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On Computational Aspects of Tchebichef Polynomials for Higher Polynomial Order

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ABSTRACT Tchebichef polynomials (TPs) and their moments are widely used in signal processing due to their remarkable performance in signal analysis, feature extraction, and compression capability. The common problem of the TP is that the coefficients computation is prone to numerical instabilities when the polynomial order becomes large. In this paper, a new algorithm is proposed to compute the TP coefficients (TPCs) for higher polynomial order by combining two existing recurrence algorithms: the three-term recurrence relations in the n-direction and x-direction. First, the TPCs are computed for $x, n = 0, 1, \ldots, (N/2) - 1$ using the recurrence in the x-direction. Second, the TPCs for $x = 0, 1, \ldots, (N/2) - 1$ and $n = (N/2), (N/2) + 1, \ldots, N - 1$ based on n and x directions are calculated. Finally, the symmetry condition is applied to calculate the rest of the coefficients for $x = (N/2), (N/2) + 1, \ldots, N - 1$. In addition to the ability of the proposed algorithm to reduce the numerical propagation errors, it also accelerates the computational speed of the TPCs. The performance of the proposed algorithm was compared to that of existing algorithms for the reconstruction of speech and image signals taken from different databases. The performance of the TPCs computed by the proposed algorithm was also compared with the performance of the discrete cosine transform coefficients for speech compression systems. Different types of speech quality measures were used for evaluation. According to the results of the comparative analysis, the proposed algorithm makes the computation of the TP superior to that of conventional recurrence algorithms when the polynomial order is large.

INDEX TERMS Orthogonal polynomials, recurrence algorithm, Tchebichef polynomials, numerical propagations errors.

I. INTRODUCTION

Different types of orthogonal polynomials have gained importance in the field of speech and image analysis [1]. From a cursory review of the literature, it can be seen that orthogonal polynomials have been the center of research attention since they are used in many applications such as: Face Recognition [2], hiding information [3], edge detection [4], data compression [5], image retrieval [6], and visual pattern recognition [7], [8]. Legendre, Zernike and Pseudo-Zernike moments are formed from basis functions of continuous orthogonal polynomials, and shown great capability in feature representation [9]. However, as they are defined inside a unit circle only, the moments calculation of these polynomials necessitate coordinate transformation and continuous moment integrals approximation [9], [10]. On the other hand, discrete orthogonal moments such as Tchebichef moments [11], Hahn moments [12] and Krawtchouk moments [13] are characterized in a rectangular coordinate space [11]; therefore, there is no necessity for integral approximation and coordinate space transformation [14]. The discrete Tchebichef transform (DTT) is considered one of the significant discrete orthogonal transforms. It can be generated using the TP basis functions, and it is utilized for time-moment transformation [15]. DTT has a significant energy compaction attribute as discrete cosine transform (DCT) [16].

The TP is a function of three parameters: the length of the signal (N), the polynomial order (n), and the signal index (x). The computation of TP coefficients can be implemented using the three-term recurrence algorithm in which the new TP coefficient is recursively estimated using the previous two coefficients. In [11], a framework of recurrence algorithm in the n-direction was proposed to compute the TPCs. In this approach, the TPCs of the nth order for all values of the xth index are estimated based on polynomial values of the $n-1$ and $n-2$ orders. However, when the signal size becomes large, the TPCs computation exhibits numerical instabilities because the squared norm of the scaled TP assumes small values [9]. On the other hand, the x-direction recurrence algorithm was proposed [9] to solve the problem of TPCs when the moment order becomes high. In this approach, the TPCs of the xth index for all values of the nth order are estimated based on the polynomial values of $x-1$ and $x-2$ positions. Although the x-direction approach improves the computation accuracy, there is a limitation to this method as it becomes unable to compute the polynomial values for very high polynomial order. The instabilities are due to the initial values utilized in computing the TPCs, which become zeros at high order.

There has been attention paid to develop the computation of TMs based on the aforementioned algorithms. In [17] a recursive algorithm to compute Tchebichef moments based on Clenshaw's formula was presented. Shu *et al.* [18] proposed an efficient method for computing TMs based on properties of TPs to aggregate with image block representation algorithm for binary and gray-scale images. A method to compute TM through geometric moments based on digital filters was introduced in [19]. In [20] a TM and its inverse transform based on Z-transform was given.

Both of the aforementioned algorithms generated numerical instability and were therefore unable to handle very high signal sizes. Motivated by this issue, this study introduces a new algorithm to tackle this problem.

The paper is organized as follows: in section II, the basic computation aspects of the TP are presented. Section III illustrates the types of three-term recurrence algorithms. The proposed algorithm is explained in section IV. An experimental study to evaluate the proposed algorithm performance is presented in section V. Finally, section VI provides the conclusion.

