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Decentralized Fixed-Order Piecewise Affine Dynamic Output Feedback Controller Design for Discrete-Time Nonlinear Large-Scale Systems

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ABSTRACT This paper proposes a novel decentralized robust $\mathscr{H}_\infty$ fixed-order dynamic output feedback (DOF) controller design approach for discrete-time nonlinear large-scale systems via Takagi-Sugeno fuzzy-affine models. By a state-input augmentation method and piecewise quadratic Lyapunov functions, some sufficient conditions for decentralized fixed-order piecewise affine DOF controller synthesis are given. It is shown that by some convexification techniques, the controller gains can be obtained by solving a set of linear matrix inequalities. Two simulation examples are carried out to verify the effectiveness of the proposed design method.

INDEX TERMS Large-scale systems, fuzzy-affine models, robust control, dynamic output feedback.

I. INTRODUCTION

Large-scale systems (LSSs), which consist of a group of nonlinear subsystems with interconnections, widely exist in practical applications such as industrial processes, spacecraft systems, communication networks, and electrical power grids [1], [2]. The strong interconnections in LSSs bring many difficulties in system analysis and control. During the past years, a number of significant results have been reported [3]–[9].

On another research frontier, last few decades have witnessed fast developments of fuzzy logic control (FLC) technique from academic studies to industrial applications [10], [11]. FLC has been recognized as a very efficient approach to control highly complex nonlinear plants or even nonanalytic systems. Among various fuzzy control strategies, the Takagi-Sugeno (T-S) fuzzy-model-based method has attracted tremendous attention from the control community [12]–[20]. Through fuzzy membership functions, a set of local linear or affine models are smoothly connected to approximate nonlinear systems to arbitrary degrees of accuracy in any convex compact region [21], [22]. In recent years, there have appeared some results on analysis and synthesis for fuzzy-model-based nonlinear large-scale systems [23]–[29]. To mention a few, [25] designed a decentralized parallel distributed compensation (PDC) fuzzy controller based on a common Lyapunov function. Liu and Zhang [28] studied the stability analysis of continuous-time fuzzy large-scale systems with time-varying delays and parameter uncertainties. In [29], a decentralized fuzzy observer-based output feedback control method was proposed for large-scale nonlinear systems.

It is worth mentioning that the aforementioned results for fuzzy-model-based large-scale systems were mainly derived in the framework of common quadratic Lyapunov function (CQLF). To reduce the conservatism, more recently there have also been some results on analysis and synthesis for LSSs based on piecewise quadratic Lyapunov functions (PQLFs). Zhang *et al.* [30] and Zhang and Feng [31] addressed the problem of fuzzy state feedback controller design for both continuous-time and discrete-time large-scale systems on the basis of piecewise quadratic Lyapunov functions. Nevertheless, these results were obtained based on fuzzy systems with linear local models, while fuzzy dynamic models with offset terms have been shown with substantially enhanced function approximation competence [17]. Furthermore, most existing results on decentralized controller design for fuzzy LSSs are in state feedback form while few attention has been focused on the general output feedback control case. To the authors' best knowledge, the problem of fixed-order dynamic output feedback controller design for fuzzy-affine-model-based large-scale nonlinear systems has not been fully investigated and remains important and challenging, which motivates us for this study.

In this paper, we propose a decentralized robust $\mathscr{H}_\infty$ fixed-order dynamic output feedback (DOF) controller design approach for discrete-time nonlinear large-scale systems based on piecewise quadratic Lyapunov functions. Specifically, the nonlinear subsystems of LSSs are represented by T-S fuzzy models with affine terms. Through utilizing a state-input augmentation method, the closed-loop system is firstly transformed into a descriptor fuzzy affine system, which eliminates the couplings between the piecewise affine controller gains and system matrices. Based on PQLFs and some convexification techniques, sufficient conditions in terms of a set of linear matrix inequalities (LMIs) are obtained.

The rest of this paper is organized as follows. The large-scale fuzzy affine system model description and decentralized DOF controller design problem formulation are given in Section II. Section III presents the main results for controller analysis and synthesis. Simulation examples are shown in Section IV to verify the feasibility and effectiveness of the proposed approach. Finally, the conclusions are given in Section V.

Notations: The notations used throughout this paper are standard. $\mathscr{R}^n$ denotes the n-dimensional Euclidean space. A real symmetric matrix $P > 0(\geq 0)$ denotes P being positive definite (positive semidefinite). $\mathbf{I}_n$ and $\mathbf{0}_{m\times n}$ are used to denote the $n \times n$ identity matrix and $m \times n$ zero matrix, respectively. The subscripts n and $m \times n$ are omitted when the size is not relevant or can be determined from the context. The short hand diag $\{X_1, X_2......X_l\}$ denotes a block diagonal matrix, with diagonal blocks being the matrices $X_1, X_2......X_l$. $\mathtt{Sym}\{A\}$ is the shorthand notation for $A + A^{\mathrm{T}}$. The notation $\star$ is used to indicate the terms that can be induced by symmetry.

II. MODEL DESCRIPTION AND PROBLEM FORMULATION

A. T-S FUZZY AFFINE LARGE-SCALE DYNAMIC MODELS

Consider the following discrete-time nonlinear large-scale system S consisting of N interconnected nonlinear subsystems $S_i, i = 1, 2, \cdots, N$. Each nonlinear subsystem can be represented by T-S fuzzy affine dynamic models as follows,

Plant Rule $\mathscr{R}_i^l$: IF $\zeta_{i1}(t)$ is $\mathscr{F}_{i1}^l$ and $\zeta_{i2}(t)$ is $\mathscr{F}_{i2}^l$ and $\cdots$ and $\zeta_{i\varphi}(t)$ is $\mathscr{F}_{i\varphi}^l$, THEN

$$\begin{cases} x_i(t+1) = A_{il}x_i(t) + a_{il} + B_{il}u_i(t) + D_{i1l}w_i(t) \\ \qquad\qquad + \sum\limits_{\substack{n=1 \\ n\neq i}}^{N} C_{ni}x_n(t) \\ y_i(t) = H_{il}x_i(t) + D_{i2l}w_i(t) \\ z_i(t) = L_{il}x_i(t) + N_{il}u_i(t), \; l \in \mathscr{L}_i := \{1, 2, \cdots, r_i\}, \end{cases} \tag{1}$$

where $\mathscr{R}_i^l$ denotes the l-th fuzzy inference rule for the i-th nonlinear subsystem; r_i is the number of inference rules; $\mathscr{F}_{i\phi}^l(\phi = 1, 2, \cdots, \varphi)$ are fuzzy sets; $\zeta_i(t) := [\zeta_{i1}(t), \zeta_{i2}(t), \cdots, \zeta_{i\varphi}(t)]$ are the premise variables for the i-th subsystem S_i; $i \in \mathcal{N} := \{1, 2, \cdots, N\}$; $x_i(t) \in \Re^{n_{xi}}$ is the state; $u_i(t) \in \Re^{n_{ui}}$ is the control input; $z_i(t) \in \Re^{n_{zi}}$ is the regulated output; C_{ni} represents the interconnection terms between the n-th subsystem and the i-th subsystem. The disturbance $w_i(t)$ is assumed to be norm-bounded. $(A_{il}, a_{il}, B_{il}, D_{i1l}, C_{ni}, H_{il}, D_{i2l}, L_{il}, N_{il})$ denotes the l-th fuzzy local model of the subsystem S_i.

Remark 1: It can be seen that the system models described in (1) are in fact affine systems rather than linear models. An additional offset term a_{il} is involved. It is noted that this type of model is more powerful for approximation of nonlinear systems.

Then let $\mu_{il}[\zeta_i(t)]$ denote the normalized membership function of the inferred fuzzy set $\mathscr{F}_i^l := \prod_{\phi=1}^{\varphi} \mathscr{F}_{i\phi}^l(t)$ and

$$\mu_{il}[\zeta_i(t)] := \frac{\prod_{\phi=1}^{\varphi} \mu_{il\phi}\left[\zeta_{i\phi}(t)\right]}{\sum_{\alpha=1}^{r_i} \prod_{\phi=1}^{\varphi} \mu_{i\alpha\phi}\left[\zeta_{i\phi}(t)\right]} \geq 0,$$
$$\sum_{l=1}^{r_i} \mu_{il}[\zeta_i(t)] = 1, \tag{2}$$

where $\mu_{il\phi}\left[\zeta_{i\phi}(t)\right]$ is the grade of membership of $\zeta_{i\phi}(t)$ in $\mathscr{F}_{i\phi}^l$. In the sequel, the argument of $\mu_{il}[\zeta_i(t)]$ will be dropped for the situations without ambiguity, i.e., $\mu_{il} := \mu_{il}[\zeta_i(t)]$ for brevity.

Through utilizing center-average defuzzifier, product-fuzzy inference, and singleton fuzzifier, we obtain the following global T-S fuzzy model,

$$\begin{cases} x_i(t+1) = A_i(\mu_i)x_i(t) + a_i(\mu_i) + B_i(\mu_i)u_i(t) \\ \qquad\qquad + D_{i1}(\mu_i)w_i(t) + \sum\limits_{\substack{n=1 \\ n\neq i}}^{N} C_{ni}x_n(t) \\ y_i(t) = H_i(\mu_i)x_i(t) + D_{i2}(\mu_i)w_i(t) \\ z_i(t) = L_i(\mu_i)x(t) + N_i(\mu_i)u_i(t), \end{cases} \tag{3}$$

where

$$\begin{cases} A_i(\mu_i) = \sum\limits_{l=1}^{r_i} \mu_{il}A_{il}, & a_i(\mu_i) = \sum\limits_{l=1}^{r_i} \mu_{il}a_{il}, \\ B_i(\mu_i) = \sum\limits_{l=1}^{r_i} \mu_{il}B_{il}, & D_{i1}(\mu_i) = \sum\limits_{l=1}^{r_i} \mu_{il}D_{i1l}, \\ L_i(\mu_i) = \sum\limits_{l=1}^{r_i} \mu_{il}L_{il}, & N_i(\mu_i) = \sum\limits_{l=1}^{r_i} \mu_{il}N_{il}, \\ H_i(\mu_i) = \sum\limits_{l=1}^{r_i} \mu_{il}H_{il}, & D_{i2}(\mu_i) = \sum\limits_{l=1}^{r_i} \mu_{il}D_{i2l}. \end{cases} \tag{4}$$

In this paper, we aim to deal with the fixed-order dynamic output feedback controller design problem of fuzzy large-scale system in (1) via piecewise quadratic Lyapunov functions. Because the fuzzy rules induce a polyhedral partition of the system state-space, the global model in (3) can be viewed as a convex combination of local models in individual regions. Similar to [14], the premise-variable space can be decomposed into two kinds of regions: crisp regions and fuzzy regions. The crisp region is the region that possesses only one rule. That is, for some l, $\mu_{il}[\zeta_i(t)] = 1$ and all other membership functions are zero. The system dynamics in the crisp region are governed by the l-th fuzzy local model of (1). The fuzzy regions denote the regions where $0 < \mu_{il}[\zeta_i(t)] < 1$ and the system dynamics are characterized by a convex combination of some local models.

