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Reduced-Order Observer-Based Consensus for Multi-Agent Systems With Time Delay and Event Trigger Strategy

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ABSTRACT This paper investigates the reduced-order observer-based consensus problem of multi-agent system with time delay and event trigger strategy. First, a multi-step algorithm is presented to construct a reduced-order observer for each agent. Then, a novel push-based event-triggered control strategy, which based on the reduced-order observer and the relative outputs of neighboring agents is proposed. Under this control strategy, a sufficient condition for the consensus of multi-agent systems is obtained by using the integral inequality technique and matrix theory. Moreover, the estimation value of the output time delay is also obtained and the Zeno-behavior of triggering time sequences is excluded. Finally, two simulations about the multi-agent system are provided to illustrate the correctness of theoretical results.

INDEX TERMS Reduced-order observer-based, output time delay, push-based distributed event-triggered, multi-agent systems consensus.

I. INTRODUCTION

Recently, the cooperative control of multi-agent systems (MASs) has received attractive attentions due to its wide applications, such as congestion control in sensor networks, robot control, distributed computation and so on [1]–[4]. One of the major problems in cooperative control of MAS is to design control strategy such that the state of all agents reach a common value. This problem is called consensus. Up to now, researchers have proposed a number of consensus control protocols to solve the consensus problem for multi-agent system with and without time delay [5], [6].

In many practical systems, due to the cost and other factors, the state cannot be obtained directly, which means the control protocol based on the state is not available. In order to obtain the agent state, observer is proposed, which can construct the state based on the input and output. Consequently, there are many works to investigate the observer design [7]–[10]. In [7], the authors design two types of distributed observer and observer-based consensus protocols for leader-following discrete-time multi-agent systems. In [8], the authors investigate distributed observer-based stabilization problem of multi-agent systems. In [9], by considering two logic switches, the authors propose a new general functional observer scheme for three linear systems with unknown inputs. In [10], the authors improve the general functional observers for two linear systems by reducing the observer order.

Based on the proposed observers, numerous consensus protocols for multi-agent system are designed [11]–[16]. In [11], the authors investigate the consensus problem of linear multi-agent systems based on centralized and distributed observer-based control strategies. In [12], the authors present an algorithm which can construct a full-order observer to guarantee the consensus of multi-agent systems. In [13], by using the relative outputs and inputs of neighboring agents, the authors establish distributed and truncated reduced-order observer which can be applicable to continuous-time and discrete-time multi-agent systems. In [14], the authors investigate the tracking consensus problem of linear multi-agent systems under a networked detectability condition with reduced-order protocols. In [15], the authors study the multi-agent consensus problem with general linear dynamics via the distributed reduced-order observer-based protocols under directed switching topology. In [16], the authors investigate the consensus problem of linear multi-agent systems with reduced-order observer-based protocols.

From the above literatures, it can be seen there are two type observers: the full-order observer and the reduced-order observer. Compared with the full-order observer, the reduced-order observer only need local information to construct the agent state, which implies it needs less computational cost than full-order observer. That is to say, the computational complexity of the observer is reduced while the conservativeness of the multi-agent system is improved. However, all existing control strategies for reduced-order observer-based multi-agent system are continuous. Continuous control strategies lead to frequent communication among nodes, which cause the network congestion and waste the network resources. In order to overcome the conservativeness of continuous strategies, the event-triggered scheme is proposed [17]–[25], where updates are only determined by certain events that triggered depending on the nodes dynamic behaviors. In [17], the authors propose a distributed event-triggered control strategy for first order multi-agent system. In [18], by using event-triggered strategy, the authors study the consensus problem of multiple double-integrator multi-agent systems under fixed topology and switching topology. In [19]–[23], the consensus problems of general linear multi-agent system with different event-triggered control strategy are investigated. In [24], the authors investigate event-triggered sampled-data consensus problem for distributed multi-agent system with directed graph. In [25], the authors model the switching of network topologies as a Markov process and propose a novel event-triggered strategy. However, there are few works on the reduced-order observer-based consensus problem of multi-agent system with event trigger strategy and output time delay. So the observation provides us the motivation of this paper to design a new reduced-order observer-based event-triggered consensus strategy for multi-agent system with output time delay.

In this paper, we investigate the reduced-order observer-based consensus problem of multi-agent system with output time delay and event trigger strategy. The primary contributions of this paper as following : (I) A multi-step algorithm is designed to construct a reduced-order observer for multi-agent systems with output time delay. (II) A novel push-based event-triggered function is designed. Under this control strategy, a sufficient condition for the consensus of multi-agent systems is obtained by using the integral inequality technique and matrix theory.

The rest of this paper is organized as follows. Some necessary definitions, preliminary results of graph theory and model description are given in Section 2. In Section 3, the main results of this paper are presented. Two simulation examples about the multi-agent system are given to illustrate the effectiveness of the analytical result in Section 4.

Notations: Throughout this paper, R^n and $R^{m\times n}$ denote the n-dimensional Euclidean space and the set of real $m \times n$ dimensions matrix, respectively. $\|\chi\|$ indicates the Euclidean norm for vector χ. Denote $\|A\| = \sqrt{\lambda_{\max}\left(A^T A\right)}$ the induced 2-norm for matrix A, where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of matrix $(\cdot)$ and the superscript T means transpose for real matrices. $\otimes$ represents the Kronecker product. I_m denotes the $m \times m$ dimensional identity matrix. Let $\mathbf{1}_n$ ($\mathbf{0}_n$) be a column vector with n elements being 1 (0). For a square non-singular matrix X, X^{-1} denotes its inverse matrix, X^T and X^H represent its transport matrix and conjugate transpose matrix, respectively.

