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On the Performance of Optical Wireless Links Over Random Foggy Channels

MAGED ABDULLAH ESMAIL[1,2], (Student Member, IEEE), HABIB FATHALLAH[1,2,3], (Senior Member, IEEE), AND MOHAMED-SLIM ALOUINI[4], (Fellow, IEEE)

[1]Electrical Engineering Department, King Saud University, Riyadh 11421, Saudi Arabia
[2]KACST Technology Innovation Center in Radio Frequency and Photonics center, Riyadh 11421, Saudi Arabia
[3]Computer Department, College of Science of Bizerte, University of Carthage, Carthage 1054, Tunisia
[4]Computer, Electrical and Mathematical Science and Engineering Division, King Abdullah University of Science and Technology, Thuwal 23955, Saudi Arabia

Corresponding author: M. A. Esmail (abdullahmaged@gmail.com)

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ABSTRACT Fog and dust are used to be considered as major performance degrading factors for free space optic (FSO) communication links. Despite the number of field measurements, performed in foggy environments during the last decades, most of the proposed channel attenuation models are deterministic, i.e., assumed the channel attenuation constant over time. Stochastic behavior of the channel is still under-studied. In this paper, we investigate the probabilistic behavior of the FSO channel in fog and develop a new statistical model for the signal attenuation. Moreover, we derive a probability distribution function (PDF) for the channel state. Using this PDF, we study the FSO system performance considering various metrics including average signal-to-noise ratio, average bit error rate, channel capacity, and probability. of outage Closed form expressions are derived for the average SNR and outage probability. We found acceptable performance with moderate and light fog. However, under thick and dense fog, the system performance poorly deteriorates. Finally, we derived closed form expressions for the average attenuation-distance product and the link availability that will potentially be very helpful for network design and planning.

INDEX TERMS FSO, Fog, PDF, channel state, channel capacity, attenuation distribution model, outage probability.

I. INTRODUCTION

In contrast to radio frequency (RF), free space optics (FSO) has access to a huge bandwidth which can be a promising solution to the RF spectrum scarcity problem [1]. As such, the interest in FSO channel modelling and performance study has increased rapidly during the last decade. This is due to the improvement in laser technology industry and the exponential growth in bandwidth demand [2], [3].

Fog in literature is considered as a limiting factor for FSO links that decreases the visibility to few meters, due to very high attenuation, that may exceed hundreds of dBs/km under heavy fog [3]–[5]. This high attenuation reduces the link availability. Most of the work on foggy channel modeling assumes the signal attenuation introduced by fog to be deterministic. For instance, a pioneering work in modeling fog channel was presented in [4]. This model relates the signal attenuation to the visibility range and signal wavelength. Subsequently, some other models have been proposed to improve this visibility based model [6].

To the best of our knowledge, the authors of [7] have been the first in literature to consider the randomness of the attenuation, hence considering the FSO link as a random channel. In that work, field measurements of continental fog were used to develop a probabilistic model for the channel where Wake by distribution was found to fit the observed measurements.

In [8], we developed a new deterministic channel model, for signal attenuation in fog that is based on the visibility range. This model outperformed a number of known models in literature. Moreover, we investigated the FSO system performance using a number of key performance metrics including signal-to-noise ratio (SNR), bit error rate (BER), received power, link length, capacity, reach distance, and detector type. Furthermore, we presented some initial results on foggy channel modeling as a random system. Note that so far, most of the work in the literature considered foggy channel as deterministic. In [8], we developed Johnson SB based probability density function (PDF) as a model for the random FSO channel and studied numerically the system performance in terms of average BER and channel capacity. Because Johnson SB distribution is a four parameters PDF, this made the channel study mathematically very complex and the derivation of useful closed-form expressions unfortunately very difficult.

In this work, we propose the Gamma distribution as a simplified but acceptable model for the signal attenuation in foggy weather. Then, using this model and assuming on-off keying (OOK) signaling, we evaluate and investigate the performance of FSO system using some key standard performance metrics including: (1) the average SNR, (2) average BER, (3) channel capacity, and (4) outage probability. Some closed-form expressions are derived and system performance analyzed using a number of selected numerical examples. Furthermore, in this paper we introduce and derive closed-form expressions for the average attenuation-distance product (ADP) and link availability, as useful and practical tools for network operators to properly design and plan FSO links operating in foggy environments.

The paper is organized as follow: Section II presents the derived signal attenuation model in fog. Section III studies the FSO system model and derives expressions for some performance metrics. Section IV analyzes and discusses the obtained results. In Sections V and VI, we discuss some hints and thoughts about potential research improvements of our proposed model and conclude the paper, respectively.

II. FOG ATTENUATION PROBABILITY DENSITY FUNCTION

In this section, we analyze statistically the signal attenuation in foggy weather in order to find a suitable PDF. To achieve this goal, a set of seven measurements obtained from the literature [9]–[12] are used. The measurements were taken at different wavelengths in the near infrared band and cover different types of fog. Our ultimate objective is to develop a statistical model for the channel, that enables the prediction of the signal attenuation in foggy conditions, and provide future telecommunication operators a new planning tool to design and engineer their links.

