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SER Optimization of OFDM Based AF Relaying in the Presence of AWGGN

DONG QIN[1,2], YUHAO WANG[1], (Senior Member, IEEE), AND FUHUI ZHOU[1,2], (Member, IEEE)

[1]School of Information Engineering, Nanchang University, Nanchang 330031, China

[2]Postdoctoral Research Station of Environmental Science and Engineering, Nanchang University, Nanchang 330031, China

Corresponding authors: D. Qin (qindong@seu.edu.cn) and F. Zhou (zhoufuhui@ncu.edu.cn)

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ABSTRACT This paper studies joint power allocation and subcarrier pairing strategies for an orthogonal frequency division multiplexing-based amplify and forward relay system. The optimization target function is symbol error rate (SER) in the context of an additive white generalized Gaussian noise (AWGGN), which encompasses numerous noise types. For instance, the familiar additive white Gaussian noise (AWGN) is just a special case of AWGGN. At a given data rate, the SER performance is optimized under total and individual power constraints, respectively. Simulation results demonstrate superior performance gap through our adaptive resource allocation approach over equal power allocation as well as fixed subcarrier pairing strategies. The impacts of practical issues on the SER performance, such as relay location and unequal power budgets at a source and a relay, are further discussed.

INDEX TERMS OFDM, SER, AWGGN.

I. INTRODUCTION

Error probability is an important reliable measure in a wireless communication system. The exact and approximate error probability formulas have been investigated in OFDM relay systems over various fading channel models. Specifically, a closed-form bit error rate (BER) formula was presented in [1], where multiple decode and forward (DF) relays existed in Rayleigh fading channels. An approximate BER was provided in [2] for threshold digital relaying over log normal fading channels. When beamforming was used, an asymptotic symbol error probability expression was derived in [3] for a multiple input multiple output (MIMO) AF relay system over arbitrary channel fading models. An exact symbol error probability was analyzed in [4] for opportunistic AF relaying over Rayleigh fading channels.

On the other hand, some authors choose error probability as the objective function for bit and power loading. For instance, at a target SER in selection DF and AF relaying, the transmit power consumption was minimized in [5]. Power loading was derived in [6] based on the minimization of vector error rate in multicast OFDM based AF relaying. A joint bit and power loading was proposed in [7] to minimize BER under total power constraint in an AF cooperative OFDM system. Furthermore, [8] extended the method of [7] and optimized an upper bound on BER in a multi-relay DF network.

The concept of subcarrier pairing in OFDM relay systems is another promising technique, which can improve ergodic capacity and error rate performance. Approximate bit error rate formulas were derived in [9], [10] for an OFDM AF relay system with ordered subcarrier pairing technique. A suboptimal subcarrier pairing based on ordering method was developed in [11] to minimize the uncoded bit error rate. But common to [9]–[11] is the absence of power allocation consideration.

All of the above literature on error rate analysis assumes that a noise obeys a Gaussian distribution. Although it's true in most cases, other noise types can't be ruled out. [12]–[14] pointed out that in an underwater communication channel and some sensor networks, the noise should be modeled as AWGGN instead of AWGN. And the exact SER over extended generalized-K channels subject to AWGGN was offered in [12]–[14]. But they didn't optimize the error rate.

As far as we know, the present literature hasn't yet chosen the SER subject to AWGGN as an objective function. To address this problem, we pursue the SER minimization problem under AWGGN for the design of an adaptive

OFDM based AF relay system. A joint power allocation and subcarrier pairing algorithm is proposed subject to total and individual power constraints, respectively. Under fixed transmission rate at each subcarrier assumption, we first calculate the optimal power allocation coefficients and then pair subcarriers. Different from conventional water-filling power allocation or its variation for throughput maximization and power consumption minimization, our power allocation form is of the Lambert structure. This result is especially noticeable and interesting for the case with AWGGN, where the optimal power processing wasn't known before. Because AWGGN is a more general noise model, the SER performance under some common noise types can be easily obtained, such as well-known AWGN and Laplacian noise, as long as we get the closed-form solution of power allocation. Therefore, our proposed joint resource allocation algorithm is suitable for a broader scenario. Simulation results demonstrate that our proposed algorithm outperform equal power allocation as well as fixed subcarrier pairing schemes.