II. TCHEBICHEF POLYNOMIALS AND MOMENTS

A. THE ORTHOGONAL DISCRETE TCHEBICHEF FUNCTIONS

The nth order of the orthogonal form of the scaled Tchebichef polynomials $t_n(x)$ is given by [9], [11]

$$t_n(x) = \sqrt{\frac{w(x)}{\rho(n)}}\,(1-N)_n\,{}_3F_2(-n,-x,1+n;1,1-N;1) \tag{1}$$

where $w(x)=1$ is the weight function of the TP and $\rho(n) = (2n)!\binom{N+n}{2n+1}$ is the squared norm of the TP. Therefore, (1) can be rewritten to be:

$$t_n(x) = \frac{(1-N)_n}{\sqrt{(2n)!\binom{N+n}{2n+1}}}\,{}_3F_2(-n,-x,1+n;1,1-N;1) \tag{2}$$

where $n, x = 0, 1, 2, \ldots, N-1, N > 0$ and $(a)_k$ is the Pochhammer symbol (ascending factorial symbol) [21].

$\binom{a}{b}$ is defined as the binomial coefficients $= \frac{a!}{b!(a-b)!}$. ${}_3F_2(.)$ is the generalized hypergeometric functions and it is represented by a hypergeometric series. It is defined as:

$${}_3F_2(-n,-x,1+n;1,1-N;1) = \sum_{k=0}^{\infty}\frac{(-n)_k(-x)_k(1+n)_k}{(1)_k(1-N)_k k!} \tag{3}$$

The set of $t_n(x)$ with a unit weight, satisfies the following orthogonality condition [11]

$$\sum_{n=0}^{N-1} t_n(x)t_m(x) = \delta_{mn} \tag{4}$$

where δ_{nm} is known as Kronecher delta and symbolizes the orthonormal representation of orthogonal polynomials, and defined by:

$$\delta_{nm} = \begin{cases} 1, & n = m \\ 0, & n \neq m \end{cases}$$

B. THE DISCRETE TCHEBICHEF MOMENTS

In general, moments set can be defined as scalar quantities which are an efficient and superior data descriptor [1]. They are used efficiently to represent signal information without redundancy and to reveal small changes in the signal intensity [22]. The lower-order moments contain the most energy of the signal [15], whereas the higher-order moments contain the signal details (high frequency components) [15]. For a 1D signal function $f(x)$ with a length of N samples, the Tchebichef orthogonal moments set Ψ_n can be defined as follows [1]:

$$\Psi_n = \sum_{x=0}^{N-1} t_n(x) f(x), \quad n = 0, 1, \ldots, M \text{ and } 0 \leqslant M \leqslant N-1 \tag{5}$$

where M is the maximum order of the moments used for signal representation. To reconstruct the signal, the inverse transformations of the Tchebichef moment can be applied as follows:

$$f(x) \cong \sum_{n=0}^{M-1} \Psi_n t_n(x), \quad x = 0, 1, \ldots, N-1 \tag{6}$$

For a 2D signal $f(x,y)$ with a size of $N \times N$, the discrete Tchebichef moment Ψ_{nm} is defined as

$$\Psi_{nm} = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} t_n(x)\,t_m(y) f(x,y), \quad n, m = 0, 1, \ldots, M \tag{7}$$

To reconstruct the signal, the inverse transformations of the Tchebichef moment can be applied as follows:

$$f(x,y) \cong \sum_{n=0}^{M-1}\sum_{m=0}^{M-1} \Psi_{nm}\,t_n(x)\,t_m(y), \quad x, y = 0, 1, \ldots, N-1 \tag{8}$$

III. THE RECURRENCE ALGORITHMS

The TPCs can be arranged in a 2D array with n and x parameters and the size of this array is $N \times N$, where n and x refer to the polynomial order and the signal time index respectively. Because of the time-consuming and numerical instabilities of computing the TPC values which requires the hypergeometric series and gamma functions, the three-term recurrence algorithms are considered very useful tools for the calculation [23]. The recurrence algorithm can be implemented in two directions: in the direction of parameter n and in the direction of parameter x. In this section, the two algorithms are presented.

A. THE THREE TERM RECURRENCE ALGORITHMS IN THE n-DIRECTION

The computation of TPCs coefficients can be done using the three-term recurrence relations in n-direction [11]:

$$t_n(x) = \beta_1\, t_{n-1}(x) + \beta_2\, t_{n-2}(x) \tag{9}$$

where $n = 2, 3, \ldots, N-1, x = 0, 1, \ldots, N-1$,

$$\beta_1 = \frac{3-N}{n}\sqrt{\frac{4n^2-1}{N^2-n^2}}$$

$$\beta_2 = \frac{1-n}{n}\sqrt{\frac{2n+1}{2n-3}}\sqrt{\frac{N^2-(n-1)^2}{N^2-n^2}} \tag{10}$$

The initial conditions for the above recurrence are $t_0(x) = \frac{1}{\sqrt{N}}$ and $t_1(x) = (2x+1-N)\sqrt{\frac{3}{N(N^2-1)}}$.