II. PRELIMINARIES

A. ALGEBRAIC GRAPH THEORY

The communication topology among these agents is introduced by an interaction digraph (directed graph). Let $\zeta = (\nu, \varepsilon, \Delta)$ represent a digraph with set of vertices $\nu = \{1, 2, \ldots, N\}$ and the set of edges $\varepsilon \subseteq \nu \times \nu$. $\Delta = \left(a_{ij}\right)_{N\times N}$ is the adjacency matrix where a_{ij} represents weight of edge (i, j), there $a_{ij} > 0$ if $(i, j) \in \varepsilon$ and $a_{ij} = 0$, otherwise. When refers to $a_{ij} > 0$, it denotes that agent i can receive the information from agent j, but not vice versa. For an edge (i, j), node i is called parent node, node j is child node, and i is a neighbor of j. In this paper, we also assume that there are not exist self-loops or parallel edges in the communication topology. Then, the Laplacian matrix $L = \left(l_{ij}\right) \in R^{N\times N}$ is defined, where $l_{ij} = -a_{ij} \leq 0$, $i \neq j$; $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij} \geq 0$. The in-degree of agent is defined as $d_i = \sum_{j=1}^{N} a_{ij}$, as we can obtain the Laplacian matrix is $L = D - \Delta$ where $D = diag\left(d_1, d_2, \ldots, d_N\right)$. If there exist a sequence of edge of form $(i, j_1), (j_1, j_2), \ldots, (j_m, j)$ in a directed graph which beginning with i and ending with j, then the node j is said to be reachable from node i in directed graph. Especially, if there is a directed path to any different nodes in the directed graph, as we can say that the directed graph is strongly connected. In addition, the Laplacian matrix L has a simple zero eigenvalue and all the other eigenvalues have positive real parts if and only if the directed graph associated with L has a directed spanning tree [26].

B. MODEL DESCRIPTION

The multi-agent system is described as follows:

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) \\ y_i(t) = Cx_i(t-\tau), \quad i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where $x_i(t) \in R^n$, $u_i(t) \in R^p$, $y_i(t) \in R^q$ are the state, control and output of the ith agent, respectively. $A \in R^{n\times n}$, $B \in R^{n\times p}$ and $C \in R^{q\times n}$ are constant matrices and C is assumed to have full row rank, $\tau > 0$ is the output time delay. The controller is designed as follows:

$$\begin{aligned} u_i(t) = {} & cKQ_1 \sum_{j\in N_i} a_{ij}\left(y_i\left(t_k^i\right) - y_j\left(t_{k'}^j\right)\right) \\ & + cKQ_2 \sum_{j\in N_i} a_{ij}\left(\hat{x}_i\left(t_k^i\right) - \hat{x}_j\left(t_{k'}^j\right)\right), \end{aligned} \tag{2}$$

where $\hat{x}_i(t) \in R^{n-q}$ is the observer state, $Q_1 \in R^{n\times q}$, $Q_2 \in R^{n\times(n-q)}$ are given by $\left[\, Q_1 \; Q_2 \,\right] = \begin{bmatrix} C \\ T \end{bmatrix}^{-1}$ and $K \in R^{p\times n}$

is the feedback gain matrix to be designed, c is a coupling strength.

C. OBSERVER DESIGN

Compared with full-order observer, the reduced-order observer only need local information to construct the agent state, which means it need less computational cost. Inspired by [16], the reduced-order observer of the system (1) is given by:

$$\dot{\hat{x}}_i(t) = F\hat{x}_i(t) + Gy_i(t) + TBu_i(t), \tag{3}$$

where $\hat{x}_i(t) \in R^{n-q}$ is the observer state, $F \in R^{(n-q)\times(n-q)}$ is Hurwitz and has no eigenvalues in common with those of A, $G \in R^{(n-q)\times q}$, $T \in R^{(n-q)\times n}$ is the unique solution to the following Sylvester equation: $TA - FT = GC$.

Definition 1 [27]: For system (1), if there exist $u_i(t) \in U$ such that for any initial value $x_i(0)$,

$$\lim_{t\to\infty} \left\|x_j(t) - x_i(t)\right\| = 0, \quad i, j = 1, 2, \ldots, N, \tag{4}$$

then we say the system (1) is consensusable with respect to U.

III. MAIN RESULT

In this section, we give the main result of this paper. First, we give the algorithm to construct the reduced-order observer. Then, consensus analysis of the multi-agent systems is presented.

Assumption 1: Matrix $F \in R^{(n-q)\times(n-q)}$ is Hurwitz which has no eigenvalues in common with A and matrix $T \in R^{(n-q)\times n}$ is the unique solution to the following Sylvester equation:

$$TA - FT = GC. \tag{5}$$

In the following, we give the algorithm to construct the reduced-order observer.

Algorithm 1: Under the Assumption 1 that A, C, T, F and G satisfy the Sylvester equation: $TA - FT = GC$ and that (A, B, C) is controllable and observable, the reduced-order observer-based consensus protocol (2) can be designed as follows:

(1) Choose a Hurwitz matrix F having no eigenvalues in common with those of A. Select G such that (F, G) is stabilizable.

(2) Solve Sylvester equation (5) to get a solution T, which satisfies that $\begin{bmatrix} C \\ T \end{bmatrix}$ is nonsingular. Then, compute matrices Q_1 and Q_2 by $\begin{bmatrix} Q_1 & Q_2 \end{bmatrix} = \begin{bmatrix} C \\ T \end{bmatrix}^{-1}$.

(3) Solve the following algebraic Riccati equation (ARE):

$$A^TP + PA - PBB^TP = -Q_c, \quad Q_c > 0 \tag{6}$$

to get one solution $P > 0$ where the Q_c must be symmetric. Then, choose the matrix $K = -B^TP$.

(4) Select the coupling strength $c \geq 1/\left(2\min_{\lambda_i\neq 0}\{Re(\lambda_i)\}\right)$, where λ_i is the ith eigenvalue of Laplacian matrix L.

(5) Choose $\gamma > 0$ and $k \geq 1$ to satisfy the equation: $\tau < [\gamma - k(\alpha_3 + \alpha_4)]/k(\alpha_1 + \alpha_2)$.