II. SYSTEM MODEL

Consider that a source node S communicates with a destination node D via a relay node R in a half duplex mode. In the first hop, S emits signals while R and D listen. In the second hop, R forwards the amplified signals to D while S keeps silent. All signal transmission occurs on N parallel subcarriers in an OFDM mode. Denote $h_{s,i}$, $h_{r,i}$ and $h_{d,i}$ as the channel coefficients over subcarrier i in S→R link, R→D link and S→D link, respectively. The signal on subcarrier i in the first hop may be changed to subcarrier j in the second hop and j isn't necessarily equal to i. It's called a subcarrier pair (i, j). D combines the signals emitted by S in the first hop and the amplified signals forwarded by R in the second hop based on the maximum ratio approach. The instantaneous signal to noise ratio (SNR) on subcarrier pair (i, j) is given by

$$\begin{aligned}\gamma_{i,j} &= p_{s,i}c_i + \frac{p_{s,i}a_i p_{r,j}b_j}{1 + p_{s,i}a_i + p_{r,j}b_j} \\ &\approx p_{s,i}c_i + \frac{p_{s,i}a_i p_{r,j}b_j}{p_{s,i}a_i + p_{r,j}b_j} \end{aligned} \tag{1}$$

where $p_{s,i}$ and $p_{r,j}$ are transmit powers on subcarrier i at S and subcarrier j at R, respectively. $a_i = |h_{s,i}|^2/\sigma_r^2$, $b_j = |h_{r,j}|^2/\sigma_d^2$ and $c_i = |h_{d,i}|^2/\sigma_d^2$. σ_r^2 and σ_d^2 are noise variances at R and D, respectively. The approximation in (1), initially introduced in [15], is very tight in medium to high SNR region and has been widely used [16], [17]. By this approximate operation, $\gamma_{i,j}$ becomes a joint concave function on $p_{s,i}$ and $p_{r,j}$.

Assume that the signal is corrupted by AWGGN. So the SER in this noise model is expressed by

$$P_e = \sum_{i=1}^{N}\sum_{j=1}^{N} \rho_{i,j} u Q_\alpha\left(\sqrt{v\gamma_{i,j}}\right) \tag{2}$$

where $\rho_{i,j}$ is a binary indicator variable for subcarrier pairing. If subcarrier i in the first hop is coupled with subcarrier j in the second hop, $\rho_{i,j} = 1$ while $\rho_{i,j} = 0$ otherwise. u and v are constants dependent on the specific modulation scheme. $Q_\alpha(\cdot)$ is a generalized Gaussian Q function and is defined as [12]

$$Q_\alpha(x) = \frac{1}{2\Gamma(1/\alpha)}\Gamma\left[\frac{1}{\alpha}, |\Lambda_0 x|^\alpha\right] \tag{3}$$

where $\Gamma(\cdot)$ and $\Gamma(\cdot,\cdot)$ are Gamma function [18, eq. (8.310.1)] and upper incomplete Gamma function [18, eq. (8.350.2)], respectively. The auxiliary parameter Λ_0 is formulated as $\Lambda_0 = \sqrt{\Gamma(3/\alpha)/\Gamma(1/\alpha)}$. Note that $Q_\alpha(\cdot)$ is a generalization of the traditional Q function. It can be seen from (3) that $Q_\alpha(x)$ reduces to the famous Gaussian Q function and AWGGN becomes well-known AWGN when $\alpha = 2$. Besides, when $\alpha = 1$, AWGGN degenerates to a Laplace noise. So AWGGN can cover a wide variety of noise types via different parameters α.

III. RESOURCE ALLOCATION SUBJECT TO TOTAL POWER CONSTRAINT

In this section, we are interested in studying the SER optimization problem in the presence of AWGGN, when total power constraint is imposed. Before dealing with this problem, we first maximize instantaneous SNR $\gamma_{i,j}$ under total power constraint $p_{s,i} + p_{r,j} = p_{i,j}$ on subcarrier pair (i, j). Based on the available results in [19], $\gamma_{i,j}$ can be written as a simple form $\gamma_{i,j} = \gamma_{i,j}^{eq} p_{i,j}$. $\gamma_{i,j}^{eq}$ is an equivalent channel gain and is dictated by

$$\gamma_{i,j}^{eq} = \begin{cases} \dfrac{b_j\left(d_{i,j}+c_i\right)^2}{\left(d_{i,j}+b_j\right)^2} & \text{if } c_i < b_j \\ c_i & \text{otherwise} \end{cases} \tag{4}$$

where $d_{i,j} = \sqrt{a_i b_j - a_i c_i + b_j c_i}$. The corresponding transmit powers at S and R are given by

$$p_{s,i} = \begin{cases} \dfrac{b_j\left(d_{i,j}+c_i\right)}{d_{i,j}\left(d_{i,j}+b_j\right)} p_{i,j} & \text{if } c_i < b_j \\ p_{i,j} & \text{otherwise} \end{cases} \tag{5}$$

$$p_{r,j} = \begin{cases} \dfrac{a_i\left(b_j-c_i\right)}{d_{i,j}\left(d_{i,j}+b_j\right)} p_{i,j} & \text{if } c_i < b_j \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Then this SER optimization problem can be modeled as

$$\begin{aligned} \min_{\{\rho_{i,j}, p_{i,j}\}} \; & P_e = \sum_{i=1}^{N}\sum_{j=1}^{N} \rho_{i,j} u Q_\alpha\left(\sqrt{v p_{i,j}\gamma_{i,j}^{eq}}\right) \\ \text{s.t.} \; & \sum_{i=1}^{N}\rho_{i,j} = 1, \sum_{j=1}^{N}\rho_{i,j} = 1, \forall i, j \\ & \sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{i,j}p_{i,j} \le P_T, p_{i,j} \ge 0, \forall i, j \end{aligned} \tag{7}$$

where P_T represents total available power. The first constraint condition in (7) means exclusive pairing constraint and indicates that one subcarrier in the first hop subcarrier can be matched with one and only one subcarrier in the second hop.

