

By submitting into the bound on $\lambda(T)$ in Lemma 2, we have

$$E[\lambda^d(T)] \leq 5k\log(\theta n_sp)\left(\frac{4\sqrt{\pi}WT}{\Delta(1-\theta/2\pi)}\sqrt{n_sp}\right) + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_sp). \qquad \square$$

In the following analysis, we consider the data packets transmitted from relays to destinations, i.e., the packets will be transmitted in multihop fashion within a delay constraint D. We will calculate the upper bound on the expected number of bits under the relaying scheme.

Lemma 4: Under the conditions of the protocol model and DTDR model, we consider the two-dimensional random walk fast mobility, when packets have to be transmitted from relays to their destinations with a delay constraint D, then we have

$$E[\lambda^r(T)] \leq 5k\log(\theta n_sp)\left(\frac{4\sqrt{\pi}WT(p+1)}{\Delta(1-\theta/2\pi)}\sqrt{n_sD}\right) + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_sp).$$

Proof: In the proof of this Lemma, we assume that every vehicle can be a relay node. So we can use inequality (2) to bound the maximum number of bits at relay nodes. Let $H(B)$ denote the minimum distance between the relay node carrying bit B and one of the p destinations in a multicast session under the conditions of the directional antenna beamwidth θ and a delay constraint D. Follow the analysis in Lemma 3, we calculate the probability that a packet can hit all of its destinations in one of D time slots is $1-(1-\frac{\theta^2\pi L^2}{4\pi^2S^2})^{Dp}$, for any $L \in [0, S/\sqrt{\pi})$ and based on the properties of random walk in [13], we have

$$P(H(B) \leq L) \leq 1-\left(1-\frac{\theta^2\pi L^2}{4\pi^2S^2}\right)^{\frac{99}{10}S^2pD} \leq \frac{99}{40\pi}\theta^2L^2p \leq \theta^2L^2pD,$$

which implies

$$E\left[\sum_{B\in\lambda^r(T)}1_{H(B)\leq L}\right] \leq n_s(p+1)WT\theta^2L^2pD.$$

Furthermore, we know that the total direct transmission distance that all bits are travelled is less or equal to the transmission distance by multihop transmissions. Hence, we have

$$\sum_{B=1}^{B[T]}1_{\alpha_B\leq L} \leq \sum_{B\in\Lambda^r[T]}1_{H(B)\leq L},$$

and

$$E[B[T]] = E\left[\sum_{B=1}^{B[T]}1_{\alpha_B\leq L}\right] + E\left[\sum_{B=1}^{B[T]}1_{\alpha_B>L}\right] \leq E\left[\sum_{B\in\Lambda^r[T]}1_{H(B)\leq L}\right] + E\left[\sum_{B=1}^{B[T]}1_{\alpha_B>L}\right] \leq n_s(p+1)WT\theta^2L^2pD + E\left[\sum_{B=1}^{B[T]}1_{\alpha_B>L}\right].$$

Then, using the Cauchy-Schwarz inequality, we have

$$\left(\sum_{B=1}^{B[T]}\alpha_B\right)^2 \leq \left(\sum_{B=1}^{B[T]}1\right)\left(\sum_{B=1}^{B[T]}(\alpha_B)^2\right) \leq B[T]\frac{8WT}{\theta\Delta^2},$$

which implies that

$$\sqrt{\frac{8WT}{\theta\Delta^2}}\sqrt{E[B[T]]} \geq E\left[\sqrt{\frac{8WT}{\theta\Delta^2}B[T]}\right] \geq E\left[\sum_{B=1}^{B[T]}\alpha_B\right] \geq LE\left[\sum_{B=1}^{B[T]}1_{\alpha_B>L}\right] \geq L(E[B[T]] - n_s(p+1)WT\theta^2L^2pD),$$

since the inequality holds for $L \in [0, S/\sqrt{\pi})$, and choose $L = \sqrt{\frac{E[B[T]]}{2\pi\theta WTn_sp(p+1)D}}$, we have

$$\frac{4\sqrt{\pi}WT}{\Delta(1-\theta/2\pi)}\sqrt{n_sp(p+1)D} \geq E[B[T]],$$

we know that $n = n_s(1+p)$ in the multicast transmission model, thus, the above inequality can be further simplified as

$$\frac{4\sqrt{\pi}WT}{\Delta(1-\theta/2\pi)}\sqrt{npD} \geq E[B[T]].$$

Submitting to Lemma 2, we can finally have

$$E[\lambda^r(T)] \leq 5k\log(\theta n_sp)\left(\frac{4\sqrt{\pi}WT}{\Delta(1-\theta/2\pi)}\sqrt{npD}\right) + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_sp). \qquad \square$$