Thus, we denote $\{\mathscr{S}_{ij}\}_{j\in\mathcal{I}_i}$ to be the premise variable space partition for the i-th subsystem, where $\mathcal{I}_i$ stands for the set of region indices. With the state-space partition, one can reformulate the subsystem S_i into the following piecewise fuzzy affine model,

$$\begin{cases} x_i(t+1) = \mathcal{A}_{ij}x_i(t) + \mathrm{a}_{ij} + \mathcal{B}_{ij}u_i(t) + \mathcal{D}_{i1j}w_i(t) \\ \qquad\qquad + \sum_{\substack{n=1\\n\neq i}}^{N} C_{ni}x_n(t) \\ y_i(t) = \mathcal{H}_{ij}x_i(t) + \mathcal{D}_{i2j}w_i(t) \\ z_i(t) = \mathcal{L}_{ij}x_i(t) + \mathcal{N}_{ij}u_i(t),\ \zeta_i(t) \in \mathscr{S}_{ij}, j \in \mathcal{I}_i, i \in \mathcal{N}, \end{cases} \tag{5}$$

where

$$\begin{cases} \mathcal{A}_{ij} := \sum_{m\in\mathcal{N}_i(j)} \mu_{im}A_{im}, & \mathrm{a}_{ij} := \sum_{m\in\mathcal{N}_i(j)} \mu_{im}a_{im}, \\ \mathcal{B}_{ij} := \sum_{m\in\mathcal{N}_i(j)} \mu_{im}B_{im}, & \mathcal{D}_{i1j} := \sum_{m\in\mathcal{N}_i(j)} \mu_{im}D_{i1m}, \\ \mathcal{L}_{ij} := \sum_{m\in\mathcal{N}_i(j)} \mu_{im}L_{im}, & \mathcal{N}_{ij} := \sum_{m\in\mathcal{N}_i(j)} \mu_{im}N_{im}, \\ \mathcal{H}_{ij} := \sum_{m\in\mathcal{N}_i(j)} \mu_{im}H_{im}, & \mathcal{D}_{i2j} := \sum_{m\in\mathcal{N}_i(j)} \mu_{im}D_{i2m}, \end{cases} \tag{6}$$

with $0 < \mu_{im} < 1$ and $\sum_{m\in\mathcal{N}_i(j)} \mu_{im} = 1$. The set $\mathcal{N}_i(j) := \{m|\mu_{im}[\zeta_i(t)] > 0,\ m \in \mathscr{L}_i,\ \zeta_i(t) \in \mathscr{S}_{ij},\ j \in \mathcal{I}_i,\ i \in \mathcal{N}\}$ describes the indices for the local models used in the interpolation within each local region $\mathscr{S}_{ij}$. Furthermore, $\mathcal{I}_i$ can be divided into two parts: one part is $\mathcal{I}_{i0}$ which refers to the index set of regions containing the origin, while $\mathcal{I}_{i1}$ is the index set of regions otherwise. Note that for all $j \in \mathcal{I}_{i0}$, $\mathrm{a}_{ij} = 0$. It is easy to see that $\mathcal{I}_{i0}$ contains only one element for a crisp region.

For future use, a new set Ω_i is introduced to describe all possible region transitions for the i-th subsystem,

$$\Omega_i := \{(j, s)|\zeta_i(t) \in \mathscr{S}_{ij}, \zeta_i(t+1) \in \mathscr{S}_{is}, j, s \in \mathcal{I}_i\}. \tag{7}$$

It is assumed in this paper that each polyhedral region $\mathscr{S}_{ij}$ can be outer approximated by an ellipsoid R_{ij} [21], i.e., there exist matrices F_{ij} and f_{ij} such that

$$\mathscr{S}_{ij} \subseteq R_{ij}, \quad R_{ij} := \{x_i|\|F_{ij}x_i + f_{ij}\| \leq 1\}. \tag{8}$$

This covering is very useful when $\mathscr{S}_{ij}$ are slab regions. Because of this case, the parameters F_{ij} and f_{ij} are guaranteed to exist, and the covering is exact, i.e.,$\mathscr{S}_{ij} \subseteq R_{ij}$ and $R_{ij} \subseteq \mathscr{S}_{ij}$. Specifically, if the polyhedral regions $\mathscr{S}_{ij}$ are slabs of the following form,

$$\mathscr{S}_{ij} = \{x_i|\alpha_{ij} \leq \theta_{ij}^{\mathrm{T}}x_i \leq \beta_{ij}\}, \quad j \in \mathcal{I}_i, \tag{9}$$

where $\alpha_{ij}, \beta_{ij} \in \mathscr{R}$, $\theta_{ij} \in \mathscr{R}^{n_{xi}}$, and then each slab region can be exactly described by a degenerate ellipsoid as in (9) with

$$F_{ij} = \frac{2\theta_{ij}^{\mathrm{T}}}{\beta_{ij} - \alpha_{ij}}, \quad f_{ij} = -\frac{\beta_{ij} + \alpha_{ij}}{\beta_{ij} - \alpha_{ij}}. \tag{10}$$

Then we have the following relationship for each ellipsoid region:

$$\begin{bmatrix} x_i(t) \\ 1 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} F_{ij}^{\mathrm{T}}F_{ij} & F_{ij}^{\mathrm{T}}f_{ij} \\ \star & f_{ij}^{\mathrm{T}}f_{ij} - 1 \end{bmatrix} \begin{bmatrix} x_i(t) \\ 1 \end{bmatrix} \leq 0, \quad j \in \mathcal{I}_i. \tag{11}$$

B. DECENTRALIZED FIXED-ORDER DYNAMIC OUTPUT FEEDBACK CONTROLLER

For the fuzzy large-scale system (5) in each region, we consider the following decentralized piecewise fixed-order dynamic output feedback (DOF) controller,

$$\begin{cases} x_{ci}(t+1) = A_{cij}x_{ci}(t) + a_{cij} + B_{cij}y_i(t) \\ u_i(t) = K_{cij}x_{ci}(t) + k_{cij} + D_{cij}y_i(t), j \in \mathcal{I}_i, i \in \mathcal{N}, \end{cases} \tag{12}$$

where $x_{cij} \in \Re^{n_{ci}}$, $0 \leq n_{ci} \leq n_{xi}$ is the controller states, $A_{cij} \in \Re^{n_{ci}\times n_{ci}}$, $B_{cij} \in \Re^{n_{ci}\times n_{yi}}$, $K_{cij} \in \Re^{n_{ui}\times n_{ci}}$, $D_{cij} \in \Re^{n_{ui}\times n_{yi}}$, $a_{cij} \in \Re^{n_{ci}}$, and $k_{cij} \in \Re^{n_{ui}}$ are controller gains to be determined. Note that $a_{cij} = 0$ and $k_{cij} = 0$ for $j \in \mathcal{I}_{i0}$. It is also worth mentioning that when $n_{ci} = n_{xi}$, (12) refers to a full-order dynamic output feedback (DOF) controller. While a reduced-order controller is characterized as $n_{ci} < n_{xi}$. In particular, (12) reduces to a static output feedback controller if $n_{ci} = 0$.

Applying the DOF controller (12) to system (5), one can obtain the following closed-loop system,

$$\begin{cases} x_i(t+1) = \mathcal{A}_{ij}x_i(t) + \mathrm{a}_{ij} + \mathcal{B}_{ij}u_i(t) + \mathcal{D}_{i1j}w_i(t) \\ \qquad\qquad + \sum_{\substack{n=1\\n\neq i}}^{N} C_{ni}x_n(t) \\ x_{ci}(t+1) = A_{cij}x_{ci}(t) + a_{cij} + B_{cij}\mathcal{H}_{ij}x_i(t) \\ \qquad\qquad + B_{cij}\mathcal{D}_{i2j}w_i(t) \\ 0 \cdot u_i(t+1) = K_{cij}x_{ci}(t) + k_{cij} - u_i(t) + D_{cij}\mathcal{H}_{cij}x_i(t) \\ \qquad\qquad + D_{cij}\mathcal{D}_{i2j}w_i(t) \\ z_i(t) = \mathcal{L}_{ij}x_i(t) + \mathcal{N}_{ij}u_i(t),\ \zeta_i(t) \in \mathscr{S}_{ij}, j \in \mathcal{I}_i,\ i \in \mathcal{N}. \end{cases} \tag{13}$$

Define $\bar{x}_i(t) = [\,x_i^{\mathrm{T}}(t)\,x_{ci}^{\mathrm{T}}(t)\,u_i^{\mathrm{T}}(t)]^{\mathrm{T}}$ and reformulate system (13) into the following descriptor form,

$$\begin{cases} E \cdot \bar{x}_i(t+1) = \bar{\mathcal{A}}_{ij}\bar{x}_i(t) + \bar{\mathrm{a}}_{ij} + \bar{\mathcal{D}}_{ij}w_i(t) \\ \qquad\qquad + J\sum_{\substack{n=1\\n\neq i}}^{N} C_{ni}x_n(t) \\ z_i(t) = \bar{\mathcal{L}}_{ij}\bar{x}_i(t),\ \zeta_i(t) \in \mathscr{S}_{ij}, j \in \mathcal{I}_i, i \in \mathcal{N}, \end{cases} \tag{14}$$

where

$$\begin{cases} E = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \\ \bar{\mathcal{A}}_{ij} = \begin{bmatrix} \mathcal{A}_{ij} & \mathbf{0} & \mathcal{B}_{ij} \\ B_{cij}\mathcal{H}_{ij} & A_{cij} & \mathbf{0} \\ D_{cij}\mathcal{H}_{ij} & K_{cij} & -\mathbf{I} \end{bmatrix}, \\ \bar{\mathrm{a}}_{ij} = \begin{bmatrix} \mathrm{a}_{ij} \\ a_{cij} \\ k_{cij} \end{bmatrix}, \quad \bar{\mathcal{D}}_{ij} = \begin{bmatrix} \mathcal{D}_{i1j} \\ B_{cij}\mathcal{D}_{i2j} \\ D_{cij}\mathcal{D}_{i2j} \end{bmatrix}, \\ J = \begin{bmatrix} \mathbf{I}_{n_x} \\ \mathbf{0}_{n_c \times n_x} \\ \mathbf{0}_{n_u \times n_x} \end{bmatrix}, \quad \bar{\mathcal{L}}_{ij} = \begin{bmatrix} \mathcal{L}_{ij} & \mathbf{0} & \mathcal{N}_{ij} \end{bmatrix}, \end{cases} \tag{15}$$

and specifically, we have $x_i(t) = J \cdot \bar{x}_i(t)$.

It is worth mentioning that in this paper, through applying the x-u augmentation approach, the controller gains have been decoupled from the system matrices. It will be shown in the sequel that this feature enables one to design the fixed-order piecewise affine dynamic output feedback controller in convex optimization framework based on linear matrix inequalities.

The objective of this paper is to design a fixed-order DOF controller in the form of (12) such that the resulting closed-loop system is asymptotically stable with a guaranteed robust $\mathscr{H}_\infty$ performance γ. To be more specific, for a prescribed disturbance attenuation level $\gamma > 0$, design a DOF controller (12) such that the induced l_2-norm of the operator form w to the regulated output z is less than γ,

$$\|z(t)\|_2 < \gamma \|w(t)\|_2 \tag{16}$$

under zero initial condition for all nonzero $w_i(t) \in l_2[0, \infty]$, where the regulated output $z(t) := [z_1^{\mathrm{T}}(t), z_2^{\mathrm{T}}(t), \cdots, z_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, and the disturbance $w(t) := [w_1^{\mathrm{T}}(t), w_2^{\mathrm{T}}(t), \cdots, w_N^{\mathrm{T}}(t)]^{\mathrm{T}}$.

III. MAIN RESULTS

In this section, based on piecewise Lyapunov functions and some convexification techniques, some new results will be proposed to the decentralized robust $\mathscr{H}_\infty$ fixed-order dynamic output feedback controller design for large-scale system (1).

A. FIXED-ORDER DOF CONTROLLER ANALYSIS AND SYNTHESIS

Theorem 2: Consider the large-scale fuzzy system in (1). For a given scalar $\varepsilon_0 > 0$, the closed-loop system in (14) is asymptotically stable with a guaranteed $\mathscr{H}_\infty$ robust performance γ, if there exist matrices $0 < P_{ij} = P_{ij}^T \in \Re^{(n_{xi}+n_{ci}+n_{ui})\times(n_{xi}+n_{ci}+n_{ui})}$, $G_{1ij1}, G_{2ij1} \in \Re^{n_{xi}\times n_{xi}}$, $G_{1ij2}, G_{2ij2} \in \Re^{n_{ci}\times n_{xi}}$, $G_{1ij3}, G_{2ij3} \in \Re^{n_{ui}\times n_{xi}}$, $G_{1ij4} \in \Re^{n_{ci}\times n_{ci}}$, $G_{1ij5} \in \Re^{n_{ui}\times n_{ui}}$, $G_{4ij1} \in \Re^{n_{wi}\times n_{xi}}$, $\bar{A}_{cij} \in \Re^{n_{ci}\times n_{ci}}$, $\bar{B}_{cij} \in \Re^{n_{ci}\times n_{yi}}$, $\bar{K}_{cij} \in \Re^{n_{ui}\times n_{ci}}$, $\bar{D}_{cij} \in \Re^{n_{ui}\times n_{yi}}$, for $j \in \mathcal{I}_i$, $G_{3ij1} \in \Re^{1\times n_{xi}}$, $\bar{a}_{cij} \in \Re^{n_{ci}}$, $\bar{k}_{ij} \in \Re^{n_{ui}}$, for $j \in \mathcal{I}_{i1}$, scalars $\varepsilon_{inj} > 0$, $i, n \in \mathcal{N}$, $n \neq i$, and $\lambda_{ij} < 0, j \in \mathcal{I}_{i1}$ such that the following linear matrix inequalities hold,