For this purpose, we use the Kolmogorov-Smirnov (K-S) goodness of fit test to examine and compare different statistical distributions. The K-S test compares the experimental data's cumulative distribution function (CDF) with the theoretical CDF [13]. We have examined high number of potential PDF functions including non-negative, bounded, and unbounded continuous distributions. We identified Johnson SB distribution that fits well the signal attenuation of all measurements with high significance level and reported this in [8]. The Johnson SB distribution is given by

$$f(\alpha)=\frac{\delta}{\lambda\sqrt{2\pi}\,c(1-c)}\exp\left(-\frac{1}{2}\left(\eta+\delta\ln\left(\frac{c}{1-c}\right)\right)^{2}\right), \quad (1)$$

where $c \equiv \alpha - \xi/\lambda$ [14], [15]. The parameters $\lambda > 0$, and ξ are referred as the scale and location parameters, respectively while η and $\delta > 0$ are both called shape parameters with the domain $\xi \leq \alpha \leq \xi+\lambda$. Although the Johnson SB distribution fits well the fog attenuation data, it is difficult to analyze the system performance mathematically using such distribution because (*i*) it has a complicated dependence on α and (*ii*) it depends on four different parameters. Therefore, in this work, we use instead the Gamma distribution that has a good fitting over the signal attenuation data as shown in Fig. 1(a) with significance level higher than 0.01. The Gamma distribution is given by

$$f(\alpha) = \frac{\alpha^{k-1}}{\beta^k\Gamma(k)}e^{-\alpha/\beta}, \quad 0 \leq \alpha < +\infty, \quad (2)$$

where $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ is the gamma function. The other two non-negative parameters, $k > 0$, $\beta > 0$, are the shape parameter, and the scale parameter, respectively. In comparison with the Johnson SB distribution, the Gamma PDF has a slightly lower fitting significance level but depends only on two parameters. Hence, this overcomes the analytical/computational intractability that comes with the Johnson SB distribution.

To simplify studying the system performance, the visibility range is divided into four regions according to the international visibility code: dense fog ($V < 50$ m), thick fog ($50 < V < 200$ m), moderate fog ($200 < V < 500$ m), and light fog ($V < 1000$ m) [16]. For each visibility range value, we calculated the average value of k and β as listed in Table I. We notice that as the fog density increases, the attenuation mean value ($k \times \beta$) in dB/km similarly increases. For example, the mean value for moderate fog is 66.2 dB/km whereas for thick fog is 138 dB/km. Fig. 1(a) shows the data histogram and also the fitting for both distributions for the different types of fog. We can clearly notice in this figure the good match between both distributions. Using K-S goodness of fit test, the Johnson SB distribution fits well the measurements with 0.2 significance level. The Gamma distribution shows the same significance level under dense fog and 0.1, 0.05,

TABLE 1. Gamma parameters' values based on visibility range for different types of FOG.

Fog type	Light	Moderate	Thick	Dense
Visibility range (m)	500-1000	200-500	50-200	0-50
Scale parameter (β)	13.12	12.06	23.00	11.91
Shape parameter (k)	2.32	5.49	6.00	36.05

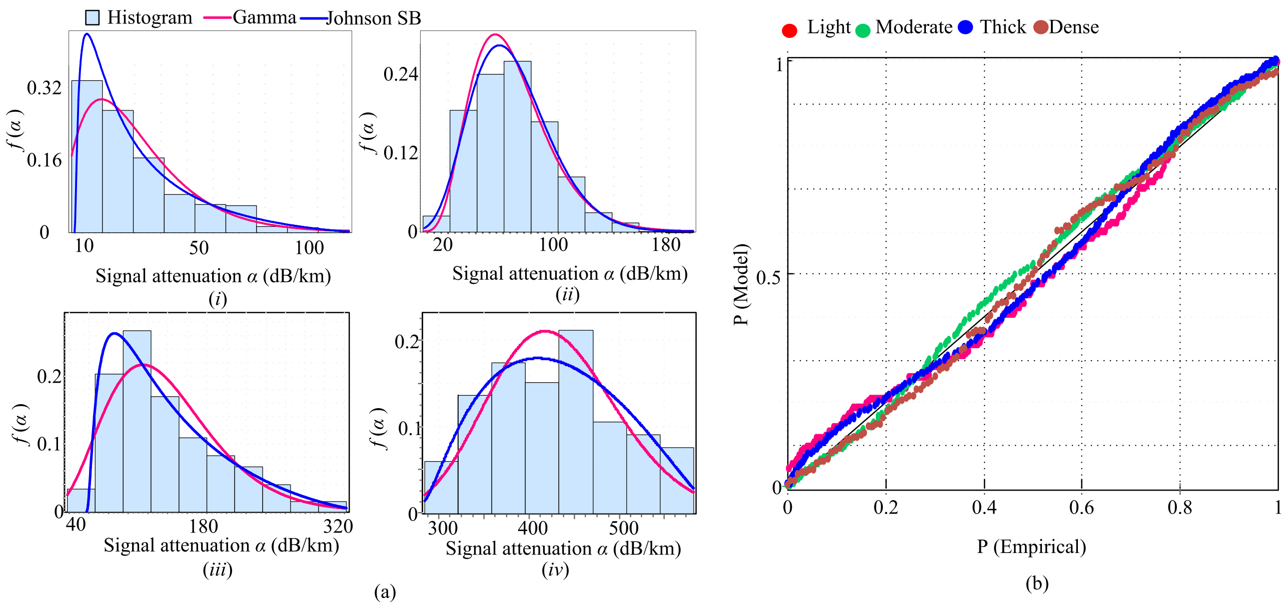

FIGURE 1. (a) Signal attenuation measurements fitting using Johnson SB and Gamma distributions for four types of fog: (*i*) light fog, (*ii*) moderate fog, (*iii*) thick fog, and (*iv*) dense fog, (b) P-P plot of the signal attenuation for attenuation measurements based on fog type.

and 0.01 under moderate, light, and thick fog, respectively. The significance level is the probability of rejecting the null hypothesis (data follows the specific distribution) given that it is true. The 0.05 and 0.01 (5% and 1%) values for significance level are typically used for most applications [14]. A significance level of 0.05 (0.01) implies that it is acceptable to have a 5% (1%) probability of incorrectly rejecting the null hypothesis.