Theorem 1: Under the two-dimensional random walk fast mobility model, the aggregate multicast capacity of VANETs with directional antennas and a delay constraint D is

$$\lambda(n) = O\left(\frac{\log p \log(\theta n_s p)}{1-\theta/2\pi}(\sqrt{npD}+\sqrt{n_s p})\right).$$

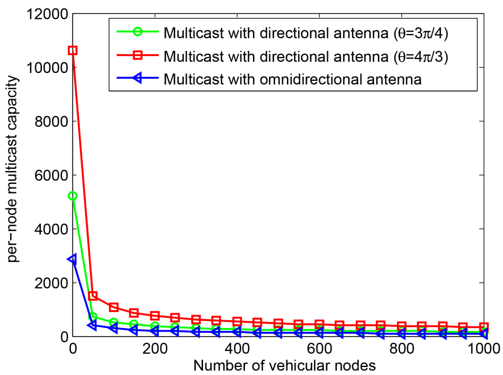

FIGURE 5. Multicast capacity ($p > 1$) with the directional antenna under the 2D random walk fast mobility model vs. the multicast capacity with the omnidirectional antenna under the 2D-i.i.d fast mobility model.

Remark 1: From the Lemma 3 and Lemma 4, we have obtained the aggregate multicast throughput capacity of a VANET when all vehicular nodes follow the two-dimensional random walk fast mobility model. We can find that the throughput capacity by using relays dominates the throughput by direct transmissions. Furthermore, if D is large enough, the aggregate multicast throughput capacity can be simplified as $O\left(\frac{\log p \log(\theta n_s p)}{1-\theta/2\pi}\sqrt{npD}\right)$. Since we know that there are n vehicles, the maximum throughput per vehicle is $O(\frac{\log p \log(\theta n_s p)}{1-\theta/2\pi}\sqrt{\frac{pD}{n}})$ under the fast mobility model.

Specially, when the beamwidth θ tends to 2π and the multicast destination p tends to 1, we find that the multicast capacity of pre-node trends to $O(\sqrt{\frac{D}{n}})$, which means the unicast capacity with omnidirectional antenna model under the random walk model in [13] can be regarded as the special case of our result. The numerical simulations are shown in Fig. 5 and Fig. 6. We compare the obtained throughput capacity of pre-node with the existing results. In Table II, we list the essential numerical simulation parameters.

The first case that when the number of destinations $p > 1$ is plotted in Fig. 5, we find that the directional antenna can improve the multicast capacity compared with [25]. In addition, Fig. 5 shows that the multicast capacity increases as the beamwith increases. This can be explained as a larger beamwidth of a directional antenna equipped in the vehicle can lead to an increase in the transmission probability between transmitter-receiver pairs. Hence, the upper bound on multicast capacity can be improved. The second case that when the number of destinations trends to 1 is shown in Fig. 6, which can be approximately considered as the unicast transmission. We find our result is better than [13] by adopting the directional transmission and directional reception (DTDR) model.

TABLE 2. Simulation parameters.

Parameters	Value
The maximum number of vehicles n	1000
The beamwidth of a directional antenna θ	$\frac{3\pi}{4}$, $\frac{4\pi}{3}$
The tolerant delay of the packets D	1000
The maximum multicast sessions n_s	20

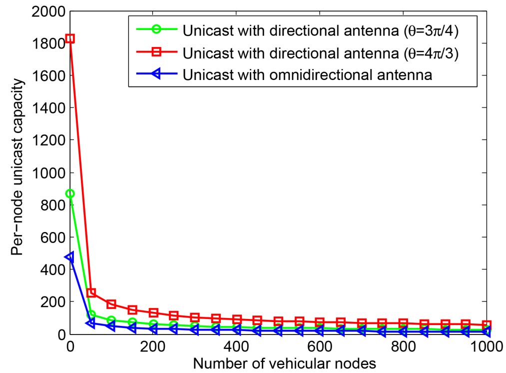

FIGURE 6. Unicast capacity ($p \to 1$) with the directional antenna under the 2D random walk fast mobility model vs. the unicast capacity with the omnidirectional antenna under the 2D random walk fast mobility model.