$$\begin{bmatrix} \Theta_{ijsm} & \mathcal{G}_{ij}J\mathscr{C}_i \\ \star & -\mathscr{M}_{ij} \end{bmatrix} < 0,$$
$$m \in \mathcal{N}_i(j), \quad j \in \mathcal{I}_{i0}, (j, s) \in \Omega_i, \ i \in \mathcal{N}, \tag{17}$$

$$\begin{bmatrix} \bar{\Theta}_{ijsm} & \bar{\mathcal{G}}_{ij}J\mathscr{C}_i \\ \star & -\mathscr{M}_{ij} \end{bmatrix} < 0,$$
$$m \in \mathcal{N}_i(j), \quad j \in \mathcal{I}_{i1}, (j, s) \in \Omega_i, \ i \in \mathcal{N}, \tag{18}$$

where

$$\Theta_{ijsm} = \begin{bmatrix} P_{is} - G_{1ij} - G_{1ij}^{\mathrm{T}} & \Theta_{ijm}^{(12)} & \Theta_{ijm}^{(13)} \\ \star & \Theta_{ijm}^{(22)} & \Theta_{ijm}^{(23)} \\ \star & \star & \Theta_{ijm}^{(33)} \end{bmatrix},$$

$$\bar{\Theta}_{ijsm} = \begin{bmatrix} P_{is} - G_{1ij} - G_{1ij}^{\mathrm{T}} & \bar{\Theta}_{ijm}^{(12)} & \bar{\Theta}_{ijm}^{(13)} & \bar{\Theta}_{ijm}^{(14)} \\ \star & \bar{\Theta}_{ijm}^{(22)} & \bar{\Theta}_{ijm}^{(23)} & \bar{\Theta}_{ijm}^{(24)} \\ \star & \star & \bar{\Theta}_{ijm}^{(33)} & \bar{\Theta}_{ijm}^{(34)} \\ \star & \star & \star & \bar{\Theta}_{ijm}^{(44)} \end{bmatrix},$$

$$G_{1ij} = \begin{bmatrix} G_{1ij1} & \hat{H}_1 G_{1ij4} & \hat{H}_2 G_{1ij5} \\ G_{1ij2} & \delta_1 G_{1ij4} & 0 \\ G_{1ij3} & 0 & \rho_1 G_{1ij5} \end{bmatrix},$$

$$G_{2ij} = \begin{bmatrix} G_{2ij1} & \delta_2 \hat{H}_1 G_{1ij4} & \rho_2 \hat{H}_2 G_{1ij5} \\ G_{2ij2} & \delta_3 G_{1ij4} & 0 \\ G_{2ij3} & 0 & \rho_3 G_{1ij5} \end{bmatrix},$$

$$G_{3ij} = \begin{bmatrix} G_{3ij1} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad G_{4ij} = \begin{bmatrix} G_{4ij1} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\hat{H}_1 = \begin{bmatrix} \mathbf{I}_{n_{ci}} \\ \mathbf{0}_{(n_{xi}-n_{ci})\times n_{ci}} \end{bmatrix}, \quad \hat{H}_2 = \begin{bmatrix} \mathbf{I}_{n_{ui}} \\ \mathbf{0}_{(n_{xi}-n_{ui})\times n_{ui}} \end{bmatrix},$$

$$\mathcal{G}_{ij} = \begin{bmatrix} G_{1ij}^{\mathrm{T}} & G_{2ij}^{\mathrm{T}} & G_{4ij}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$

$$\bar{\mathcal{G}}_{ij} = \begin{bmatrix} G_{1ij}^{\mathrm{T}} & G_{2ij}^{\mathrm{T}} & G_{3ij}^{\mathrm{T}} & G_{4ij}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$

$$\mathscr{C}_i = [C_{i1} \cdots C_{i(i-1)}, C_{i(i+1)}, \cdots C_{iN}],$$
$$\mathscr{M}_{ij} = \mathrm{diag}\{\varepsilon_{i1j}\mathbf{I}, \cdots, \varepsilon_{i(i-1)j}\mathbf{I}, \varepsilon_{i(i+1)j}\mathbf{I}, \cdots, \varepsilon_{iNj}\mathbf{I}\},$$

$$\Theta_{ijm}^{(12)} = \Theta_{ijm}^{(121)} - G_{2ij}^{\mathrm{T}},$$

$$\Theta_{ijm}^{(121)} = \begin{bmatrix} G_{1ij1}A_{im} + \hat{H}_1 \bar{B}_{cij} H_{im} + \hat{H}_2 \bar{D}_{cij} H_{im} & \hat{H}_1 \bar{A}_{cij} + \hat{H}_2 \bar{K}_{cij} & G_{1ij1}B_{im} - \hat{H}_2 G_{1ij5} \\ G_{1ij2}A_{im} + \delta_1 \bar{B}_{cij} H_{im} & \delta_1 \bar{A}_{cij} & G_{1ij2}B_{im} \\ G_{1ij3}A_{im} + \rho_1 \bar{D}_{cij} H_{im} & \rho_1 \bar{K}_{cij} & G_{1ij3}B_{im} - \rho_1 G_{1ij5} \end{bmatrix},$$

$$\Theta_{ijm}^{(13)} = \Theta_{ijm}^{(131)} - G_{4ij}^{\mathrm{T}},$$

$$\Theta_{ijm}^{(131)} = \begin{bmatrix} G_{1ij1}D_{i1m} + \hat{H}_1 \bar{B}_{cij} D_{i2m} + \hat{H}_2 \bar{D}_{cij} D_{i2m} \\ G_{1ij2}D_{i1m} + \delta_1 \bar{B}_{cij} D_{i2m} \\ G_{1ij3}D_{i1m} + \rho_1 \bar{D}_{cij} D_{i2m} \end{bmatrix},$$

$$\Theta_{ijm}^{(22)} = -E^{\mathrm{T}} P_{ij} E + \bar{L}_{im}^{\mathrm{T}} \bar{L}_{im} + \varepsilon_0 (N-1) J J^{\mathrm{T}} + \mathtt{Sym}\left\{\Theta_{ijm}^{(221)}\right\},$$

$$\Theta_{ijm}^{(221)} = \begin{bmatrix} G_{2ij1}A_{im} + \delta_2\hat{H}_1\bar{B}_{cij}H_{im} + \rho_2\hat{H}_2\bar{D}_{cij}H_{im} & \delta_2\hat{H}_1\bar{A}_{cij} + \rho_2\hat{H}_2\bar{K}_{cij} & G_{2ij1}B_{im} - \rho_2\hat{H}_2G_{1ij5} \\ G_{2ij2}A_{im} + \delta_3\bar{B}_{cij}H_{im} & \delta_3\bar{A}_{cij} & G_{2ij2}B_{im} \\ G_{2ij3}A_{im} + \rho_3\bar{D}_{cij}H_{im} & \rho_3\bar{K}_{cij} & G_{2ij3}B_{im} - \rho_3G_{1ij5} \end{bmatrix},$$
$$\Theta_{ijm}^{(23)} = \Theta_{ijm}^{(231)} + \left\{\Theta_{ijm}^{(232)}\right\}^{\mathrm{T}},$$
$$\Theta_{ijm}^{(231)} = \begin{bmatrix} G_{2ij1}D_{i1m} + \delta_2\hat{H}_1\bar{B}_{cij}D_{i2m} + \rho_2\hat{H}_2\bar{D}_{cij}D_{i2m} \\ G_{2ij2}D_{i1m} + \delta_3\bar{B}_{cij}D_{i2m} \\ G_{2ij3}D_{i1m} + \rho_3\bar{D}_{cij}D_{i2m} \end{bmatrix},$$
$$\Theta_{ijm}^{(232)} = \begin{bmatrix} G_{4ij1}A_{im} & 0 & G_{4ij1}B_{im} \end{bmatrix},$$
$$\Theta_{ijm}^{(33)} = -\gamma^2\mathbf{I} + G_{4ij1}D_{i1m} + D_{i1m}^{\mathrm{T}}G_{4ij1}^{\mathrm{T}},$$
$$\bar{\Theta}_{ijm}^{(12)} = \Theta_{ijm}^{(12)},$$
$$\bar{\Theta}_{ijm}^{(13)} = \bar{\Theta}_{ijm}^{(131)} - G_{3ij}^{\mathrm{T}},$$
$$\bar{\Theta}_{ijm}^{(131)} = \begin{bmatrix} G_{1ij1}a_{im} + \hat{H}_2\bar{k}_{cij} + \hat{H}_1\bar{a}_{cij} \\ G_{1ij2}a_{im} + \delta_1\bar{a}_{cij} \\ G_{1ij3}a_{im} + \rho_1\bar{k}_{cij} \end{bmatrix},$$
$$\bar{\Theta}_{ijm}^{(14)} = \Theta_{ijm}^{(13)},$$
$$\bar{\Theta}_{ijm}^{(22)} = -E^{\mathrm{T}}P_{ij}E + \bar{L}_{im}^{\mathrm{T}}\bar{L}_{im} + \varepsilon_0(N-1)JJ^{\mathrm{T}} + \lambda_{ij}JF_{ij}^{\mathrm{T}}F_{ij}J^{\mathrm{T}} + \mathrm{Sym}\left\{\Theta_{ijm}^{(221)}\right\},$$
$$\bar{\Theta}_{ijm}^{(23)} = \lambda_{ij}JF_{ij}^{\mathrm{T}}f_{ij} + \bar{\Theta}_{ijm}^{(231)} + \left\{\bar{\Theta}_{ijm}^{(232)}\right\}^{\mathrm{T}},$$
$$\bar{\Theta}_{ijm}^{(231)} = \begin{bmatrix} G_{2ij1}a_{im} + \delta_2\hat{H}_2\bar{k}_{cij} + \rho_2\hat{H}_1\bar{a}_{cij} \\ G_{2ij2}a_{im} + \delta_3\bar{a}_{cij} \\ G_{2ij3}a_{im} + \rho_3\bar{k}_{cij} \end{bmatrix},$$
$$\bar{\Theta}_{ijm}^{(232)} = \begin{bmatrix} G_{3ij1}A_{im} & 0 & G_{3ij1}B_{im} \end{bmatrix},$$
$$\bar{\Theta}_{ijm}^{(24)} = \Theta_{ijm}^{(23)},$$
$$\bar{\Theta}_{ijm}^{(33)} = \lambda_{ij}(f_{ij}^{\mathrm{T}}f_{ij} - 1) + G_{3ij1}a_{im} + a_{im}^{\mathrm{T}}G_{3ij1}^{\mathrm{T}},$$
$$\bar{\Theta}_{ijm}^{(34)} = G_{3ij1}D_{i1m} + a_{ijm}^{\mathrm{T}}G_{4ij1}^{\mathrm{T}},$$
$$\bar{\Theta}_{ijm}^{(44)} = -\gamma^2\mathbf{I} + G_{4ij1}D_{i1m} + D_{i1m}^{\mathrm{T}}G_{4ij1}^{\mathrm{T}}. \tag{19}$$

Moreover, the controller gains can be obtained as follows

$$A_{cij} = G_{1ij4}^{-1}\bar{A}_{cij}, \quad B_{cij} = G_{1ij4}^{-1}\bar{B}_{cij},$$
$$a_{cij} = G_{1ij4}^{-1}\bar{a}_{cij}, \quad K_{cij} = G_{1ij5}^{-1}\bar{K}_{cij},$$
$$D_{cij} = G_{1ij5}^{-1}\bar{D}_{cij}, \quad k_{cij} = G_{1ij5}^{-1}\bar{k}_{cij}. \tag{20}$$

Proof: Note that the condition (17) for $j \in \mathcal{I}_{i0}$ is a special case of the condition (18) for $j \in \mathcal{I}_{i1}$. Without loss of generality, in the following, the proof of the more complex case $j \in \mathcal{I}_{i1}$ is presented. Consider the following piecewise Lyapunov function,

$$V(t) = \sum_{i=1}^{N} V_i(t) = \sum_{i=1}^{N} \bar{x}_i^{\mathrm{T}}(t)E^{\mathrm{T}}P_{ij}E\bar{x}_i(t), \tag{21}$$

where $0 < P_{ij} = P_{ij}^{\mathrm{T}} \in \Re^{(n_{xi}+n_{ci}+n_{ui})\times(n_{xi}+n_{ci}+n_{ui})}$, $j \in \mathcal{I}_{i1}$, $i \in \mathcal{N}$, are positive definite symmetric Lyapunov matrices.