Before giving consensus analysis of the multi-agent systems, some Lemmas are presented in the following.

Lemma 1 [28]: The Kronecker product has the following properties: for any matrices A, B and C with appropriate dimensions,

(1) $(A + B) \otimes C = A \otimes C + B \otimes C$;

(2) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$.

Lemma 2 [30]: Zero is an eigenvalue of L with $\mathbf{1}$ and a nonnegative vector $r^T \in R^{1\times N}$, respectively, as the corresponding right and left eigenvectors, and all nonzero eigenvalues have positive real parts. Furthermore, zero is a simple eigenvalue of L if and only if the graph ζ has a directed spanning tree.

Lemma 3 [29]: For any $t > t_0$, there exist constants $k \geq 1$ and $\rho > 0$ when all the eigenvalues of J are in the open left-half plane, such that

$$\left\|e^{J(t-t_0)}\right\| \leq ke^{-\rho(t-t_0)}, \quad t \geq t_0 \tag{7}$$

By defining the original measurement error function of each agent i as

$$e_i(t) = x_i\left(t_k^i\right) - x_i(t), \quad t \in \left[t_k^i, t_{k+1}^i\right),$$
$$\hat{e}_i(t) = \hat{x}_i\left(t_k^i\right) - \hat{x}_i(t), \quad t \in \left[t_k^i, t_{k+1}^i\right),$$

the control strategy (2) can be rewritten as

$$\begin{aligned} u_i(t) = cKQ_1C\sum_{j\in N_i} a_{ij}\big(&x_i(t-\tau) - x_j(t-\tau) \\ &+ e_i(t-\tau) - e_j(t-\tau)\big) \\ + cKQ_2\sum_{j\in N_i} a_{ij}&\left(\hat{x}_i(t) - \hat{x}_j(t) + \hat{e}_i(t) - \hat{e}_j(t)\right) \end{aligned} \tag{8}$$

Let $\eta_i(t) = \left[x_i^T(t)\ \hat{x}_i^T(t)\right]^T$, $\delta_i(t) = \left[e_i^T(t), \hat{e}_i^T(t)\right]^T$, define a sequence of triggering time instants $\{t_k^i\}$, $k = 1, 2, \ldots$, for each agent i, which can be expressed as:

$$t_{k+1}^i = \inf\left\{t : t > t_k^i, f_i(t) > 0\right\}, \tag{9}$$

where $1 > \beta_1 > 0$, $\beta_2 > 0$, $\gamma > 0$ and trigger function

$$\begin{aligned} f_i(\delta_i(t), \eta_i(t), t) = &\left\|\sum_{j=1}^N a_{ij}\left(\delta_i(t) - \delta_j(t)\right)\right\| \\ &- \beta_1\left\|\sum_{j=1}^N a_{ij}\left(\eta_i\left(t_k^i\right) - \eta_j\left(t_{k'}^j\right)\right)\right\| \\ &- \beta_2 e^{-\gamma(t-t_0)}. \end{aligned} \tag{10}$$

Using the Kronecker product of matrix, the multi-agent systems (1) with respect to the control (2) can be rewritten as follows:

$$\dot{\eta}(t) = M_1\eta(t) + M_2\eta(t-\tau) + V_1\delta(t) + V_2\delta(t-\tau), \tag{11}$$

where $\eta(t) = \left[\eta_1^T(t), \ldots, \eta_N^T(t)\right]^T$,

$$\delta(t) = \left[\delta_1^T(t), \ldots, \delta_N^T(t)\right]^T,$$

$$M_1 = \begin{pmatrix} I_N \otimes A & cL \otimes BKQ_2 \\ 0 & (I_N \otimes F) + (cL \otimes TBKQ_2) \end{pmatrix},$$

$$M_2 = \begin{pmatrix} cL \otimes BKQ_1C & 0 \\ (I_N \otimes GC) + (cL \otimes TBKQ_1C) & 0 \end{pmatrix},$$

$$V_1 = \begin{pmatrix} 0 & cL \otimes BKQ_2 \\ 0 & cL \otimes TBKQ_2 \end{pmatrix},$$

$$V_2 = \begin{pmatrix} cL \otimes BKQ_1C & 0 \\ cL \otimes TBKQ_1C & 0 \end{pmatrix}.$$

Theorem 1: Assume that the communication topology of these agents has a directed spanning tree, the pair (A, B) is controllable and the pair (A, C) is observable, the consensus of the multi-agent systems with output time delay is achieved if the parameters in (2) and (3) are selected according to Algorithm 1 and the triggering function is designed as (10) where $\beta_1 \in (0, 1)$, $\beta_2 > 0$, $\rho > \gamma > 0$. Furthermore, the Zeno-behavior is excluded in the closed-loop system.

Proof: For (11), using the Newton-Leibnitz formula $\eta(t-\tau) = \eta(t) - \int_{t-\tau}^{t} \dot{\eta}(s)\,ds$, system (11) can be rewritten as:

$$\dot{\eta}(t) = (M_1 + M_2)\eta(t) - M_2 \int_{t-\tau}^{t} \begin{bmatrix} M_1\eta(s) + M_2\eta(s-\tau) \\ +V_1\delta(s) + V_2\delta(s-\tau) \end{bmatrix} ds + V_1\delta(t) + V_2\delta(t-\tau). \quad (12)$$