In this algorithm, the polynomial coefficients of the nth order for all values of the xth index are computed using the polynomial values of the previous $n-1$ and $n-2$ orders. This set is inconvenient for polynomial coefficients computing when the signal length (i.e. parameter N) becomes long, as it can be easily verified when the value of $t_n(x)$ grows for N^n [11], [23]. Since the n-direction recurrence algorithm cannot handle any size in excess of 81, it results in the sight going toward using the three-term recurrence algorithms in x-direction.

B. THE THREE TERM RECURRENCE ALGORITHM IN THE x-DIRECTION

The three-term recurrence algorithm in the x-direction is defined as [9]

$$t_n(x) = \alpha_1\, t_n(x-1) + \alpha_2\, t_n(x-2) \tag{11}$$

where $n = 1, 2, \ldots, N-1,$ and $x = 2, 3, \ldots, \frac{N}{2}-1$

$$\alpha_1 = \frac{-n(n+1)-(2x-1)(x-N-1)-x}{x(N-x)}$$

$$\alpha_2 = \frac{(x-1)(x-N-1)}{x(N-x)} \tag{12}$$

The initial values for the above relations can be obtained by

$$t_n(0) = -\sqrt{\frac{N-n}{N+n}}\sqrt{\frac{2n+1}{2n-1}}\, t_{n-1}(0), \quad n = 1, 2, \ldots, N-1$$

$$t_n(1) = \left(1 + \frac{n(1+n)}{1-N}\right) t_n(0), \quad n = 0, 1, \ldots, N-1 \tag{13}$$

where $t_0(0) = \frac{1}{\sqrt{N}}$. The TPCs of the xth index for all values of the nth order are estimated using the polynomial values of the previous $x-1$ and $x-2$ positions. It can be seen from (11) that the recurrence relation can be ended at $x = \frac{N}{2}-1$ and the following symmetry property can be used to calculate the polynomial coefficients for the second half of the polynomial array ($x = \frac{N}{2}, \frac{N}{2}+1, \ldots, N-1, n = 0, 1, \ldots, N-1$)

$$t_n(N-1-x) = (-1)^n\, t_n(x) \tag{14}$$

The conventional recurrence algorithm in x-direction of discrete Tchebichef orthogonal polynomial computation can deal with order reaches to approximately N=1095 samples. However, in real situations the length of the signal could be very long. Therefore, a more robust method is required to deal with signals of large sizes. Motivated by this idea, in this study a new algorithm to compute the TP values based on the two traditional recurrence algorithms is proposed as presented in the following section.

IV. THE PROPOSED ALGORITHM

A. COMPUTATION PROBLEM OF THE TP RECURRENCE ALGORITHM IN THE x-DIRECTION

Fig. 1a shows the plot of the TPCs array computed using (11-13) for $N = 1000$. Obviously, there are two dimensions, the horizontal axis represents the TPCs in terms of x parameter (signal index), whereas the vertical axis represents the TPCs in terms of n parameter (order of the polynomial). From this figure, it is obvious that TPCs distribution is represented by a half of an oval shape and the values outside this region are approximately zero (the values are less than 10^{-5}).

Now we discuss the case for larger values of N. Fig. 1b displays the polynomial array for $N = 1400$. It can be seen that, the values of the TPCs approach zero when the polynomial order (n) is greater than $\sim$1300. The numerical propagation of error is due to the initial value in the first column where $x = 0$. Fig. 1 indicates that the algorithm in the x-direction is unable to generate TPCs for large values of N.

B. THE PROPOSED THREE-TERM RECURRENCE ALGORITHM

In this study, a new algorithm to compute the TPCs is proposed for high polynomial order. The new algorithm is based on the integration of two traditional recurrence relations (the x-direction algorithm and n-direction algorithm) in a sequential manner. The theoretical calculation flow of the proposed algorithm will be as follows:

Step 1: Compute the TPCs values for the first quarter of the polynomial array, where $x, n = 0, 1, \ldots, \frac{N}{2}-1$:

(1) Find $t_n(0)$ and $t_n(1)$ using (13).
(2) Find the rest of the TPCs values using the three-term recurrence algorithms in the x-direction from (11).