Based on (21), the closed-loop system in (14) is asymptotically stable with a robust $\mathscr{H}_\infty$ performance γ under zero initial condition with nonzero disturbance $w_i(t) \in l_2[0, \infty]$, if the following inequality holds for $(j, s) \in \Omega_i$, $j \in \mathcal{I}_{i1}$, $i \in \mathcal{N}$,

$$\sum_{i=1}^{N}\{\bar{x}_i^{\mathrm{T}}(t+1)E^{\mathrm{T}}P_{is}E\bar{x}_i(t+1) - \bar{x}_i^{\mathrm{T}}(t)E^{\mathrm{T}}P_{ij}E\bar{x}_i(t) + z_i^{\mathrm{T}}(t)z_i(t) - \gamma^2 w_i^{\mathrm{T}}(t)w_i(t)\} < 0. \tag{22}$$

Define the following augmented vector,

$$\xi_i(t) = [\bar{x}_i^{\mathrm{T}}(t+1)E^{\mathrm{T}} \quad \bar{x}_i^{\mathrm{T}}(t) \quad 1 \quad w_i^{\mathrm{T}}(t)]^{\mathrm{T}}, \tag{23}$$

and then inequality (22) holds with $(j, s) \in \Omega_i$, if

$$\xi_i^{\mathrm{T}}(t)\begin{bmatrix} P_{is} & 0 & 0 & 0 \\ \star & -E^{\mathrm{T}}P_{ij}E + \bar{\mathcal{L}}_{ij}^{\mathrm{T}}\bar{\mathcal{L}}_{ij} & 0 & 0 \\ \star & \star & 0 & 0 \\ \star & \star & \star & -\gamma^2\mathbf{I} \end{bmatrix}\xi_i(t) < 0. \tag{24}$$

Based on the space partition (11) and utilizing S-procedure, the following inequality (25) implies (24),

$$\xi_i^{\mathrm{T}}(t)\begin{bmatrix} P_{is} & 0 & 0 & 0 \\ \star & -E^{\mathrm{T}}P_{ij}E + \bar{\mathcal{L}}_{ij}^{\mathrm{T}}\bar{\mathcal{L}}_{ij} & 0 & 0 \\ \star & \star & 0 & 0 \\ \star & \star & \star & -\gamma^2\mathbf{I} \end{bmatrix}\xi_i(t) + \lambda_{ij}\begin{bmatrix} x_i(t) \\ 1 \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} F_{ij}^{\mathrm{T}}F_{ij} & F_{ij}^{\mathrm{T}}f_{ij} \\ \star & f_{ij}^{\mathrm{T}}f_{ij} - 1 \end{bmatrix}\begin{bmatrix} x_i(t) \\ 1 \end{bmatrix} < 0, \tag{25}$$

with $\lambda_{ij} < 0$. Noticing $x_i(t) = J \cdot \bar{x}_i(t)$, then yields,

$$\xi_i^{\mathrm{T}}(t)\Lambda_{ijs}\xi_i(t) < 0, \quad (j, s) \in \Omega_i, j \in \mathcal{I}_{i1}, i \in \mathcal{N}, \tag{26}$$

where

$$\Lambda_{ijs} = \begin{bmatrix} P_{is} & 0 & 0 & 0 \\ \star & \Lambda_{ijs}^{(1)} & \lambda_{ij}JF_{ij}^{\mathrm{T}}f_{ij} & 0 \\ \star & \star & \lambda_{ij}(f_{ij}^{\mathrm{T}}f_{ij} - 1) & 0 \\ \star & \star & \star & -\gamma^2\mathbf{I} \end{bmatrix}, \tag{27}$$

and $\Lambda_{ijs}^{(1)} = -E^{\mathrm{T}}P_{ij}E + \bar{\mathcal{L}}_{ij}^{\mathrm{T}}\bar{\mathcal{L}}_{ij} + \lambda_{ij}JF_{ij}^{\mathrm{T}}F_{ij}J^{\mathrm{T}}$.

On the basis of Lemma 8 and Tchebyshev's inequality shown in the appendix, it follows from the closed-loop system (14) that the following equality holds for any matrices $\bar{\mathcal{G}}_{ij}$

$$0 = 2\sum_{i=1}^{N}\xi_i^{\mathrm{T}}(t)\bar{\mathcal{G}}_{ij}\Big\{-E\cdot\bar{x}_i(t+1) + \bar{\mathcal{A}}_{ij}\bar{x}_i(t) + \bar{\mathrm{a}}_{ij} + \bar{\mathcal{D}}_{ij}w_i(t) + J\sum_{\substack{n=1\\n\neq i}}^{N}C_{ni}x_n(t)\Big\}$$
$$\leq 2\sum_{i=1}^{N}\xi_i^{\mathrm{T}}(t)\bar{\mathcal{G}}_{ij}\Big\{-E\cdot\bar{x}_i(t+1) + \bar{\mathcal{A}}_{ij}\bar{x}_i(t) + \bar{\mathrm{a}}_{ij} + \bar{\mathcal{D}}_{ij}w_i(t)\Big\}$$

$$
\begin{aligned}
&+\sum_{i=1}^{N}\sum_{\substack{n=1\\n\neq i}}^{N}\Big\{\varepsilon_{nij}^{-1}\xi_i^{\mathrm T}(t)\bar{\mathcal G}_{ij}JC_{ni}C_{ni}^{\mathrm T}J^{\mathrm T}\bar{\mathcal G}_{ij}^{\mathrm T}\xi_i(t)\\
&\qquad\qquad+\varepsilon_{nij}x_n^{\mathrm T}(t)x_n(t)\Big\}\\
&\le 2\sum_{i=1}^{N}\xi_i^{\mathrm T}(t)\bar{\mathcal G}_{ij}\mathscr A_{ij}\xi_i(t)+\sum_{i=1}^{N}\varepsilon_0(N-1)\bar x_i^{\mathrm T}(t)JJ^{\mathrm T}\bar x_i(t)\\
&+\sum_{i=1}^{N}\sum_{\substack{n=1\\n\neq i}}^{N}\varepsilon_{inj}^{-1}\xi_i^{\mathrm T}(t)\bar{\mathcal G}_{ij}JC_{in}C_{in}^{\mathrm T}J^{\mathrm T}\bar{\mathcal G}_{ij}^{\mathrm T}\xi_i(t),
\end{aligned}\tag{28}
$$

where $\mathscr A_{ij}=[-\mathbf I\quad \bar{\mathcal A}_{ij}\quad \bar{\mathrm a}_{ij}\quad \bar{\mathcal D}_{ij}]$, and scalar parameters $\varepsilon_{inj}>0$, $\varepsilon_0\ge\varepsilon_{inj}$, $n\neq i$, for $(j,s)\in\Omega_i$, $j\in\mathcal I_{i1}$, $i,n\in\mathcal N$.

Combining (26) and (28), yields,

$$
\sum_{i=1}^{N}\xi_i^{\mathrm T}(t)\Big\{\Pi_{ijs}+\mathrm{sym}\{\bar{\mathcal G}_{ij}\mathscr A_{ij}\}+\sum_{\substack{n=1\\n\neq i}}^{N}\varepsilon_{inj}^{-1}\bar{\mathcal G}_{ij}JC_{in}C_{in}^{\mathrm T}J^{\mathrm T}\bar{\mathcal G}_{ij}^{\mathrm T}\Big\}\xi_i(t)<0,\tag{29}
$$

where

$$
\Pi_{ijs}=\begin{bmatrix}P_{is}&0&0&0\\ \star&\Pi_{ijs}^{(1)}&\lambda_{ij}JF_{ij}^{\mathrm T}f_{ij}&0\\ \star&\star&\lambda_{ij}(f_{ij}^{\mathrm T}f_{ij}-1)&0\\ \star&\star&\star&-\gamma^2\mathbf I\end{bmatrix},
$$

$$
\begin{aligned}
\Pi_{ijs}^{(1)}&=-E^{\mathrm T}P_{ij}E+\bar{\mathcal L}_{ij}^{\mathrm T}\bar{\mathcal L}_{ij}+\lambda_{ij}JF_{ij}^{\mathrm T}F_{ij}J^{\mathrm T}\\
&\quad+\varepsilon_0(N-1)JJ^{\mathrm T}.
\end{aligned}\tag{30}
$$

Applying Schur complements, one has

$$
\begin{bmatrix}\Pi_{ijs}+\mathrm{sym}\{\bar{\mathcal G}_{ij}\mathscr A_{ij}\}&\bar{\mathcal G}_{ij}J\mathscr C_i\\ \star&-\mathscr M_{ij}\cdot\mathbf I\end{bmatrix}<0,\tag{31}
$$

where $\mathscr C_i=[C_{i1}\cdots C_{i(i-1)},C_{i(i+1)},\cdots C_{iN}]$, $\mathscr M_{ij}=\mathrm{diag}\{\varepsilon_{i1j}\mathbf I,\cdots,\varepsilon_{i(i-1)j}\mathbf I,\varepsilon_{i(i+1)j}\mathbf I,\cdots,\varepsilon_{iNj}\mathbf I\}$, with $\varepsilon_{inj}>0$, $i,n\in\mathcal N$, $n\neq i$, and $\bar{\mathcal G}_{ij}=[G_{1ij}^{\mathrm T}\quad G_{2ij}^{\mathrm T}\quad G_{3ij}^{\mathrm T}\quad G_{4ij}^{\mathrm T}]^{\mathrm T}$.

Expanding the fuzzy-basis functions, the following inequality implies (31),

$$
\begin{bmatrix}\Xi_{ijsm}+\mathrm{sym}\{\bar{\mathcal G}_{ij}\mathbb A_{ijm}\}&\mathcal G_{ij}J\mathscr C_i\\ \star&-\mathscr M_{ij}\cdot\mathbf I\end{bmatrix}<0,\tag{32}
$$

where

$$
\Xi_{ijsm}=\begin{bmatrix}P_{is}&0&0&0\\ \star&\Xi_{ijm}^{(1)}&\lambda_{ij}JF_{ij}^{\mathrm T}f_{ij}&0\\ \star&\star&\lambda_{ij}(f_{ij}^{\mathrm T}f_{ij}-1)&0\\ \star&\star&\star&-\gamma^2\mathbf I\end{bmatrix},
$$

$$
\begin{aligned}
\Xi_{ijm}^{(1)}&=-E^{\mathrm T}P_{ij}E+\bar L_{im}^{\mathrm T}\bar L_{im}+\lambda_{ij}JF_{ij}^{\mathrm T}F_{ij}J^{\mathrm T}\\
&\quad+\varepsilon_0(N-1)JJ^{\mathrm T},
\end{aligned}
$$

$$
\begin{aligned}
\mathbb A_{ijm}&=\begin{bmatrix}-\mathbf I&\bar A_{ijm}&\bar a_{ijm}&\bar D_{ijm}\end{bmatrix},\\
\bar A_{ijm}&=\begin{bmatrix}A_{im}&0&B_{im}\\ B_{cij}H_{im}&A_{cij}&0\\ D_{cij}H_{im}&K_{cij}&-\mathbf I\end{bmatrix},\\
\bar a_{ijm}&=\begin{bmatrix}a_{im}\\ a_{cij}\\ k_{cij}\end{bmatrix},\\
\bar D_{ijm}&=\begin{bmatrix}D_{i1m}\\ B_{cij}D_{i2m}\\ D_{cij}D_{i2m}\end{bmatrix},\\
\bar L_{im}&=\begin{bmatrix}L_{im}&0&N_{im}\end{bmatrix}.
\end{aligned}\tag{33}
$$

Note that the piecewise affine controller gains are not involved in the first row of the closed-loop matrices $\bar A_{ijm}$, $\bar a_{ijm}$, and $\bar D_{ijm}$. For the numerical tractability of the controller synthesis conditions, the slack variable matrices G_{1ij}, G_{2ij}, G_{3ij}, G_{4ij} are given as,

$$
\begin{aligned}
G_{1ij}&=\begin{bmatrix}G_{1ij1}&\hat H_1G_{1ij4}&\hat H_2G_{1ij5}\\ G_{1ij2}&\delta_1G_{1ij4}&0\\ G_{1ij3}&0&\rho_1G_{1ij5}\end{bmatrix},\\
G_{2ij}&=\begin{bmatrix}G_{2ij1}&\delta_2\hat H_1G_{1ij4}&\rho_2\hat H_2G_{1ij5}\\ G_{2ij2}&\delta_3G_{1ij4}&0\\ G_{2ij3}&0&\rho_3G_{1ij5}\end{bmatrix},\\
G_{3ij}&=\begin{bmatrix}G_{3ij1}&\mathbf 0&\mathbf 0\end{bmatrix},\\
G_{4ij}&=\begin{bmatrix}G_{4ij1}&\mathbf 0&\mathbf 0\end{bmatrix},\\
\hat H_1&=\begin{bmatrix}\mathbf I_{n_{ci}}\\ \mathbf 0_{(n_{xi}-n_{ci})\times n_{ci}}\end{bmatrix},\\
\hat H_2&=\begin{bmatrix}\mathbf I_{n_{ui}}\\ \mathbf 0_{(n_{xi}-n_{ui})\times n_{ui}}\end{bmatrix},
\end{aligned}\tag{34}
$$

where δ_1, δ_2, δ_3, ρ_1, ρ_2, ρ_3 are scalar parameters.