B. TWO-DIMENSIONAL RANDOM WALK SLOW MOBILITY MODEL

In this section, we investigate the upper bound on multicast capacity of the two-dimensional random walk slow mobility model. From Section II, we know that the mobility of nodes is much slower than the transmission of packets under slow mobility model. Hence, in a time slot, the packets delivery can use multihop transmissions. Let h_B denote the number of hops bit B travels to destinations in the time slot. The Euclidean distance bit B travels in a time slot denoted by $L(B)$. And in hop h for $1 \leq h \leq h_B$, the packets transmission radius is α_B^h. Then, we have the following lemmas.

Lemma 5: For any vehicular mobility model, the following inequalities hold under the directional antenna model and the protocol model:

$$\sum_{B=1}^{B[T]}\sum_{h=1}^{h_B} 1 \leq n_s(p+1)WT, \quad (5)$$

$$\sum_{B=1}^{B[T]}\sum_{h=1}^{h_B} \frac{\Delta^2}{16}(\alpha_B^h)^2 \leq \frac{2WT}{\theta}. \quad (6)$$

Lemma 6: Under the conditions of the protocol model and DTDR model, we consider the two-dimensional random walk slow mobility model, when the packets are directly transmitted from source vehicle to their destinations, then

$$E[\lambda^d(T)] \leq 5k\log(\theta n_s p)\left(\frac{8\pi n_s}{\Delta(1-\theta/2\pi)}\right)^{\frac{2}{3}} WT(p(p+1))^{\frac{1}{3}} + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_s p).$$

Proof: Using Cauchy-Schwarz inequality and inequality (4) and (5), we have

$$\left(\sum_{B=1}^{B[T]}\sum_{h=1}^{h_B}\alpha_B^h\right)^2 \le \left(\sum_{B=1}^{B[T]}\sum_{h=1}^{h_B}1\right)\left(\sum_{B=1}^{B[T]}\sum_{h=1}^{h_B}(\alpha_B^h)^2\right)$$
$$\le n_s(p+1)WT\sum_{B=1}^{B[T]}\sum_{h=1}^{h_B}(\alpha_B^h)^2$$
$$\le n_s(p+1)WT\frac{32WT}{\theta\Delta^2},$$

And $\sum_{h=1}^{h_B}\alpha_B^h \ge L(B)$, we have

$$\frac{4\sqrt{2}WT}{\Delta}\sqrt{\frac{n_s(p+1)}{\theta}} \ge \sum_{B=1}^{B[T]}L(B).$$

Similar to Lemma 3, we have

$$E[B[T]] = E\left[\sum_{B=1}^{B[T]}1_{\alpha_B\le L}\right] + E\left[\sum_{B=1}^{B[T]}1_{\alpha_B> L}\right]$$
$$\le \theta^2L^2Tn_spW + E\left[\sum_{B=1}^{B[T]}1_{\alpha_B> L}\right].$$

applying the Jensen's inequality, we obtain that

$$E\left[\frac{4\sqrt{2}WT}{\Delta}\sqrt{\frac{n_s(p+1)}{\theta}}\right] \ge E\left[\sum_{B=1}^{B[T]}L(B)\right]$$
$$\ge LE\left[\sum_{B=1}^{B[T]}1_{L(B)>L}\right]$$
$$\ge L(E[B[T]] - \theta^2L^2Tn_spW),$$

let $L=\sqrt{\frac{E[B[T]]}{2\pi\theta WTn_sp}}$, we obtain that

$$\left(\frac{8\pi n_s}{\Delta(1-\theta/2\pi)}\right)^{\frac{2}{3}} WT\,(p(p+1))^{\frac{1}{3}} \ge E[B[T]],$$

submitting to Lemma 2, we can finally have

$$E[\lambda^d(T)] \le 5k\log(\theta n_sp)\left(\frac{8\pi n_s}{\Delta(1-\theta/2\pi)}\right)^{\frac{2}{3}} WT(p(p+1))^{\frac{1}{3}} + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_sp).$$ □

Lemma 7: Under the conditions of the protocol model and DTDR model, we consider the two-dimensional random walk slow mobility model, when the packets have to be transmitted from relays to their destinations with a delay constraint D, then we have

$$E[\lambda^r(T)] \le 5k\log(\theta n_sp)\left(\frac{8\pi n_s(p+1)}{\Delta(1-\theta/2\pi)}\right)^{\frac{2}{3}} WT(Dp)^{\frac{1}{3}} + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_sp).$$

Proof: The proof process is similar with the packets transmitted from relays to destinations in the two-dimensional random walk fast mobility model. □