Because ζ contains a directed spanning tree, it follows from Lemma 2 that zero is a simple eigenvalue of L and all other eigenvalues have positive real parts. Let $U \in R^{N\times N}$ be a unitary matrix such that $U^TLU = \Lambda = \begin{bmatrix} 0 & 0 \\ 0 & \Delta \end{bmatrix}$, where the diagonal entries of Δ are the nonzero eigenvalues of L. Since the right and left eigenvectors corresponding to the zero eigenvalue of L are, respectively, $\mathbf{1}$ and r^T, we can choose $U = \left[\frac{1}{\sqrt{N}} Y_1\right]$, $U^T = \begin{bmatrix} r^T \\ Y_2 \end{bmatrix}$, with $Y_1 \in R^{N\times(N-1)}$, $Y_2 \in R^{(N-1)\times N}$, $r^T \in R^N$ is a nonnegative vector such that $r^TL = 0$ and $r^T\mathbf{1} = 1$. Let $\dot{\upsilon}(t) = (M_1 + M_2)\upsilon(t)$ and $\varepsilon \triangleq \left[\varepsilon_1^T, \varepsilon_2^T, \ldots, \varepsilon_N^T\right]^T = \left(U^T \otimes I_{2n-q}\right)\upsilon$. Then $\dot{\upsilon}(t) = (M_1 + M_2)\upsilon(t)$ can be rewritten as:

$$\dot{\varepsilon}(t) = \begin{pmatrix} c_1 & c_3 \\ c_2 & c_4 \end{pmatrix} \varepsilon(t), \quad (13)$$

where

$$c_1 = I_N \otimes A + c(\Lambda \otimes BKQ_1C),$$

$$c_2 = (I_N \otimes GC) + c(\Lambda \otimes TBKQ_1C),$$

$$c_3 = c(\Lambda \otimes BKQ_2), c_4 = (I_N \otimes F) + c(\Lambda \otimes TBKQ_2).$$

Equivalently, for $i = 2, \ldots, N$, system (13) can be rewritten as follows:

$$\dot{\varepsilon}_i(t) = \begin{pmatrix} A + c\lambda_iBKQ_1C & c\lambda_iBKQ_2 \\ GC + c\lambda_iTBKQ_1C & F + c\lambda_iTBKQ_2 \end{pmatrix} \varepsilon_i(t). \quad (14)$$

Multiplying the left and right sides of the matrix in (14) by $Q = \begin{bmatrix} I & 0 \\ -T & I \end{bmatrix}$ and $Q^{-1} = \begin{bmatrix} I & 0 \\ T & I \end{bmatrix}$, respectively, we can obtain:

$$Q\begin{pmatrix} A + c\lambda_iBKQ_1C & c\lambda_iBKQ_2 \\ GC + c\lambda_iTBKQ_1C & F + c\lambda_iTBKQ_2 \end{pmatrix} Q^{-1} = \begin{pmatrix} A + c\lambda_iBK & c\lambda_iBKQ_2 \\ 0 & F \end{pmatrix}. \quad (15)$$

Form steps (3) and (4) in Algorithm 1, we can obtain that there exists a $P > 0$ satisfying

$$\begin{aligned} P(A + c\lambda_iBK) + (A + c\lambda_iBK)^T P &= PA + A^TP - 2c\lambda_iPBB^TP \\ &\le AP + PA^T - PBB^TP \\ &< 0, \quad i = 2, \ldots, N. \end{aligned} \quad (16)$$

That is, $A + c\lambda_iBK$, $i = 2, \ldots, N$ are Hurwitz. Therefore, the $N-1$ systems in (14) are exponentially stable, implying that system $\dot{\upsilon}(t) = (M_1 + M_2)\upsilon(t)$ is exponentially stable, *i.e.*, the consensus problem is solved.

Then, by (13), (14), (15) and (16), then we can obtain that all the eigenvalues of $M_1 + M_2$ are in the open left-half plane. Then, using the variation of parameter formula, we can get

$$\begin{aligned} \eta(t) &= e^{(M_1+M_2)(t-t_0)}\eta(t_0) - \int_{t_0}^{t} e^{(M_1+M_2)(t-\theta)} \\ &\times \Big\{ M_2 \int_{\theta-\tau}^{\theta} M_1\eta(s)\,ds + M_2\eta(s-\tau)\,ds + V_1\delta(s)\,ds \\ &\quad + V_2\delta(s-\tau)\,ds + V_1\delta(\theta) + V_2\delta(\theta-\tau) \Big\} d\theta. \end{aligned} \quad (17)$$

Since (A, B) is stabilizable and the communication topology has a directed spanning tree, we can obtain:

$$\begin{aligned} \|\eta(t)\| &\le ke^{-\rho(t-t_0)}\|\eta(t_0)\| + k\int_{t_0}^{t} e^{-\rho(t-\theta)} \\ &\times \Big\{ \|M_2\| \int_{\theta-\tau}^{\theta} \|M_1\|\,\|\eta(s)\|\,ds \\ &\quad + \|M_2\|\,\|\eta(s-\tau)\|\,ds + \|V_1\|\,\|\delta(s)\|\,ds \\ &\quad + \|V_2\|\,\|\delta(s-\tau)\|\,ds + \|V_1\|\,\|\delta(\theta)\| \\ &\quad + \|V_2\|\,\|\delta(\theta-\tau)\| \Big\} d\theta. \end{aligned} \quad (18)$$

From event trigger condition (10) and notice it can always guarantee $f_i(\delta_i(t), \eta_i(t), t) \leq 0$, we can obtain

$$\|L \otimes \delta(t)\| \leq \frac{\beta_1}{1-\beta_1}\|L \otimes \eta(t)\| + \frac{N\beta_2}{1-\beta_1}e^{-\gamma(t-t_0)}. \quad (19)$$

That is to say,

$$\|\delta(t)\| \leq \frac{\beta_1}{1-\beta_1}\|\eta(t)\| + \frac{N\beta_2}{\|L\|(1-\beta_1)}e^{-\gamma(t-t_0)}. \quad (20)$$