Define

$$
\begin{aligned}
\bar A_{cij}&=G_{1ij4}A_{cij},\quad \bar B_{cij}=G_{1ij4}B_{cij},\\
\bar a_{cij}&=G_{1ij4}a_{cij},\quad \bar K_{cij}=G_{1ij5}K_{cij},\\
\bar D_{cij}&=G_{1ij5}D_{cij},\quad \bar k_{cij}=G_{1ij5}k_{cij}.
\end{aligned}\tag{35}
$$

Substituting the matrices defined in (34) into (32), together with consideration of (35), lead to (18). In addition, the conditions in (18) imply that $\delta_1G_{1ij4}+\delta_1G_{1ij4}^{\mathrm T}-P_{is4}>0$ and $\rho_1G_{1ij5}+\rho_1G_{1ij5}^{\mathrm T}-P_{is6}>0$, which implies that G_{1ij4} and G_{1ij5} are invertible. The controller gains can be obtained via (20). The proof is completed. ■

Remark 3: Note that the results given in Theorem 2 are derived based on a piecewise-affine DOF controller as in (12). However, to further reduce the design conservatism, one should notice that the approach in this paper can be extended to synthesize a piecewise-fuzzy-affine controller as follows,

$$\begin{cases} x_{ci}(t+1) = \sum_{m\in\mathscr{N}_i(j)} \mu_{im}A_{cijm}x_{ci}(t) \\ \qquad + \sum_{m\in\mathscr{N}_i(j)} \mu_{im}a_{cijm} \\ \qquad + \sum_{m\in\mathscr{N}_i(j)} \mu_{im}B_{cijm}y_i(t) \\ u_i(t) = \sum_{m\in\mathscr{N}_i(j)} \mu_{im}K_{cijm}x_{ci}(t) \\ \qquad + \sum_{m\in\mathscr{N}_i(j)} \mu_{im}k_{cijm} \\ \qquad + \sum_{m\in\mathscr{N}_i(j)} \mu_{im}D_{cijm}y_i(t), \\ \qquad\qquad j\in\mathcal{I}_i, i\in\mathcal{N}. \end{cases} \tag{36}$$

B. A SPECIAL RESULT FOR STATIC OUTPUT FEEDBACK CONTROLLER DESIGN

Note that when $n_{ci} = 0$, i.e., the controller order is zero, the fixed-order controller (12) reduces to a decentralized static output feedback (SOF) controller in the following form,

$$u_i(t) = D_{cij}y_i(t) + k_{cij}, j\in\mathcal{I}_i, i\in\mathcal{N}, \tag{37}$$

where $D_{cij} \in \Re^{n_{ui}\times n_{yi}}$, and $k_{cij} \in \Re^{n_{ui}}$ are controller gains to be determined. Note that $k_{cij} = 0$ for $j \in \mathcal{I}_{i0}$.

Define $\bar{x}_i(t) = [\, x_i^{\mathrm{T}}(t)\ u_i^{\mathrm{T}}(t)]^{\mathrm{T}}$ and the corresponding closed-loop system is

$$\begin{cases} E\cdot\bar{x}_i(t+1) = \bar{\mathcal{A}}_{ij}\bar{x}_i(t) + \bar{\mathrm{a}}_{ij} + \bar{\mathcal{D}}_{ij}w_i(t) \\ \qquad + J_1\sum_{\substack{n=1\\ n\neq i}}^{N} C_{ni}x_n(t) \\ z_i(t) = \bar{\mathcal{L}}_{ij}\bar{x}_i(t),\ \zeta_i(t)\in\mathscr{S}_{ij}, j\in\mathcal{I}_i, i\in\mathcal{N}, \end{cases} \tag{38}$$

where

$$\begin{cases} E = \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}, \\ \bar{\mathcal{A}}_{ij} = \begin{bmatrix}\mathcal{A}_{ij} & \mathcal{B}_{ij}\\ D_{cij}\mathcal{H}_{ij} & -\mathbf{I}\end{bmatrix}, \\ \bar{\mathrm{a}}_{ij} = \begin{bmatrix}\mathrm{a}_{ij}\\ k_{cij}\end{bmatrix}, \quad \bar{\mathcal{D}}_{ij} = \begin{bmatrix}\mathcal{D}_{i1j}\\ D_{cij}\mathcal{D}_{i2j}\end{bmatrix}, \\ J_1 = \begin{bmatrix}\mathbf{I}_{n_x}\\ \mathbf{0}_{n_u\times n_x}\end{bmatrix}, \quad \bar{\mathcal{L}}_{ij} = \begin{bmatrix}\mathcal{L}_{ij} & \mathcal{N}_{ij}\end{bmatrix}. \end{cases} \tag{39}$$

Specifically, $x_i(t) = J_1\cdot\bar{x}_i(t)$. On the basis of Theorem 2, the SOF controller synthesis conditions can be given in the following corollary.

Corollary 4: Consider the large-scale fuzzy system in (1). For a given scalar $\varepsilon_0 > 0$, the closed-loop system in (14) is asymptotically stable with a robust $\mathscr{H}_\infty$ disturbance attenuation level γ, if there exist matrices $0 < P_{ij} = P_{ij}^{\mathrm{T}} \in \Re^{(n_{xi}+n_{ui})\times(n_{xi}+n_{ui})}$, $G_{1ij1}, G_{2ij1} \in \Re^{n_{xi}\times n_{xi}}$, $G_{1ij3}, G_{2ij3} \in \Re^{n_{ui}\times n_{xi}}$, $G_{1ij5} \in \Re^{n_{ui}\times n_{ui}}$, $G_{4ij1} \in \Re^{n_{wi}\times n_{xi}}$, $\bar{D}_{cij} \in \Re^{n_{ui}\times n_{yi}}$, for $j\in\mathcal{I}_i$, $G_{3ij1} \in \Re^{1\times n_{xi}}$, $\bar{k}_{ij} \in \Re^{n_{ui}}$, for $j\in\mathcal{I}_{i1}$, scalars $\varepsilon_{inj} > 0$, $i, n\in\mathcal{N}$, $n\neq i$, and $\lambda_{ij} < 0$, $j\in\mathcal{I}_{i1}$ such that the following linear matrix inequalities hold,

$$\begin{bmatrix}\Upsilon_{ijsm} & \mathcal{G}_{ij}J_1\mathscr{C}_i\\ \star & -\mathscr{M}_{ij}\cdot\mathbf{I}\end{bmatrix} < 0,$$
$$m\in\mathscr{N}_i(j),\quad j\in\mathcal{I}_{i0}, (j,s)\in\Omega_i,\ i\in\mathcal{N}, \tag{40}$$

$$\begin{bmatrix}\bar{\Upsilon}_{ijsm} & \bar{\mathcal{G}}_{ij}J_1\mathscr{C}_i\\ \star & -\mathscr{M}_{ij}\cdot\mathbf{I}\end{bmatrix} < 0,$$
$$m\in\mathscr{N}_i(j),\quad j\in\mathcal{I}_{i1}, (j,s)\in\Omega_i,\ i\in\mathcal{N}, \tag{41}$$

where

$$\Upsilon_{ijsm} = \begin{bmatrix} P_{is}-G_{1ij}-G_{1ij}^{\mathrm{T}} & \Upsilon_{ijm}^{(12)} & \Upsilon_{ijm}^{(13)}\\ \star & \Upsilon_{ijm}^{(22)} & \Upsilon_{ijm}^{(23)}\\ \star & \star & \Upsilon_{ijm}^{(33)}\end{bmatrix},$$

$$\bar{\Upsilon}_{ijsm} = \begin{bmatrix} P_{is}-G_{1ij}-G_{1ij}^{\mathrm{T}} & \bar{\Upsilon}_{ijm}^{(12)} & \bar{\Upsilon}_{ijm}^{(13)} & \bar{\Upsilon}_{ijm}^{(14)}\\ \star & \bar{\Upsilon}_{ijm}^{(22)} & \bar{\Upsilon}_{ijm}^{(23)} & \bar{\Upsilon}_{ijm}^{(24)}\\ \star & \star & \bar{\Upsilon}_{ijm}^{(33)} & \bar{\Upsilon}_{ijm}^{(34)}\\ \star & \star & \star & \bar{\Upsilon}_{ijm}^{(44)}\end{bmatrix},$$

$$G_{1ij} = \begin{bmatrix} G_{1ij1} & \hat{H}_2G_{1ij5}\\ G_{1ij3} & \rho_1G_{1ij5}\end{bmatrix},$$
$$G_{2ij} = \begin{bmatrix} G_{2ij1} & \rho_2\hat{H}_2G_{1ij5}\\ G_{2ij3} & \rho_3G_{1ij5}\end{bmatrix},$$
$$G_{3ij} = \begin{bmatrix} G_{3ij1} & \mathbf{0}\end{bmatrix},\quad G_{4ij} = \begin{bmatrix} G_{4ij1} & \mathbf{0}\end{bmatrix},$$
$$\hat{H}_2 = \begin{bmatrix}\mathbf{I}_{n_{ui}} & \mathbf{0}_{(n_{xi}-n_{ui})\times n_{ui}}\end{bmatrix}^{\mathrm{T}},$$
$$\mathcal{G}_{ij} = \begin{bmatrix} G_{1ij}^{\mathrm{T}} & G_{2ij}^{\mathrm{T}} & G_{4ij}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},$$
$$\bar{\mathcal{G}}_{ij} = \begin{bmatrix} G_{1ij}^{\mathrm{T}} & G_{2ij}^{\mathrm{T}} & G_{3ij}^{\mathrm{T}} & G_{4ij}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},$$
$$\mathscr{C}_i = [C_{i1}\cdots C_{i(i-1)}, C_{i(i+1)}\cdots C_{iN}],$$
$$\mathscr{M}_{ij} = \mathrm{diag}\{\varepsilon_{i1j}\mathbf{I},\cdots,\varepsilon_{i(i-1)j}\mathbf{I},\varepsilon_{i(i+1)j}\mathbf{I},\cdots,\varepsilon_{iNj}\mathbf{I}\},$$
$$\Upsilon_{ijm}^{(12)} = \Upsilon_{ijm}^{(121)} - G_{2ij}^{\mathrm{T}},$$
$$\Upsilon_{ijm}^{(121)} = \begin{bmatrix} G_{1ij1}A_{im}+\hat{H}_2\bar{D}_{cij}H_{im} & G_{1ij1}B_{im}-\hat{H}_2G_{1ij5}\\ G_{1ij3}A_{im}+\rho_1\bar{D}_{cij}H_{im} & G_{1ij3}B_{im}-\rho_1G_{1ij5}\end{bmatrix},$$
$$\Upsilon_{ijm}^{(13)} = \Upsilon_{ijm}^{(131)} - G_{4ij}^{\mathrm{T}},$$
$$\Upsilon_{ijm}^{(131)} = \begin{bmatrix} G_{1ij1}D_{i1m}+\hat{H}_2\bar{D}_{cij}D_{i2m}\\ G_{1ij3}D_{i1m}+\rho_1\bar{D}_{cij}D_{i2m}\end{bmatrix},$$
$$\Upsilon_{ijm}^{(22)} = -E^{\mathrm{T}}P_{ij}E + \bar{L}_{im}^{\mathrm{T}}\bar{L}_{im} + \varepsilon_0(N-1)J_1J_1^{\mathrm{T}} + \mathtt{Sym}\left\{\Upsilon_{ijm}^{(221)}\right\},$$
$$\Upsilon_{ijm}^{(221)} = \begin{bmatrix} G_{2ij1}A_{im}+\rho_2\hat{H}_2\bar{D}_{cij}H_{im} & G_{2ij1}B_{im}-\rho_2\hat{H}_2G_{1ij5}\\ G_{2ij3}A_{im}+\rho_3\bar{D}_{cij}H_{im} & G_{2ij3}B_{im}-\rho_3G_{1ij5}\end{bmatrix},$$
$$\Upsilon_{ijm}^{(23)} = \Upsilon_{ijm}^{(231)} + \left(\Upsilon_{ijm}^{(232)}\right)^{\mathrm{T}},$$
$$\Upsilon_{ijm}^{(231)} = \begin{bmatrix} G_{2ij1}D_{i1m}+\rho_2\hat{H}_2\bar{D}_{cij}D_{i2m}\\ G_{2ij3}D_{i1m}+\rho_3\bar{D}_{cij}D_{i2m}\end{bmatrix},$$
$$\Upsilon_{ijm}^{(232)} = \begin{bmatrix} G_{4ij1}A_{im} & G_{4ij1}B_{im}\end{bmatrix},$$
$$\Upsilon_{ijm}^{(33)} = -\gamma^2\mathbf{I} + G_{4ij1}D_{i1m} + D_{i1m}^{\mathrm{T}}G_{4ij1}^{\mathrm{T}},$$
$$\bar{\Upsilon}_{ijm}^{(12)} = \Upsilon_{ijm}^{(12)},$$

