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Energy-Efficient Power Control Algorithms in Massive MIMO Cognitive Radio Networks

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ABSTRACT To achieve the maximum network energy efficiency (EE) and guarantee the fairness of EE among cognitive users (CUs), respectively, in the massive multiple-input multiple-output cognitive radio network, we investigate two power optimization problems: network EE optimization problem (NEP) and fair EE optimization problem (FEP) under a practical power consumption model. Because of the fractional nature of EE and the interference, both NEP and FEP are non-convex and NP-hard. To tackle these issues, we propose two energy-efficient power control algorithms, in which we decompose NEP/FEP into two steps, and solve them with an alternating iterative optimization scheme. Specifically, in the first step, for an initial transmit power, the maximum network EE/fair EE is achieved by the bisection method based on fractional programming; then, with the achieved EE, in the second step, the adapted optimal transmit power can be obtained by an efficient iterative algorithm based on sequential convex programming. These two steps are performed alternately until the stop conditions are reached. Numerical results confirm the fast convergence of these proposed algorithms and demonstrate their effectiveness with high network EE and well fairness of EE among CUs. Furthermore, it is illustrated that, under a practical power consumption model, more cognitive base station antennas would cause some loss of network EE but bring some improvements on the network spectral efficiency (SE), whereas higher circuit power consumption would reduce the network EE but only slightly affect the network SE.

INDEX TERMS Cognitive radio, energy efficiency, fractional programming, massive MIMO, power control, spectral efficiency, sequential convex programming.

I. INTRODUCTION

Nowadays, spectral efficiency (SE), which evaluates how effectively limited available spectrum resources are utilized, and energy efficiency (EE), which measures how efficiently energy resources are consumed, are two key performance metrics for the next generation (5G) wireless communications. It is foreseen that by 2020 there will be more than 50 billion devices [1] connected through cellular networks to implement the ubiquitous communications, and the data traffic is anticipated to increase by 1000 times over the next ten years [2]. This indicates that higher SE is required in future communications. However, obtaining such a large capacity by simply scaling up the transmit power is clearly impossible. The reason is that it would lead to excess emission of greenhouse gas and electromagnetic pollution, along with an unmanageable energy demand. Hence, a sharp improvement on EE, at a similar power consumption level as present, is considered to be an effective way to achieve such goal.

Cognitive radio (CR) [3] has been a hot topic in the field of wireless telecommunications over the past decades, for its superiority on substantially enhancing the spectrum utilization. Lots of researches [4]–[6] have confirmed that power control is an efficient and effective method to improve the performance of SE and EE in CR systems. In parallel, massive MIMO is deemed to be a promising candidate technology of 5G, for its predominance in boosting SE and EE with low complexity [7]. In massive MIMO systems, though the aggressive multiplexing gain can be obtained with equal power allocation scheme [8], power control among users can help to harvest all the benefits brought by massive antenna arrays [9]–[12]. To meet the high service requirements of future communications, CR networks (CRNs) with massive antenna arrays at the transceivers, i.e., massive MIMO CRNs [13], will be a potential development trend, for their remarkable advantages on SE and EE. Even though some work [14] has explored the design and analysis of massive MIMO CRNs, there has been little work concerning power control problem in it. However, considering its notable superiority, power control technology in massive MIMO CRNs is very worthy of attention and study. This is the motivation of this work.

According to the open literature, it is widely recognized that high network EE is a key indicator of the technological advance in future communication networks [15], especially for massive MIMO systems with a large number of base station (BS) antennas. In addition to that, the fairness among cognitive users (CUs) in CRNs is an inevitable problem, which guarantees their individual quality-of-service (QoS) when they share the spectrum. While in massive MIMO systems, the large-scale fading still remains, and it would directly cause the unfairness between users near and far from the BS. Therefore, in this paper, we aim at maximizing the network EE of massive MIMO CRNs and guaranteeing the fairness of EE among CUs through optimizing the uplink transmit power.

II. RELATED WORKS

As an essential cognitive technology, power control has been studied for many years. Various objectives [4]–[6] can be achieved through designing suitable transmit power strategy. Different from the existing system-wise energy-efficient designs, [4] derives an optimal user-wise energy-efficient power allocation scheme, which dramatically improves the EE of cognitive femto users. The tradeoff between the sensing quality and EE is studied in a CRN [5], concluding that the network SE and network EE can be together enhanced via power control and sensing bandwidth adjustment. Taking a fairness constraint into account, [6] considers a noncooperative power control game for EE maximization in a multiuser CRN.

In massive MIMO systems, power control among users has been considered as a necessary and essential tool to take full advantage of massive antenna arrays [9]–[12]. In [9], power control is applied as an effective way to minimize the uplink power consumption with maximum sum SE in multi-cell massive MIMO systems. While it is noted that all the above power models only consider the transmit power consumption, which tend to achieve higher SE and better EE performance with more BS antennas. However, in practical massive MIMO systems, since the size of hardware systems increases, the effect of circuit power consumption would be gradually aggravated by the factor of BS antennas number. As a result, it would bring nonnegligible negative impacts on massive MIMO systems [10]. [11] points out that the EE of massive MIMO systems depends heavily on the circuit power consumption. Hence, a new realistic power consumption model is proposed [12], where the number of BS antennas and transmit power are individually optimized to investigate how they affect the EE. In sharp contrast to the common belief, the optimal transmit power is found to increase with the BS antennas number, which means that the circuit power consumption is an important design parameter for high EE massive MIMO systems.

All these related works have shed light on the power control algorithm design of optimizing the EE in the context of 5G networks [16]–[23]. To achieve high global EE for multi-cell massive MIMO systems, a power control algorithm under the assumption of equal power allocation is provided in [20]. For the MIMO CRN, [21] proposes both distributed and centralized EE optimization algorithms based on the augmented Lagrangian multiplier method. Considering the limited feedback resource in CRNs, [22] gives an adaptive efficient resource allocation for the multiuser MIMO rateless-coded CRN with QoS provisioning. However, they do not account for rate requirements of users. Thus the resulting users' rates may be fairly low. To fill this gap, [23] develops a unified framework with minimum rate constraints for EE optimization in both centralized and distributed networks, but it indirectly achieves the optimal power control strategy by means of changing variables and a logarithmic approximation of the achievable rate. Note that the distributed algorithm (user-centric) suffers a system-wide performance gap with respect to a centralized one (network-centric), as demonstrated in [21], [23], and [24], due to the users' selfishly competitive fashion. Nevertheless, there is little work concerning energy-efficient power optimization problems in practical massive MIMO CRNs. Our previous conference work [36] gives the outline of an energy-efficient power optimization scheme without changing variables for the EE fairness among CUs. In this paper, we extend the work in [36], and further investigate into the case with the aim of achieving higher system-wide EE performance. Furthermore, the energy-efficient power optimization problems to a practical massive MIMO CRN are reformulated by taking into account the effect brought by the circuit power consumption on the system. The major contributions are summarized as follows:

