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# Fuzzy Logic Controllers for Specialty Vehicles Using a Combination of Phase Plane Analysis and Variable Universe Approach

# LISHU QIN $^{1,3}$ , JIANJUN HU $^{2}$ , HONGXING LI $^{1}$ , AND WANG CHEN $^{2}$

<sup>1</sup> School of control Science and Engineering, Dalian University of Technology, Dalian 116024, China <sup>2</sup>China North Vehicle Research Institute, Beijing 100072, China <sup>3</sup>College of Mechanical Engineering, Dalian University, Dalian 116622, China Corresponding author: W. Chen (imchenwang@foxmail.com)

**ABSTRACT** For the control request of specialty vehicles, this paper combines a phase plane analysis with the variable universe fuzzy control technique. Moreover, we develop a new systemic design strategy for the adaptive fuzzy logic controller. With the phase plane analysis, the complete rule base that consists of few key rules can be objectively established, which avoids the irrationality introduced into the process via the subjectivity of the designer of the fuzzy controllers. Furthermore, with the variable universe, the design requirements of the membership functions can be relaxed. Meanwhile, the accuracy of the performance of the fuzzy logic controller can be enhanced despite the limited number of fuzzy rules in the rule base. Based on the Lyapunov stability theory, the stability of the close-loop system can be guaranteed. Simulation results of a double inverted pendulum demonstrate the feasibility of the design strategy for the adaptive fuzzy logic controller, which simplifies the design of the fuzzy logic controller and ensures control. The application of this design strategy can significantly lighten the burden for fuzzy logic controller designers and shorten the development period of the fuzzy logic controller.

**INDEX TERMS** Fuzzy logic controller, phase plane analysis, specialty vehicle control, variable universe.

#### **I. INTRODUCTION**

Specialty vehicles are vehicles that have specialty functions, such as harvesters, agitator trucks, armored vehicles, and tanks. Most specialty vehicles have complex mechanical structures. The control request of the specialty vehicles is notably different from general vehicles. For example, for the armored vehicle, the control request includes flexibility, sealing, and high performance.

First, the specialty-vehicle market requires that the products are flexible to satisfy various requirements with a quick turn-around time, such as the options of a broad range of sensor connectivity. In other words, various sensor types in industry-standard housing and easy-to-install sensors are in high demand. Second, in the specialty-vehicle market, there must be versatile and dependable switch enclosures and connector designs to protect electrical conductors against harmful damage from the surrounding elements. Finally, specialty vehicles demand durable, reliable products for maximizing machine uptime, increasing productivity and reducing warranty costs.

One of the most important technologies to satisfy the control request for specialty vehicles is the fuzzy control technology [1]. Fuzzy control research based on the fuzzy set theory [2] was initiated by Mamdani [3]. Both the algebraic model and the linguistic model for fuzzy control were proposed by Braae and Rutherford [4]. The algebraic model cannot directly address the rules of the fuzzy logic controller. The fuzzy control system developments were enhanced using the t-norm and t-conorms, which were introduced by Schweitzer and Sklar [5]. Wang [6] proposed an adaptive fuzzy system, where a training algorithm adjusts the parameters of fuzzy systems using numerical input-output pairs. Li [7] formulated the variable universe idea, whose core idea is that the universe contracts according to the decrease in error.

Fuzzy logic controllers have been applied to various control processes such as vehicle control [1], water quality control [9], nuclear reactor control [10] and automobile transmission control [11]. Although there is extensive literature concerning various applications of fuzzy logic controllers, there have been few systematic procedures to design

fuzzy logic controllers until now. The most straightforward approach is to subjectively define the rules and membership functions by studying a human-operated or controlled system of an existing controller and testing the design for the proper output. Then, the rules and membership functions are adjusted if the design fails the tests. Thus, the design of fuzzy logic controllers is a burdensome task for practitioners. These shortcomings significantly limit the practical use of fuzzy logic controllers.

The rule base is the core of the fuzzy logic controller. It reflects the intelligence of a fuzzy logic controller [4], [13]. However, in practice, obtaining the complete rule base is always difficult [14]. The gradient descent method was proposed to adjust the rule base of the Takagi-Sugeno fuzzy system [17]. This method was applied to the rule base with the single-noteput [18]. In [19], the Hooke-Jeeves pattern search algorithm was proposed, which is used to adjust the fuzzy logic controller with nine or 25 rules. Paper [20] introduces the fuzzy expanded form of a neural network model: the Kohonen self-organizing map, where the center point of the fuzzy sets was adjusted, and the fuzzy rules were initialized. In [21], using Lyapunov stability criteria, the symbols of the two-state variables of the second-order system were tested, and the fuzzy control rules were subsequently extracted. These fuzzy controller design methods have some success but remain considerably difficult for engineers who lack sufficient professional knowledge. In contrast, this paper presents an intuitional and simple method to extract fuzzy rules.

Another important consideration in designing a fuzzy logic controller is the formation of membership functions. It is well known that the bionic algorithm to solve optimization problems is currently a hot topic, where researchers investigate the genetic algorithm (GA) [23], [24], colony algorithm (ACO) [25], artificial neural networks (ANN) [26], artificial immune systems (AIS) [27], particle swarm optimization (PSO) [28], etc. However, there are some drawbacks to the optimization algorithms. For example, what type of optimization algorithm should be selected is always a lack of theoretical directions. Additionally, because of the subjective perceptions of the optimization algorithm users, the selection of the initial parameters can often significantly affect the optimization results. In addition, for the membership functions of a fuzzy logic controller, there are often plenty of parameters that must be optimized. That case will lead to greater increases in the computing burden, which hinders practical applications in engineering. Meanwhile, it is commonly difficult for us to obtain the global optimal solution. However, the algorithm of the fuzzy logic controller that we always use can be a type of interpolation algorithm [29], [30], which indicates that the accuracy of a fuzzy logic controller mainly depends on the number of effective fuzzy rules in the validity discourse universe. Thus, when the discourse universe contracts, the amount of membership functions relatively increases. In other words, the number of fuzzy rules increases, and the control effect improves. According to the

aforementioned discussion, a new design strategy for the adaptive fuzzy logic controller combined with the variable universe approach and phase plane analysis is proposed in this paper.