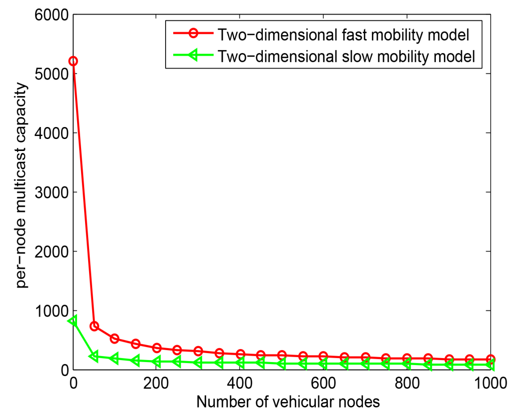

FIGURE 7. The multicast capacity of per-node under the 2D fast mobility model vs. the multicast capacity of per-node under the 2D slow mobility model.

Combining Lemma 6 and Lemma 7, we can obtain the following Theorem.

Theorem 2: Under the two-dimensional random walk slow mobility model, the aggregate multicast capacity of VANETs with directional antennas and a delay constraint D is

$$\lambda(n) = O\left(\log p\log(\theta n_sp)(\frac{n_s(p+1)}{1-\theta/2\pi})^{\frac{2}{3}}(p^{\frac{1}{3}}+(Dp)^{\frac{1}{3}})\right).$$

Remark 2: Since we know that $n = n_s(p+1)$ and the throughput capacity by using relays dominates the throughput by direct transmissions, the throughput capacity can be further simplified as $O\left(\log p\log(\theta n_sp)(\frac{n}{1-\theta/2\pi})^{\frac{2}{3}}(Dp)^{\frac{1}{3}}\right)$. Hence, under the slow mobility model, the maximum throughput per-node is $O(\frac{\log p\log(\theta n_sp)}{(1-\theta/2\pi)^{\frac{2}{3}}}\sqrt[3]{\frac{Dp}{n}})$. As shown in Fig. 7, it compares the multicast capacity in Theorem 1 with the result in Theorem 2, which indicates the multicast capacity of the two-dimensional slow mobility becomes smaller than the two-dimensional fast mobility with the same delay constraint and the directional beamwidth. This can be explained as follows. Assume that the speed of a vehicle in fast mobility is V_f and the speed of a vehicle in slow mobility is V_s, $0 < V_s \ll V_f$. For the same transmission radius, the time required for the vehicle with the speed of V_f moving to the receiver and transmitting the packets is much smaller than a given delay D. In this case, the multicast capacity of a vehicle is denoted by $\lambda_f(n)$. For the slow mobility model, the time required for the vehicle with the speed of V_s is much larger than a given delay D. Hence, some packets can't be delivered to the destinations within the delay D. In this case, the multicast capacity is denoted by $\lambda_s(n)$. When the speed of a vehicle is $V_s < V < V_f$, the multicast capacity of per-node is between $\lambda_f(n)$ and $\lambda_s(n)$, as shown in Fig. 8.

C. ONE-DIMENSIONAL RANDOM WALK FAST MOBILITY MODEL

In this section, we will study the one-dimensional random walk mobility model with fast mobiles. We first give two

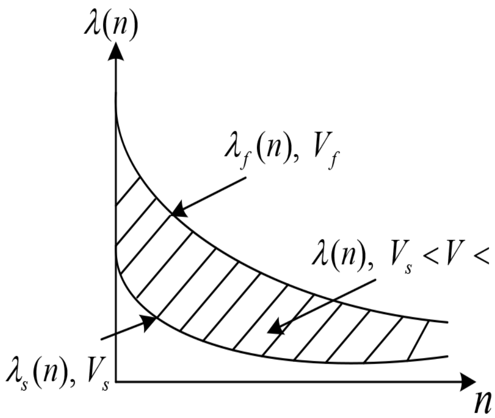

FIGURE 8. The multicast capacity $\lambda_f(n)$ of a vehicle when the speed is V_f vs. the multicast capacity $\lambda_s(n)$ of a vehicle when the speed is V_s.

lemmas on the direct transmission and the packets transmitted by relays. Then, we will obtain the maximum throughput capacity with the delay constraint. Furthermore, we find the throughput capacity in this case is larger than the two-dimensional fast mobility model.