- We investigate two optimization problems of maximizing the network EE/fair EE, i.e., the network EE optimization problem (NEP) and fair EE optimization problem (FEP), in a practical massive MIMO CRN by means of uplink power control. Wherein we give a detailed description of the system model and problem formulation, following a more realistic power consumption model compared with some available work.
- After the theoretic analysis of these constrained optimization problems, it is found that they are nonconvex nonlinear fractional programming and NP-hard. Based on the fractional programming [25], [26] and sequential convex programming [27], [28], we address these prob-

lems with an alternating iterative optimization scheme, and present the detailed algorithm procedures accordingly. It should be noted that, due to the rate-dependent item in the power consumption model, the methods in [23], [24], [28] are not applicable to the cases in our work. Besides, though considering the rate-dependent item, the [12, Lemma 3] is not suitable in our cases, for the presence of interference in the system.

- Moreover, the convergence and complexity of these proposed algorithms in the practical massive MIMO CRN are analyzed. At last, we make simulations and demonstrate their effectiveness by comparing the results with those of other transmission schemes [29], [30]. Apart from that, the impacts of cognitive base station (CBS) antennas number and the circuit power consumption on the network EE/SE and the transmit power of these methods are further investigated.

The remaining of this paper is organized as follows. In Section III, we give the system model and a practical power consumption model for the massive MIMO CRN. NEP and FEP with their individual energy-efficient power control algorithmic solution are presented in Section IV and Section V, respectively. Numerical results are described in Section VI followed by conclusions in Section VII. Throughout the paper, we use the following notations. Boldface uppercase letters denote matrices or sets, and boldface lower-case letters denote column vectors or sets. The superscripts $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, and $\|\cdot\|$ denotes the standard Euclidean norm.

III. SYSTEM MODEL AND PROBLEM FORMULATION

In this paper, we assume that the channel usage pattern of primary users (PUs) is fairly static over time and CUs are allowed to share the licensed spectrum of PUs in an underlay coexistence mechanism, if their caused interference to PUs is below a threshold. In this section, we first present an underlay massive MIMO CRN, and then formulate a general constrained EE optimization problem to it.

A. SYSTEM MODEL

As depicted in Fig. 1, the massive MIMO CRN consists of a primary network and a multiuser massive MIMO cognitive network. The primary network contains a multi-antenna primary base station (PBS) and a single-antenna PU. Within its communication area, one CBS equipped with M antennas and K single-antenna CUs compose the cognitive network, where $M \gg K$. All CUs are assumed to share the same time-frequency resources with the PU. So, when they simultaneously communicate with their own BS, there must exist mutual interference among them.

B. SIGNAL TRANSMISSION

Let $\mathbf{x} = [x_1, x_2, \cdots, x_K]^{\mathrm{T}}$ and $\mathbf{p} = [p_1, p_2, \cdots, p_K]^{\mathrm{T}}$, where $x_k = \sqrt{p_k} s_k$ is the signal transmitted from the k-th CU, p_k is its corresponding transmit power and s_k denotes the data

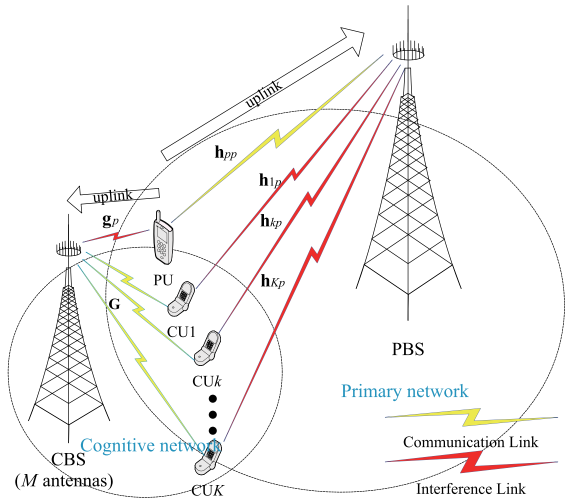


FIGURE 1. System model.

symbol with $E\{|s_k|^2\} = 1$. Similarly, $x_p = \sqrt{p_p} s_p$ is the transmitted signal from the PU with transmit power p_p and data symbol s_p.

After the data are sent off, the $M \times 1$ received vector at the CBS can be given as

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{g}_p x_p + \mathbf{n}, \tag{1}$$

where $\mathbf{G}$ represents the $M \times K$ channel matrix between the CBS and CUs, $\mathbf{g}_p \triangleq [g_p, \ldots, g_p]^T$ is the $M \times 1$ channel vector between the CBS and the PU, and $\mathbf{n}$ is an $M \times 1$ vector of additive white Gaussian noises (AWGNs) with zero mean and covariance matrix $\sigma^2 \mathbf{I}_M$ at the CBS antennas, with $\mathbf{I}_M$ an identity matrix of dimension M.

$\mathbf{G}$ incorporates the effects of small-scale fading and large-scale fading. In particular, its k-th column, i.e. $\mathbf{g}_k \triangleq [\mathbf{G}]_k$, can be written as $\mathbf{g}_k = \mathbf{h}_k \sqrt{\beta_k}$, where $\mathbf{h}_k$ and $\sqrt{\beta_k}$ are the $M \times 1$ small-scale fading vector and the large-scale fading coefficient, respectively, between the CBS and the k-th CU. Similarly, $\mathbf{g}_p = \mathbf{h}_p \sqrt{\beta_p}$ describes the channel state between the CBS and the PU with small-scale fading information $\mathbf{h}_p$ and large-scale fading information $\sqrt{\beta_p}$.

Note that each small-scale fading coefficient h_j ($j \in \{\forall k, p\}$) is assumed to be independent identically distributed (i.i.d.) random variable (RV) with zero mean and unit variance, and the large-scale fading captures the geometric attenuation and shadow fading with $\beta_j = z_j/(d_j/d_h)^{\theta}$, where z_j is a log-normal random variable with standard deviation σ_{shadow}, d_j and d_h denote the distance from user j to the CBS and the cell-hole radius from which users are excluded [7], respectively, and θ is the path-loss exponent. Since the distances between all users and the CBS are much larger than the antenna spacing, β_j is assumed to only depend on j and to be constant over many coherence time intervals [8].