The paper is organized as follows. Brief statements about the control system and the variable universe fuzzy system are provided in Sections II and III. In Section IV, a simple method based on the phase plane to extract the initial fuzzy rules, which will be used to construct the variable universe adaptive fuzzy logic controller, is proposed. The experimental results of the designed adaptive fuzzy logic controller for specialty vehicles are shown in Section V. Finally, Section VI provides the conclusion.

#### **II. PROBLEM FORMULATION**

In this study, the control system of specialty vehicles is a nonlinear system. Consider the following *n* th-order nonlinear system,

$$
\mathbf{x}^{(n)} = f\left(x, \dot{x}, \cdots, x^{(n-1)}\right) + bu, \mathbf{y} = x \tag{1}
$$

where  $f$  is an unknown nonlinear continuous function,  $b$  is an unknown control gain, *u* is the control input, and *y* is the output. Assume that the state vector  $\hat{x} = (x_1, x_2, \dots, x_n)^T =$  $(x, \dot{x}, \dots, x^{(n-1)})^T$  is available for measurement. Let  $r(t)$  be a reference input of the system. Define error e (t) =  $r(t) - y(t)$ and error vector  $\hat{\mathbf{e}} = (e_1, \dot{\mathbf{e}}, \cdots, e^{(n-1)})^T$ . Our goal is to force the system output  $y(t)$  to follow the reference input  $r(t)$ , i.e., the equation  $\lim_{h \to 0} \hat{e} = 0$  should be satisfied. Select a feedback gain vector  $\hat{\mathbf{k}} = (k_1, k_2, \dots, k_n)$ , and construct a polynomial equation  $s^n + k_1 s^{n-1} + \cdots + k_n = 0$ . If all coefficients  $k_i$ are selected to make the polynomial Hurwitz, the main goal of the control task is achieved. The Hurwitz polynomial is a polynomial whose roots (zeros) are located in the left halfplane of the complex plane or on the imaginary axis, that is, the real part of every root is zero or negative.

If function *f* and control gain *b* are known, the control law is,

$$
\mathbf{u}^* = (1/b) \left[ -f(x) + r^{(n)} + \hat{\mathbf{k}}^T \hat{\mathbf{e}} \right].
$$
 (2)

Applied to the nonlinear system (1), the equation  $e^{(n)}$  +  $k_1e^{(n-1)} + \cdots + k_ne = 0$  is set up. However, f and b are unknown, so control law (2) cannot be ascertained. Our purpose is to design an adaptive fuzzy logic controller to approximate control law (2).

#### **III. VARIABLE UNIVERSE FUZZY LOGIC CONTROLLER** A. FUZZY SYSTEM

Generally, a multi-output system can be separated into a group of single-output systems. Thus, for simplicity, we consider only a multi-input, single-output fuzzy system here. Let  $X_i = [-E_i, E_i], (i = 1, 2, \cdots, n)$  be the universe of the input variable  $x_i$ ,  $(i = 1, 2, \dots, n)$ , and let  $Y = [-U, U]$ be the universe of output variable *y*. The fuzzifier performs a mapping from a crisp input  $x_i$  to a fuzzy set  $A_{x_i}$  in  $X_i$ ,

where  $A_{x_i}$  is the label of the fuzzy set, such as "small", "medium", "large", etc. The fuzzy rule base consists of a collection of fuzzy IF-THEN rules. Assume that there are *M* rules, and the 1th rule is the following: if  $x_1$  is  $A_{x_1}^l$  and  $x_2$ is  $A_{x_2}^l$  and  $\cdots$  and  $x_n$  is  $A_{x_n}^l$ , then y is  $B^l$ ,  $l = 1, 2, \dots$ , M, where  $x_i$ ,  $(i = 1, 2, \dots, n)$  and *y* are the crisp input and output of the fuzzy system, respectively, and  $A_{x_i}^l$  and  $B^l$  are labels of fuzzy sets in  $X_i$  and  $Y$ , respectively. The fuzzy inference performs a mapping from fuzzy sets in X*<sup>i</sup>* to fuzzy sets in *Y* based on the fuzzy IF-THEN rules in the fuzzy rule base. The defuzzifier maps the fuzzy sets in *Y* to a crisp value in *Y* . Here, we use the sum-product inference and center-average defuzzifier. Thus, the fuzzy system can be expressed as,

$$
y(x) = \frac{\sum_{l=1}^{M} y^{l} \prod_{i=1}^{n} \mu_{A_{x_{i}}}^{l}(x_{i})}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{A_{x_{i}}}^{l}(x_{i})}
$$
(3)

where  $y(x)$  is the crisp output of the fuzzy system,  $\mu_{A_{x_i}}^l(x_i)$  is the membership degree of input  $x_i$  to fuzzy set  $A_{x_i}^l$ , and  $y^l$  is the point where the membership function of fuzzy set  $B^l$  achieves its maximum value.