$$\begin{aligned}
\bar{\Upsilon}_{ijm}^{(13)} &= \bar{\Upsilon}_{ijm}^{(131)} - G_{3ij}^{\mathrm{T}},\\
\bar{\Upsilon}_{ijm}^{(131)} &= \begin{bmatrix} G_{1ij1}a_{im} + \hat{H}_2\bar{k}_{cij} \\ G_{1ij3}a_{im} + \rho_1\bar{k}_{cij} \end{bmatrix},\\
\bar{\Upsilon}_{ijm}^{(14)} &= \Upsilon_{ijm}^{(13)},\\
\bar{\Upsilon}_{ijm}^{(22)} &= -E^{\mathrm{T}}P_{ij}E + \bar{L}_{im}^{\mathrm{T}}\bar{L}_{im} + \varepsilon_0(N-1)J_1J_1^{\mathrm{T}}\\
&\quad +\lambda_{ij}J_1F_{ij}^{\mathrm{T}}F_{ij}J_1^{\mathrm{T}} + \mathrm{Sym}\left\{\Upsilon_{ijm}^{(221)}\right\}^{\mathrm{T}},\\
\bar{\Upsilon}_{ijm}^{(23)} &= \lambda_{ij}J_1F_{ij}^{\mathrm{T}}f_{ij} + \bar{\Upsilon}_{ijm}^{(231)} + \left\{\bar{\Upsilon}_{ijm}^{(232)}\right\}^{\mathrm{T}},\\
\bar{\Upsilon}_{ijm}^{(231)} &= \begin{bmatrix} G_{2ij1}a_{im} + \rho_2\hat{H}_2\bar{k}_{cij} \\ G_{2ij3}a_{im} + \rho_3\bar{k}_{cij} \end{bmatrix},\\
\bar{\Upsilon}_{ijm}^{(232)} &= \begin{bmatrix} G_{3ij1}A_{im} & G_{3ij1}B_{im} \end{bmatrix},\\
\bar{\Upsilon}_{ijm}^{(24)} &= \Upsilon_{ijm}^{(23)},\\
\bar{\Upsilon}_{ijm}^{(33)} &= \lambda_{ij}(f_{ij}f_{ij} - 1) + G_{3ij1}a_{im} + a_{im}^{\mathrm{T}}G_{3ij1}^{\mathrm{T}},\\
\bar{\Upsilon}_{ijm}^{(34)} &= G_{3ij1}D_{i1m} + a_{im}^{\mathrm{T}}G_{4ij1}^{\mathrm{T}},\\
\bar{\Upsilon}_{ijm}^{(44)} &= -\gamma^2\mathbf{I} + G_{4ij1}D_{i1m} + D_{i1m}^{\mathrm{T}}G_{4ij1}^{\mathrm{T}}.
\end{aligned} \tag{42}$$

Moreover, the controller gains can be obtained as follows,

$$D_{cij} = G_{1ij5}^{-1}\bar{D}_{cij}, \quad k_{cij} = G_{1ij5}^{-1}\bar{k}_{cij}. \tag{43}$$

Proof: Condition (40) is a special case of condition (41). Without loss of generality, in the following, the more complex case $j \in \mathcal{I}_{i1}$ will be considered. Noticing that $n_{ci} = 0$, and specify the matrix $\bar{\mathcal{G}}_{ij} = \begin{bmatrix} G_{1ij}^{\mathrm{T}} & G_{2ij}^{\mathrm{T}} & G_{3ij}^{\mathrm{T}} & G_{4ij}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ with

$$\begin{cases}
G_{1ij} = \begin{bmatrix} G_{1ij1} & \hat{H}_2G_{1ij5} \\ G_{1ij3} & \rho_1G_{1ij5} \end{bmatrix},\\
G_{2ij} = \begin{bmatrix} G_{2ij1} & \rho_2\hat{H}_2G_{1ij5} \\ G_{2ij3} & \rho_3G_{1ij5} \end{bmatrix},\\
G_{3ij} = \begin{bmatrix} G_{3ij1} & \mathbf{0} \end{bmatrix},\\
G_{4ij} = \begin{bmatrix} G_{4ij1} & \mathbf{0} \end{bmatrix},
\end{cases} \tag{44}$$

where $\hat{H}_2 = \begin{bmatrix} \mathbf{I}_{n_{ui}} & \mathbf{0}_{(n_{xi}-n_{ui})\times n_{ui}} \end{bmatrix}^{\mathrm{T}}$, and ρ_1, ρ_2, ρ_3 are scalar parameters. Then rest derivation procedures can be conducted in similar ways as the proof of Theorem 2. ■

Remark 5: Compared with conventional approach for fixed-order DOF controller design, a descriptor system approach has been proposed to deal with the difficulty that strong couplings existing in piecewise affine controller gains and fuzzy affine dynamic models with interconnections. With the augmentation of system states and control inputs, this decoupling feature enables one to synthesize the fixed-order piecewise affine DOF controller in a unified framework based on linear matrix inequalities.

IV. SIMULATION EXAMPLES

In this section, two examples are given to verify the effectiveness of the proposed design method.

Example 6: Consider a discrete-time fuzzy-affine large-scale system with two interconnected subsystems as follows,

Plant Rule $\mathscr{R}_i^l$: IF $x_{i2}(t)$ is $\mathscr{F}_{i1}^l$, THEN

$$\begin{cases}
x_i(t+1) = A_{il}x_i(t) + a_{il} + B_{il}u_i(t) + D_{i1l}w_i(t)\\
\qquad\qquad + \sum_{n=1, n\neq i}^{N} C_{ni}x_n(t)\\
y_i(t) = H_{il}x_i(t) + D_{i2l}w_i(t)\\
z_i(t) = L_{il}x_i(t) + N_{il}u_i(t), l = \{1,2,3\}, \ i = \{1,2\},
\end{cases} \tag{45}$$

and the system matrices are given as,

subsystem S_1:

$$\begin{cases}
\begin{bmatrix} A_{11} \mid a_{11} \end{bmatrix} = \left[\begin{array}{cc|c} 1.001 & -0.009 & 0 \\ 0.121 & -0.7 & 0.05 \end{array}\right],\\
\begin{bmatrix} A_{12} \mid a_{12} \end{bmatrix} = \left[\begin{array}{cc|c} 1.003 & 0.01 & 0 \\ 0.179 & 0.5 & 0 \end{array}\right],\\
\begin{bmatrix} A_{13} \mid a_{13} \end{bmatrix} = \left[\begin{array}{cc|c} 1.000 & 0.01 & 0 \\ 0.1729 & 0.6 & -0.05 \end{array}\right],\\
\begin{bmatrix} B_{11} \mid B_{12} \mid B_{13} \end{bmatrix} = \left[\begin{array}{c|c|c} 0.05 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.06 \end{array}\right],\\
\begin{bmatrix} D_{111} \mid D_{112} \mid D_{113} \end{bmatrix} = \left[\begin{array}{c|c|c} 0.01 & 0.02 & 0.01 \\ 0.01 & 0.01 & 0.02 \end{array}\right],\\
C_{21} = \begin{bmatrix} 0 & 0 \\ 0.016 & 0 \end{bmatrix},\\
\begin{bmatrix} H_{11}^{\mathrm{T}} \mid H_{12}^{\mathrm{T}} \mid H_{13}^{\mathrm{T}} \end{bmatrix} = \left[\begin{array}{c|c|c} 0.965 & 1 & 0.5 \\ 0.1 & 0 & 0 \end{array}\right],\\
\begin{bmatrix} D_{121} \mid D_{122} \mid D_{123} \end{bmatrix} = \begin{bmatrix} 0.02 \mid 0.02 \mid 0.03 \end{bmatrix},\\
L_{1l} = \begin{bmatrix} 0.898 & 0 \end{bmatrix},\\
N_{1l} = 0.5, \ l = \{1,2,3\},
\end{cases} \tag{46}$$

and subsystem S_2:

$$\begin{cases}
\begin{bmatrix} A_{21} \mid a_{21} \end{bmatrix} = \left[\begin{array}{cc|c} 1.002 & 0.01 & 0 \\ 0.101 & -0.67 & 0.03 \end{array}\right],\\
\begin{bmatrix} A_{22} \mid a_{22} \end{bmatrix} = \left[\begin{array}{cc|c} 1.001 & -0.013 & 0 \\ 0.05 & 0.8 & 0 \end{array}\right],\\
\begin{bmatrix} A_{23} \mid a_{23} \end{bmatrix} = \left[\begin{array}{cc|c} 1.000 & 0.02 & 0 \\ 0.17 & -0.5 & -0.03 \end{array}\right],\\
\begin{bmatrix} B_{21} \mid B_{22} \mid B_{23} \end{bmatrix} = \left[\begin{array}{c|c|c} 0.1 & 0.08 & 0.1 \\ 0.08 & 0.06 & 0.1 \end{array}\right],\\
\begin{bmatrix} D_{211} \mid D_{212} \mid D_{213} \end{bmatrix} = \left[\begin{array}{c|c|c} 0.01 & 0.01 & 0.01 \\ 0.01 & 0.015 & 0.01 \end{array}\right],\\
C_{12} = \begin{bmatrix} 0 & 0 \\ 0.018 & 0 \end{bmatrix},\\
\begin{bmatrix} H_{21}^{\mathrm{T}} \mid H_{22}^{\mathrm{T}} \mid H_{23}^{\mathrm{T}} \end{bmatrix} = \left[\begin{array}{c|c|c} 0.9 & 0.8 & 0.6 \\ 0.1 & 0.5 & 0.2 \end{array}\right],\\
\begin{bmatrix} D_{221} \mid D_{222} \mid D_{223} \end{bmatrix} = \begin{bmatrix} 0.03 \mid 0.01 \mid 0.02 \end{bmatrix},\\
L_{2l} = \begin{bmatrix} 0.898 & 0 \end{bmatrix}, \ N_{2l} = 0.5, \ l = \{1,2,3\}.
\end{cases} \tag{47}$$

The normalized membership functions of subsystems S_1 and S_2 are demonstrated in Fig. 1. Through the space partition, the premise-variable space in each subsystem can be divided into three regions,

$$\begin{aligned}\mathscr{S}_{i1} &= \{x_i(t)| - d_{i2} < x_{i2}(t) \le -d_{i1}\},\\ \mathscr{S}_{i2} &= \{x_i(t)| - d_{i1} < x_{i2}(t) \le d_{i1}\},\\ \mathscr{S}_{i3} &= \{x_i(t)| d_{i1} < x_{i2}(t) \le d_{i2}\},\end{aligned} \tag{48}$$

where $d_{i1} = 50$, $d_{i2} = 300$. It is noticed from Fig. 1 that $\mathscr{S}_{i2}$ is a crisp region, and $\mathscr{S}_{i1}$ and $\mathscr{S}_{i3}$ are fuzzy regions. Note that $\theta_{ij}^{\mathrm{T}} = [0 \quad 1]$, for $j \in \mathcal{I}_i$, $i \in \mathcal{N}$, and the parameters of the degenerate ellipsoid are given as,

$$\begin{aligned}F_{i1} &= \frac{2\theta_{ij}^{\mathrm{T}}}{d_{i2} - d_{i1}},\ f_{i1} = \frac{d_{i2} + d_{i1}}{d_{i2} - d_{i1}},\\ F_{i2} &= \frac{1}{d_{i1}},\ f_{i2} = 0,\\ F_{i3} &= \frac{2\theta_{ij}^{\mathrm{T}}}{d_{i2} - d_{i1}},\ f_{i3} = -\frac{d_{i2} + d_{i1}}{d_{i2} - d_{i1}},\ i \in \{1, 2\}.\end{aligned} \tag{49}$$