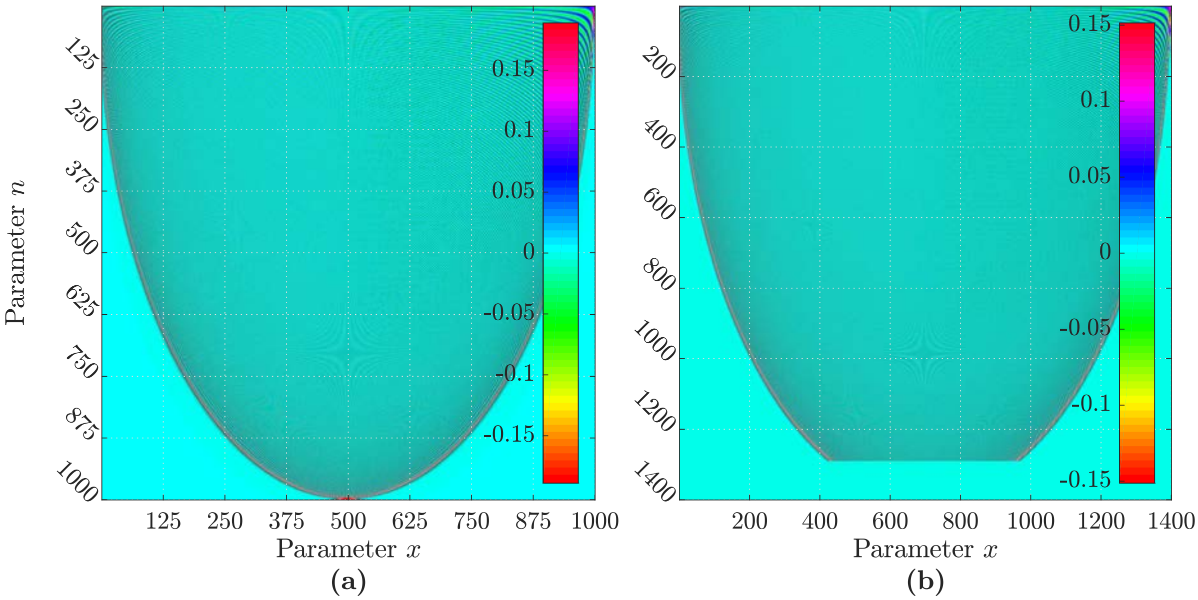

Fig. 1. **Computation of TPC values using the recurrence algorithm in x-direction for (a) $N = 1000$ (b) $N = 1400$. It can be seen that the coefficients tend to zero values for parameter $n > 1300$.**

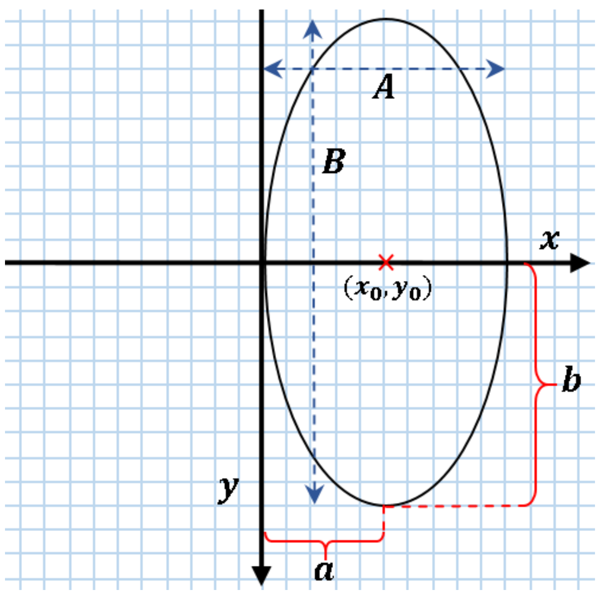

Fig. 2. **Vertical oval shape.**

Step 2: Compute the TPCs values in the range where $x = 0, 1, \ldots, \frac{N}{2} - 1$ and $n = \frac{N}{2}, \frac{N}{2} + 1, \ldots, N - 1$

(1) Find the location of parameter x, (l_x) using the mathematical formula of the lower half of the vertical oval shape shown in Fig. 2.
The standard form equation of an oval shape subject to the vertical axis is given by:

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$

where $a = A/2$ and $b = B/2$. As explained in the previous subsection, we derive the equation of the TPCs values shape where: $l_x = x, n = y, x_0 = N/2$, $y_0 = 0, a = N/2$, and $b = N$. The solution of the problem is to find the equation of l_x as a function of n for the lower half of the oval shape.

$$\frac{(l_x - N/2)^2}{(N/2)^2} + \frac{(n)^2}{N^2} = 1$$
$$\frac{(l_x - N/2)^2}{(N/2)^2} = 1 - \frac{(n)^2}{N^2}$$
$$(l_x - N/2)^2 = (N/2)^2 - (n/2)^2$$
$$(l_x - N/2) = \pm\sqrt{(N/2)^2 - (n/2)^2}$$
$$l_x = N/2 \pm \sqrt{(N/2)^2 - (n/2)^2}$$
$$l_x = \begin{cases} N/2 - \sqrt{(N/2)^2 - (n/2)^2}, \\ \text{for left half} \\ N/2 + \sqrt{(N/2)^2 - (n/2)^2}, \\ \text{for right half} \end{cases}$$

for our case, the equation for the left half is used.

$$l_x = 0.5\,N - \sqrt{(0.5\,N)^2 - (0.5\,n)^2} \tag{15}$$

(3) Find the TPCs values using n-direction recurrence (9) for the elements at $x = l_x, l_x + 1, \ldots, N/2 - 1$ and $n = \frac{N}{2}, \frac{N}{2} + 1, \ldots, N - 1$).