If we fix the membership functions of  $A_{x_i}^l$  and let  $y^l$  be adjustable parameters, then (3) can be written as,

$$
y(x) = \theta^T \xi(x) \tag{4}
$$

where  $\theta = (y^1, \dots, y^M)^T$  is a parameter vector, and  $\xi(x) =$  $(\xi^1(x), \cdots, \xi^M(x))^T$  is defined as,

$$
\xi^{1}(\mathbf{x}) = \frac{\prod_{i=1}^{n} \mu_{A_{x_i}}^{l}(x_i)}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{A_{x_i}}^{l}(x_i)}.
$$
\n(5)

The fuzzy system in the form of (4) is proven in [34] to be a universal approximator if its parameters are suitable.

# B. VARIABLE UNIVERSE ADAPTIVE FUZZY LOGIC **CONTROLLER**

One advantage of a traditional fuzzy logic controller is that it can use the information of various systems to accomplish the control tasks. However, the control precision is commonly not high. Adaptive fuzzy logic controllers are used to solve this problem. The fuzzy logic controller based on the variable universe [7] is a type of adaptive fuzzy logic controller. On the premise of the identical number of membership functions, the parameters of the membership functions can change according to the changes in control error. When the domain of discourse contracts, the control rules are relatively added.

The variable universe implies that the input and output universes  $X_i$  and  $Y$  can change according to the change of variables  $x_i \in X_i$  and  $y \in Y$ , respectively. Thus, the universes are denoted by,

$$
X_i(x_i) = [-\alpha_i(x_i) E_i, \alpha_i(x_i) E_i],
$$
  
\n
$$
Y(y) = [-\beta(y) U, \beta(y) U]
$$
\n(6)



**FIGURE 1.** Change of the discourse domain. (a) Expansion of the discourse domain. (b) Original discourse domain. (c) Contraction of the discourse domain.

where  $\alpha_i(x_i)$ ,  $(i = 1, 2, \dots, n)$  and  $\beta(y)$  are called contraction-expansion factors of universes  $X_i$ ,  $(i$  $1, 2, \cdots, n$  and *Y*.

In Fig. 1, (a) shows the expansion of the discourse domain, (b) shows the original discourse domain, and (c) represents the contraction of the discourse domain.

Then, according to fuzzy system (4), the new variable universe adaptive fuzzy control (VUAFLC) can be rewritten as,

$$
y(x, \beta) = \beta \theta^{T} \xi(x, \alpha)
$$
 (7)

where  $\theta = (y^1, \dots, y^M)^T$  is a parameter vector, but  $\xi$  (x,  $\alpha$ ) = ( $\xi^1$  (x,  $\alpha$ ),  $\cdots$ ,  $\xi^M$  (x,  $\alpha$ ))<sup>T</sup> should be re-defined as,

$$
\xi^{1}(\mathbf{x}, \alpha) = \frac{\prod_{i=1}^{n} \mu_{A_{x_i}}^{l}(x_i/\alpha_i(x_i))}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{A_{x_i}}^{l}(x_i/\alpha_i(x_i))}.
$$
 (8)

Because *f* and *b* in (2) are unknown, we cannot obtain control law (2). Here, we should design controller *u* to approximate  $u^*$  in (2),

$$
u(t) = u_c(t) + u_s(t).
$$
 (9)

Let  $u_c$  (t) be the adaptive fuzzy logic controller with the form  $(7)$ , and let  $u<sub>s</sub>(t)$  be the compensator. Substituting  $(9)$ into (1), we obtain,

$$
x^{(n)} = f(\hat{x}) + b[u_c(t) + u_s(t)].
$$
 (10)

We combined (10) and (2); after several manipulations, the error equation of the nonlinear system can be expressed as,

$$
e^{(n)} = -\hat{k}^T \hat{e} + b(u^* - u_c - u_s). \tag{11}
$$

Here, for the concrete control task, the adaptive fuzzy logic controller should be,

$$
u_c(t) = y(e, \beta) = \beta \theta^T \xi(e, \alpha),
$$
  

$$
\xi(e, \alpha) = (\xi^1(e, \alpha), \cdots, \xi^M(e, \alpha))^T,
$$

$$
\xi^{1}(e,\alpha) = \frac{\prod_{i=1}^{n} \mu_{A_{x_i}}^{l}(e_i/\alpha_i(e_i))}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{A_{x_i}}^{l}(e_i/\alpha_i(e_i))}
$$

If we let,

$$
A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -k_n & -k_{n-1} & -k_{n-2} & \cdots & -k_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b \end{bmatrix}
$$

then Eq. (11) can be rewritten as,

$$
\dot{\hat{e}} = A\hat{e} + B(u^* - u_c - u_s). \tag{12}
$$

.

*Theorem 1:* If we select the compensator,

$$
\mathbf{u}_{s}(t) = \begin{cases} 0, |\hat{\mathbf{e}}^{T}PB|(1/b(f_{0} + |r^{(n)}| + |\hat{\mathbf{k}}^{T}\hat{\mathbf{e}}| + b|u_{c}|)) \\ sgn(\hat{\mathbf{e}}^{T}PB)(1/b(f_{0} + |r^{(n)}| + |\hat{\mathbf{k}}^{T}\hat{\mathbf{e}}| + b|u_{c}|)), \\ obterwise \\ 0. \end{cases}
$$
(13)

where  $P$  and  $Q$  are parameters that satisfy the Lyapunov equation  $A^T P + PA = -Q$ . Clearly, *A* is a stable matrix. Then, the error equation (12) is a globally asymptotically stable system.