It is assumed that the CBS has a good knowledge of the channel information. After receiving signals, each CBS antenna would detect the desired signal with the maximum ratio combining (MRC) linear processor, which corresponds to multiplying $\mathbf{y}$ with $\mathbf{g}_k^H$ to extract the intended signal x_k from interference and noise. Then the processed data transmitted from CU k at the CBS is

$$r_k = \mathbf{g}_k^H \mathbf{g}_k \sqrt{p_k} s_k + \sum_{i=1, i\neq k}^{K} \mathbf{g}_k^H \mathbf{g}_i \sqrt{p_i} s_i + \mathbf{g}_k^H \mathbf{g}_p \sqrt{p_p} s_p + \mathbf{g}_k^H \mathbf{n}. \quad (2)$$

In this paper, we model the interference and noise terms as additive Gaussian noise independent of s_k and further assume that the channel is ergodic [8]. Then, with the same methodology in [8], we develop a lower bound of ergodic achievable uplink rate of CU k at the CBS,

$$\begin{aligned} R_k &\geq \tilde{R}_k \\ &= \log_2(1+E[\frac{|\mathbf{g}_k^H \mathbf{g}_k|^2 p_k}{\sum_{i=1,i\neq k}^{K} |\mathbf{g}_k^H \mathbf{g}_i|^2 p_i + |\mathbf{g}_k^H \mathbf{g}_p|^2 p_p + \|\mathbf{g}_k\|^2 \sigma^2}]) \\ &= \log_2(1 + \frac{(M-1)\beta_k p_k}{\sum_{i=1,i\neq k}^{K} \beta_i p_i + \beta_p p_p + \sigma^2}). \end{aligned} \quad (3)$$

Note that this lower bound will be taken as one performance metric in the following to evaluate the SE of CU k, i.e. $SE_k \triangleq \tilde{R}_k$ [8]. Thus the network SE (the overall throughput) can be expressed as [8]

$$SE_{tot} \triangleq \sum_{k=1}^{K} \tilde{R}_k = \sum_{k=1}^{K} \log_2(1+\frac{(M-1)\beta_k p_k}{\sum_{i=1,i\neq k}^{K} \beta_i p_i + \beta_p p_p + \sigma^2}). \quad (4)$$

On the other hand, in CR, the cognitive network is required to preserve the performance of the primary network. Intuitively, the interference caused by all the CUs to the PU should be below its interference temperature. Therefore, we should have the following interference temperature constraint on the CUs:

$$\sum_{k=1}^{K} \alpha_k p_k \leq T, \quad (5)$$

where α_k is the large-scale fading coefficient between the PBS and the k-th CU, and T denotes the tolerated maximal peak interference level, i.e., interference temperature threshold, at the PU.

C. POWER CONSUMPTION MODEL

Since the power model with ideal hardware [8], which only considers the radiated power, might be misleading in designing practical networks, we shall introduce a more realistic power model for massive MIMO CRNs to derive a more accurate EE metric.

In this section, we consider not only the radiated power consumed for transmitting but also the power dissipated in the other circuit blocks of the CBS and CUs. Thus, for the uplink transmission in massive MIMO CRNs, the total power consumption of the whole network can be modeled as [12], [16], [20]

$$PC_{tot} = P_{amp} + P_{cir}. \quad (6)$$

Specifically, P_{amp} is the power consumption at all the power amplifiers of CUs,

$$P_{amp} = \frac{1}{\varepsilon} \sum_{k=1}^{K} p_k, \quad (7)$$

where ε is the drain efficiency of power amplifiers. And P_{cir} is modeled as the power consumption in the processing circuit on both the transmitter and receiver sides [12], [16], [20],

$$P_{cir} = M\rho_c + \varrho \sum_{k=1}^{K} \tilde{R}_k + \xi, \quad (8)$$

where $\rho_c \triangleq \varsigma(P_{LP}+P_{BB}+P_{CBS})$ denotes the effective circuit power consumption per CBS antenna, ς reflects the impacts of cooling, direct-current to direct-current (DC-DC) power supply and main supply at the CBS, P_{LP} is the linear processing power consumed by the MRC detector at each CBS antenna, P_{BB} and P_{CBS} are the other baseband processing power consumption at each antenna and the power consumed at the circuit components of each CBS antenna (e.g., converters, mixers, and filters), respectively, $\varrho \sum_{k=1}^{K} \tilde{R}_k$ denotes the circuit power consumption that grows in proportion to the uplink data rate with a constant scaling factor ϱ [12], [16], and ξ is a static circuit power consumption term independent of M (but might scale with the number of CUs), for example it includes the fixed power consumption at the CBS and the circuit power consumption at K CUs, but it is mainly determined by the former in a communication process [12], [20]. Therefore, ξ can be approximated as a constant. Thus, from the perspective of each CU, the total power consumption for its uplink data transmission, which includes the power consumption on both the transmitter and receiver sides, can be approximately expressed as follows:

$$PC_k \triangleq \frac{1}{\varepsilon} p_k + M\rho_c + \varrho \tilde{R}_k + \xi. \quad (9)$$

D. ENERGY EFFICIENCY

In general, EE is measured in bits/Joule and a common definition of EE in communication systems is the ratio between the SE (sum-rate in bits/channel use) and the total power expended (in Joules/channel use) [8]. Hence, combined with the above practical power consumption model, the expressions of network EE and EE of CU k [29] can be written as:

$$\eta_{tot} = \frac{SE_{tot}}{PC_{tot}}, \quad \eta_k = \frac{SE_k}{PC_k}. \quad (10)$$

In practical applications, we can assume that SE_{tot}, PC_{tot}, SE_k, $PC_k > 0$.

E. PROBLEM STATEMENT

A general constrained energy-efficient power optimization problem in massive MIMO CRNs can be defined mathematically as:

$$\begin{aligned} &\max_{\mathbf{p}} \ U(\mathbf{p}) \\ &s.t.\ C1\colon \sum\nolimits_{k=1}^{K} \alpha_k p_k \le T \\ &\qquad C2\colon R_k(\mathbf{p}) \ge r_k^{req}, \quad \forall k \\ &\qquad C3\colon 0 \le p_k \le p_k^{max}, \quad \forall k. \end{aligned} \tag{11}$$

The goal is to optimize the transmit power of CUs to maximize a given utility function $U(\mathbf{p})$ under the PU interference temperature constraint $C1$ and the QoS requirements $C2$ for CUs with their transmit power constraints $C3$. Wherein $C2$ imposes the minimum data rate requirement for each CU; p_k^{max} in $C3$ is the peak transmit power of CU k.