The goodness of fit of a specific distribution to some measurements can also be validated graphically using the P-P plot. P-P plot is a visual/graphical method that compares the empirical CDF values against the theoretical CDF values [17]. It shows how well the proposed distribution fits the observed data. If the selected theoretical CDF is the correct model, the plot will be approximately diagonal line. In Fig. 1(b), we show the P-P plot for the proposed Gamma model with the measurements data based on different fog intensities. It can be seen that the plot follows the linear diagonal line with small deviation, reflecting the goodness of Gamma distribution, as a model for random signal attenuation in fog.

III. FSO SYSTEM MODEL

The FSO system under study in this paper is assumed to use intensity modulation direct detection (IM/DD) technique. We use OOK modulation because its simplicity of implementation. The propagated signal is degraded by fog and corrupted in the receiver by additive white Gaussian (AWGN) noise. After direct detection, the received signal is expressed as [18]

$$y = h_a R x + n, \tag{3}$$

where h_a, R, x, and n are the random channel state, detector responsivity, binary transmitted signal intensity, and additive white Gaussian noise (AWGN), respectively. In general, the channel in FSO systems is subject to three main impairments: atmospheric loss, pointing errors, and atmospheric turbulence. In this paper, the atmospheric loss, h_a is raised by fog condition. The effect of atmospheric turbulence is negligible because fog and turbulence has a strong inverse correlation especially under strong turbulence [18]. Furthermore, we assume a good alignment so that the effects of pointing errors are neglected. Since FSO is a high speed transmission system with data rate in Gbps, we assume that the foggy channel is a slow-fading channel. The symbol duration (in nanoseconds) is very much smaller than the channel coherence time, which is found to be in milliseconds in FSO channels [1], [18].

The instantaneous received electrical SNR for OOK signaling and-slow fading channel is expressed by [18]

$$\gamma(h) = \frac{2P_t^2 R^2 h_a^2}{\sigma_n^2} = \gamma_o h_a^2, \tag{4}$$

where P_{t} is the optical average transmitted power, $\sigma_n^2$is the AWGN variance, and $\gamma_o = 2P_t^2R^2/\sigma_n^2$ is the Gaussian SNR.

From the Beer-Lambert law which describes the relation between the signal attenuation and the link length, the channel state in foggy conditions is given by [18]

$$h_a = \exp(-\alpha_p l), \tag{5}$$

where α_p is the attenuation coefficient in km^{-1} and l is the link length in km. The signal attenuation in dB/km is related to the attenuation coefficient by [19]

$$\alpha = 4.343\alpha_p \ (\mathrm{dB/km}). \tag{6}$$

From (5) and (6), the signal attenuation as a function of h_a can be expressed as

$$\alpha = -\frac{4.343}{l} \ln(h_a) \text{ (dB/km)}. \tag{7}$$

The atmospheric channel state distribution can be obtained from the signal attenuation by [20]

$$f_{h_a}(h) = \sum_i f_\alpha(\alpha) \left| \frac{d\alpha}{dh_a} \right| \Bigg|_{\alpha=\alpha_i}, \tag{8}$$

where

$$\left| \frac{dh_a}{d\alpha} \right| = 0.23\, l \exp(-0.23\alpha l) = \frac{l\, h_a}{4.343}. \tag{9}$$

The index i in (8) represents the number of solutions for the variable α over the interval of interest ($0 < h_a \leq 1$). In our case, we have only one solution represented by (7). Substituting (2), (7) and (9) in (8), the PDF of the foggy channel h_a is obtained

$$f_{h_a}(h_a) = \frac{z^k}{\Gamma(k)} \left[\ln\left(\frac{1}{h_a}\right) \right]^{k-1} h_a^{z-1}, \tag{10}$$

where $0 < h_a \leq 1$ and $z = 4.343/(\beta\, l)$.

A. AVERAGE SIGNAL-TO-NOISE RATIO

Signal-to-noise ratio is a common performance measure for communication system. For digital communications, the BER that is intimately related to the SNR, is the ultimate performance measure tool. Using (8) and (10), the probability distribution function of the SNR is found to be

$$f_\gamma(\gamma) = \frac{z^k}{2\Gamma(k)\sqrt{\gamma\gamma_o}} \left[\ln\left(\frac{1}{\sqrt{\gamma/\gamma_o}}\right) \right]^{k-1} \left(\sqrt{\frac{\gamma}{\gamma_o}}\right)^{z-1}, \tag{11}$$

where $0 < \gamma \leq \gamma_o$. The average SNR in slow-fading channels is expressed by [21]

$$\overline{\gamma} = \int_0^\infty \gamma\, f_\gamma(\gamma) d\gamma. \tag{12}$$

Thus substituting (11) in (12), the average SNR is given by

$$\overline{\gamma} = \frac{z^k}{2\Gamma(k)} \int_{0^+}^{\gamma_o} \left[\ln\left(\frac{1}{\sqrt{\gamma/\gamma_o}}\right) \right]^{k-1} \left(\sqrt{\frac{\gamma}{\gamma_o}}\right)^{z} d\gamma. \tag{13}$$

Let $x = \sqrt{\gamma/\gamma_o}$, then (13) can be rewritten as

$$\overline{\gamma} = \frac{\gamma_o z^k}{\Gamma(k)} \int_0^1 \left[\ln\left(\frac{1}{x}\right) \right]^{k-1} x^{z+1} dx. \tag{14}$$

Using the following identity [[22] - pp.552]

$$\int_0^1 \left(\ln \frac{1}{x} \right)^{\mu-1} x^{\nu-1} dx = \frac{1}{\nu^\mu} \Gamma(\mu), \tag{15}$$

the average SNR in (13) ends up with a closed form expression given by

$$\overline{\gamma} = \frac{\gamma_o}{(1 + 0.46\beta l)^k}. \tag{16}$$

The obtained average SNR depends mainly on the transmission link length which shows exponential decay as the link length increases. For long link length, the average SNR value approaches zero. Similarly, as fog density increases from light to dense, the signal attenuation increases, hence degrading the average SNR.