*Proof:* Let the energy function be,

$$
V_1(\hat{e}) = \frac{1}{2} \hat{e}^T P \hat{e}.
$$

Using (11), (12), (2) and the derivative of  $V_1(\hat{e})$ , we obtain,

$$
\dot{V}_1(\hat{e}) = -\frac{1}{2} \hat{e}^T Q \hat{e} + \hat{e}^T P B (u^* - u_c - u_s)
$$
\n
$$
\leq -\frac{1}{2} \hat{e}^T Q \hat{e}
$$
\n
$$
+ \left| \hat{e}^T P B \right| \left( \frac{1}{b} \left( f_0 + \left| r^{(n)} \right| + \left| \hat{k}^T \hat{e} \right| + b |u_c| \right) \right)
$$
\n
$$
- \hat{e}^T P B u_s.
$$

Notice that we selected  $u_s$  in the form of  $(13)$  to obtain,

$$
\dot{V}_1\left(\hat{\mathbf{e}}\right) \leq -\frac{1}{2}\hat{\mathbf{e}}^T Q \hat{\mathbf{e}} \leq 0.
$$

Thus, system (12) is an asymptotically stable system. This completes the proof.

#### C. DESIGN OF THE ADAPTIVE LAW

In paper [35], factor  $\beta(t)$  that acts on the inference consequents is designed using the integral regulation principle to improve the robustness of the control system. For convenience, we define,

$$
\mathbf{u}_c(t) = \mathbf{y}(e,\beta) = \beta \theta^T \xi(e,\alpha) = \beta(t) u_{Fc}(t). \tag{14}
$$

According to [35], we know that  $\beta$  is bounded. We use  $M_\beta$  to act as the bound. Then, we obtain  $|\beta| \le M_\beta$ . We define the optimal  $\beta$  as  $\beta^*$ , which satisfies the condition,

$$
\beta^* = argmin\{\left\|u_c\left(t\right) - \mathbf{u}^*\right\|, |\beta| \le M_\beta. \tag{15}
$$

Then, we make a residual error function,

$$
\mathcal{X}(t) = u_c^*(t) - u^*(t) = \beta^*(t) u_{Fc}(t) - u^*(t). \quad (16)
$$

Considering  $(14)$ ,  $(15)$  and  $(16)$ , the error equation  $(12)$  can be rewritten as,

$$
\dot{\hat{e}} = A\hat{e} + B[u_c^*(t) - u^*(t)] - Bu_s(t) - B_X(t)
$$
  
=  $A\hat{e} + B\varphi(t)u_{Fc}(t) - Bu_s(t) - B_X(t)$  (17)

where  $\varphi$  (t) =  $\beta^* - \beta$ . Then, we define another energy function,

$$
V_2 (e) = \frac{1}{2} \hat{e}^T P \hat{e} + \frac{1}{2\gamma} \varphi^2 (t)
$$
 (18)

 $\gamma$  is a constant. We can select the adaptive law as follows,

$$
\dot{\varphi}(t) = \begin{cases}\n-\gamma \hat{e}^* u_{FC}(t), & \left(|\beta| = M_\beta \text{ and } \hat{e}^* u_{FC}(t) \beta \le 0\right) \\
0, & \text{if } |\beta| \ge M_\beta \text{ and } \hat{e}^* u_{FC}(t) \beta < 0\n\end{cases}
$$

where  $\hat{e}^* = \hat{e}^T PB$ . If the residual error  $X(t)$  is integrable, i.e.,  $\int_0^\infty X^2(t) dt < \infty$ , then  $\lim_{\underline{t}\to\infty} ||\hat{e}|| = 0$ .

By noting the equation  $A^T P + PA = -Q$  and (18), the derivative of  $V_2(e)$  is,

$$
\dot{V}_2 = -\frac{\hat{e}^T Q \hat{e}}{2} + \frac{\varphi(t) \dot{\varphi}(t)}{\gamma}
$$
  
+  $\hat{e}^T PB (\varphi(t) u_{FC}(t) - u_s(t) + X(t))$   
=  $-\frac{\hat{e}^T Q \hat{e}}{2} - \hat{e}^T PB (u_s(t) + X(t))$   
+  $\frac{\varphi(t)}{\gamma} (\dot{\varphi}(t) + \gamma \hat{e}^T PB u_{FC}(t)).$ 

From the definition of  $u_s$ , we know that  $\hat{e}^T P B u_s \geq 0$ . Because  $\chi(t)$  is a residual error, as long as  $\chi(t)$  is sufficiently small, we obtain,

$$
-\frac{\hat{\mathbf{e}}^T Q \hat{\mathbf{e}}}{2} - \hat{\mathbf{e}}^T P B \left( u_s\left(t\right) + X\left(t\right) \right) \leq 0.
$$

In other words,

$$
\dot{V}(\hat{e}) \le -\frac{1}{2}\lambda_{min}(Q) \left\|\hat{e}\right\|^2 \n- \frac{1}{2} \left( \left\|\hat{e}\right\|^2 + 2\hat{e}^T PBX(t) + \left\|PBX(t)\right\|^2 \right) \n\le -\frac{1}{2} \left( \lambda_{min}(Q) - 1 \right) \left\|\hat{e}\right\|^2 + \frac{1}{2} \left\|PBX(t)\right\|^2
$$

where  $\lambda_{min}(Q)$  is the smallest eigenvalue of the positive definite matrix Q; because Q is arbitrary, we can select an adequate Q such that  $\lambda_{min}(Q) > 1$ . After integrating both sides of the above equation, we have,

$$
\int_0^\infty \left\| \hat{\mathbf{e}}(\tau) \right\|^2 d\tau \leq \frac{2}{\lambda_{min}(Q) - 1} |PB|^2 \int_0^\infty \chi^2(\tau) d\tau
$$

$$
+ \frac{2(|V(0)| + |V(\infty)|)}{\lambda_{min}(Q) - 1.}
$$

Using the Barbalat theorem [36], we obtain  $\lim_{t \to \infty} \|\hat{e}(t)\| = 0.$ 

 $\lim_{n \to \infty}$   $\lim_{n \to \infty}$  adaptive fuzzy control is a stable adaptive control.