Lemma 8: Under the conditions of the protocol model and DTDR model, we consider the one-dimensional random walk fast mobility model, when the packets are directly transmitted from sources to their destinations, then we have

$$E[\lambda^d(T)] \leq 5k\log(\theta n_s p)WT\theta^{\frac{1}{3}}\left(\frac{2\sqrt{2}\pi n_s p}{\Delta(1-\theta/2\pi)}\right)^{\frac{2}{3}} + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_s p).$$

Proof: In this case, each node moves along a straight line. If the mobile trajectory of sources and destinations are vertical to each other, then $D(s_i, t) < L$ holds only if the two vehicles are in the same square with side length $2L$ at some time slot t. Thus, $2L < 1$, we can have

$$P(D(s_i,t) \leq L) \leq 1 - (1 - \frac{\theta^2 4L^2}{4\pi^2 S^2})^{\frac{33}{10}S^2 p}.$$

If the mobile trajectory of sources and destinations are parallel to each other, we have

$$P(D(s_i,t) \leq L) \leq 1 - (1 - \frac{\theta^2 2L}{4\pi^2 S})^{\frac{33}{10}Sp}.$$

Thus, we have assumed that sources and destinations are the same type nodes in Section II, for $L \leq 1/2$, we conclude that

$$P(D(s_i,t) \leq L) \leq 1 - (1 - \frac{\theta^2 2L}{4\pi^2 S})^{\frac{33}{10}Sp} \leq \frac{\theta^2 Lp}{2}.$$

which implies

$$E\left[\sum_{t=1}^{T}\sum_{i=1}^{n_s} 1_{D(s_i,t)\leq L,\text{covered by } s_i}\right] \leq \frac{\theta^2 Lp}{2}n_s T,$$

we further obtain,

$$E[B[T]] = E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B \leq L}\right] + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right] \leq \frac{\theta^2 Lp}{2}n_s TW + E\left[\sum_{B=1}^{B[T]} 1_{\alpha_B > L}\right].$$

Then, using Cauchy-Schwarz inequality, we have

$$\left(\sum_{B=1}^{B[T]}\alpha_B\right)^2 \leq \left(\sum_{B=1}^{B[T]}1\right)\left(\sum_{B=1}^{B[T]}(\alpha_B)^2\right) \leq B[T]\frac{8WT}{\theta\Delta^2},$$

and applying Jensen's inequality,

$$\sqrt{\frac{8WT}{\theta\Delta^2}}\sqrt{E[B[T]]} \geq E\left[\sqrt{\frac{8WT}{\theta\Delta^2}B[T]}\right] \geq E\left[\sum_{B=1}^{B[T]}\alpha_B\right] \geq LE\left[\sum_{B=1}^{B[T]}1_{\alpha_B > L}\right] \geq L(E[B[T]] - \frac{\theta^2 Lp}{2}n_s TW),$$

let $L = \frac{E[B[T]]}{\pi\theta n_s pTW}$, we have

$$WT\theta^{\frac{1}{3}}\left(\frac{2\sqrt{2}\pi n_s p}{\Delta(1-\theta/2\pi)}\right)^{\frac{2}{3}} \geq E[B[T]].$$

Finally, we obtain that

$$E[\lambda^d(T)] \leq 5k\log(\theta n_s p)WT\theta^{\frac{1}{3}}\left(\frac{2\sqrt{2}\pi n_s p}{\Delta(1-\theta/2\pi)}\right)^{\frac{2}{3}} + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_s p). \quad \square$$

Lemma 9: Under the conditions of the protocol model and DTDR model, we consider the one-dimensional random walk fast mobility model, when the packets have to be transmitted from relays to their destinations with a delay constraint D, then we have

$$E[\lambda^r(T)] \leq 5k\log(\theta n_s p)WT\theta^{\frac{1}{3}}\left(\frac{2\sqrt{2}\pi n_s p(p+1)D}{\Delta(1-\theta/2\pi)}\right)^{\frac{2}{3}} + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_s p).$$

Proof: The proof of this Lemma is similar to Lemma 4 and Lemma 8. $\square$

Theorem 3: Under the one-dimensional random walk fast mobility model, the aggregate multicast capacity of

VANETs with directional antennas and a delay constraint D is

$$\lambda(n) = O\left(\frac{\log p\log(\theta n_s p)\theta^{\frac{1}{3}}}{(1-\theta/2\pi)^{\frac{2}{3}}}\left((n_s p)^{\frac{2}{3}}+(n_s p(p+1)D)^{\frac{2}{3}}\right)\right).$$

Remark 3: Since the throughput by using relays dominates the throughput by direct transmissions, the aggregate throughput capacity under the one-dimensional fast mobility model can be further simplified as $O\left(\frac{\log p\log(\theta n_s p)\theta^{\frac{1}{3}}}{(1-\theta/2\pi)^{\frac{2}{3}}}(npD)^{\frac{2}{3}}\right)$. And the maximum multicast throughput capacity of per-node is $O\left(\frac{\log p\log(\theta n_s p)}{(1-\theta/2\pi)^{\frac{2}{3}}}\sqrt[3]{\frac{\theta p^2 D^2}{n}}\right)$.