Step 3: Compute the TPCs values in the range where $n = \frac{N}{2}, \frac{N}{2} + 1, \ldots, N - 1$ and $x = l_x - 1, l_x - 2, \ldots, l_x - 12$ using the x-direction recurrence formula (11).

Note that, l_x is shifted by 12 to ensure that the polynomial coefficients are less than 10^{-5}.

As a summary, the mathematical formula of the proposed algorithm based on the combination of the recurrence in

$$t_n(x) = \begin{cases} \alpha_1 t_n(x-1) + \alpha_2 t_n(x-2), & \text{for } 0 < n < N/2-1 \text{ and } 2 < x < N/2-1 \\ \beta_1 t_{n-1}(x) + \beta_2 t_{n-2}(x), & \text{for } N/2 < n < N-1 \text{ and } l_x < x < N/2-1 \\ \alpha_1 t_n(x-1) + \alpha_2 t_n(x-2), & \text{for } N/2 < n < N-1 \text{ and } l_x < x < l_x - 12 \end{cases} \tag{16}$$

$n-$ and $x-$directions can be represented by (16), as shown at the top of this page.

Step 4: Compute the coefficients values for the second half of the polynomial array where $n = 0, 1, \ldots, N-1$ and $x = \frac{N}{2}, \frac{N}{2}+1, \ldots, N-1$ using the symmetry condition property defined in (14).

For more clarification, the procedure of the aforementioned steps is demonstrated in Fig. 3.

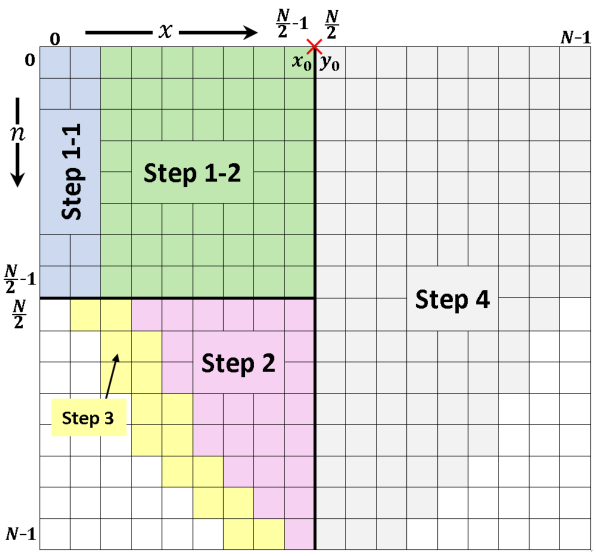

Fig. 3. Procedure steps of the proposed method.

Fig.4 shows the 3D plot of the TPCs computed by the proposed method for $N = 2000$. It is clear that the shape of the TPCs array is a square with a size of 2000×2000, and the non-zero polynomial coefficients (greater than 10^{-5}) are located inside an oval shape.

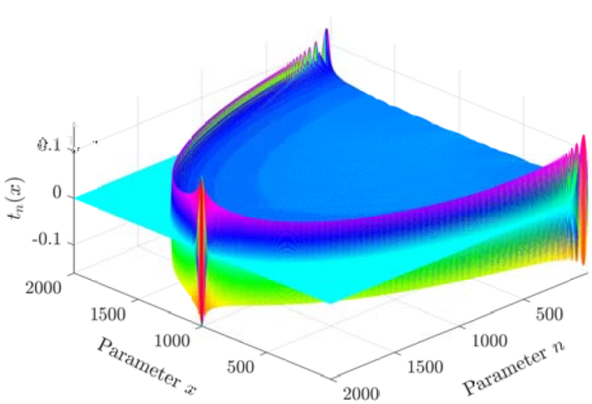

Fig. 4. 3D plot of the TPCs computed by the proposed algorithm for $N = 2000$.

V. EXPERIMENTAL RESULTS

This section illustrates the performance evaluation of the proposed method in terms of the computational cost, signal reconstruction, and a comparison with DCT.

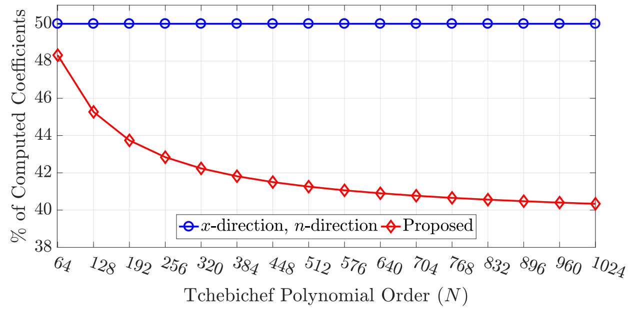

Fig. 5. The percentage of TPCs Computed for different values of *N*.