By (18) and (20), one can obtain:

$$\begin{aligned}\|\eta(t)\| \leq\ & ke^{-\rho(t-t_0)}\|\eta(t_0)\| \\ & + k\int_{t_0}^{t} e^{-\rho(t-\theta)} \\ & \times \left\{\int_{\theta-\tau}^{\theta}[\alpha_1\|\eta(s)\| + \alpha_2\|\eta(s-\tau)\|]\,ds \right. \\ & \left. + \alpha_3\|\eta(\theta)\| + \alpha_4\|\eta(\theta-\tau)\|\right\}d\theta \\ & + k\alpha_5\left[e^{-\gamma(t-t_0)} - e^{-\rho(t-t_0)}\right]. \end{aligned} \quad (21)$$

where

$$\begin{aligned}\alpha_1 &= \|M_1\|\|M_2\| + \frac{\|M_2\|\|V_1\|\beta_1}{1-\beta_1}, \\ \alpha_2 &= \|M_2\|\|M_2\| + \frac{\|M_2\|\|V_2\|\beta_1}{1-\beta_1}, \\ \alpha_3 &= \frac{\|V_1\|\beta_1}{1-\beta_1}, \quad \alpha_4 = \frac{\|V_2\|\beta_1}{1-\beta_1}, \\ \alpha_5 &= \frac{\left(\frac{\|V_1\|N\beta_2}{1-\beta_1} - \frac{\|M_2\|\|V_1\|N\beta_2}{\gamma\|L\|(1-\beta_1)}\right)}{(\rho-\gamma)} + \frac{e^{2\gamma\tau}\left(\frac{\|M_2\|\|V_2\|N\beta_2}{\gamma\|L\|(1-\beta_1)}\right)}{(\rho-\gamma)} \\ &\quad + \frac{e^{\gamma\tau}\left(\frac{\|V_2\|N\beta_2}{1-\beta_1} + \frac{\|M_2\|\|V_1\|N\beta_2}{\gamma\|L\|(1-\beta_1)} - \frac{\|M_2\|\|V_2\|N\beta_2}{\gamma\|L\|(1-\beta_1)}\right)}{(\rho-\gamma)}. \end{aligned}$$

Assume that $\gamma < \rho$ (the same way holds for $\gamma > \rho$), by (21), one has

$$\begin{aligned}\|\eta(t)\| \leq\ & k(\|\eta(t_0)\| + \alpha_5)e^{-\gamma(t-t_0)} \\ & + k\int_{t_0}^{t} e^{-\gamma(t-\theta)} \\ & \times \left\{\int_{\theta-\tau}^{\theta}\alpha_1\|\eta(s)\|\,ds + \alpha_2\|\eta(s-\tau)\|\,ds \right. \\ & \left. \times\ \alpha_3\|\eta(\theta)\| + \alpha_4\|\eta(\theta-\tau)\|\right\}d\theta. \end{aligned} \quad (22)$$

In the following, we show that if there is a $\lambda \in (0, \gamma)$ satisfying

$$\frac{k\left[\left(\alpha_1 + \alpha_2 e^{\lambda\tau}\right)\left(e^{\lambda\tau} - 1\right) + \lambda\alpha_3 + \lambda\alpha_4 e^{\lambda\tau}\right]}{\lambda(\gamma-\lambda)} < 1. \quad (23)$$

Such that the following inequality holds for any $\xi > 1$

$$\|\eta(t)\| < \xi k(\|\eta(t_0)\| + \alpha_5)e^{-\lambda(t-t_0)} \triangleq v(t), \quad t \geq t_0. \quad (24)$$

First, we prove the existence of λ in (24). Let $f(\lambda) = k\left(\alpha_1 + \alpha_2 e^{\lambda\tau}\right)\left(e^{\lambda\tau} - 1\right) + k\alpha_3\lambda + k\lambda\alpha_4 e^{\lambda\tau} + \lambda^2 - \lambda\gamma$, we can obtain $f(0) = 0$ and $f'(0) = k(\alpha_1 + \alpha_2)\tau + k(\alpha_3 + \alpha_4) - \gamma$. When $\tau < [\gamma - k(\alpha_3 + \alpha_4)]/k(\alpha_1 + \alpha_2) = \tau_0$, which implies that there is a $\lambda \in (0, \gamma)$ such that $f(\lambda) < 0$, that is to say, (23) holds on.

Second, we prove the (24).

If (24) does not set up for any $t \in [t_0 - \tau, t^*)$, there must exist a $t^* > t_0$ such that $\|\eta(t^*)\| = v(t^*)$ and $\|\eta(t)\| < v(t)$. Then, by (21), we can get

$$\begin{aligned} v(t^*) =\ & \|\eta(t^*)\| \\ <\ & \xi k(\|\eta(t_0) + \alpha_5\|) \\ & \times \left\{e^{-\gamma(t^*-t_0)} + k\int_{t_0}^{t^*} e^{-\gamma(t^*-\theta)} \right. \\ & \times \left[\frac{\left(\alpha_1 + \alpha_2 e^{\lambda\tau}\right)\left(e^{\lambda\tau} - 1\right)}{\lambda} + \alpha_3 + \alpha_4 e^{\lambda\tau}\right] \\ & \left. \times e^{-\lambda(\theta-t_0)}d\theta\right\} \\ =\ & \xi k(\|\eta(t_0) + \alpha_5\|) \\ & \times \left\{e^{-\gamma(t^*-t_0)} \right. \\ & + \frac{k\left[\left(\alpha_1 + \alpha_2 e^{\lambda\tau}\right)\left(e^{\lambda\tau} - 1\right) + \lambda\alpha_3 + \lambda\alpha_4 e^{\lambda\tau}\right]}{\lambda(\gamma-\lambda)} \\ & \left. \times \left(e^{-\lambda(t^*-t_0)} - e^{-\gamma(t^*-t_0)}\right)\right\} \\ <\ & \xi k(\|\eta(t_0) + \alpha_5\|)e^{-\lambda(t^*-t_0)} = v(t^*) \end{aligned} \quad (25)$$

The contradiction of (25) shows that (24) is valid for any $\xi > 1$, Let $\xi \to 1$, one has

$$\|\eta(t)\| \leq e^{-\gamma(t-t_0)}, \quad t \geq t_0 \quad (26)$$

which implies the reduced-order observer-based output feedback push-based event-triggered consensus for multi-agent systems with output time delay can be achieved exponentially.