A. POWER ALLOCATION

At given subcarrier pairing, the problem (7) is a standard convex programming problem as proved in the Appendix. Factoring power constraint into the objective function, the Lagrangian is formulated as

$$\begin{aligned} L &= \sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{i,j}uQ_\alpha\left(\sqrt{vp_{i,j}\gamma_{i,j}^{eq}}\right) \\ &\quad + \lambda\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{i,j}p_{i,j} - P_T\right) \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{i,j}\left[uQ_\alpha\left(\sqrt{vp_{i,j}\gamma_{i,j}^{eq}}\right) + \lambda p_{i,j}\right] - \lambda P_T \end{aligned} \tag{8}$$

where λ is a nonnegative dual variable. Employing Karush-Kuhn-Tucker (KKT) [20] conditions for L yields

$$\frac{\partial L}{\partial p_{i,j}} = \lambda - \frac{u\alpha\Lambda_0\sqrt{vp_{i,j}\gamma_{i,j}^{eq}}}{4\Gamma(1/\alpha)\,p_{i,j}}e^{-\left(\sqrt{vp_{i,j}\gamma_{i,j}^{eq}}\Lambda_0\right)^\alpha} = 0 \tag{9}$$

After some mathematical rearrangement, the closed-form expression of power allocation is written as

$$p_{i,j}^{\star} = \frac{1}{v\gamma_{i,j}^{eq}}\left\{\frac{1}{\alpha\Lambda_0^\alpha}W\left[\alpha\left(\frac{uv\alpha\Lambda_0^2\gamma_{i,j}^{eq}}{4\Gamma(1/\alpha)\lambda}\right)^\alpha\right]\right\}^{\frac{2}{\alpha}} \tag{10}$$

where $W(\cdot)$ is the Lambert function [7]. It should be emphasized that different from traditional water-filling power allocation or its variation on rate maximization and power consumption minimization, the power assignment on the SER optimization is in the form of Lambert function as in (10). Especially, this is the case where no explicit optimal power expression was known before. Now we begin to study two special examples.

(1) Laplacian noise: when $\alpha = 1$, AWGGN degenerates to a Laplace noise and (10) becomes

$$p_{i,j}^{\star} = \frac{1}{2v\gamma_{i,j}^{eq}}W^2\left(\frac{uv\gamma_{i,j}^{eq}}{2\lambda}\right) \tag{11}$$

(2) AWGN: as one of the most common noise models, AWGN has appeared in many literature. AWGGN reduces to AWGN for $\alpha = 2$ and (10) becomes

$$p_{i,j}^{\star} = \frac{1}{v\gamma_{i,j}^{eq}}W\left[\left(\frac{uv\gamma_{i,j}^{eq}}{2\sqrt{2\pi}\lambda}\right)^2\right] \tag{12}$$

As a double check, (12) is exactly the same as [7, eq. (35)], which reflects the correctness of (10) from one side.

B. SUBCARRIER PAIRING

Inserting (10) into (8) leads to

$$L = \sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{i,j}\left[uQ_\alpha\left(\sqrt{vp_{i,j}^{\star}\gamma_{i,j}^{eq}}\right) + \lambda p_{i,j}^{\star}\right] - \lambda P_T \tag{13}$$

There is only one variable $\rho_{i,j}$ left in (13). This is the maximum matching problem in graph theory, which has been solved in [21]. Finally, λ can be determined by bisection method to meet power constraint condition.

IV. RESOURCE ALLOCATION SUBJECT TO INDIVIDUAL POWER CONSTRAINTS

Sometimes, a total power constraint over S and R is forbidden due to their distributed characteristics. Thus, we aim to find joint power allocation and subcarrier pairing subject to individual power constraints at S and R for practical scenario. Under this assumption, the optimization problem is given by

$$\begin{aligned} &\min_{\{\rho_{i,j},p_{s,i},p_{r,j}\}} \sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{i,j}uQ_\alpha\left(\sqrt{v\gamma_{i,j}}\right) \\ &\text{s.t.} \sum_{i=1}^{N}\rho_{i,j} = 1, \sum_{j=1}^{N}\rho_{i,j} = 1, \forall i,j \\ &\quad \sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{i,j}p_{s,i} \le P_S, p_{s,i} \ge 0, \forall i,j \\ &\quad \sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{i,j}p_{r,j} \le P_R, p_{r,j} \ge 0, \forall i,j \end{aligned} \tag{14}$$

where P_S and P_R are available power budgets at S and R, respectively.