A. COMPUTATIONAL COST COMPARISON

For the proposed method, it can be seen that not all TPC values needed to be scanned and computed. Only the TPC values located inside the half of the oval shape are required to be generated. Fig. 5 shows the percentage ratio of the computed coefficients out of the total number of coefficients as a function of parameter N for the proposed method and the two existing algorithms (the three-term recurrence algorithms in the n and x-directions). Note that, the total number of polynomial coefficients in the polynomial array is $N \times N$. The percentage ratio of the computed coefficients out of the total number of coefficients is calculated using the following formula:

$$\%Ratio = \frac{Total\ computed\ coefficients}{N \times N} \times 100 \tag{17}$$

As shown in Fig. 5, the proposed method computes less number of TPCs than other algorithms. This property affects positively to reduce the numerical propagations error and speed up the process of recurrence relations. The two existing methods always compute 50% of the coefficients, and the other 50% of polynomial values can be generated using the property of symmetry of TP. For example, when $N = 512$, the total number of coefficients is 262144, only 108670 (41.45%) of the coefficients are needed to be computed by the proposed algorithm, whereas the other methods compute 131072 (50%) coefficients.

B. EXPERIMENTAL RESULTS FOR SPEECH SIGNAL

In this experiment, thirty clean (noise-free) speech signals taken from the standard NOIZEUS [24] database were used for evaluation. Each signal was divided into frames using a

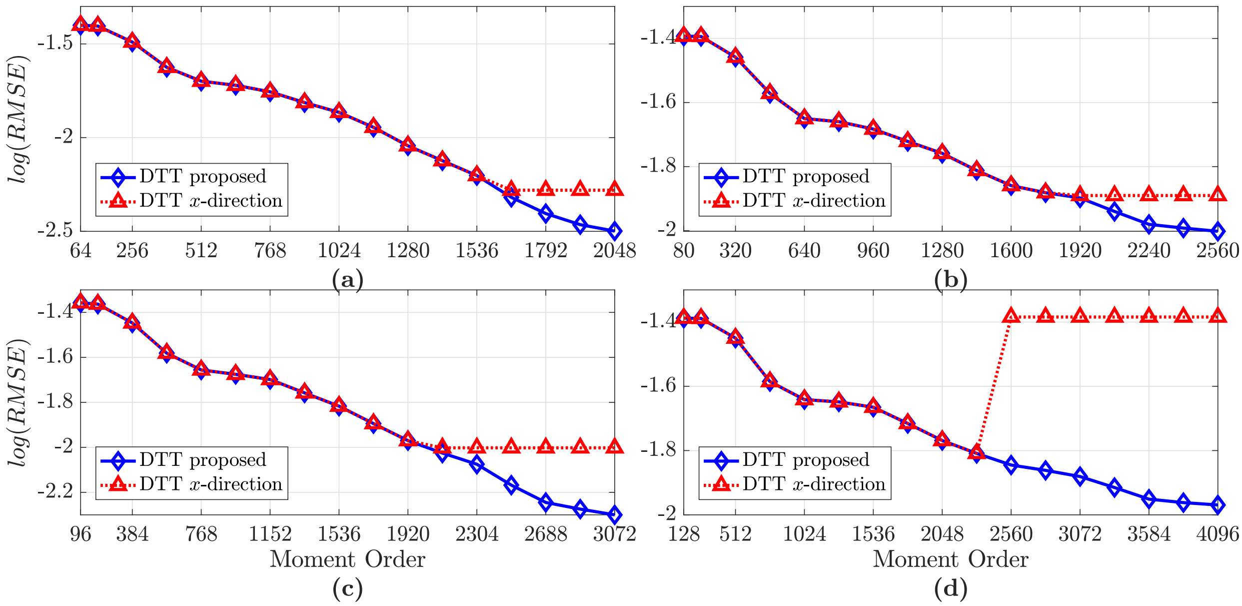

Fig. 6. A comparison of RMSE measure for x-direction and proposed algorithms with different frame size (a) N=2048 (b) N=2560 (c) N=3072 (d) N=4096. Note that the RMSE measure acheived by the *n*-direction recurrence is excluded from the figure as it gives unpractical values.

Hamming window with 50% overlap among adjacent frames. Each frame with a length of N samples was transformed into moment domain with a limited order of moments, M, using TPCs generated by the proposed method. Equation 5 was used for the transformation. The frames were reconstructed using the inverse formula given in (6). Finally, the reconstructed frames were combined using add-overlap technique [1] to reconstruct the speech signal. The root-mean-squared-error (RMSE) between the original and the estimated signals was reported. Different values of frame sizes, N were tested. The same procedure was repeated for the TPCs generated by the x-direction algorithm. Fig. 6 shows the averaged RMSE of the thirty signals as a function of moment order for the three algorithms. The n-direction algorithm was not used in this experiment as it fails to reconstruct the speech signal when the order is greater than $\sim$ 80 points.

From Fig. 6, it can be observed that the proposed algorithm outperforms the other algorithms in terms of RMSE measure. It is clear that the RMSE decreases gradually as a function of the moment order M until it reaches zero when M equals to the signal size, N. For the x-direction algorithm, it can be seen that this algorithm has the ability to reconstruct the audio signal with some error until a specific value of moment orders. For example, when N=4096 the x-direction algorithm failed to reconstruct the signal for $M > 2048$ as shown in Fig. 6d. On the other hand, the proposed algorithm produces less RMSE as a function of M which proves its superior ability to deal with very large frame size.