To obtain good performance of network EE and well fairness of EE among CUs in massive MIMO CRNs, we choose the network EE and the EE of the worst-case CU, or simply 'fair EE' for short, to be the utility function $U(.)$, respectively. Then we formulate two optimization problems, i.e., NEP and FEP, in Section IV and Section V, respectively. Given the fractional nature of EE and the presence of interference, the network EE and the fair EE do not have concave numerators. In other words, NEP and FEP are both nonconvex and NP-hard, which indicates that it is very difficult to find global solutions of NEP and FEP with affordable complexity.

IV. NETWORK EE OPTIMIZATION PROBLEM (NEP)

In this section, the main objective is to maximize the network EE by optimizing the transmit power of CUs. In order to overcome the nonconvexity of NEP, we decompose NEP into two steps and address them alternately with an alternating iterative optimization scheme, based on fractional programming for the maximum network EE and sequential convex programming for the optimal transmit power.

A. FIRST-STEP FOR MAXIMUM NETWORK EE

By substituting the network EE η_{tot} in (10) into (11), NEP can be formulated as

$$\begin{aligned} &\max_{\mathbf{p}} \ \eta_{tot}(\mathbf{p}) \\ &s.t.\ C1\colon \sum\nolimits_{k=1}^{K} \alpha_k p_k \le T \\ &\qquad C2\colon R_k(\mathbf{p}) \ge r_k^{req}, \quad \forall k \\ &\qquad C3\colon 0 \le p_k \le p_k^{max}, \quad \forall k. \end{aligned} \tag{12}$$

Due to that the objective function in (12) is a ratio of two real-valued functions, and both $SE_{tot}(\mathbf{p})$ and $PC_{tot}(\mathbf{p})$ are differentiable, (12) is a nonlinear differentiable fractional programming problem [25]. Unfortunately, the interference in the system makes the numerator of network EE a nonconcave function of transmit power. Thus the fractional programming tool fails to globally maximize the network EE [24]. Nevertheless, [26] proposes an efficient and effective way to resolve such problem.

Based on [25], we first transform (12) into its equivalent subtractive form to facilitate the algorithm development. Let η_{tot}^{opt} and $\mathbf{p}^{opt}$ represent the maximum network EE and its correspondingly optimal transmit power of CUs in (12), respectively. Then we express η_{tot}^{opt} as

$$\eta_{tot}^{opt} = \frac{SE_{tot}(\mathbf{p}^{opt})}{PC_{tot}(\mathbf{p}^{opt})} = \max_{\mathbf{p}\in C1,C2,C3} \frac{SE_{tot}(\mathbf{p})}{PC_{tot}(\mathbf{p})}, \tag{13}$$

and let

$$F(\eta_{tot}) = \max_{\mathbf{p}\in C1,C2,C3} [SE_{tot}(\mathbf{p}) - \eta_{tot} PC_{tot}(\mathbf{p})]. \tag{14}$$

Theorem 1 provides a basis for the transformation of (12).

Theorem 1: For any network EE η_{tot}, the optimal transmit power $\mathbf{p}^{opt}$ is achieved if, and only if:

$$\begin{aligned} F(\eta_{tot}^{opt}) &= \max_{\mathbf{p}\in C1,C2,C3} [SE_{tot}(\mathbf{p}) - \eta_{tot}^{opt} PC_{tot}(\mathbf{p})] \\ &= SE_{tot}(\mathbf{p}^{opt}) - \eta_{tot}^{opt} PC_{tot}(\mathbf{p}^{opt}) = 0, \end{aligned} \tag{15}$$

for $SE_{tot}(\mathbf{p}) > 0$ and $PC_{tot}(\mathbf{p}) > 0$.

Proof: Theorem 1 can be proved with the similar approach in [26].

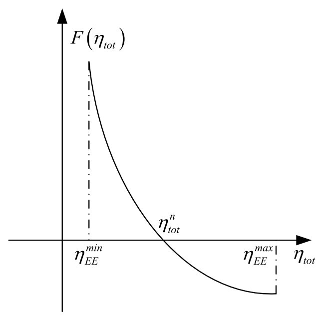


FIGURE 2. Sketch of $F(\eta_{tot})$.

Since $F(\eta_{tot})$ is convex, continuous and strictly decreasing in η_{tot} [26], as shown in Fig. 2, η_{tot}^{opt} must satisfy $F(\eta_{tot}^{opt}) = 0$, and one can use the bisection method [31] to find the optimal η_{tot}^{opt}. Its correspondingly optimal transmit power $\mathbf{p}^{opt}$ can be obtained by solving the following optimization problem:

$$\begin{aligned} &\max_{\mathbf{p}} \ SE_{tot}(\mathbf{p}) - \eta_{tot}^{n} PC_{tot}(\mathbf{p}) \\ &s.t.\ C1, C2, C3. \end{aligned} \tag{16}$$

The specific procedure is given in Algorithm 1.

B. SECOND-STEP FOR $\mathbf{p}^{opt}$

Due to the presence of interference, the objective function of (16) in Algorithm 1 remains nonconcave. Hence, (16) is a very difficult nonconvex problem. Meanwhile, (16) is NP-hard as will be shown later. To tackle this difficulty, inspired by [32], we introduce sequential convex programming and develop an efficient iterative algorithm for $\mathbf{p}^{opt}$.

Algorithm 1: Main Algorithm Procedure for Maximizing Network EE in NEP

1: **Initialization**
- Set iteration index $n = 0$, the maximum iteration number N_{max} and the termination precision $\Delta > 0$.
- Set η_{EE}^{min} and η_{EE}^{max}, such that $\eta_{EE}^{min} \leq \eta_{tot}^{opt} \leq \eta_{EE}^{max}$.

2: **Repeat**

3: $\eta_{tot}^{n} = (\eta_{EE}^{min} + \eta_{EE}^{max})/2$.

4: solve the optimization problem (16) for η_{tot}^{n} and obtain the optimal transmit power $\mathbf{p}^{n}$.

5: **if** $|F(\eta_{tot}^{n})| = \left|SE_{tot}(\mathbf{p}^{n}) - \eta_{tot}^{n} PC_{tot}(\mathbf{p}^{n})\right| \leq \Delta$, **then**

$$\mathbf{p}^{opt} = \mathbf{p}^{n} \text{ and } \eta_{tot}^{opt} = \frac{SE_{tot}(\mathbf{p}^{n})}{PC_{tot}(\mathbf{p}^{n})},$$

break.

else

if $F(\eta_{tot}^{n}) < 0$, **then**

$\eta_{EE}^{max} = \eta_{tot}^{n}$.

else

$\eta_{EE}^{min} = \eta_{tot}^{n}$.

end if

end if

6: Set $n := n + 1$ for each CU.