A. POWER ALLOCATION

The Lagrangian associated with (14) is constructed by

$$\begin{aligned} L &= \sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{i,j}uQ_\alpha\left(\sqrt{v\gamma_{i,j}}\right) + \beta_1\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{i,j}p_{s,i} - P_S\right) \\ &\quad + \beta_2\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{i,j}p_{r,j} - P_R\right) \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{i,j}\left[uQ_\alpha\left(\sqrt{v\gamma_{i,j}}\right) + \beta_1 p_{s,i} + \beta_2 p_{r,j}\right] \\ &\quad - \beta_1 P_S - \beta_2 P_R \end{aligned} \tag{15}$$

where $\boldsymbol{\beta} = (\beta_1, \beta_2)$ is a nonnegative dual vector. Similarly, using KKT conditions for (15) leads to

$$\begin{aligned} \frac{\partial L}{\partial p_{s,i}} &= \beta_1 - \frac{u\left[b_j^2 c_i p_{r,j}^2 + a_i^2 c_i p_{s,i}^2\right.}{4\Gamma(1/\alpha)\sqrt{p_{s,i}\left(a_i b_j p_{r,j} + b_j c_i p_{r,j} + a_i c_i p_{s,i}\right)}} \\ &\quad \times \frac{\left.+a_i b_j p_{r,j}\left(b_j p_{r,j} + 2c_i p_{s,i}\right)\right]\alpha\Lambda_0\sqrt{v}}{\left(b_j p_{r,j} + a_i p_{s,i}\right)^{3/2}} \\ &\quad \times e^{-\left(\sqrt{\frac{p_{s,i}\left(a_i b_j p_{r,j} + b_j c_i p_{r,j} + a_i c_i p_{s,i}\right)v}{b_j p_{r,j} + a_i p_{s,i}}}\Lambda_0\right)^\alpha} = 0 \end{aligned} \tag{16}$$

$$\frac{\partial L}{\partial p_{r,j}} = \beta_2 - \frac{u a_i^2 b_j \alpha p_{s,i}^{3/2} \sqrt{v} \Lambda_0}{4\Gamma(1/\alpha)\left(b_j p_{r,j} + a_i p_{s,i}\right)^{3/2}} \times \frac{1}{\sqrt{a_i b_j p_{r,j} + b_j c_i p_{r,j} + a_i c_i p_{s,i}}} \times e^{-\left(\sqrt{\frac{p_{s,i}\left(a_i b_j p_{r,j} + b_j c_i p_{r,j} + a_i c_i p_{s,i}\right) v}{b_j p_{r,j} + a_i p_{s,i}}}\Lambda_0\right)^{\alpha}} = 0 \quad (17)$$

After some mathematical manipulations, $p_{s,i}$ and $p_{r,j}$ are computed by

$$p_{s,i}^{\star} = \frac{b_j f_{i,j} + a_i}{\left(b_j a_i f_{i,j} + b_j c_i f_{i,j} + a_i c_i\right) v} \times \left\{ \frac{1}{\alpha \Lambda_0^{\alpha}} W\left[\alpha \left(\frac{u v \alpha \Lambda_0^2 a_i^2 b_j}{4\Gamma(1/\alpha)\beta_2\left(b_j f_{i,j} + a_i\right)^2}\right)^{\alpha}\right]\right\}^{\frac{2}{\alpha}} \quad (18)$$

$$p_{r,j}^{\star} = f_{i,j} p_{s,i} \quad (19)$$

where

$$f_{i,j} = \begin{cases} \dfrac{a_i\left[\sqrt{\beta_2\left(b_j c_i \beta_1 - a_i c_i \beta_2 + a_i b_j \beta_1\right)} - c_i \beta_2\right]}{\beta_2 b_j \left(a_i + c_i\right)} & \text{if } b\beta_1 > c\beta_2 \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

Note that $f_{i,j} = 0$ inside (20) corresponds to the direct path on subcarrier pair (i, j). In other words, S will communicate directly with D without R's assistance if the channel condition of S→D link is better than that of R→D link. Note that (18) is the same as (10) for $f_{i,j} = 0$, which also proves the correctness of (18) from another point of view. Then, let's take a look at two special examples.

(1) Laplacian noise: when $\alpha = 1$, (18) reduces to

$$p_{s,i}^{\star} = \frac{b_j f_{i,j} + a_i}{2\left(b_j a_i f_{i,j} + b_j c_i f_{i,j} + a_i c_i\right) v} \times W^2\left[\left(\frac{u v a_i^2 b_j}{2\beta_2\left(b_j f_{i,j} + a_i\right)^2}\right)\right] \quad (21)$$

(2) AWGN: when $\alpha = 2$, (18) becomes

$$p_{s,i}^{\star} = \frac{b_j f_{i,j} + a_i}{\left(b_j a_i f_{i,j} + b_j c_i f_{i,j} + a_i c_i\right) v} \times W\left[\left(\frac{u v a_i^2 b_j}{2\sqrt{2\pi}\beta_2\left(b_j f_{i,j} + a_i\right)^2}\right)^2\right] \quad (22)$$

B. SUBCARRIER PAIRING

Plugging (18) and (19) into (15) yields

$$L = \sum_{i=1}^{N}\sum_{j=1}^{N} \rho_{i,j}\left[u Q_{\alpha}\left(\sqrt{v \gamma_{i,j}^{\star}}\right) + \beta_1 p_{s,i}^{\star} + \beta_2 p_{r,j}^{\star}\right] - \beta_1 P_S - \beta_2 P_R \quad (23)$$

Similarly, there is one binary variable $\rho_{i,j}$ in (23). This problem has been solved in [21]. The dual vector $\boldsymbol{\beta}$ has to be searched to meet individual power constraints.