C. EXPERIMENTAL RESULTS OF IMAGES

In this experiment, 10 images taken from the well-known Live Image database [25] were used for performance evaluation of the proposed algorithms. Each image was converted

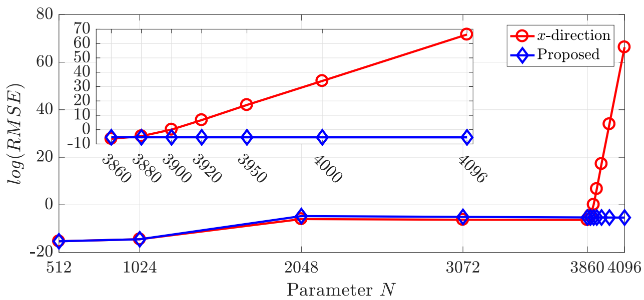

Fig. 7. A comparison between *x*-direction algorithm and proposed algorithm for 2-D signal.

into a gray scale and then resized to different sizes $512 \times 512, 1024 \times 1024, \ldots,$ and 4096×4096 points. For each size, the image was transformed into moment domain using DTT. Thereafter, the image was reconstructed using full moment order. The RMSE between the original and reconstructed images was reported. Fig. 7 shows the average RMSE of the 10 images as a function of image size. The experiment was performed for the proposed and the x-direction algorithms. The n-direction algorithm was not used in this experiment as it fails to reconstruct the image for image size greater than $\sim$ 80 $\times$ 80 points. Fig. 7 shows the plot of the RMSE values as a function of the image size. It is clear that the x-direction algorithm fails to reconstruct the image when its size is greater than $\sim$3900 $\times$ 3900 points. On the other hand, the proposed algorithm gives a high level of stability for a large size of 2-D signals.

D. PERFORMANCE COMPARISON BETWEEN DTT AND DCT

In order to compare the performance of the DTT computed by the proposed method with that of other discrete transforms

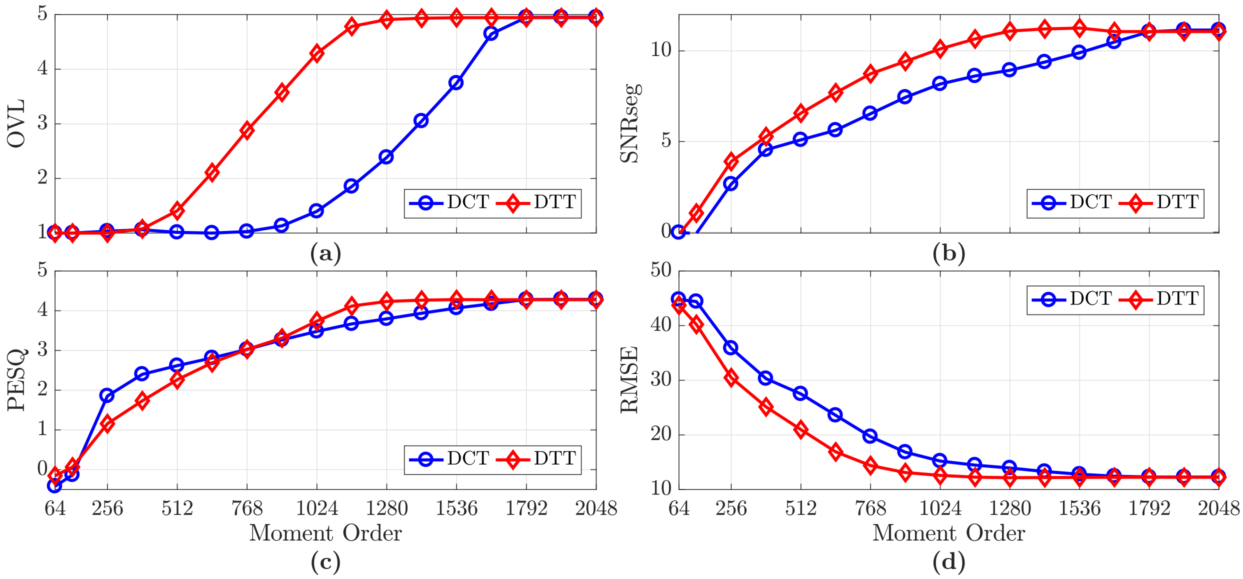

Fig. 8. Plots of quality scores as a function of number of coefficients used for reconstruction, (a) OVL, (b) SNRseg, (c) PESQ, and (d) RMSE. Note that, the frame size is 2048 samples.