The aim is to develop a decentralized fixed-order dynamic output feedback controller in the form of (12) such that the corresponding closed-loop fuzzy-affine large-scale system is asymptotically stable with a robust $\mathscr{H}_\infty$ performance γ. Given the parameter $\varepsilon_0 = 1$, and applying Theorem 2 with $\rho_1 = \rho_3 = 1$, $\rho_2 = 3$, $\delta_1 = 4$, $\delta_2 = 1$, and $\delta_3 = -1$, one obtains the feasible solution with $\mathscr{H}_\infty$ performance $\gamma_{\min} = 0.2483$ for the full-order (2-order) DOF controller. The corresponding controller gains are

$$\left\{\begin{aligned}&\left[A_{c11} \mid a_{c11} \mid B_{c11}\right]\\ &\quad = \left[\begin{array}{cc|c|c} 0.6051 & 0.0549 & -0.1261 & -0.2307\\ -1.7982 & -0.0912 & -0.4931 & -0.8517\end{array}\right],\\ &K_{c11} = \left[-1.2193 \quad -0.1898\right],\ k_{c11} = 0.0145,\\ &D_{c11} = -3.2456,\\ &\left[A_{c12} \mid a_{c12} \mid B_{c12}\right]\\ &\quad = \left[\begin{array}{cc|c|c} 0.2090 & 0.0213 & 0 & -0.0700\\ -0.7362 & 0.1813 & 0 & -1.2717\end{array}\right],\\ &K_{c12} = \left[0.8117 \quad 0.1410\right],\quad k_{c12} = 0,\\ &D_{c12} = -2.4837,\\ &\left[A_{c13} \mid a_{c13} \mid B_{c13}\right]\\ &\quad = \left[\begin{array}{cc|c|c} 0.0900 & 0.0098 & 0.0363 & -0.0487\\ 0.9034 & 0.2525 & -0.2617 & -1.8325\end{array}\right],\\ &K_{c13} = \left[0.9847 \quad 0.1851\right],\quad k_{c13} = -0.1423,\\ &D_{c13} = -2.4124,\end{aligned}\right. \tag{50}$$

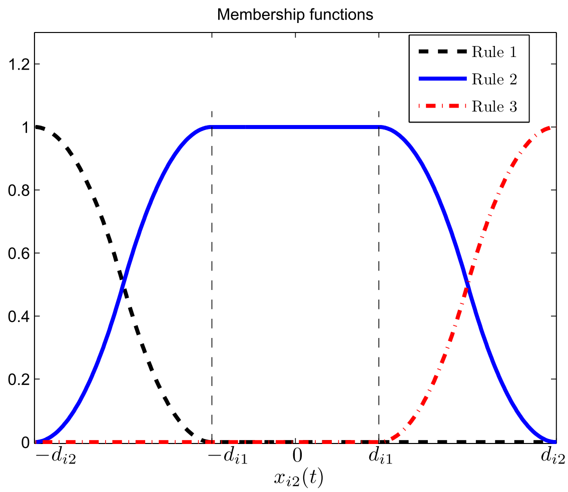

FIGURE 1. Membership functions in Example 6.

for subsystem S_1 and

$$\left\{\begin{aligned}&\left[A_{c21} \mid a_{c21} \mid B_{c21}\right]\\ &\quad = \left[\begin{array}{cc|c|c} 0.2914 & -0.0197 & -0.0745 & -0.4349\\ -0.6207 & 0.3578 & 0.4868 & -0.3689\end{array}\right],\\ &K_{c21} = \left[0.7955 \quad -0.0200\right],\quad k_{c21} = -0.1180,\\ &D_{c21} = -1.6087,\\ &\left[A_{c22} \mid a_{c22} \mid B_{c22}\right]\\ &\quad = \left[\begin{array}{cc|c|c} 0.2795 & -0.0168 & 0 & -0.4818\\ -1.3648 & 0.4359 & 0 & -1.1873\end{array}\right],\\ &K_{c22} = \left[0.3963 \quad 0.0037\right],\quad k_{c22} = 0,\\ &D_{c22} = -2.1224,\\ &\left[A_{c23} \mid a_{c23} \mid B_{c23}\right]\\ &\quad = \left[\begin{array}{cc|c|c} 0.4860 & -0.0233 & 0.0274 & -0.3159\\ -0.8733 & 0.3068 & -0.4627 & -0.8928\end{array}\right],\\ &K_{c23} = \left[0.1572 \quad 0.1615\right],\quad k_{c23} = -0.2632,\\ &D_{c23} = -2.8849,\end{aligned}\right. \tag{51}$$

for subsystem S_2.

The $\mathscr{H}_\infty$ performance is $\gamma_{\min} = 0.2487$ for the reduced-order (1-order) DOF controller with gains given as follows,

$$\left\{\begin{aligned}&\left[A_{c11} \mid a_{c11} \mid B_{c11}\right]\\ &\quad = \left[0.4678 \mid -0.1719 \mid -0.2966\right],\\ &\left[K_{c11} \mid k_{c11} \mid D_{c11}\right]\\ &\quad = \left[-1.1186 \mid -0.0012 \mid -3.2089\right],\\ &\left[A_{c12} \mid a_{c12} \mid B_{c12}\right]\\ &\quad = \left[0.1014 \mid 0 \mid -0.1104\right],\\ &\left[K_{c12} \mid k_{c12} \mid D_{c12}\right]\\ &4pt \quad = \left[0.8973 \mid 0 \mid -2.4882\right],\\ &\left[A_{c13} \mid a_{c13} \mid B_{c13}\right]\\ &\quad = \left[0.0700 \mid -0.0201 \mid -0.1072\right],\\ &\left[K_{c13} \mid k_{c13} \mid D_{c13}\right]\\ &\quad = \left[1.0260 \mid -0.0012 \mid -2.4409\right],\end{aligned}\right. \tag{52}$$

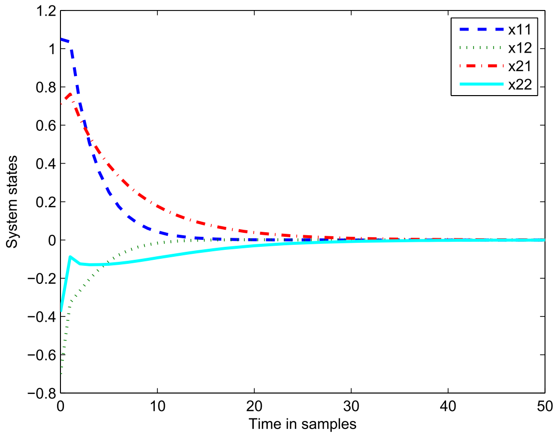

FIGURE 2. Trajectories of the closed-loop system states in Example 6.

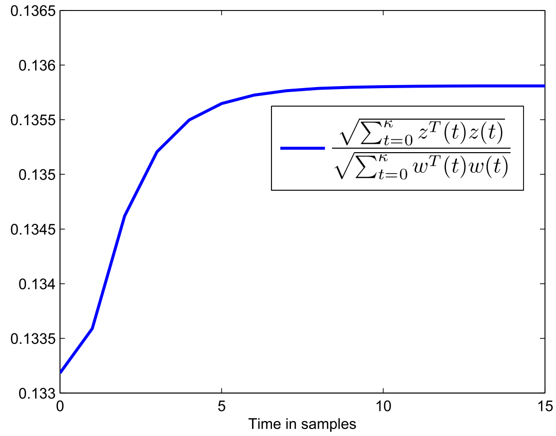

FIGURE 3. Response of the ratio $\frac{\sqrt{\sum_{t=0}^{\kappa} z^T(t)z(t)}}{\sqrt{\sum_{t=0}^{\kappa} w^T(t)w(t)}}$ in Example 6.

for subsystem S_1 and

$$\begin{cases} \left[A_{c21} \mid a_{c21} \mid B_{c21}\right] \\ \qquad = \left[0.3781 \mid -0.4407 \mid -0.3812\right], \\ \left[K_{c21} \mid k_{c21} \mid D_{c21}\right] \\ \qquad = \left[0.9116 \mid -0.6669 \mid -1.3532\right], \\ \left[A_{c22} \mid a_{c22} \mid B_{c22}\right] \\ \qquad = \left[0.3688 \mid 0 \mid -0.4384\right], \\ \left[K_{c22} \mid k_{c22} \mid D_{c22}\right] \\ \qquad = \left[0.4459 \mid 0 \mid -2.0195\right], \\ \left[A_{c23} \mid a_{c23} \mid B_{c23}\right] \\ \qquad = \left[0.4922 \mid 0.1117 \mid -0.2994\right], \\ \left[K_{c23} \mid k_{c23} \mid D_{c23}\right] \\ \qquad = \left[0.3863 \mid -1.7521 \mid -2.8480\right], \end{cases} \tag{53}$$

for subsystem S_2.

In addition, via Corollary 4, we can obtain feasible solution with $\gamma_{\min} = 0.4956$ for the static output feedback controller with gains given as follows,

$$\begin{cases} \left[k_{c11} \mid D_{c11}\right] = \left[-0.0210 \mid -4.3453\right], \\ \left[k_{c12} \mid D_{c12}\right] = \left[0 \mid -4.1588\right], \\ \left[k_{c13} \mid D_{c13}\right] = \left[-0.0911 \mid -5.1514\right], \\ \left[k_{c21} \mid D_{c21}\right] = \left[0.0049 \mid -2.4088\right], \\ \left[k_{c22} \mid D_{c22}\right] = \left[0 \mid -3.3238\right], \\ \left[k_{c23} \mid D_{c23}\right] = \left[0.0203 \mid -2.7442\right], \end{cases} \tag{54}$$

for subsystems S_1 and S_2, respectively.

It is easy to see that a better disturbance attenuation level can be obtained with a higher order controller.

To verify the effectiveness of the designed controllers, simulations are carried out. For brevity, we only demonstrate the simulation results for the full-order controller case. Given the initial condition $x_{10} = [1.0500 \quad -0.7000]^{\mathrm{T}}$, $x_{20} = [0.7125 \quad -0.3750]^{\mathrm{T}}$, and the external disturbances $w_1 = w_2 = 135\mathrm{e}^{-2t} \cdot \sin(2t)$, the state trajectories of closed-loop subsystems S_1 and S_2 are shown in Fig. 2. Under zero initial conditions, the $\mathscr{H}_\infty$ performance is shown in Fig. 3. The ratio $\frac{\sqrt{\sum_{t=0}^{\kappa} z^{\mathrm{T}}(t)z(t)}}{\sqrt{\sum_{t=0}^{\kappa} w^{\mathrm{T}}(t)w(t)}}$ is about 0.1357, which is lower than the minimum disturbance attenuation level $\gamma_{\min} = 0.2483$. Thus, the obtained decentralized controller is able to stabilize the fuzzy large-scale system with satisfactory performance.

To further validate the effectiveness of the proposed approach, in the sequel, we consider another simulation example.

Example 7: Consider a large-scale nonlinear continuous stirred tank reactor (CSTR) system with two interconnected subsystems as follows,

TABLE 1. Nominal parameters of CSTR system in Example 7.

parameter	nominal value	parameter	nominal value
F	100L/min	C_p	0.239J/gK
C_{a0}	1mol/L	$-\Delta H$	5×10^4J/mol
T_{fs}	350K	E/R	8750K
V	100L	k_0	7.2×10^{10} 1/min
ρ	1000g/L	UA	5×10^4 J/minK

$$\begin{cases} \dot{x}_{i1} = b_0(T_{fs} - x_{i1}) + b_1 k_0 \exp(-\frac{b_2}{x_{i1}})x_{i2} \\ \qquad -b_3 x_{i1} + b_3 u_i + b_0 w_i + \Phi_{i1} \\ \dot{x}_{i2} = b_0(C_{a0} - x_{i2}) - k_0 \exp(-\frac{b_2}{x_{i1}})x_{i2} + \Phi_{i2}, \end{cases} \tag{55}$$

where the system states x_{i1} and x_{i2} denote the reactor temperature and the concentration, respectively. u_i is the temperature of the coolant stream, which is system control input and w_i is external disturbance. The nominal plant parameters in this simulation are given in Table I, and the parameters $b_0 = \frac{F}{V}$, $b_1 = \frac{-\Delta H}{\rho C_p}$, $b_2 = \frac{E}{R}$, $b_3 = \frac{UA}{V\rho C_p}$.