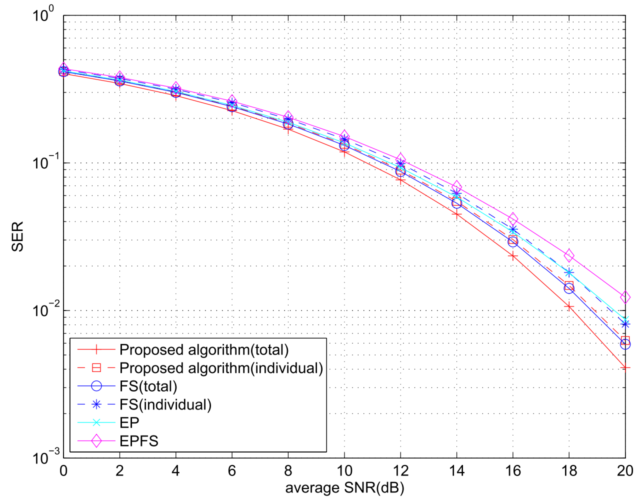

FIGURE 1. Average SER subject to Laplacian noise.

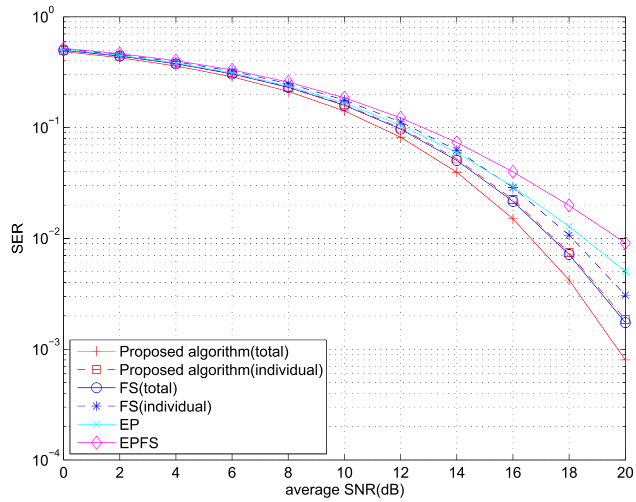

FIGURE 2. Average SER subject to AWGN.

V. SIMULATION RESULTS

In this section, the SER performances of 4-ary pulse amplitude modulation (4PAM) modulation are studied by Monte Carlo simulations. Other sophisticated modulation choices can be depicted in a similar way. The channel impulse response of each link is generated according to COST207 typical urban model with path powers of {-3, 0, -2, -6, -8, -10}dB and path delays of {0, 0.2, 0.5, 1.6, 2.3, 5.0}μs [22]–[24]. The number of subcarriers is set as $K = 32$ and each subcarrier experiences flat fading. Unless otherwise specified, $P_S = P_R = P_T/2$ is assumed. The average SNR per subcarrier is defined as $P_T/\left(2K\sigma^2\right)$, where $\sigma^2 = \sigma_r^2 = \sigma_d^2$. For comparative observations, we propose some suboptimal approaches following a stepwise method by different allocation and pairing combinations.

- Equal power allocation (EP): each subcarrier equally uses the power.
- Fixed subcarrier pairing (FS): the first hop and the second hop use the same subcarrier. In other words, the subcarrier pair is (i, i) for $i = 1, 2, \ldots, K$.

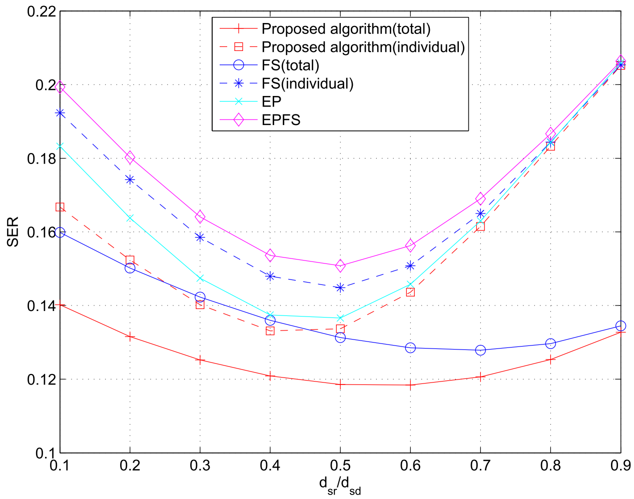

FIGURE 3. Laplacian noise: the effect of relay location on the SER, SNR=10dB.