As shown in Fig. 9, it compares the multicast capacity in Theorem 1 with the numerical result in Theorem 3, which indicates the multicast throughput capacity in the one-dimensional fast mobility model is larger than the two-dimensional fast mobility model with the same delay constraint and the directional beamwidth. In the order sense, the maximum multicast capacity of per-node in the two-dimensional fast model and the one-dimensional fast model is $O(\sqrt{\frac{Dp}{n}})$ and $O(\sqrt[3]{\frac{D^2p^2}{n}})$, respectively. Both of them are nondecreasing functions, which means the multicast capacity is larger in the one-dimensional fast model under the given delay D. Moreover, for the one-dimensional random walk mobility network, the location of destinations that the source can choose becomes less compared with the two-dimensional mobility network. In other words, the probability that the vehicle selects the destination locations in the next time slot becomes larger in the one-dimensional fast mobility model than the two-dimensional fast mobility model. In brief, the figure suggests that our theoretical results coincide with the intuitive thinking accurately.

D. ONE-DIMENSIONAL RANDOM WALK SLOW MOBILITY MODEL

In this section, we will investigate multicast capacity of the one-dimensional random walk slow mobility model. Similar to above deducing process, we will give two lemmas on the multicast throughput capacity under the conditions that packets transmitted from the source to destinations directly and from relays to destinations, respectively. Then, we will derive the maximum aggregate multicast throughput capacity of a VANET with a delay constraint. The multicast capacity in this case becomes comparatively smaller as a result of the low rate of the packets transmission.

Lemma 10: Under the conditions of the protocol model and DTDR model, we consider the one-dimensional random walk slow mobility model, when the packets are directly transmitted from sources to their destinations, then we have

$$E[\lambda^d(T)] \leq 5k\log(\theta n_s p)WT(\theta(p+1))^{\frac{1}{4}}\left(\frac{4\sqrt{2}\pi p}{\Delta(1-\theta/2\pi)}\right)^{\frac{1}{2}}n_s^{\frac{3}{4}} + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_s p).$$

Proof: Recall that $L(B)$ denotes the Euclidean distance that bit B travels in time slot t and α_B^h denotes the the packets transmission radius in hop h. Assume that the mobile trajectory of sources and destinations are parallel to each other, which is same as the one-dimensional fast mobility model. Then, we can obtain that

$$P(L(B)\leq L) \leq 1-(1-\frac{\theta^2 2L}{4\pi^2 S})^{\frac{33}{10}Sp} \leq \frac{\theta^2 Lp}{2}.$$

Using Cauchy-Schwarz inequality and Lemma 5, we have

$$\left(\sum_{B=1}^{B[T]}\sum_{h=1}^{h_B}\alpha_B^h\right)^2 \leq \left(\sum_{B=1}^{B[T]}\sum_{h=1}^{h_B}1\right)\left(\sum_{B=1}^{B[T]}\sum_{h=1}^{h_B}(\alpha_B^h)^2\right) \leq n_s(p+1)WT\sum_{B=1}^{B[T]}\sum_{h=1}^{h_B}(\alpha_B^h)^2 \leq n_s(p+1)WT\frac{32WT}{\theta\Delta^2},$$

and $\sum_{h=1}^{h_B}\alpha_B^h \geq L(B)$, we have

$$\frac{4\sqrt{2}WT}{\Delta}\sqrt{\frac{n_s(p+1)}{\theta}} \geq \sum_{B=1}^{B[T]}L(B).$$

Similar with the proof of Lemma 6, we have

$$E[B[T]] = E\left[\sum_{B=1}^{B[T]}1_{\alpha_B\leq L}\right]+E\left[\sum_{B=1}^{B[T]}1_{\alpha_B>L}\right] \leq \frac{\theta^2 Lp}{2}n_sTW + E\left[\sum_{B=1}^{B[T]}1_{\alpha_B>L}\right].$$