In the following, we eliminate Zeno-behavior in the closed-loop systems. Compute the upper-right-hand Dini derivative of $\|\delta_i(t)\|$ over interval $[t_k^i, t_{k+1}^i)$, we derive that

$$\begin{aligned} D^+\left\|\dot{\delta}_i(t)\right\| \leq\ & \left\|\dot{\delta}_i(t)\right\| \leq \|\dot{\eta}(t)\| \\ =\ & \|M_1\|\|\eta(t)\| + \|M_2\|\|\eta(t-\tau)\| \\ & + \|V_1\|\|\delta(t)\| + \|V_2\|\|\delta(t-\tau)\| \end{aligned} \quad (27)$$

Noticing that (20), one can obtain that

$$\begin{aligned}\|\delta(t-\tau)\| \leq\ & \frac{\beta_1}{1-\beta_1}\|\eta(t-\tau)\| \\ & + \frac{N\beta_2}{\|L\|(1-\beta_1)}e^{-\gamma(t-\tau-t_0)}. \end{aligned} \quad (28)$$

By (26), (27) and (28), one can obtain that

$$D^+\|\delta_i(t)\| \leq \alpha_6 e^{-\gamma(t-t_0)} + \alpha_7 e^{-\gamma(t-\tau-t_0)}, \quad (29)$$

where $\alpha_6 = \|M_1\| + \frac{\beta_1}{1-\beta_1}\|V_1\| + \frac{N\beta_2}{\|L\|(1-\beta_1)}\|V_1\|$, $\alpha_7 = \|M_2\| + \frac{\beta_1}{1-\beta_1}\|V_2\| + \frac{N\beta_2}{\|L\|(1-\beta_1)}\|V_2\|$.

For (20) and $\delta_i\left(t_k^i\right) = 0$ that $\|\delta_i(t)\| \leq \frac{\alpha_6}{\gamma}\left[e^{-\gamma\left(t_k^i-t_0\right)} - e^{-\gamma(t-t_0)}\right] + \frac{\alpha_7}{\gamma}\left[e^{-\gamma\left(t_k^i-\tau-t_0\right)} - e^{-\gamma(t-\tau-t_0)}\right]$, $t \in \left[t_k^i, t_{k+1}^i\right)$.

The next event will not be triggered until function (6) crosses zero, *i.e.*,

$$\begin{aligned}\frac{\beta_1}{(1-\beta_1)N}\left\|\eta_i\left(t_{k+1}^i\right)\right\| + \frac{\beta_2}{\|L\|(1-\beta_1)}e^{-\gamma\left(t_{k+1}^i-t_0\right)} &= \left\|\delta_i\left(t_{k+1}^i\right)\right\| \\ &\leq \frac{\alpha_6}{\gamma}\left[e^{-\gamma\left(t_k^i-t_0\right)} - e^{-\gamma\left(t_{k+1}^i-t_0\right)}\right] \\ &\quad + \frac{\alpha_7}{\gamma}\left[e^{-\gamma\left(t_k^i-\tau-t_0\right)} - e^{-\gamma\left(t_{k+1}^i-\tau-t_0\right)}\right]. \qquad (30)\end{aligned}$$

Denote $T_k^i = t_{k+1}^i - t_k^i$, by (30), we have

$$\alpha_8 e^{-\gamma T_k^i} \leq \frac{\alpha_6}{\gamma}\left[1 - e^{-\gamma T_k^i}\right] + \frac{\alpha_7}{\gamma}e^{\gamma\tau}\left[1 - e^{-\gamma T_k^i}\right], \qquad (31)$$

where $\alpha_8 = \frac{\beta_2}{\|L\|(1-\beta_1)}$ and $\gamma \in (0, \lambda)$.

By (31), we can get

$$T_k^i \geq \ln\left(\frac{\alpha_6}{\gamma} + \frac{\alpha_7}{\gamma}e^{\gamma\tau} \Big/ \alpha_8 + \frac{\alpha_6}{\gamma} + \frac{\alpha_7}{\gamma}e^{\gamma\tau}\right) \Big/ -\gamma. \qquad (32)$$

Obviously, $\frac{\alpha_6}{\gamma} + \frac{\alpha_7}{\gamma}e^{\gamma\tau} \Big/ \alpha_8 + \frac{\alpha_6}{\gamma} + \frac{\alpha_7}{\gamma}e^{\gamma\tau} < 1$ and $\ln\left(\frac{\alpha_6}{\gamma} + \frac{\alpha_7}{\gamma}e^{\gamma\tau} \Big/ \alpha_8 + \frac{\alpha_6}{\gamma} + \frac{\alpha_7}{\gamma}e^{\gamma\tau}\right) < 0$, we can easily derive that $T_k^i > 0$ by (32) for any i. which implies that the Zero-behavior is exclude for any agent i. The proof of Theorem 1 is completed.#

Remark 1: Compare with some existing works, the model and consensus algorithm in this paper are more general, especially suitable for practical applications. For $\tau = 0$, the system (1) can be rewritten as

$$\begin{cases}\dot{x}_i(t) = Ax_i(t) + Bu_i(t) \\ y_i(t) = Cx_i(t), \quad i = 1, 2, \ldots, N,\end{cases}$$

which is same as the model of [16]. So, the model of [16] can be considered as a special case of our model. Moreover, that method used in [16] cannot solve the consensus problem of the system (1). For $Q_1 = 0$, $Q_2 \in R^{n\times n}$, then (2) can be rewritten as full order observer control protocol:

$$u_i(t) = aKQ_2\sum_{j\in N_i} a_{ij}\left(\hat{x}_i(t) - \hat{x}_j(t)\right),$$

which is proposed in [12]. So, our protocol can be regarded as an extension of [12].