## **IV. INITIAL RULE-BASED DESIGN IN THE PHASE PLANE**

It is crucial that the VUAFLC requires the appropriate initial fuzzy rules. Unfortunately, there is little literature on the method to reasonably obtain a group of initial fuzzy rules. However, the general principles to design fuzzy rules should be that in the premise of satisfying the completeness of the rule base, the number of rules should be as few as possible to simplify the design and implementation of the fuzzy logic controllers.

Here, to improve the practical application of VUAFLC, we provide a method to extract the fuzzy rules based on the phase plane analysis.



**FIGURE 2.** Commonly used fuzzy control system.

To a fuzzy logic controller, the most common inputs are error e and change-in-error è; e is equal to r  $(t) - y(t)$ , as shown in Fig. 2.

For the motion characteristics of the control system, the most direct representations are error  $e$  and change-in-error  $\dot{e}$ , which can be directly reflected on the phase plane. According to this thinking, we can design the reasonable initial fuzzy rules. The initial rule base design method can also be extended to other rules, which is the future direction of our research.



**FIGURE 3.** Typical step response of the control system.

To make the discussion more concrete, we can use the step response of the control system from the perspective of the time-domain for the illustration, as shown in Fig. 3. For example, the reference input is "1", and the tolerance steadystate error is  $\pm 10\%$ .

Let  $\pm e_0$  and  $\pm \dot{e}_0$  represent the zero bands of error e and change-in-error  $\dot{e}$ . Then, when the control system is running, there are three possible cases of error  $e$  and change-in-error  $\dot{e}$ ,

$$
\begin{cases}\n e \leq -e_0, \\
 -e_0 < e < e_0 \\
 e \geq e_0\n\end{cases} \text{ and } \begin{cases}\n \dot{e} \leq -\dot{e}_0, \\
 -\dot{e}_0 < \dot{e} < \dot{e}_0 \\
 \dot{e} \geq \dot{e}_0\n\end{cases}
$$



**FIGURE 4.** Movement of the system response.

Thus, regardless of time, the response of the system can be summarized in one of the following nine cases. As shown in Fig. 4, AB, AC, AD, LM, LN, LO, GH, GI and GJ express the nine cases. The full red line is the setting value, and the two dotted blue lines represent the zero band of error e. For example, point ''R'' in Fig. 3 can be expressed using state AB in Fig. 4. For the state, the response of the system is in the positive-overshoot condition and tends to continue to deviate from the expected state. This case is e  $\leq -e_0$ ,  $\dot{\text{e}} \leq -\dot{\text{e}}_0$ . Thus, if we want to adjust, we should let the controller provide the ''much reduction'' directive. In addition, point ''P'' in Fig. 3 corresponds to the working state represented by GJ in Fig. 4. For this state, the overshoot is negative; meanwhile, this tendency continues. This case is  $e \ge e_0, \dot{e} \ge \dot{e}_0$ . To inhibit this trend, the controller should give a maximum energy supplement to the system. Then, the nine states can be summarized in Table 1.

**TABLE 1.** Complete nine cases of the system response.

	AB	AC	AD	LМ	LΝ
e	$e \leq -e_0$	$e \leq -e_0$	$e \leq -e_0$	$ e  < e_0$	$ e  < e_0$
ė	$\dot{e} \leq -\dot{e}_0$	$ e  < \dot{e}_0$	$\dot{e} \geq \dot{e}_0$	$\dot{e} \leq -\dot{e}_0$	$ e  < e_0$
	LΟ	GH.	GІ	GJ.	
e	$ e  < e_0$	$e > e_0$	$e > e_0$	$e > e_0$	
e	$\dot{e} \geq \dot{e}_0$	$\dot{e} \leq -\dot{e}_0$	$ e  < \dot{e}_0$	$\dot{e} \geq \dot{e}_0$	

Moreover, the nine cases completely divide the phase plane, which consists of error e and change-in-error  $\dot{e}$ . Moreover, the nine areas represent all working situations of the control systems. As shown in Fig. 5, the two vertical dotted lines and the two horizontal dotted lines represent the zero bands of error e and change-in-error  $\dot{e}$ , respectively.

This analysis shows that the output selection of the controller is based on error e and change-in-error  $\dot{e}$ , which will be the fuzzy rules of the fuzzy controllers that we will use, as shown in Table 2.

The positive or negative sign indicates the direction of the output of the fuzzy logic controller, and the number of positive or negative signs indicates the strength of the output of the fuzzy logic controller. Now, we can easily construct our VUAFLC.