7: **until** $n > N_{max}$

At first, based on that SE_{tot} can be expressed as a difference of two concave functions with respect to $\mathbf{p}$, we rearrange the objective function in (16) as

$$SE_{tot}(\mathbf{p}) - \eta_{tot}^{n} PC_{tot}(\mathbf{p}) = f(\mathbf{p}) - h(\mathbf{p}), \tag{17}$$

where

$$\begin{aligned} f(\mathbf{p}) = &(1 - \eta_{tot}^{n}\varrho) \\ &\times \sum_{k=1}^{K} \log_2(\sum_{i=1}^{K} \beta_i p_i + \beta_p p_p + \sigma^2 + (M-2)\beta_k p_k) \\ &- \eta_{tot}^{n}[\frac{1}{\varepsilon}\sum_{k=1}^{K} p_k + M\rho_c + \xi], \end{aligned} \tag{18}$$

and

$$h(\mathbf{p}) = (1 - \eta_{tot}^{n}\varrho) \sum_{k=1}^{K} \log_2(\sum_{i=1, i\neq k}^{K} \beta_i p_i + \beta_p p_p + \sigma^2). \tag{19}$$

To achieve a constraint convex set of NEP, we transform the nonconvex constraint $C2$ into its equivalent convex linear form $C2'$ as follows:

$$\begin{aligned} C2' : (M-1)\beta_k p_k + (1 - 2^{r_k^{req}})(\sum_{i=1, i\neq k}^{K} \beta_i p_i + \beta_p p_p + \sigma^2) \\ \geq 0, \quad \forall k \end{aligned} \tag{20}$$

Now, (16) can be recast as

$$\begin{aligned} &\max_{\mathbf{p}} f(\mathbf{p}) - h(\mathbf{p}) \\ &s.t.\ C1, C2', C3. \end{aligned} \tag{21}$$

It should be noted that the constraint set formed by $C1$, $C2'$ and $C3$ is convex. Besides, [33] defines that a continuous function $f: [l, u] \to \mathbf{R}$ is a sigmoidal function if it is either convex, concave, or convex for $x \leq z \in [l, u]$ and concave for $x \geq z$ for some parameter $z \in \mathbf{R}$. So, both $f(\mathbf{p})$ and $-h(\mathbf{p})$ are the sum of a series of sigmoidal functions, and thus (21) is a problem that maximizes the sum of a series of sigmoidal functions over a constraint convex set. Hence, from [33], (21) is a sigmoidal programming. However, based on the theory of sigmoidal programming, (21) is NP-hard and NP-hard to approximate [33]. Similarly, (16) is NP-hard and NP-hard to approximate. Therefore, it is difficult to address (16) directly in polynomial time in the globally optimal sense.

To circumvent the problem, we shall borrow the idea of sequential convex programming. According to [27], the basic idea of sequential convex programming is to find local optima of a difficult problem of maximizing objective functions, through solving a sequence of easier approximate problems by standard methods. Once suitable approximations have been found, sequential convex programming can obtain a first-order optimal solution of the original problem [27], [28] with affordable complexity, requiring only the solution of convex approximate problems but satisfying theoretical optimality claims [28]. Therefore, finding the suitable approximations is the most critical issue in this approach.

Suppose $\mathbf{p}^{l}$ be the value of $\mathbf{p}$ in iteration l. The first-order Taylor approximation of $h(\mathbf{p})$ at $\mathbf{p}^{l}$ can be written as

$$h(\mathbf{p}^{l}) + \nabla h^{T}(\mathbf{p}^{l})(\mathbf{p} - \mathbf{p}^{l}), \tag{22}$$

where $\nabla h(\mathbf{p})$ is the gradient of $h(\mathbf{p})$ at $\mathbf{p}$, and is given by

$$\nabla h(\mathbf{p}) = \sum_{k=1}^{K} \frac{\mathbf{e}_k}{\sum_{i=1, i\neq k}^{K} \beta_i p_i + \beta_p p_p + \sigma^2}, \tag{23}$$

where $\mathbf{e}_k$ is a K-dimensional column vector with $\mathbf{e}_k(k) = 0$, and $\mathbf{e}_k(i) = \frac{(1 - \eta_{tot}^{n}\varrho)\beta_i}{\ln 2}, i \neq k$.

Since $h(\mathbf{p})$ is concave and differentiable over the constraint convex set, the first-order Taylor approximation (22) is in fact a global over-estimator of $h(\mathbf{p})$, i.e., $h(\mathbf{p}) \leq h(\mathbf{p}^{l}) + \nabla h^{T}(\mathbf{p}^{l})(\mathbf{p} - \mathbf{p}^{l})$ [31]. The inequality shows that from the local information (i.e., $h(\mathbf{p}^{l})$, and $\nabla h^{T}(\mathbf{p}^{l})$), we can derive global information (i.e., a global over-estimator of it) [31]. Since the logarithmic structure of $h(\mathbf{p})$, it is not very sensitive to changes in $\mathbf{p}$. Hence, $h(\mathbf{p})$ can be well approximated by its first-order Taylor approximation in a fairly large neighborhood of $\mathbf{p}^{l}$ [32].

Consequently, the following transformation can provide a well approximated lower bound maximization for (21):

$$\begin{aligned} &\max_{\mathbf{p}} f(\mathbf{p}) - [h(\mathbf{p}^{l}) + \nabla h^{T}(\mathbf{p}^{l})(\mathbf{p} - \mathbf{p}^{l})] \\ &s.t.\ C1, C2', C3. \end{aligned} \tag{24}$$

Since the above objective function is concave over the constraint convex set, (24) is a standard convex optimization problem. Thus the first-order optimal solution $\mathbf{p}^{opt}$ of (16) can be efficiently achieved by solving its convex approximate problem (24) with available convex software packages in polynomial time. The detailed description of this procedure is represented in Algorithm 2.

Algorithm 2: Iterative Algorithm Procedure for $\mathbf{p}^{opt}$

1: **Initialization**
- Set iteration index $l = 0$, and the termination precision $\delta > 0$.
- Set $\mathbf{p}^0$, calculate $I^0 = f(\mathbf{p}^0) - h(\mathbf{p}^0)$.

2: **Repeat**
3: Solve the optimization problem (24) to obtain the optimal transmit power $\mathbf{p}^{opt}$.
4: Set $l = l + 1$, and $\mathbf{p}^l = \mathbf{p}^{opt}$.
5: Calculate $I^l = f(\mathbf{p}^l) - h(\mathbf{p}^l)$.
6: **until** $\left|I^l - I^{l-1}\right| \leq \delta$.