IV. SIMULATION EXAMPLE

In this section, we provide two examples about the robotic system to illustrate the theoretical result.

Example 1: Consider the network of multi-agent systems (1) with six agents, the topology of this netwrok is shown in Fig.1.

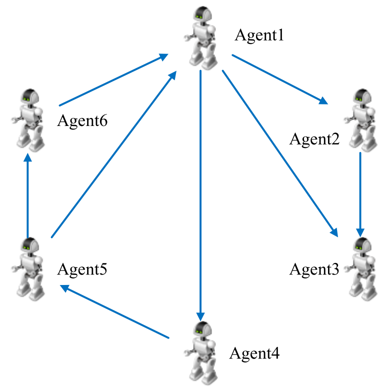

FIGURE 1. The directed communication topology of six agents.

The dynamic of each robot is as follows:

$$\begin{cases}\dot{x}_i(t) = \begin{bmatrix}-2 & 1 \\ -0.5 & -1\end{bmatrix}x_i(t) + \begin{bmatrix}1 \\ 1\end{bmatrix}u_i(t) \\ y_i(t) = \begin{bmatrix}1 & 0.9\end{bmatrix}x_i(t-\tau), \quad i = 1, 2, \ldots, N.\end{cases}$$

It is easy to see $A = \begin{bmatrix}-2 & 1 \\ -0.5 & -1\end{bmatrix}$, $B = \begin{bmatrix}1 \\ 1\end{bmatrix}$, $C = \begin{bmatrix}1 & 0.9\end{bmatrix}$.

Now, we construct the reduced order observer based on the Algorithm 1, which is as follows:

(1) According to step (1), we choose $F = -2$. It is obviously that F is a Hurwitz matrix and have no eigenvalues in common with A. Select $G = -1$ such that $(F, G) = \begin{bmatrix}-2 & -1\end{bmatrix}$ is stabilizable.
(2) According to step (2), by solving the Sylvester equation (5), we can get $T = \begin{bmatrix}-2.9 & 2\end{bmatrix}$, $Q_1 = \begin{bmatrix}0.4338 & 0.6291\end{bmatrix}^T$ and $Q_2 = \begin{bmatrix}-0.1952 & 0.2169\end{bmatrix}^T$, respectively.
(3) By using LMI toolbox in MATLAB, we can solve the algebraic Riccati equation (ARE) to get one solution P and choose the matrix $K = -B^TP = \begin{bmatrix}-0.2249 & -0.3905\end{bmatrix}$ according to step (3).
(4) We choose coupling strength $c = 1$ according to step (4).
(5) By simple calculation based on step (5), we can obtain $\alpha_1 = 8.85$, $\alpha_2 = 39.30$, $\alpha_3 = 1.07$, $\alpha_4 = 12.09$, $\beta_1 = 0.8$, $\beta_2 = 20$, $\gamma = 40.15$, $k = 1$ and $\tau_0 = 0.56$.
(6) Finally, the reduced-order observer about the robotic system is as follows:

$$\dot{\hat{x}}_i(t) = -2\hat{x}_i(t) - y_i(t) + \begin{bmatrix}-2.9 & 2\end{bmatrix} \times \begin{bmatrix}1 \\ 1\end{bmatrix}u_i(t).$$

Moreover, the initial state of each agent is randomly generated in the interval $[-3, 3]$.

Choosing $\tau = 0.5 < \tau_0 = 0.56$, from the Fig.2 it can be observed the multi-agent systems consensus is achieved. Choosing $\tau = 0.6 > \tau_0 = 0.56$, from the Fig.3 it can be observed the multi-agent systems consensus is not achieved. Fig.4 shows the control inputs of all agents. Fig.5 shows the event triggering times of six agents.

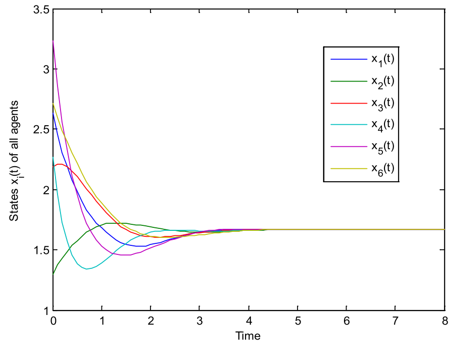

FIGURE 2. The evolution of every agent's observer states using Theorem1 with $\tau = 0.5$.

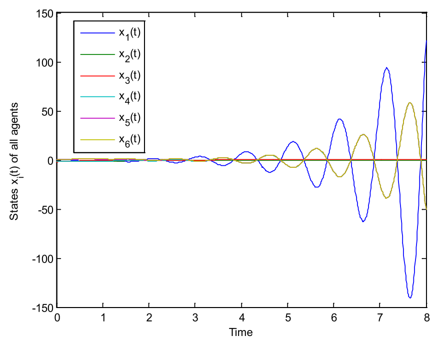

FIGURE 3. The evolution of every agent's observer states using Theorem1 with $\tau = 0.6$.

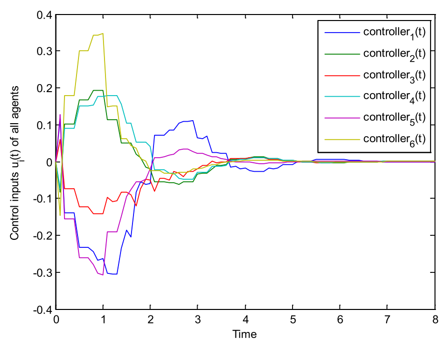

FIGURE 4. The control inputs update by Theorem 1.

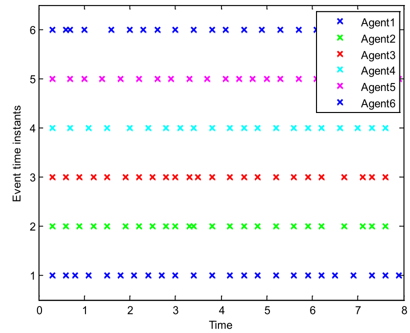

FIGURE 5. Event time instants for every agent.