B. AVERAGE BIT-ERROR-RATE (BER)

The average BER is one of the most revealing performance criteria about the nature of the system behavior. The average BER for OOK modulation and equiprobable symbols is given by [23]

$$\overline{P_e} = \int_0^\infty f_{h_a}(h_a)\, Q\left(\sqrt{2\gamma_o h_a^2}\right) dh_a, \tag{17}$$

where $Q(.)$ is the Gaussian Q function. Substituting (10) in (17) yields

$$\overline{P_e} = \frac{z^k}{\Gamma(k)} \int_0^1 \left[\ln\left(\frac{1}{h_a}\right) \right]^{k-1} h_a^{z-1} Q\left(\sqrt{2\gamma_o h_a^2}\right) dh_a. \tag{18}$$

Using integration by parts, (18) can be simplified to yield

$$\overline{P_e} = Q\left(\sqrt{2\gamma_o}\right) + \frac{\sqrt{\gamma_o/\pi}}{\Gamma(k)} \int_0^1 \Gamma\left(k, z \ln \frac{1}{h_a}\right) e^{-\gamma_o h_a^2} dh_a, \tag{19}$$

where $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$ is the incomplete Gamma function. The first term in (19) approaches zero and can be neglected because the Gaussian SNR is high.

C. ERGODIC CHANNEL CAPACITY

The ergodic channel capacity of FSO system is a random variable that depends on the SNR which is mathematically given as [24]

$$\overline{C} = \int_0^\infty \log_2(1 + \gamma_o h_a^2) f_{h_a}(h_a) dh_a. \tag{20}$$

Substituting (11) in (20) yields

$$\overline{C} = \frac{z^k}{\Gamma(k)} \int_0^1 \log_2(1 + \gamma_o h_a^2) \left[\ln\left(\frac{1}{h_a}\right) \right]^{k-1} h_a^{z-1} dh, \tag{21}$$

where the integral in (21) can be computed numerically. In the next section, we see that for $P_t = 22$ dBm, $\overline{C} \geq 10$ b/s/Hz can be obtained under thick fog, moderate fog, and light fog but constrained to different link lengths: 200 m, 500 m, and 1 km, respectively.

D. OUTAGE PROBABILITY

Outage probability is the probability that the average SNR drops below a specific threshold value, γ_{th}. It is expressed mathematically by [20]

$$P_{\text{out}} = P(\gamma \leq \gamma_{th}). \tag{22}$$

Substituting (4) in (22) yields

$$P_{\text{out}} = P(\gamma_o h_a^2 \leq \gamma_{th}) = P(h_a \leq h_o), \tag{23}$$

where $h_o = \sqrt{\gamma_{th}/\gamma_o}$. Hence, the probability of outage is the CDF of the channel state. Substituting (10) that represents the channel state in (23) yields

$$P_{\text{out}} = \frac{z^k}{\Gamma(k)} \int_0^{h_o} \left[\ln\left(\frac{1}{h_a}\right)\right]^{k-1} h_a^{z-1}\, dh_a. \tag{24}$$

Let $x = \ln(1/h_a)$, then (24) can be rewritten as

$$P_{\text{out}} = \frac{z^k}{\Gamma(k)} \int_{1/\sqrt{h_o}}^{\infty} x^{k-1} e^{-zx}\, dx. \tag{25}$$

Then using the following identity [[22] -pp.348]

$$\int_u^{\infty} x^{v-1} e^{\mu x} dx = \mu^{-v}\Gamma(v, \mu u), \quad u > 0,\ \mu > 0, \tag{26}$$

we obtain an expression for the probability of outage in FSO channels under fog conditions as follows

$$P_{\text{out}} = \frac{1}{\Gamma(k)} \Gamma\left(k, \frac{4.343}{\beta l} \ln(\sqrt{\gamma_o/\gamma_{th}})\right). \tag{27}$$

The outage probability in (27) increases as the SNR threshold, γ_{th} and/or the link length increases as expected. When $\gamma_{th} \to \infty$, $P_{\text{out}} \to 1$.

E. AVERAGE ATTENUATION-DISTANCE PRODUCT

In this subsection, we define the term average attenuation-distance product (ADP). It is introduced to define the limit of an FSO system. A network planner can consider the tradeoff between the average attenuation introduced by fog and the maximum reach distance between the FSO nodes. A high product value means more challenging channel which may lead to a link drop. The average ADP is given by

$$\overline{\text{ADP}} = \text{E}\{\alpha \times l\} \text{ (dB)}, \tag{28}$$

where E{.} is the statistical mean value. Since α is a random variable, we use (2) to derive the average ADP which is given by

$$\overline{\text{ADP}} = l\beta k \text{ (dB)}, \tag{29}$$

which depends on the average signal attenuation (βk) and the transmission link length.

F. LINK AVAILABILITY

The link availability can be defined as the percentage of time, when the data transmission bit rate is more than its required value. Equivalently, the availability can be defined as the probability that additional power losses caused by atmospheric effects are less than the link margin. The link margin is expressed as

$$LM(dB) = Pt(dBm) - Ps(dBm) - Lp(dB), \tag{30}$$

where P_s, and Lp are the receiver sensitivity, and the propagation and system loss, respectively. The propagation and system loss is given by

$$LP = 10 \log\left[(D_2/(D_1 + \theta_t L))^2 \tau_t \tau_r\right], \text{ (dB)} \tag{31}$$

where D_1 and D_2 are the transmitter and receiver aperture diameter, θ_t is the full transmitting divergence angle, L is the link length, τ_t is the transmitter optical efficiency, τ_r is the receiver optical efficiency.