It is noted that, since the maximum value of (24) is upper bounded due to the power constraints $C3$, the iterative procedure is guaranteed to converge (*Proof:* See Appendix A), and the obtained optimal transmit power would converge to a stationary point of (24), i.e., (16), with any feasible initial value (*Proof:* See Appendix B).

To sum up, we can address the NEP (12) by repeating the following procedure, solving (24) to get the optimal transmit power $\mathbf{p}^{opt}$ for a current η_{tot}^n and updating η_{tot}^n with the bisection method, until we reach the optimal $\eta_{tot}^{opt} \geq 0$ satisfying $|F(\eta_{tot}^{opt})| \leq \Delta$. This algorithm is implemented at the CBS. After the CBS achieves the optimal transmit power for CUs by the proposed algorithm, it broadcasts the allocated transmit power to CUs at each transmission block.

Next, we make a brief analysis about the computational complexity of the proposed algorithm for NEP. On the one hand, the bisection method adopted in the first-step needs $\lceil \log_2(\frac{\eta_{EE}^{max} - \eta_{EE}^{min}}{\Delta}) \rceil$ iterations before obtaining the optimal η_{tot}^{opt} with error tolerance of Δ [31]. From Algorithm 1, we know that it is bounded by N_{max}. On the other hand, the optimal transmit power $\mathbf{p}^{opt}$ can be achieved with the polynomial complexity $O(K^3)$ in the second-step. Putting these facts together, we can conclude that the proposed algorithm has an affordable polynomial complexity, which makes it applicable to practical systems.

V. FAIR EE OPTIMIZATION PROBLEM (FEP)

Although the system can achieve a high network-side EE in NEP, it gives rise to the unfairness among CUs in terms of individual EE. To be specific, in NEP, the system simply improves the network EE by allocating more power to CUs in good channel conditions sacrificing CUs in bad channel conditions. This appears unreasonable in practical CRNs. Given this, in this section we explore the fairness of EE among CUs in massive MIMO CRNs.

Inspired by [29], in which max-min fairness among users is achieved through maximizing the EE of the user in the worst-case, we choose the fair EE be the objective function of FEP. Considering the nonconvexity of FEP, like in NEP, we decompose the FEP into two steps and address them alternately with an alternating iterative optimization scheme, on the basis of the interplay of fractional programming for the fair EE maximization and sequential convex programming for the optimal transmit power.

A. FIRST-STEP FOR MAXIMUM FAIR EE

In this section, the utility function $U(\mathbf{p})$ of (11) corresponds to the fair EE ($\min_k \eta_k$). As a result, FEP becomes

$$\begin{aligned} \max_{\mathbf{p}} \ & \min_{k} \ \eta_k(\mathbf{p}) \\ s.t. \ & C1: \sum\nolimits_{k=1}^{K} \alpha_k p_k \leq T \\ & C2: R_k(\mathbf{p}) \geq r_k^{req}, \quad \forall k \\ & C3: 0 \leq p_k \leq p_k^{max}, \quad \forall k. \end{aligned} \tag{25}$$

It can be easily proved that (25) is nonlinear differentiable fractional programming [25] and the numerator of η_k is nonconcave. Therefore, fractional programming tool can not globally maximize the fair EE [24]. Similar to IV-A, we utilize the methods in [26] to overcome this difficulty.

Firstly, we replace the objective function in (25) with its equivalent subtractive form [25]. Let η_k^* and $\mathbf{p}^*$ be the maximum fair EE and its correspondingly optimal transmit power of CUs in (25), respectively. Then, η_k^* can be expressed as

$$\eta_k^* = \min_k \frac{SE_k(\mathbf{p}^*)}{PC_k(\mathbf{p}^*)} = \max_{\mathbf{p} \in C1, C2, C3} \min_k \frac{SE_k(\mathbf{p})}{PC_k(\mathbf{p})}. \tag{26}$$

Define function

$$\widetilde{F}(\eta_k) = \max_{\mathbf{p} \in C1, C2, C3} \min_k [SE_k(\mathbf{p}) - \eta_k PC_k(\mathbf{p})], \tag{27}$$

then we have the following essential theorem to perform the transformation of (25):

Theorem 2: For any fair EE η_k, the optimal transmit power $\mathbf{p}^*$ is achieved if, and only if:

$$\begin{aligned} \widetilde{F}(\eta_k^*) &= \max_{\mathbf{p} \in C1, C2, C3} \min_k [SE_k(\mathbf{p}) - \eta_k^* PC_k(\mathbf{p})] \\ &= \min_k [SE_k(\mathbf{p}^*) - \eta_k^* PC_k(\mathbf{p}^*)] = 0, \end{aligned} \tag{28}$$

for $SE_k(\mathbf{p}) > 0$ and $PC_k(\mathbf{p}) > 0$.

Proof: Please refer to the similar approach in [26].

For $\widetilde{F}(\eta_k)$ being convex, continuous and strictly decreasing in η_k [26], the maximum fair EE η_k^* must satisfy $\widetilde{F}(\eta_k^*) = 0$. Wherein η_k^* can be obtained with the bisection method [31], and the optimal transmit power $\mathbf{p}^*$ can be achieved by addressing the following optimization problem:

$$\begin{aligned} \max_{\mathbf{p}} \ & \min_k [SE_k(\mathbf{p}) - \eta_k^n PC_k(\mathbf{p})] \\ s.t. \ & C1, C2, C3. \end{aligned} \tag{29}$$

The specific algorithm procedure is presented in Algorithm 3.

Algorithm 3 : Main Algorithm Procedure for Maximizing Fair EE in FEP

1: **Initialization**
- Set iteration index $n = 0$, the maximum iteration number N_{max} and the termination precision $\Delta > 0$.
- Set η_{EE}^{min} and η_{EE}^{max}, such that $\eta_{EE}^{min} \leq \eta_k^* \leq \eta_{EE}^{max}$.

2: **Repeat**

3: $\eta_k^n = (\eta_{EE}^{min} + \eta_{EE}^{max})/2$.

4: solve the optimization problem (29) for η_k^n and obtain the optimal transmit power $\mathbf{p}^n$.

5: **if** $|\widetilde{F}(\eta_k^n)| = \left|\min_k [SE_k(\mathbf{p}^n) - \eta_k^n PC_k(\mathbf{p}^n)]\right| \leq \Delta$, **then**

$\mathbf{p}^* = \mathbf{p}^n$ and $\eta_k^* = \min_k [\frac{SE_k(\mathbf{p}^n)}{PC_k(\mathbf{p}^n)}]$,

break.

else

if $\widetilde{F}(\eta_k^n) < 0$, **then**

$\eta_{EE}^{max} = \eta_k^n$.

else

$\eta_{EE}^{min} = \eta_k^n$.

end if

end if

6: Set $n := n + 1$ for each CU.