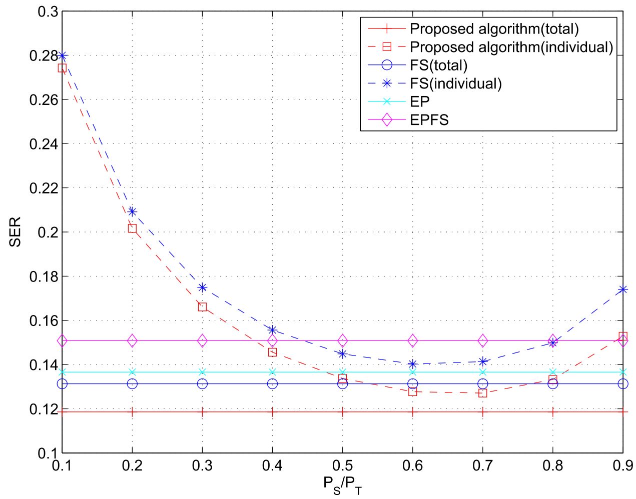

FIGURE 5. Laplacian noise: average SER versus relative power ratio, SNR=10dB.

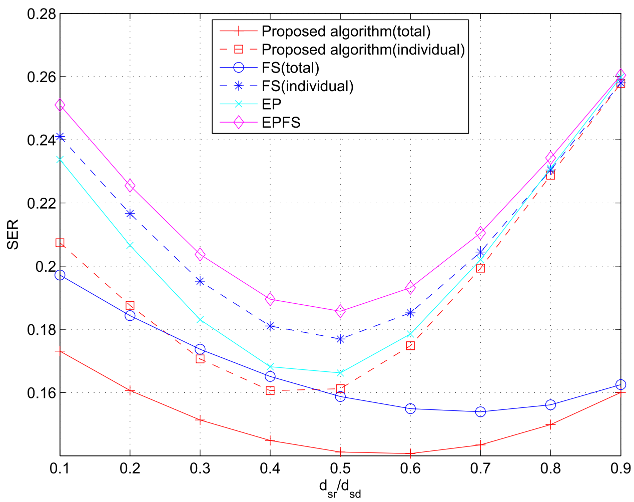

FIGURE 4. AWGN: the effect of relay location on the SER, SNR=10dB.

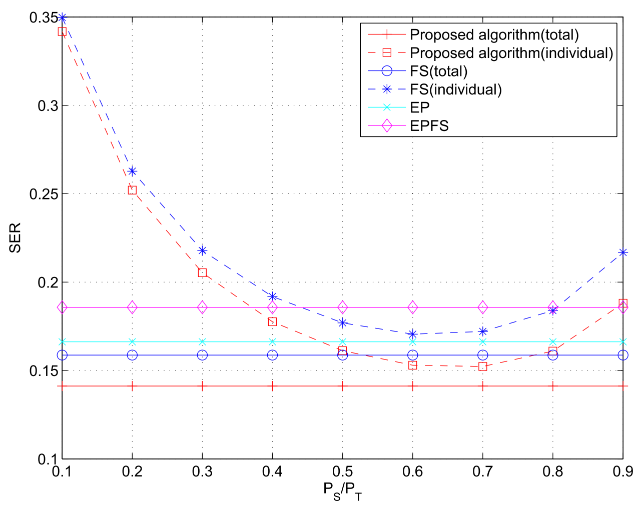

FIGURE 6. AWGN: average SER versus relative power ratio, SNR=10dB.

- Equal power allocation and fixed subcarrier pairing (EPFS): the power is evenly distributed to all subcarriers with fixed pairing technique.

As two examples of noise types, Figs.1 and 2 show SERs when received signals are corrupted by Laplacian noise and AWGN, respectively. Other noises models can be illustrated in a similar way. We clearly observe that the proposed algorithm provides the highest gain in both total and individual power constraints. This demonstrates that the optimal power allocation benefits from Lambert function instead of conventional water-filling scheme and the explicit subcarrier pairing benefits from the maximum matching method in graph theory. We found that FS significantly outperforms EPFS since the latter isn't able to extract the underlying gains of power allocation.

Next, we explicitly take into account the effect of different relay positions on the SER performances of various approaches. Assume that S, R and D lie in a straight line. R and D are disposed at distances d_{sr} and d_{sd} apart from S, respectively. The path loss exponent is set as 3. By changing R's location between S and D, Figs.3 and 4 tell us what position is the most beneficial for the relay. When the total power constraint is adopted, R should move closer to D. The reason is that S can be assigned more power than R under the total power constraint. The redundant power can compensate the distance between S and R. In the individual power constraints, each method attains its lowest SER when the relay approaches the midpoint since $P_S = P_R$ is assumed. When R is at midway, S→R link and R→D link exhibit identical channel statistical properties and therefore yield similar results.