Φ_{i1} and Φ_{i2} represent the interconnected terms of the two CSTR subsystems that $\Phi_{11} = 0.1x_{21} + 0.05x_{22}$, $\Phi_{12} = 0.1x_{22}$, $\Phi_{21} = 0.1x_{12}$, and $\Phi_{22} = 0.05x_{11} + 0.1x_{12}$. Each CSTR subsystem shares one equilibrium states as $x_o = [350, \ 0.5]^{\mathrm{T}}$, at steady input $u_o = 300$K. Choose three

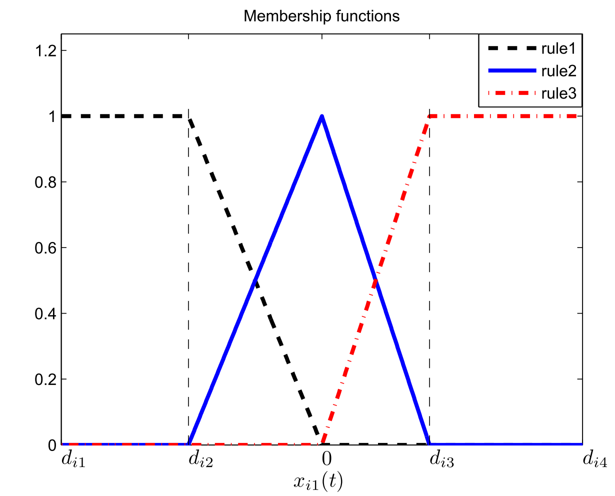

FIGURE 4. Membership functions in Example 7.

operating points $x_{o1} = [324.4 \quad 0.875]^{\mathrm{T}}$, $x_{o2} = [350 \quad 0.5]^{\mathrm{T}}$, $x_{o3} = [370.65 \quad 0.2]^{\mathrm{T}}$, and then linearize the large-scale system based on the change of coordinates $\tilde{x}_i = x_i - x_o$, $\tilde{u}_i = u_i - u_o$. With sampling period $T = 0.01$s, the nonlinear large-scale system can be represented by the following discrete-time fuzzy-affine dynamic models,

Plant Rule $\mathscr{R}_i^l$: IF $x_{i1}(t)$ is $\mathscr{F}_{i1}^l$, THEN

$$\left\{\begin{aligned} &\tilde{x}_i(t+1) = A_{il}\tilde{x}_i(t) + a_{il} + B_{il}\tilde{u}_i(t) + D_{i1l}w_i(t) \\ &\qquad\qquad + \sum_{\substack{n=1\\ n\neq i}}^{N} C_{ni}\tilde{x}_n(t) \\ &y_i(t) = H_{il}\tilde{x}_i(t) + D_{i2l}w_i(t) \\ &z_i(t) = L_{il}\tilde{x}_i(t) + N_{il}\tilde{u}_i(t),\ l = \{1,2,3\},\ \ i = \{1,2\}, \end{aligned}\right. \tag{56}$$

where

$$\left\{\begin{aligned} &\left[A_{i1} \mid a_{i1}\right] = \left[\begin{array}{cc|c} 0.9902 & 0.2909 & -0.3589 \\ -0.0001 & 0.9886 & 0.0017 \end{array}\right], \\ &\left[A_{i2} \mid a_{i2}\right] = \left[\begin{array}{cc|c} 1.0438 & 2.0919 & 0 \\ -0.0004 & 0.9800 & 0 \end{array}\right], \\ &\left[A_{i3} \mid a_{i3}\right] = \left[\begin{array}{cc|c} 1.0764 & 8.4224 & 0.9498 \\ -0.0005 & 0.9497 & -0.0045 \end{array}\right], \\ &B_{il} = \begin{bmatrix} 0.0209 \\ 0 \end{bmatrix}, \quad D_{i1l} = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}, \\ &C_{21} = 0.001 \times \begin{bmatrix} 1 & 0.5 \\ 0 & 0.1 \end{bmatrix}, \\ &C_{12} = 0.001 \times \begin{bmatrix} 0 & 1 \\ 0.5 & 1 \end{bmatrix}, \\ &H_{il} = \begin{bmatrix} 1.0 & 0 \end{bmatrix}, \quad D_{i2l} = 0.02, \quad N_{il} = 0.5, \\ &L_{il} = \begin{bmatrix} 1.0 & 0 \end{bmatrix}, \quad l = \{1,2,3\}, \quad i = \{1,2\}. \end{aligned}\right. \tag{57}$$

The normalized membership functions for each CSTR subsystem are shown in Fig. 4 with $d_{i1} = -50$, $d_{i2} = -25.6$, $d_{i3} = 20.65$, and $d_{i4} = 50$. The premise-variable space can be decomposed into three regions:

$$\begin{aligned} \mathscr{S}_{i1} &= \{x_i(t) | d_{i1} < x_{i1}(t) \leq d_{i2}\}, \\ \mathscr{S}_{i2} &= \{x_i(t) | d_{i2} < x_{i1}(t) \leq d_{i3}\}, \\ \mathscr{S}_{i3} &= \{x_i(t) | d_{i3} < x_{i1}(t) \leq d_{i4}\}. \end{aligned} \tag{58}$$

Note that $\mathscr{S}_{i1}$ and $\mathscr{S}_{i3}$ are crisp regions while $\mathscr{S}_{i2}$ is a fuzzy region.

For brevity, only the full-order DOF controller is considered in this example. Based on Theorem 2, one obtains the feasible solution with performance index $\gamma_{\min} = 0.3704$, and the controller gains are

$$\left\{\begin{aligned} &\left[A_{c11} \mid a_{c11} \mid B_{c11}\right] \\ &\quad = \left[\begin{array}{cc|c|c} 0.5901 & -0.1578 & 0.2588 & -0.1205 \\ -0.0190 & 0.6005 & -0.0016 & -0.0057 \end{array}\right], \\ &K_{c11} = \begin{bmatrix} -3.2475 & 67.8137 \end{bmatrix}, k_{c11} = 6.3415, \\ &D_{c11} = -7.9001, \\ &\left[A_{c12} \mid a_{c12} \mid B_{c12}\right] \\ &\qquad = \left[\begin{array}{cc|c|c} 0.5691 & 0.1883 & 0 & -0.1160 \\ -0.0238 & 0.5561 & 0 & -0.0052 \end{array}\right], \\ &K_{c12} = \begin{bmatrix} -3.0614 & 84.1099 \end{bmatrix}, k_{c12} = 0, \\ &D_{c12} = -8.1637, \\ &\left[A_{c13} \mid a_{c13} \mid B_{c13}\right] \\ &\quad = \left[\begin{array}{cc|c|c} 0.1743 & -0.4450 & -0.0467 & -0.0505 \\ -0.0377 & 0.4492 & -0.0061 & -0.0016 \end{array}\right], \\ &K_{c13} = \begin{bmatrix} -4.1898 & 170.5667 \end{bmatrix}, \quad k_{c13} = -14.1229, \\ &D_{c13} = -9.5756, \end{aligned}\right. \tag{59}$$

for subsystem S_1 and

$$\left\{\begin{aligned} &\left[A_{c21} \mid a_{c21} \mid B_{c21}\right] \\ &\quad = \left[\begin{array}{cc|c|c} 0.4722 & -0.0469 & 0.0269 & -0.1923 \\ -0.0271 & 0.5918 & -0.0168 & -0.0106 \end{array}\right] l, \\ &K_{c21} = \begin{bmatrix} -4.4432 & 66.2125 \end{bmatrix}, k_{c21} = 1.3006, \\ &D_{c21} = -8.9534, \\ &\left[A_{c22} \mid a_{c22} \mid B_{c22}\right] \\ &\qquad = \left[\begin{array}{cc|c|c} 0.4690 & 0.2155 & 0 & -0.1964 \\ -0.0250 & 0.4818 & 0 & -0.0083 \end{array}\right], \\ &K_{c22} = \begin{bmatrix} -4.9777 & 75.6530 \end{bmatrix}, k_{c22} = 0, \\ &D_{c22} = -9.3838, \\ &\left[A_{c23} \mid a_{c23} \mid B_{c23}\right] \\ &\quad = \left[\begin{array}{cc|c|c} 0.5033 & -0.7657 & -0.1461 & -0.1535 \\ -0.0240 & 0.4619 & -0.0114 & -0.0066 \end{array}\right], \\ &K_{c23} = \begin{bmatrix} -4.8554 & 202.7985 \end{bmatrix}, \ k_{c23} = -16.6049, \\ &D_{c23} = -12.0188, \end{aligned}\right. \tag{60}$$

for subsystem S_2.

With initial condition $\tilde{x}_{i0} = [20 \quad 0.2]^{\mathrm{T}}$, and external disturbance $w_i(t) = 450e^{-3t} \cdot \sin(5t)$, Fig. 5 shows the state responses for both subsystems S_1 and S_2. Under

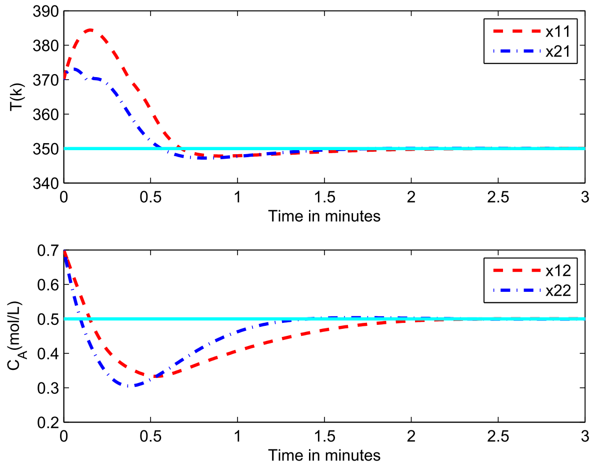

FIGURE 5. Trajectories of the closed-loop system states in Example 7.

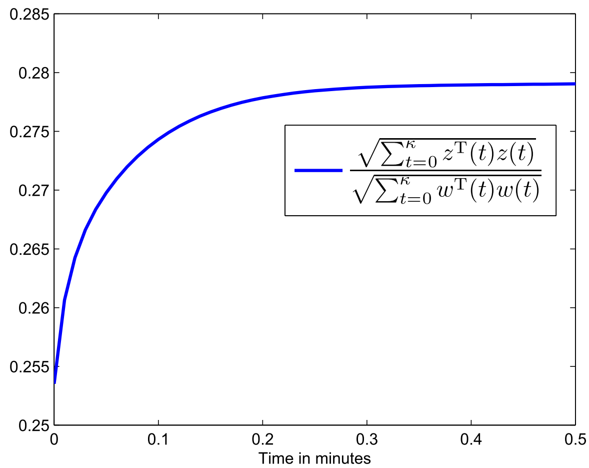

FIGURE 6. Response of the ratio $\frac{\sqrt{\sum_{t=0}^{\kappa} z^T(t)z(t)}}{\sqrt{\sum_{t=0}^{\kappa} w^T(t)w(t)}}$ in Example 7.

zero initial condition, Fig. 6 shows that the response of ratio $\frac{\sqrt{\sum_{t=0}^{k} z^{\mathrm{T}}(t)z(t)}}{\sqrt{\sum_{t=0}^{k} w^{\mathrm{T}}(t)w(t)}}$ is about 0.28, which is lower than $\gamma_{\min} = 0.3704$.

V. CONCLUSION

In this paper, a novel decentralized robust $\mathscr{H}_\infty$ fixed-order dynamic output feedback (DOF) control approach has been proposed for a class of discrete-time nonlinear large-scale systems based on T-S fuzzy-affine models. Through a descriptor system formulation and some convexification procedures, it is shown that the piecewise affine fixed-order DOF controller gains can be obtained via convex optimization. Two simulation examples have been presented to show the effectiveness of the proposed approach.

APPENDIX

Lemma 8 [21]: For two real matrices M and N with appropriate dimensions, the inequality

$$M^{\mathrm{T}}N + N^{\mathrm{T}}M \le \varepsilon M^{\mathrm{T}}M + \varepsilon^{-1}N^{\mathrm{T}}N$$

holds for a positive scalar ε.

Lemma 9 (Tchebyshev's inequality): The following inequality holds for arbitrary vectors $x_l \in \mathscr{R}^n, l = 1, 2, .., N$,

$$\left[\sum_{l=1}^{N} x_l\right]^{\mathrm{T}} \left[\sum_{l=1}^{N} x_l\right] \le N \sum_{l=1}^{N} x_l^{\mathrm{T}} x_l.$$

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