7: **until** $n > N_{max}$

B. SECOND-STEP FOR $\mathbf{p}^*$

Obviously, due to the interference item in (29), it is still difficult to handle (29) for its nonconvexity and NP-hard feature. To tackle this issue, we apply sequential convex programming [27] into developing an algorithm to iteratively and efficiently search for $\mathbf{p}^*$.

The objective function in (29) can be first rearranged into a difference of two concave functions with respect to $\mathbf{p}$ thanks to the logarithmic feature of SE_k,

$$SE_k(\mathbf{p}) - \eta_k^n PC_k(\mathbf{p}) = \widetilde{f}_k(\mathbf{p}) - \widetilde{h}_k(\mathbf{p}), \tag{30}$$

where

$$\begin{aligned}\widetilde{f}_k(\mathbf{p}) = (1 - \eta_k^n \varrho)&\\ &\times \log_2(\sum_{i=1}^{K} \beta_i p_i + \beta_p p_p + \sigma^2 + (M-2)\beta_k p_k)\\ &-\eta_k^n[\frac{1}{\varepsilon} p_k + M\rho_c + \xi],\end{aligned} \tag{31}$$

and

$$\widetilde{h}_k(\mathbf{p}) = (1 - \eta_k^n \varrho) \log_2 \left(\sum_{i=1, i\neq k}^{K} \beta_i p_i + \beta_p p_p + \sigma^2 \right). \tag{32}$$

Then we replace $C2$ with its equivalent convex linear form $C2'$ in IV-B to form a constraint convex set of FEP. Consequently, (29) can be equivalently rewritten as

$$\begin{aligned}&\max_{\mathbf{p}} \ \min_k [\widetilde{f}_k(\mathbf{p}) - \widetilde{h}_k(\mathbf{p})]\\ &s.t. \ C1, C2', C3.\end{aligned} \tag{33}$$

Algorithm 4: Iterative Algorithm Procedure for $\mathbf{p}^*$

1: **Initialization**
- Set iteration index $l = 0$, and the termination precision $\delta > 0$.
- Set $\mathbf{p}^0$, calculate $J^0 = \min_k [\widetilde{f}_k(\mathbf{p}^0) - \widetilde{h}_k(\mathbf{p}^0)]$.

2: **Repeat**

3: Solve the optimization problem (37) to obtain the optimal transmit power $\mathbf{p}^*$.

4: Set $l = l + 1$, and $\mathbf{p}^l = \mathbf{p}^*$.

5: Calculate $J^l = \min_k [\widetilde{f}_k(\mathbf{p}^l) - \widetilde{h}_k(\mathbf{p}^l)]$.

6: **until** $\left|J^l - J^{l-1}\right| \leq \delta$.

It is worth noting that, on account of the different minimum of $\widetilde{f}_k(\mathbf{p}) - \widetilde{h}_k(\mathbf{p})$ in each iteration, the objective function in (33) is nonsmooth. Enlightened by the concept of epigraph [31], we introduce an auxiliary variable φ to eliminate the nonsmoothness of the objective function [29]. Therefore, (33) is further transformed into

$$\begin{aligned}&\max_{\mathbf{p},\varphi} \ \varphi\\ &s.t. \ C1, C2', C3,\\ &\quad C4: \widetilde{f}_k(\mathbf{p}) - \widetilde{h}_k(\mathbf{p}) \geq \varphi, \quad \forall k.\end{aligned} \tag{34}$$

But the nonconvexity and NP-hard feature still remain. The difficulty of solving (34) lies in the convex component $-\widetilde{h}_k(\mathbf{p})$. Since $\widetilde{h}_k(\mathbf{p})$ is concave and differentiable over the constraint convex set, one can easily find the first-order Taylor approximation of $\widetilde{h}_k(\mathbf{p})$ at any $\mathbf{p}^l$ as its global over-estimator [31], i.e.,

$$\widetilde{h}_k(\mathbf{p}) \leq \widetilde{h}_k(\mathbf{p}^l) + \nabla \widetilde{h}_k^T(\mathbf{p}^l)(\mathbf{p} - \mathbf{p}^l), \tag{35}$$

where $\nabla \widetilde{h}_k(\mathbf{p})$ is the gradient of $\widetilde{h}_k(\mathbf{p})$ at $\mathbf{p}$, given by

$$\nabla \widetilde{h}_k(\mathbf{p}) = \frac{\mathbf{e}_k}{\sum_{i=1, i\neq k}^{K} \beta_i p_i + \beta_p p_p + \sigma^2}, \tag{36}$$

where $\mathbf{e}_k$ is a K-dimensional column vector with $\mathbf{e}_k(k) = 0$, and $\mathbf{e}_k(i) = \frac{(1-\eta_k^n \varrho)\beta_i}{\ln 2}, i \neq k$.

Thanks to the logarithmic structure, $\widetilde{h}_k(\mathbf{p})$ can be well approximated by its first-order Taylor approximation in a fairly large neighborhood of $\mathbf{p}^l$ [29]. Therefore, we further transform (34) into

$$\begin{aligned}&\max_{\mathbf{p},\varphi} \ \varphi\\ &s.t. \ C1, C2', C3,\\ &\quad C4: \widetilde{f}_k(\mathbf{p}) - [\widetilde{h}_k(\mathbf{p}^l) + \nabla \widetilde{h}_k^T(\mathbf{p}^l)(\mathbf{p} - \mathbf{p}^l)] \geq \varphi, \quad \forall k.\end{aligned} \tag{37}$$

At present, (37) is a smooth and standard convex optimization problem. The locally optimal transmit power can be efficiently achieved by solving (37) with interior point methods [31]. We show the detailed algorithm process in Algorithm 4 and give the following theorem:

Theorem 3: (1) The efficient iterative algorithm 4 always converges, and (2) the obtained optimal transmit power converges to a stationary point of (37), i.e., (29), with any feasible initial value.

Proof: See Appendix C.

In conclusion, we can address problem (25) by repeating the following procedure, solving its equivalent subtractive form (37) for a current η_k^n and updating η_k^n with the bisection method, until we reach the optimal $\eta_k^* \geq 0$ satisfying $|F(\eta_k^*)| \leq \Delta$. This algorithm is implemented at the CBS. After the optimal transmit power for CUs is found by the proposed algorithm, it would be broadcast by the CBS to CUs at each transmission block.