Then we study the average SER versus relative power ratio P_S/P_T, where $P_R = P_T - P_R$. Figs.5 and 6 reflect how the level of the relative power ratio between S and R affects the SER performance. It is easy to observe that S needs more

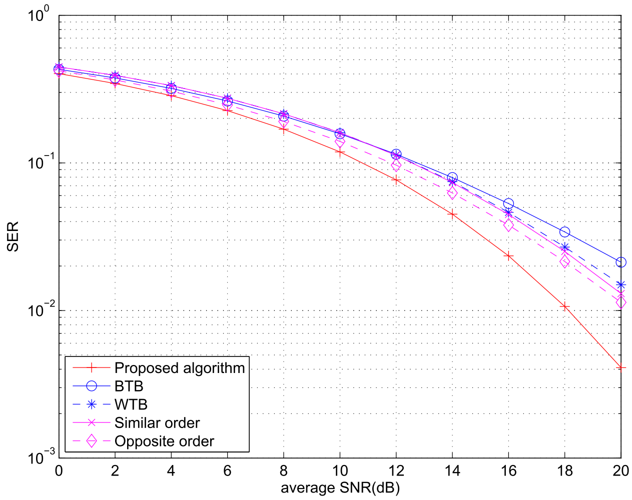

FIGURE 7. Laplacian noise: comparison of different approaches.

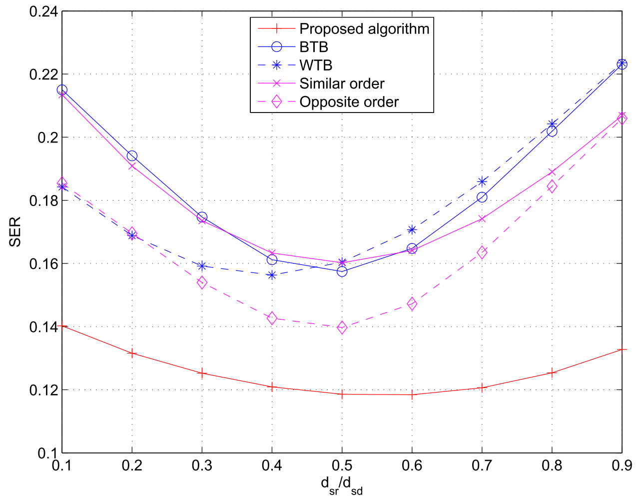

FIGURE 9. Laplacian noise: effect of relay location on the SER performance of different approaches.

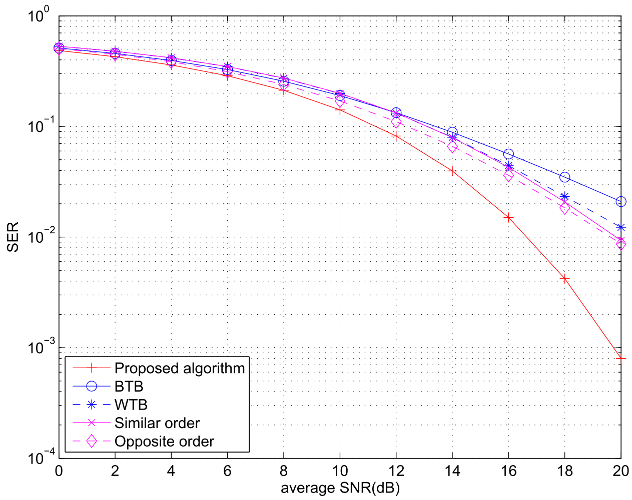

FIGURE 8. AWGN: comparison of different approaches.

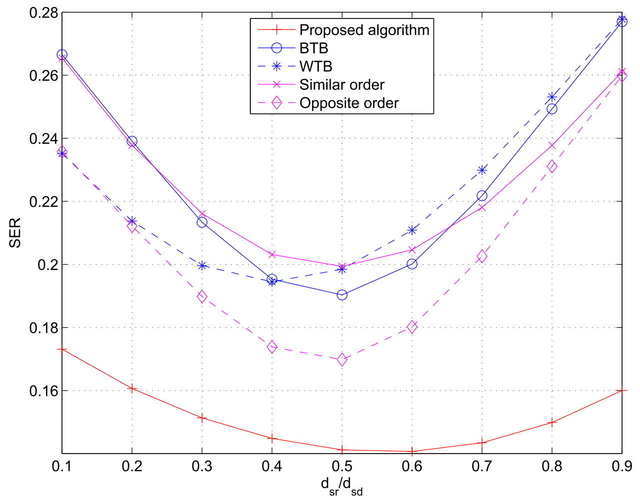

FIGURE 10. AWGN: effect of relay location on the SER performance of different approaches.

power than R because sometimes R turns off the transmitter and keeps idle when the channel quality of relay path is weak. On the other hand, the SERs of total power constraint are insensitive to such relative ratio and remain constant because the total power constraint means the overall power budget is only related to P_T instead of P_S or P_R.