**FIGURE 5.** Working situations displayed on the phase plane.

**TABLE 2.** Rule base intuitively established based on the phase plane.

	Output of the controller	Change-in-error (e)		
	้น)	$\dot{e} \leq -\dot{e}_0$	$ e  \leq -\dot{e}_0$	$\dot{e} \geq \dot{e}_0$
	$e \leq -e_0$			--
Error (e)	$ e  \leq -e_0$			
	$i \geq e_0$	$++$	+++	

#### **V. ILLUSTRATIVE EXAMPLE**

To illustrate the effect of the fuzzy logic controller that we can design using the new method above, let us consider the following example for specialty vehicles. The fuzzy rules from the analysis of the phase-plane partitions are used to construct the variable universe adaptive fuzzy logic controller. In this example, a double inverted pendulum is balanced on a cart in Fig. 6.

### A. SYSTEM DESCRIPTION

By applying Lagrange mechanics, the equation of the double inverted pendulum is [37],

$$
\mathbf{M}(\theta_1, \theta_2) \begin{bmatrix} \ddot{x} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + F(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2) \begin{bmatrix} x \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = G(\mathbf{u}, \theta_1, \theta_2)
$$
\n(19)

where the following equation is given, as shown at the bottom of this page.

**TABLE 3.** Parameters and values of the double inverted pendulum.

Symbol	The meaning of the symbol	Value
m <sub>0</sub>	Mass of the cart	1.328kg
m <sub>1</sub>	Mass of the lower pendulum	$0.220$ <sub>kg</sub>
m <sub>2</sub>	Mass of the upper pendulum	$0.187$ kg
$J_1$	Inertia of the lower pendulum around the lower joint	$0.004963kg \cdot m^2$
J <sub>2</sub>	Inertia of the upper pendulum around the upper joint	$0.004824$ kg · m <sup>2</sup>
	Friction constant of the cart	$22.92N \cdot s/m$
$f_0$ $f_1$ $f_2$	Friction constant of the lower pendulum	$0.007056N \cdot s/m$
	Friction constant of the upper pendulum	$0.002644N \cdot s/m$
d <sub>1</sub>	The length from the lower joint to the mass of the lower pendulum	0.304m
$d_2$	The length from the lower joint to the mass of the upper pendulum	0.226m
$d_3$	The length of the lower pendulum	0.49 <sub>m</sub>
$d_4$	The length of the upper pendulum	0.115m
$G_0$	Gain from the output voltage to the output of the motor that drives the cart	11.887N/V
g	Gravitational acceleration	9.8 m/s <sup>2</sup>

The symbols are defined, and the parameters are shown in Table 3.  $\theta_1$ ,  $\theta_2$  and *x* denote the angles of the lower and upper pendulum from the vertical and the position of the cart, respectively. In the neighborhood of the unstable balance point  $\theta_1 = \theta_2 = 0$ ,  $\dot{\theta}_1 = \dot{\theta}_2 = 0$ , the following linear model is obtained,

$$
\dot{X} = AX + Bu, Y = CX,
$$

where  $X = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T = [x \ \theta_1 \ \theta_2 - \theta_1 \dot{x} \ \dot{\theta}_1 \ \dot{\theta}_2 - \dot{\theta}_1]^T$ ,  $Y = [x_1 \ x_2 \ x_3]^T$ .

By substituting the parameters of Table 3, the following matrix of the coefficient is provided,

$$
A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2.596 & 0.16408 & -16.667 & 0.01718 & -0.0011 \\ 0 & 29.919 & -15.181 & 40.317 & -0.28268 & 0.00959 \\ 0 & -36.656 & 65.378 & -49.395 & 0.64301 & -0.41489 \end{bmatrix},
$$
  
\n
$$
B = \begin{bmatrix} 0 & 0 & 0 & 8.6462 & -20.914 & 25.642 \end{bmatrix}^T,
$$
  
\n
$$
C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}
$$

$$
M(\theta_1, \theta_2) = \begin{bmatrix} m_0 + m_1 + m_2 & (m_1d_1 + m_2d_3)cos\theta_1 & m_2d_2cos\theta_2 \\ (m_1d_1 + m_2d_3)cos\theta_1 & J_1 + m_2d_3^2 & m_2d_1d_3cos(\theta_2 - \theta_1) \\ m_2d_2cos\theta_2 & m_2d_2d_3cos(\theta_1 - \theta_2) & J_2 + m_2d_2^2 \end{bmatrix},
$$
  
\n
$$
F(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2) = \begin{bmatrix} f_0 & -(m_1d_1 + m_2d_3)\dot{\theta}_1sin\theta_1 & m_2d_2\dot{\theta}_2cos\theta_2 \\ 0 & f_1 + f_2 & -f_2 - m_2d_2d_3\dot{\theta}_2sin(\theta_2 - \theta_1) \\ 0 & m_2d_2d_3\dot{\theta}_2sin(\theta_1 - \theta_2) - f_2 & f_2 \end{bmatrix},
$$
  
\n
$$
G(u, \theta_1, \theta_2) = [G_0 \quad u(m_1d_1 + m_2d_3) \operatorname{gsin}\theta_1 - m_2d_2\operatorname{gsin}\theta_2]^T
$$

÷



**FIGURE 6.** Double-inverted pendulum system.



$$
e = (k_1 k_2 k_3) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad ec = (k_4 k_5 k_6) \begin{pmatrix} x_4 \\ x_5 \\ x_6 \end{pmatrix}
$$

where according to the linear quadratic optimal control theory, we use the optimal control functional index,

$$
J = \frac{1}{2} \int_0^\infty (X^T Q X + u^T R u) dt.
$$

Then,  $k_1, k_2, \dots, k_6$  can be calculated as follows: *Step 1:* Select the positive definite matrix Q and R,

$$
A = diag[1 50 300 00 0], \quad R = 0.1.
$$

*Step 2:* Solve the Riccati equation,

$$
-PA - A^T P + PBR^{-1}B^T P - Q = 0.
$$

Thus, the state feedback matrix can be calculated,

$$
KT = R-1BTP
$$
  
= -[3.1623 69.6134 152.8111 10.1146 27.5431 22.1697]

Then,

$$
(\mathbf{k}_1, \mathbf{k}_2, \cdots, \mathbf{k}_6) = \frac{K^T}{\|K\|_2}
$$

where  $||K||_2$  is equal to 171.9293, which is the 2-norm of matrix *K*. Moreover, we normalize the initial discourse domains whose range is  $[-1, 1]$ .