Then, using Jensen's inequality, we have

$$E\left[\frac{4\sqrt{2}WT}{\Delta}\sqrt{\frac{n_s(p+1)}{\theta}}\right] \geq E\left[\sum_{B=1}^{B[T]}L(B)\right] \geq LE\left[\sum_{B=1}^{B[T]}1_{L(B)>L}\right] \geq L(E[B[T]]-\frac{\theta^2 Lp}{2}n_sTW),$$

by choosing $L=\frac{E[B[T]]}{\pi\theta n_s pTW}$, we have

$$WT(\theta(p+1))^{\frac{1}{4}}\left(\frac{4\sqrt{2}\pi p}{\Delta(1-\theta/2\pi)}\right)^{\frac{1}{2}}n_s^{\frac{3}{4}} \geq E[B[T]].$$

By submitting to Lemma 2, we finally have

$$E[\lambda^d(T)] \leq 5k\log(\theta n_s p)WT(\theta(p+1))^{\frac{1}{4}}\left(\frac{4\sqrt{2}\pi p}{\Delta(1-\theta/2\pi)}\right)^{\frac{1}{2}}n_s^{\frac{3}{4}} + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_s p).$$

□

Lemma 11: Under the conditions of the protocol model and DTDR model, we consider the one-dimensional random walk slow mobility model, when the packets have to be transmitted

TABLE 3. **Comparing multicast capacity of per node with unicast capacity under different random walk mobility models.**

Random walk mobility model		θ, p	Multicast capacity vs. unicast capcity
two-dimensional	fast mobility	$\theta \to 2\pi, p \to 1$	$O(\frac{\log p \log(\theta n_s p)}{1-\theta/2\pi}\sqrt{\frac{pD}{n}}) \to O(\sqrt{\frac{D}{n}})$
	slow mobility		$O(\frac{\log p \log(\theta n_s p)}{(1-\theta/2\pi)^{\frac{2}{3}}}\sqrt[3]{\frac{Dp}{n}}) \to O(\sqrt[3]{\frac{D}{n}})$
one-dimensional	fast mobility	$\theta \to 2\pi, p \to 1$	$O(\frac{\log p \log(\theta n_s p)}{(1-\theta/2\pi)^{\frac{2}{3}}}\sqrt[3]{\frac{\theta p^2 D^2}{n}}) \to O(\sqrt[3]{\frac{D^2}{n}})$
	slow mobility		$O(\frac{\log p \log(\theta n_s p)}{(1-\theta/2\pi)^{\frac{1}{2}}}\sqrt[4]{\frac{\theta(p+1)^2 D^2}{n}}) \to O(\sqrt[4]{\frac{D^2}{n}})$

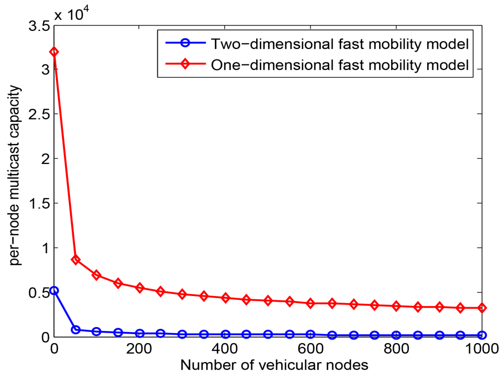

FIGURE 9. **The multicast capacity of per-node under the 2D fast mobility model vs. the multicast capacity of per-node under the 1D fast mobility model.**

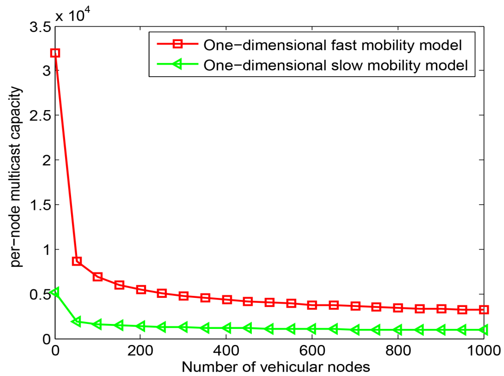

FIGURE 10. **The multicast capacity of per-node under the 1D fast mobility model vs. the multicast capacity of per-node under the 1D slow mobility model.**

from relays to their destinations with a delay constraint D, then we have

$$E[\lambda^r(T)] \leq 5k\log(\theta n_s p)WT\theta^{\frac{1}{4}}(\frac{4\sqrt{2\pi}Dp}{\Delta(1-\theta/2\pi)})^{\frac{1}{2}}(n_s(1+p))^{\frac{3}{4}} + \frac{16kWT}{\Delta^2}p(\log p)\log(\theta n_s p).$$