Using the probability density function of the signal attenuation, the availability percentage can be given by [25]

$$A_{av}\% = 100 \times \int_0^{\alpha_n} f(\alpha) d\alpha, \tag{32}$$

where $\alpha_n = LM/L$ (dB/km) is the normalized attenuation parameter. Substituting (2) in (32), and using the following identity [[22], pp. 348]

$$\int_0^u x^{v-1} e^{-\mu x} dx = \mu^{-v} (\Gamma(v) - \Gamma(v, \mu v)), \quad [\text{Re}(v) > 0], \tag{33}$$

we obtain the following closed form expression for the link availability

$$A_{av}\% = 100 \times \left[1 - \frac{\Gamma(k, \alpha_n/\beta)}{\Gamma(k)}\right], \tag{34}$$

IV. PERFORMANCE ANALYSIS AND RESULTS

In this section, we study the FSO system performance using the obtained expressions in Section III. We assume a receiver with $\sigma_n = 10^{-7}$A/Hz [15] and three FSO links' lengths: 200 m, 500 m, and 1 km. Such short range FSO links, in the order of hundreds of meters, can play a major role as a high bandwidth backhaul solution in wireless cellular cells that have similar sizes. Such cells have been exploited in current wireless networks and this cell size will reduce more in next generation 5G wireless communication systems [26]. Taking into consideration this fact, FSO is expected to be a promising technology and plays an important role in 4G/5G wireless networks.

A. SNR ANALYSIS

Fig. 2(a) illustrates the average SNR as a function of the average transmitted power. For each fog type, we plot three curves that correspond to the three different link lengths.

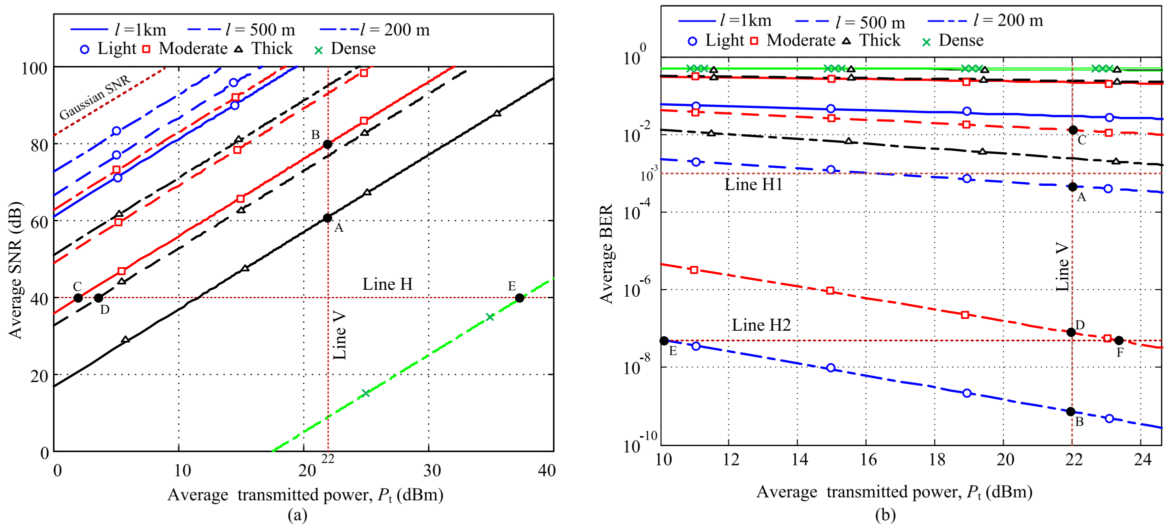

FIGURE 2. **(a) Average SNR versus the average transmitted power for different scenarios of link length and fog density, (b) Average BER versus the average transmitted power for different scenarios of link length and fog density.**

The maximum achieved average SNR is limited by the Gaussian SNR (i.e. no fog). As the link length increases, the average SNR decreases. Similarly, the average SNR decreases with the increase in fog density. However, increasing the transmitted power improves the average SNR. In Fig. 2(a), each curve is the result of subtracting the term $10k\log(1+0.46\beta l)$ from the Gaussian SNR.

If we fix the transmitted power to 22 dBm which is applied today in commercial FSO products (Line V in Fig. 2(a)) and the link distance to 1 km, we see that the FSO system can achieve average SNR=60 dB under thick fog (Point A) and 80 dB under moderate fog (Point B). For light fog, the achieved average SNR=105 dB and for dense fog, this value is less than 0 dB. This variability in the average SNR requests applying adaptive modulation techniques and adaptive power control as the fog changes from a type to another in order to improve the system performance.

To achieve an average SNR=40 dB (line H in Fig. 2(a)), we need to use shorter links as the fog density increases or use higher transmission power or both. For example, this SNR value can be achieved under moderate fog with 1 km link length (Point C) with 2.5 dBm transmitted power. For thick fog, the distance should decrease to 500 m and the needed transmitted power is 2.5 dBm (Point D). For dense fog, the distance decreased further to 200 m with higher transmission power of 36 dBm (Point E). A system under light fog needs less than 0 dBm transmission power to achieve 1 km distance.

B. BER ANALYSIS

In Fig. 2(b) we show the average BER versus the average transmitted power. We notice that under dense fog, the system performance is very poor with $\overline{P_e} \approx 0.5$ even with short link length. For thick fog, the performance is better than dense fog, but still $\overline{P_e} > 10^{-3}$ even for short link length, $l = 200$ m, and with $P_t \leq 24$ dBm. For moderate fog with $l = 200$ m, we achieve $\overline{P_e} < 10^{-5}$ for low transmitted power. Under light fog, we achieve $\overline{P_e} < 10^{-3}$ with $l = 500$ m and $P_t > 17$ dBm (see Line H1). For $\overline{P_e} < 10^{-3}$ which is considered as error free transmission [27] and with $P_t = 22$ dBm that corresponds to the transmitter power used in current commercial products, only light fog with $l = 500$ m and moderate fog with $l = 200$ m can be achieved (see Line H1 and Line V).