#### B. DESIGN OF MEMBERSHIP FUNCTIONS

Because the fuzzy logic controller is essentially an interpolator, the shape of the membership functions in the variable universe is not important. Hence, the design requirements of the membership functions can be significantly relaxed. Thus, the three membership functions of the input variables *e* and *ec* can be simply designed as triangle waves, because of the

**FIGURE 7.** Membership functions of error e and change-in-error ec in the initial universe [−1, 1].

#### **TABLE 4.** Rule base based on the composition error e and change-in-error (ec).





$$
A_1(e) = \begin{cases} 1, & e \le -1, \\ -e, & -1 < e < 0, \\ 0, & others; \end{cases}
$$
  
\n
$$
A_2(e) = \begin{cases} 1+e, & -1 < e \le 0, \\ 1-e, & 0 < e \le 1, \\ 0, & others; \end{cases}
$$
  
\n
$$
A_3(e) = \begin{cases} e, & 0 < e \le 1, \\ 1, & 1 < e, \\ 0, & others; \end{cases}
$$
  
\n
$$
B_1(ec) = \begin{cases} 1, & ec \le -1, \\ -ec, & -1 < ec < 0, \\ 0, & others; \end{cases}
$$
  
\n
$$
B_2(ec) = \begin{cases} 1+ec, & -1 < ec \le 0, \\ 1-ec, & 0 < ec \le 1, \\ 0, & others; \end{cases}
$$
  
\n
$$
B_3(ec) = \begin{cases} ec, & 0 < ec \le 1, \\ 1, & 1 < ec \\ 0, & others. \end{cases}
$$

The nine values  $0, \pm 0.2, \pm 0.4, \pm 0.6,$  and  $\pm 0.8$  in Table 4 are the peaks of the fuzzy sets of the output  $\mu$ .

#### C. DESIGN OF THE FUZZY RULE BASE

According to Table 2, the fuzzy rules based on composition error e and change-in-error ec are gained. They are summarized in Table 4.

The rule base is notably simple with only nine rules.

# D. THE FORM OF VUAFLC

The output of the VUAFLC is,

$$
u(t) = \beta(t)\omega(e, ec)
$$
  

$$
\omega(e, ec) = U \sum_{i=1}^{3} \sum_{j=1}^{3} A_i(\frac{e}{\alpha_1(e)}) B_j(\frac{ec}{\alpha_2(ec)}) u_{ij}
$$

**TABLE 5.** Initial values of the double inverted pendulum.

Variable	Η,	θ۰
Condition 1	0.10	0.10
Condition 2	$-0.25$	$-0.12$
Condition 3	0.15	$-0.08$
Condition 4	$-0.15$	0.10

where  $u_{ij}$  is the values in Table 4. The concrete form of  $u(t)$  is,

$$
u(t) = \beta(t) U[-0.8A_1 \left(\frac{e}{\alpha_1(e)}\right) B_1 \left(\frac{ec}{\alpha_2(ec)}\right)
$$
  
\n
$$
-0.6A_1 \left(\frac{e}{\alpha_1(e)}\right) B_2 \left(\frac{ec}{\alpha_2(ec)}\right)
$$
  
\n
$$
-0.4A_1 \left(\frac{e}{\alpha_1(e)}\right) B_3 \left(\frac{ec}{\alpha_2(ec)}\right)
$$
  
\n
$$
-0.2A_2 \left(\frac{e}{\alpha_1(e)}\right) B_1 \left(\frac{ec}{\alpha_2(ec)}\right)
$$
  
\n
$$
-0A_2 \left(\frac{e}{\alpha_1(e)}\right) B_2 \left(\frac{ec}{\alpha_2(ec)}\right)
$$
  
\n
$$
+0.2A_2 \left(\frac{e}{\alpha_1(e)}\right) B_3 \left(\frac{ec}{\alpha_2(ec)}\right)
$$
  
\n
$$
+0.4A_3 \left(\frac{e}{\alpha_1(e)}\right) B_1 \left(\frac{ec}{\alpha_2(ec)}\right)
$$
  
\n
$$
+0.6A_3 \left(\frac{e}{\alpha_1(e)}\right) B_2 \left(\frac{ec}{\alpha_2(ec)}\right)
$$
  
\n
$$
+0.8A_3 \left(\frac{e}{\alpha_1(e)}\right) B_3 \left(\frac{ec}{\alpha_2(ec)}\right)
$$
  
\n
$$
+0.8A_3 \left(\frac{ec}{\alpha_1(e)}\right) B_3 \left(\frac{ec}{\alpha_2(ec)}\right)
$$

where  $U = ||K||_2 = 171.9293$ ,  $\alpha_1$ , and  $\alpha_2$  are the contractionexpansion factors that act on the discourse domain of error *e* and change-in-error *ec*, respectively. We can select  $\alpha_1$  in the following form:  $\alpha_1(e) = 1 - 0.6exp(-10 \times e^2)$ ,  $\alpha_2(ec) =$  $1 - 0.5 \exp(-10 \times e c^2)$ .