To show the superiority of the proposed algorithm, we prepare to compare it with other prevailing methods. The SER results are depicted in Figs.7 and 8 with four existing schemes best to best (BTB), worst to best (WTB), similar order and opposite order. BTB (WTB) means that the best (worst) channel quality a_i of the first hop is matched with the best channel quality b_i of the second hop [9], [10]. The similar (opposite) order implies that b_i is permuted with respect to $a_i^{-1} + c_i$ in a similar (opposite) order [11]. As expected, the proposed algorithm outperforms other four schemes in all SNR regime. For example, a 2dB SNR gain can be achieved at SER=10^{-2} compared with similar and opposite order methods. More SNR can be saved compared with BTB and WTB. Because BTB and WTB overlook the channel condition of the direct link S→D. Although the similar and opposite order methods incorporates direct link, they are heuristic permutation assuming frequency flat S→R link. This assumption is always impractical because OFDM is often used in frequency selective channels.

The relationship between relative relay position and average SER is captured in Figs.9 and 10. A significant performance gain can be observed between the proposed algorithm and other four methods. This shows that the joint allocation is clearly superior to the simple heuristic method.

VI. CONCLUSION

In this paper, we have investigated adaptive power allocation and subcarrier pairing for an OFDM based AF relay system.

Different from most literature, we adopts the SER under AWGGN as the objective function. The famous Laplacian noise and AWGN are two special examples of AWGGN. Our approach obtains the optimal power allocation strategy in the form of the Lambert function and explicit subcarrier pairing technology based on the maximum matching method. Simulation results also show the superiority of our proposed algorithm over other methods.

APPENDIX
CONCAVITY AND CONVEXITY ANALYSIS OF (7)

We first introduce one theorem from [20].

Theorem 1: For composite function $f(x) = h(g(x))$, $\textbf{dom}\, f = \{x \in \textbf{dom}\, g | g(x) \in \textbf{dom}\, h\}$, f is convex if h is convex and nonincreasing, and g is concave, where ***dom*** *denotes the domain of a function.*

At the given subcarrier, (7) leaves only continuous power variables. The first and second derivatives of $Q_\alpha(x)$ $(x \geq 0)$ are

$$Q'_\alpha(x) = -\frac{\alpha \Lambda_0 e^{-(\Lambda_0 x)^\alpha}}{2\Gamma(1/\alpha)} \tag{24}$$

$$Q''_\alpha(x) = \frac{\alpha^2 (\Lambda_0 x)^{1+\alpha} e^{-(\Lambda_0 x)^\alpha}}{2\Gamma(1/\alpha)\, x^2} \tag{25}$$

Since $Q'_\alpha(x) < 0$ and $Q''_\alpha(x) \geq 0$ when $x \geq 0$, $Q_\alpha(x)$ is nonincreasing and convex in $x \geq 0$. Then $\sqrt{x}$ is concave in $x \geq 0$. Thus $Q_\alpha\left(\sqrt{x}\right)$ is convex when $x \geq 0$ by Theorem 1. Hence, the objective function in (7) is convex because it is a nonnegative weight sum of convex functions in the form of $Q_\alpha\left(\sqrt{x}\right)$. The constraints in (7) are all affine. Therefore, (7) is a standard convex programming problem. Similarly, the problem under individual power constraints can also be transformed into a convex programming problem.

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DONG QIN received the Ph.D. degree in information and communication engineering from Southeast University, Nanjing, China, in 2016. He joined the School of Information Engineering, Nanchang University, in 2016. His current research interests lie in the area of cooperative communication and OFDM techniques.

YUHAO WANG (SM'14) was born in Hubei, China, in 1977. He received the Ph.D. degree in space physics from Wuhan University, Wuhan, China, in 2006. He was a Visiting Professor with the Department of Electrical Communication Engineering, University of Calgary, Calgary, AB, Canada, in 2008, and the National Mobile Communication Research Laboratory, Southeast University, Nanjing, China, from 2010 to 2011. He is currently a Professor with the Cognition Sensor Network Laboratory, School of Information Engineering, Nanchang University, Nanchang, China. His current research interests include radio measurement and channel modeling, software-defined radio, complex environment sensing and monitoring, nonlinear signal processing, multimedia, sensor networks, and wireless communications.

FUHUI ZHOU received the Ph.D. degree in military communication from Xidian University, Xi'an, China, in 2016. He joined the School of Information Engineering, Nanchang University, in 2016. He was an International Visiting Ph.D. Student with The University of British Columbia from 2015 to 2016. His research interests focus on 5G wireless networks, green communications, cognitive radio, nonorthogonal multiple access, wireless powered networks. He has authored multiple papers in the IEEE Transactions on Wireless Communications and the IEEE GLOBECOM. He is currently serving as the Technical Program Committee Member of the IEEE GLOBECOM 2016.