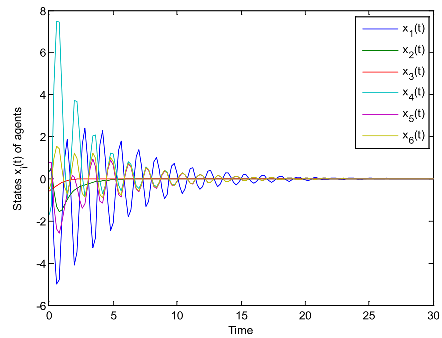

FIGURE 6. The evolution of every agent's observer states using Theorem 1 with $\tau = 0.20$.

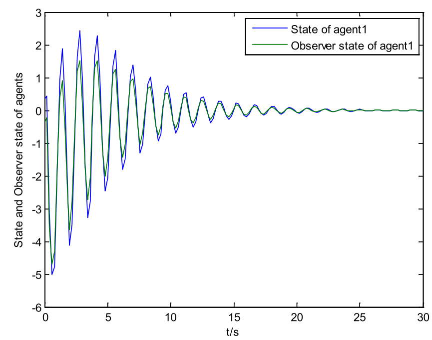

FIGURE 7. The comparison state of agent 1 and observer state of agent 1 using Theorem 1 with = 0.20.

Example 2: Comparing with Example 1 , we change more parameters in Example 2. Consider the network of multi-agent systems (1) with the following topology, which is shown in Fig.1. Let $A = \begin{bmatrix} -2 & 1 \\ -0.5 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0.9 \end{bmatrix}$.

Based on Algorithm1, corresponding parameters of the multi-agent system are determined and the specific steps are as follows:

(1) According to step (1), we choose $F = -(28/9)$, $G = -(10/9)$. It is obviously that F is a Hurwitz matrix and have no eigenvalues in common with A. Select $G = -(10/9)$ such that $(F, G) = \begin{bmatrix} -(28/9) & -(10/9) \end{bmatrix}$ is stabilizable.

(2) According to step solve Sylvester equation (5), we can get $T = [-1 \ 0]$, $Q_1 = \begin{bmatrix} 0 & 1.2 \end{bmatrix}^T$, and $Q_2 = \begin{bmatrix} -1 & 1 \end{bmatrix}^T$, respectively.

(3) By using LMI toolbox in MATLAB, we can solve the algebraic Riccati equation (ARE) to get one solution P and choose the matrix $K = -B^T P = \begin{bmatrix} -0.2249 & -0.3905 \end{bmatrix}$ according to step (3).

(4) We choose coupling strength $c = 1$ according to step (4).

(5) By simple calculation based on step (5), we can obtain α_1=5.503, α_2=1.689, α_3=2.660, α_4=7.785, $\beta_1 = 0.4$, β_2=10, γ=12.20, $k = 1$ and $\tau_0 = 0.24$.

(6) Finally, the reduced-order observer about the robotic system is as follows:

$$\begin{aligned} \dot{\hat{x}}_i(t) &= F\hat{x}_i(t) + Gy_i(t) + TBu_i(t) \\ &= -(28/9)\,\hat{x}_i(t) - (10/9)\,y_i(t) + \begin{bmatrix} -1 & 0 \end{bmatrix} \\ &\quad \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_i(t). \end{aligned}$$

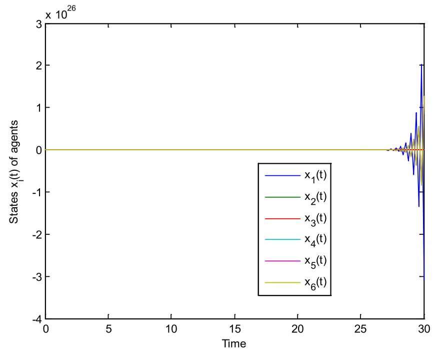

FIGURE 8. The evolution of the every agent states using Theorem 1 with $\tau = 0.28$.

where

$$u_i(t) = \begin{bmatrix} -0.2249 & -0.3905 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1.2 \end{bmatrix} \times \sum_{j \in N_i} a_{ij} \left(y_i\left(t_k^i\right) - y_j\left(t_{k'}^j\right) \right) + \begin{bmatrix} -0.2249 & -0.3905 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix} \times \sum_{j \in N_i} a_{ij} \left(\hat{x}_i\left(t_k^i\right) - \hat{x}_j\left(t_{k'}^j\right) \right)$$

Moreover, the initial state of each agent is randomly generated in the interval $[-3, 3]$.

Choosing $\tau = 0.20 < \tau_0 = 0.24$, from the Fig.6 it can be observed the multi-agent systems consensus is achieved. Fig.7 shows the state and observer state of robot 1. Choosing $\tau = 0.28 > \tau_0 = 0.24$, from the Fig.8 it can be observed the multi-agent systems consensus is not achieved.

V. CONCLUSION

Compared with the full-order observer, the reduced-order observer only need local information to construct the agent state, which means it need less computational cost than full-order observer. In this paper, the consensus problem of the reduced-order observer-based consensus in multi-agent systems with output time delay and event trigger strategy is investigated. First, a multi-step algorithm is presented to construct a reduced-order observer for each agent. Then, a novel push-based event-triggered control strategy based on the reduced-order observer and the relative outputs of neighboring agents is proposed. A sufficient condition is derived for reaching global consensus in multi-agent systems by using the integral inequality technique and matrix theory. The obtained results should be of great significance to the multi-agent system equipped with microprocessors, which have less computation and storage resources than continuously broadcasting information and frequently updating controllers. Moreover, the estimation value of the output time delay is also obtained and the Zeno-behavior of triggering time sequences is excluded. Finally, two multi-agent system simulations are provided to illustrate the correctness of theoretical results.

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