The results assure that using short links in foggy channel has high impact on the system performance. For example, using 200 m link length instead of 500 m, we are able to improve the average BER 6 orders of magnitude for light fog (from 4.6×10^{-4} to 7×10^{-10}) and similarly for moderate fog (from 1.2×10^{-2} to 7.7×10^{-8}) as shown in Fig. 2(b) (Points A-D). To achieve the same average BER, $\overline{P_e} = 5.3 \times 10^{-8}$ of light and moderate fog for 200 m link length, 13.5 dBm more transmitted power is needed (See line H2 and points E-F). However, under longer links, much more power is needed because the signal attenuation is reciprocally related to the link length.

C. ERGODIC CHANNEL CAPACITY ANALYSIS

In Fig. 3(a) we illustrate the ergodic channel capacity under foggy channel conditions. Again, the ergodic channel capacity is very low under dense fog, $\overline{C} \approx 0$b/s/Hz, resulting in very poor capacity. The capacity improves as the fog severity decreases and/or the transmitted power increases. To achieve $\overline{C} \approx 10$b/s/Hz (Line H), we need $P_t = 0$ dBm for light fog with 1 km link length (Point A). Under moderate fog, $P_t = 6$ dBm is needed to achieve the same capacity but for shorter link, $l = 500$ m (Point B). For thick fog, $P_t = 1.5$ dBm is enough to achieve the mentioned capacity but with shorter link length, $l = 200$ m (point C).

To understand better the effect of link length, we discuss the system performance using $P_t = 22$ dBm

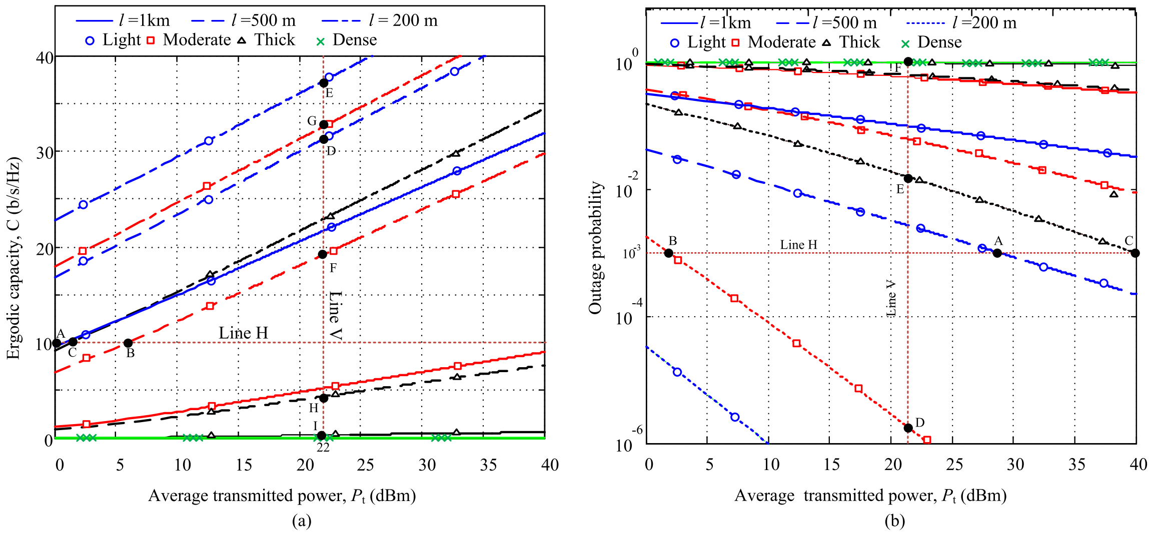

FIGURE 3. **(a) Ergodic channel capacity versus the average transmitted power for different scenarios of link length and fog density, (b) Outage probability versus the average transmitted power for different scenarios of link length and fog density, γ_{th} = 6 dB.**

which is the power used in nowadays commercial FSO transmitters. As you see in Fig. 3(a) (Line V), using 200 m instead of 500 m, the channel capacity increased 6 b/s/Hz from 31.3 b/s/Hz to 37.3 b/s/Hz for light fog and 13.3 b/s/Hz from 19.3 b/s/Hz to 32.6 b/s/Hz for moderate fog (See points D-G). For 500 m link length, the achieved capacity is 31.3 b/s/Hz, 19.3 b/s/Hz, 4.4 b/s/Hz, and 0 b/s/Hz for light, moderate, thick and dense fog, respectively (See points D, F, H, I). Again, this high variation in channel performance recalls for the use of adaptive techniques to optimize the exploitation of the channel capacity.

D. OUTAGE PROBABILITY ANALYSIS

Fig. 3(b) illustrates the outage probability versus the average transmitted power for γ_{th} = 6 dB. In general, the obtained results show that the performance of the FSO system is very poor under dense fog with $P_{out} \approx 1$ regardless of the link length. For thick fog, the outage probability improved based on the transmission distance. However, the obtained outage probability is less than 10^{-3} for P_t <40 dBm.

Using shorter link length, the outage probability improves. For $P_{out} \leq 10^{-3}$ (Line H in Fig. 3(b)), we need to transmit 28.7 dBm average power under light fog over 500 m link length (Point A). To achieve the same outage probability under moderate fog, 1.7 dBm average transmitted power is needed over 200 m link length (Point B). For thick fog, 40 dBm average transmitted power is needed over 200 m link length (Point C). Therefore, we need about 38.3 dBm additional power to keep the same probability of outage when the fog changes from moderate to thick over 200 m transmission link.