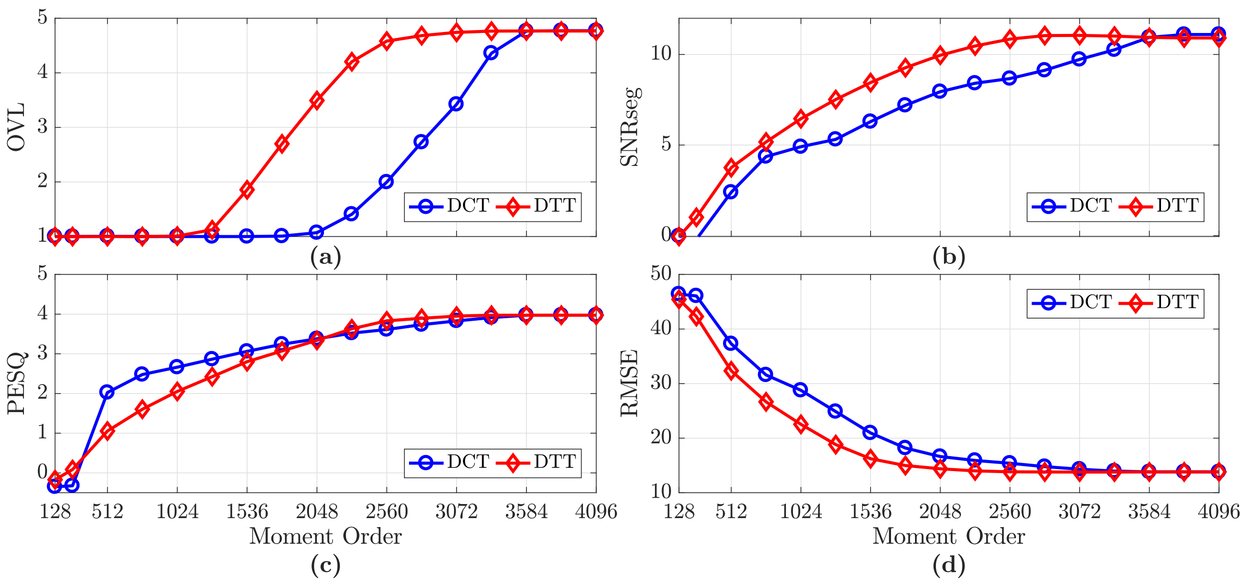

Fig. 9. Plots of quality scores as a function of number of coefficients used for reconstruction, (a) OVL, (b) SNRseg, (c) PESQ, and (d) RMSE. Note that, the frame size is 4096 samples.

such as discrete cosine transform (DCT), an experiment of speech signal compression was performed for the same thirty clean files from NOIZEUS [24] dataset. It is good to mention that DCT is considered a powerful tool since it provides a significantly higher energy compaction capability compared to other existent transforms. Each signal was divided into 2048-points frames using the Hamming window with 25% overlap, and each frame was then transformed into transform domains using DTT and DCT. For each transformed frame, a limited number of transform coefficients from the order moments were kept for reconstruction and the rest of the coefficients were set to zero. The frame was reconstructed using the inverse of transformation. All reconstructed frames were combined using the add-overlap technique to reconstruct the original signal.

Different objective measures of speech quality were employed to compare the similarity index between the original and corresponding reconstructed signals. These quality measures are: the overall quality (OVL), segmental signal-to-noise ratio (SNRseg) , perceptual evaluation of speech quality (PESQ), and RMSE [26], [27]. Note that, the higher values of OVL, SNRseg, and PESQ mean better performance in terms of speech quality. The averaged quality scores for the thirty signals were reported. The experiment was repeated for another frame size of 4096 samples. In this study the Matlab codes given in [24] were employed to compute the above mentioned speech quality scores.

Fig. 8 shows the averaged quality scores as a function of the number of transform coefficients used for reconstruction. The experiment was also repeated for the frame size of 4096 and the results are shown in Fig. 9. It is clear that the DTT computed by the proposed method achieved better compression results than DCT for all measures. For example, when the signal is reconstructed using 1024 out of 2048 coefficients (50% compression ratio) the OVL score achieved by DTT is 4.3, whereas DCT achieved OVL score of 1.4 for the same compression ratio as shown in Fig. 8a. Fig.9 shows the objective measures score used when the frame size is 2048 but for frame size of 4096. it is evident from Fig. 9b that the achieved OVL score for DTT is 3.5 while for DCT it is 1.1 for 50% compression ratio.

VI. CONCLUSION

In this paper, a new recurrence algorithm to compute the TP is proposed. The proposed method is based on a combination of two conventional algorithms which are: the three-term recurrence algorithm in the n and x directions. The proposed algorithm is able to generate the polynomial coefficients for high length of signal. Furthermore, it has the ability to reduce the computation cost of TPCs calculation which affects positively to increase the speed of polynomial coefficients calculation. A comparative study was performed to represent the promising feature and superior capability of the proposed algorithm. The proposed algorithm is found to achieve better signal reconstruction results than the two other recurrence algorithms for high polynomial orders. However, the proposed method could be modified to reduce the reconstruction error especially in 2-D signals by utilizing an adaptive shift for each signal size rather than the fixed shift which is used in the proposed method.

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