 $\beta$  (t) can be designed using the principle of weighted integral regulation,

$$
\beta(t) = \mathrm{K}_{I} \int_{0}^{t} \omega(e, ec) e^{*}(\tau) d\tau + \beta(0)
$$

where  $e^*(\tau) = (e, ec) P_n$ ,  $P_n = (p_1, p_2)^T$  is a constant vector in our simulation experiment  $\beta$  (0) = 1, K<sub>1</sub> = 5 ~ 100, and  $P_n = (5, 5)^T$ .

#### E. SIMULATION RESULTS

The stability of the double-inverted-pendulum system is much more sensitive at different pendulum angles. According to the experimental research, four types of initial conditions were set up to test the fuzzy logic controllers, i.e., (a)  $\theta_1 \geq$  $0, \theta_2 \geq 0$ ; (b)  $\theta_1 \leq 0, \theta_2 \leq 0$ ; (c)  $\theta_1 \geq 0, \theta_2 \leq 0$ ; (d)  $\theta_1 \leq 0$ ,  $\theta_2 \geq 0$ . These four types of random initial values are listed in Table 5.

The experimental simulation results and comparison with different fuzzy logic controllers are shown in Figures 8-11, including the responses of four variables  $\theta_1$ ,  $\theta_2$ ,  $\dot{\theta}_1$  (represented by the symbol  $d\theta_1$ ) and  $\dot{\theta}_2$  (e represented by the sym-



**FIGURE 8.** Comparison of different FLSs with the initial value of condition 1 in Table 5.



**FIGURE 9.** Comparison of different FLSs with the initial value of condition 2 in Table 5.

bol  $d\theta_2$ ; researchers should direct their attention primarily to these variables.

From the simulation results, we can draw several conclusions. First, using the fuzzy rules based on those in Table 4, the double inverted pendulum is successfully stabilized at the equilibrium position. Thus, the effectiveness of the fuzzy rules is proven. Second, combined with the variable universe approach, the controller's performance is significantly improved. For the case of four initial conditions, the simulation results show that the VAUFLC can better stabilize the double inverted pendulum systems with a shorter adjustment time and a smaller overshoot.

*Remark 1:* This analysis indicates that without rigorous mathematical analysis, we can successfully extract fuzzy rules through only an intuitive understanding of the error and change-in-error of the system response. Designing a fuzzy logic controller is notably simple, but the practitioners' irrationality can be avoided.

*Remark 2:* The fuzzy rules obtained using the above method are effective, considering the dynamic behavior of the systems. However, there are few fuzzy rules. We know that the control accuracy of a fuzzy logic controller significantly depends on the number of fuzzy rules.



**FIGURE 10.** Comparison of different FLSs with the initial value of condition 3 in Table 5.



**FIGURE 11.** Comparison of different FLSs with the initial value of condition 4 in Table 5.

Thus, to enhance the control ability and accuracy, we combined the few fuzzy rules with the approach of the variable universe adaptive control. Because of the continuous working of the controller, the error and change-in-error of the system become increasingly smaller. In this sense, with the domain of constant discourse contraction, although there is no change in the absolute number of fuzzy rules, the number of fuzzy rules increases in a relatively small range of the discourse domain. Obviously, the accuracy of the fuzzy logic controller can be significantly improved.

*Remark 3:* For the membership function design, because of the interpolation essence of the fuzzy logic controller and the contraction of the discourse domain, the interpolation points near the equilibrium point approach one another; thus, the type of selected membership functions is not significantly important. Generally, the commonly used triangular membership functions can be selected. Thus, the design of the fuzzy logic controller is further simplified.

#### **VI. CONCLUSION**

The main goal of this paper is to propose a systematic methodology to design a fuzzy logic controller for specialty vehicles. The design process shows that the proposed scheme is simple: because the fuzzy rules are the core of the fuzzy logic controller, the fuzzy rules were more naturally extracted, which is more conformable to an individual's intuitive thinking. The adopted error and change-inerror are used to directly describe the work status of a system, through which the system response is completely partitioned. Then, we combined the few fuzzy rules with the approach of the variable universe to improve the control effect. The simulation result of controlling the double inverted pendulum verifies the efficiency of the proposed scheme. In addition, because the fuzzy logic controller designed by the systematic method has the advantages of simplified design and good performance, the strategy will certainly provide great convenience to the designers of fuzzy logic controllers.

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LISHU QIN received the M.S. degree in control engineering and theory from the Inner Mongolia University of Science and Technology, Baotou, China, in 2004. He is currently pursuing the Ph.D. degree in control engineering and theory from Dalian University of Technology, Dalian, China. He is currently a Teacher with the College

of Mechanical Engineering, Dalian University. His research interests include intelligent control, robotics, and embedded systems.



JIANJUN HU is with the China North Vehicle Research Institute, Beijing, China. His research interests include vehicular sensor networks and data analysis.



HONGXING LI received the degree from the Department of Mathematics, Nankai University, Tianjin, China, the degree from the Department of Mathematics, Beijing Normal University, Beijing, China, and the Ph.D. degree in engineering.

He is currently a Professor and Doctoral Tutor with the School of Control Science and Engineering, Dalian University of Technology, Dalian, China. His research interests include applied mathematics, control theory and engineering, pattern

recognition, and artificial intelligence.



WANG CHEN received the Ph.D. degree in mechanical design automation from Dalian University, Dalian, China.

He is currently with the China North Vehicle Research Institute, Beijing, China. His research interests include vehicular sensor network and data analysis.

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