If we fix the link length to 200 m and the transmitted power to 22 dBm, then the system can achieve 7.8×10^{-9}, 1.6×10^{-6}, 1.1×10^{-2}, and 0.97 average outage probability for light fog, moderate fog, thick fog and dense fog, respectively (See Line V and Points D, E, and F).

E. AVERAGE ATTENUATION-DISTANCE PRODUCT ANALYSIS

The average ADP analysis is shown in Fig. 4(a). The curves show the tradeoff between the signal attenuation and distance between two nodes of an FSO system. The average ADP increases with link length and with fog density. It is constrained by the system link margin. For example, in nowadays commercial products, the average transmitted power and receiver sensitivity are 22 dBm and -34 dBm, respectively [28], [29]. This means the power budget is 56 dB neglecting the other impairments such as geometric loss. Under these specifications, the network operator can plan his link to work over 1.850 km, 840 m, 400 m, and 120 m maximum distance under light fog, moderate fog, thick, and dense fog, respectively.

F. LINK AVAILABILITY

To analyze the system availability, we need to calculate the link margin. Consider the following parameters for a practical FSO system [8]: $D_1 = 0.08$ m, $D_2 = 0.2$ m, $\theta_t = 2$ mrad, $\tau_t = 0.75$, $\tau_r = 0.75$, Ps $= -34$ dBm, and $P_t = 30$ dBm. The system availability under different fog conditions is shown in Fig. 4(b). We notice that as the normalized link margin increases (i.e. the link length decrease), the link availability improves except for dense fog where the availability is almost zero. For 1 km link length, we achieve 76% availability (respectively 18.68% and 1% availability) under light fog (respectively moderate and thick fog). If the link length decreased to 500 m, we achieve 98.97% availability (respectively 84.24% and 22.7% availability) under light fog (respectively moderate and thick fog). For short link with 200 m

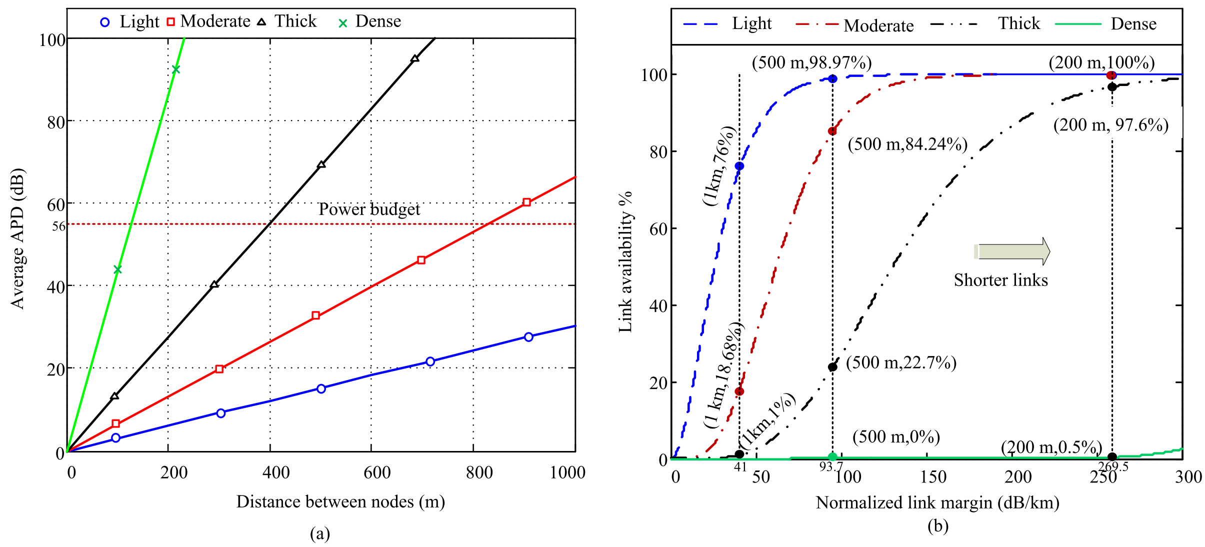

FIGURE 4. **(a) Average attenuation-distance product vs the link length for different fog types, with $P_t = 22$ dBm and $Ps = -34$ dBm, (b) FSO link availability versus the normalized link margin using different link lengths with $P_t = 30$ dBm. The dots show the link length and associated availability (L, A_{av}%) at specific α_n.**

length, we achieve 100% availability (respectively 100% and 97.59% availability) under light fog (respectively moderate and thick fog). Hence high availability can be achieved if a short link is used with high transmitted power.

V. DISCUSSION AND RESEARCH PERSPECTIVES

In this paper, we have proposed a probabilistic channel model based on the signal attenuation measurements in foggy channel. Recall that this model is obtained empirically using field measurements reported in literature. Moreover, we have derived PDF for the channel state and SNR. Using these PDF, we derived closed form expressions for several performance metrics that can help the network operator in engineering and analyzing the FSO link. In this section, we discuss other channel random impairments that can be introduced by fog. Developing empirical/analytical models of these impairments can straightforwardly serve as potential improvements for our signal attenuation model. Each of these impairments requires more special investigation, in order to develop a more complete model, for the foggy channel.

A. FORWARD SCATTERING PARAMETER

this parameter determines how much light diverges from the receiver as the beam propagates through the atmosphere as reported in [30]. The authors demonstrated that this effect increases as the link length increases so that the beam becomes highly divergent and distorted at the receiver plane. The distribution of power at that plane may become very variable in time. The receiver aperture covering only a small surface at that plane may sometimes miss to receive any power because of total divergence. In this situation that is also random, the effect of forward scattering may exceed that of attenuation. In dense fog, attenuation dominates the forward scattering. However, in light fog, forward scattering becomes the dominator effect. It is reasonable to expect that our attenuation PDF we have proposed in Section III, may be improved by integrating the forward scattering parameter effect to get more general model.