Proof: The proof of this Lemma is similar to the two-dimensional random walk slow mobility model. □

Theorem 4: Under the one-dimensional random walk slow mobility model, the aggregate multicast capacity of VANETs with directional antennas and a delay constraint D is

$$\lambda(n) = O\left(\frac{\log p \log(\theta n_s p)\theta^{\frac{1}{4}}}{(1-\theta/2\pi)^{\frac{1}{2}}}(D(p+1))^{\frac{1}{2}}(n_s(p+1))^{\frac{3}{4}}\right).$$

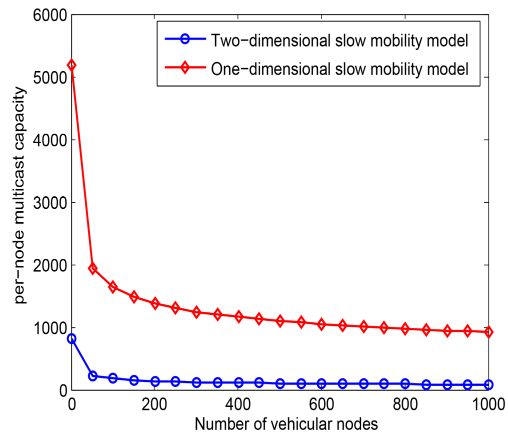

FIGURE 11. **The multicast capacity of per-node under the 2D slow mobility model vs. the multicast capacity of per-node under the 1D slow mobility model.**

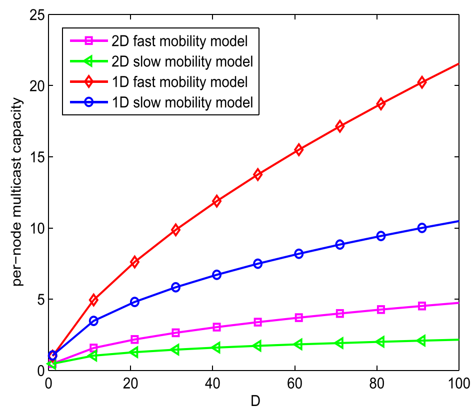

FIGURE 12. **The delay *D* vs. the multicast capacity of per-node under four different mobility models.**

Remark 4: In this case, the maximum throughput capacity of per-node is $O(\frac{\log p \log(\theta n_s p)}{(1-\theta/2\pi)^{\frac{1}{2}}}\sqrt[4]{\frac{\theta(p+1)^2D^2}{n}})$, which shows the throughput is smaller than the one-dimensional fast mobility with the same delay constraint.

Fig. 10 compares the multicast capacity in Theorem 3 with the result in Theorem 4. It also can be seen that the multicast capacity of one-dimensional fast mobility is larger than the result of the one-dimensional slow mobility. Fig. 11 compares the numerical result in Theorem 2 with Theorem 4, which also suggests that the multicast capacity of the one-dimensional slow mobility is larger than the result of the two-dimensional slow mobility model. In Fig. 12, we have plotted the numerical results from Theorem 1 to Theorem 4, to illustrate the scaling of per-node multicast capacity under the different transmission delay. We find that an increase in delay D leads to the increase in multicast capacity of per-node under four different mobility models. In particular, the vehicle obtain the largest multicast capacity under the one-dimensional fast mobility model when the transmission delay is identical.

In Table III, considering the beamwidth tends to 2π and the number of destinations tends to 1, we make a comparison between multicast capacity of per node with the directional antenna and the unicast capacity with the omnidirectional antenna under different conditions of random walk mobility model when the delay constraint is identical. We find our results unify the previous unicast throughput capacity in [13].

V. CONCLUSION

In this paper, we have investigated the multicast capacity of VANETs under the conditions of the directional antenna model and the delay constraint. We first studied the multicast capacity of VANETs for the two-dimensional random walk mobility model with two mobile time scales. Based on the results of the two-dimensional mobility model, we derived the upper bound capacity under the one-dimensional random walk mobility. We found that the multicast capacity under the one-dimensional mobility model was enhanced compared with the two-dimensional model. Since the probability that the source vehicles can obtain more information about the location of destinations is larger in the one-dimensional mobility model. Furthermore, when θ tends to 2π and p tends to 1, the multicast capacity in our work asymptotically trends to the unicast capacity with the delay constraint under the assumption of the omnidirectional antenna with high probability.

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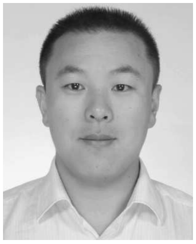

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