B. POLARIZATION DISTORTION

because of the capacity limitation in IM/DD FSO systems, researchers have theoretically and experimentally proposed polarization multiplexing of OOK and coherent FSO systems which are able to provide terabit data rate [31]. The transmission performance is sensitive to polarization distortion observed by the signal when it propagates through the foggy channel. FSO based coherent systems are sensitive to phase and polarization distortion. Investigating the polarization distortion is important in order to make appropriate characterization of its impact in foggy environment. At very high transmission speed, exceeding 10 Gbps, it is expected that PMD may affect most of the performance parameters evaluated/derived in the previous sections: SNR, BER, channel capacity, and Outage probability.

C. TIME DISPERSION

Fog induces multiple scatterings and medium refractive index dependency on the frequency, forcing the photons composing the data symbol to propagate through different trajectories and/or propagation velocities. As a consequence, the transmitted symbol spreads in time resulting in inter-symbol-interference (ISI) between successive symbols. This delay spread introduces a power penalty that degrades the system SNR and hence the overall data rate. Using numerical Mote-Carlo simulation, the authors in [32] obtained an impulse response and an RMS delay spread model under fog conditions. The results show that moderate and dense fog can introduce about 50 ps or more delay spread over 1 km link length. This delay implies negligible ISI for data rates up to 20 Gbps. But this quantity highly depends on the system geometry, e.g., field of view (FOV), source beam divergence, link length, etc. Therefore, in general, the foggy FSO link may suffer from ISI which in turn limits the system bandwidth.

D. PHASE DISTORTION

In coherent FSO systems, the information is encoded in amplitude and phase. In contrast to IM/DD systems, coherent FSO systems are very sensitive to phase variation. Multiple scattering due to fog can introduce imperfection in the propagating wave front leading to strong mismatch between the local oscillator in the receiver and the received signal. As a consequence, the system BER performance highly degrades. This random phase distortion also limits the achievable maximum modulation order (i.e. M) of the M-ary Quadratic Modulation Amplitude (M-QAM) to a low value in FSO compared to its counterpart in fiber optic communications.

E. SIGNAL WAVELENGTH DEPENDENCY

There is uncertainty about FSO signal attenuation dependency on the signal wavelength in the near infrared band. Some authors showed that the FSO signal attenuation depends on the signal wavelength where attenuation is reduced using longer wavelengths [33]. Others found that there is very little dependency, almost negligible [34]. Exploring this issue will help in improving channel modeling and also in selecting the desirable transmission wavelength. However, investigation of FSO performance in mid infrared band that has longer wavelengths in range of microns showed less attenuation than the near infrared band [35].

VI. CONCLUSION

FSO technology promises to help solving bandwidth scarcity in wireless networks. Nevertheless, in many regions around the globe, FSO is heavily affected by fog which severely reduces the FSO reliability. Fog is considered as a highly limiting factor for FSO technology spread. Predicting fog attenuation helps in designing and engineering FSO links. In this paper, we proposed a probabilistic signal attenuation model in foggy weather. A channel state model is derived and some useful closed form expressions were derived and used to investigate the system performance. Our results illustrate that FSO is a short range technology that can work over hundreds of meters' links because of the fog effect. FSO can potentially play an important role in cellular cells in 4G/5G networks as a backhaul solution. Such networks have small cells' sizes in the range of hundreds of meters and require high bandwidth backhaul links which can be offered by FSO.

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MAGED ABDULLAH ESMAIL received the B.E. degree in electronic engineering from Ibb University in 2006 and the M.Sc. degree (Hons.) in electrical engineering from KSU University in 2011, where he is currently pursuing the Ph.D. degree.

From 2009 to 2012, he joined the Prince Sultan Advanced Research Technologies Institute as a Graduate Researcher. From 2013, he joined KACST-TIC in radio frequency and photonics for the e-society as a Graduate Researcher. His research interests include fiber-optic communications, PON and long-reach PON, network management and protection, sensor networks, free space optics, and visible light communications.

HABIB FATHALLAH (S'96–M'01) received the B.S.E.E degree (Hons.) from the National Engineering School of Tunis in 1994 and the M.A. and Ph.D. degrees in electrical and computer engineering from Laval University, QC, Canada, in 1997 and 2001, respectively. He initiated the use of Bragg gratings technology for all-optical/all-fiber coding/decoding in optical CDMA systems. He was the founder of Access Photonic Networks (2001–2006) and an Adjunct Professor with the Electrical and Computer Engineering Department, Laval University, QC, Canada. He is currently with the Electrical Engineering Department, College of Engineering, King Saud University, Riyadh, Saudi Arabia, a Co-Founder and Photonics Thrust Leader of Radio Frequency and Photonics for the e-Society, KACST-TIC . His research interests include coherent optical communications systems, mode and multicore space division multiplexing, free space optical and visible light communications, long-reach PONs, hybrid fiber wireless systems, and OCDMA.

MOHAMED-SLIM ALOUINI was born in Tunis, Tunisia. He received the Ph.D. degree in electrical engineering from the California Institute of Technology, Pasadena, CA, USA, in 1998. He served as a Faculty Member with the University of Minnesota, Minneapolis, MN, USA, the Texas A&M University at Qatar, Education City, Qatar. He joined the King Abdullah University of Science and Technology, Thuwal, Saudi Arabia, as a Professor of Electrical Engineering in 2009. His current research interests include the modeling, design, and performance analysis of wireless communication systems.