Similarly, a brief analysis about the computational complexity of the proposed algorithm for FEP is made in the following. On the one hand, the bisection search for the optimal η_k^* in the first-step needs $\lceil \log_2(\frac{\eta_{EE}^{max}-\eta_{EE}^{min}}{\Delta})\rceil$ iterations to guarantee the error tolerance of Δ [31]. On the other hand, the optimal $\mathbf{p}^*$ in each iteration can be obtained by interior point methods in polynomial time with computational complexity being roughly proportional to $O(\lceil J^{\max} - J^0 - \frac{\delta}{\triangle J}\rceil \max\{K^3, F\})$ [29], [31], where $J^{\max} = \max_{\mathbf{p}} \min_k[\widetilde{f}_k(\mathbf{p}) - \widetilde{h}_k(\mathbf{p})]$, $\triangle J = \min_l[J^l - J^{l-1}]$ and F denotes the cost of evaluating the first and second derivatives of the constraint functions [31]. Therefore, it follows that the proposed algorithm to FEP is applicable to a practical implementation.

VI. SIMULATION RESULTS

In this section, we analyze the performance of these proposed energy-efficient power control algorithms in Section IV and Section V, respectively. Along with that, the impacts of the CBS antennas number M and the circuit power consumption items ξ and ρ_c on the network EE and network SE are also investigated.

TABLE 1. Simulation parameters.

Parameter	Default Value
Number of CUs, K	10
Number of CBS antennas, M	200
Path loss exponent, θ	3.8
Shadow fading standard derivation, σ_{shadow}	8 dB
Peak transmit power of each CU k, p_k^{max}	0.5 W
Transmit power of the PU, p_p	1 W
Interference temperature threshold, T	0 dB
Minimum rate requirement of each CU k, r_k^{req}	1 bits/s/Hz
Drain efficiency of power amplifiers, ε	0.38
Circuit power consumption per antenna, ρ_c	0.02 W
Constant scaling factor, ϱ	0.01 μW/bit/s
Static circuit power consumption, ξ	0.5 W
Background noise power, σ^2	5×10^{-15} W

In the numerical simulations, we consider a massive MIMO CRN with radius (from center to vertex) of 1 km, and all the single-antenna CUs are located uniformly in it. We assume that no CU gets closer to the CBS than $d_h = 100\,m$ [7], and each CU has the same minimum data rate requirement and equal peak transmit power. Here, we ignore fast fading, shadowing, and other interference. Other simulation parameters are given in Table 1, and are chosen according to some available literature [12], [13], [19]. Those parameters are used in the following simulations unless stated otherwise. In the following, we compare the proposed algorithms, marked as 'NEP' and 'FEP', respectively, with the scheme for weighted sum rate maximization (WSRM) [30], the scheme for max-min weighted data rate (MMDR) [30], and the 'PowerMax' algorithm with the peak transmit power for each CU [29], which is considered as a baseline scheme.

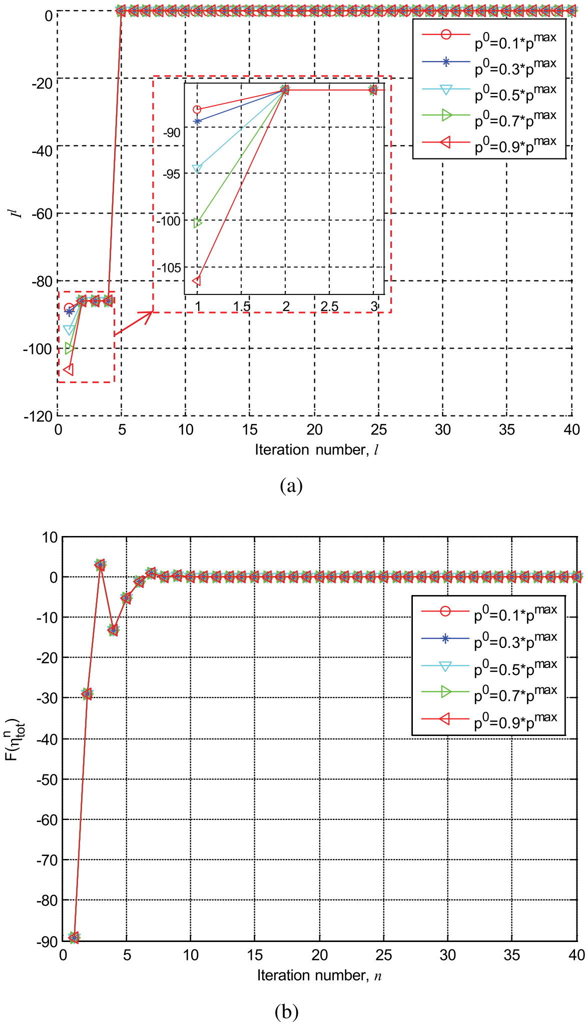
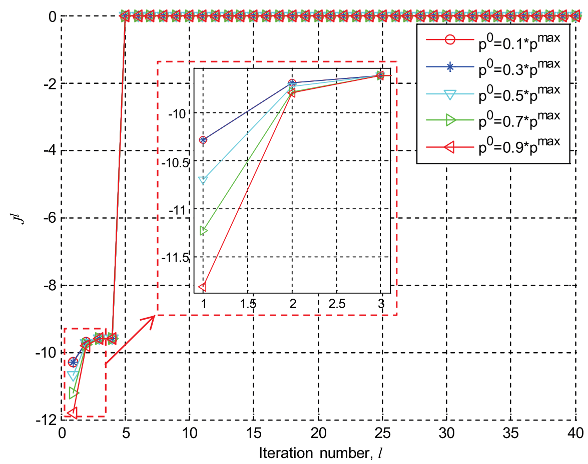
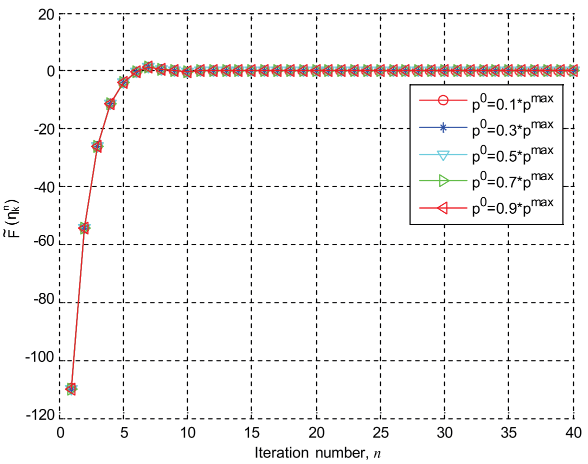


FIGURE 3. (a) Convergence evolution of the efficient iterative algorithm for $\mathbf{p}^{opt}$ with $\eta_{tot}^n = 5$ bits/Joule. (b) Convergence evolution of the bisection method for η_{tot}^{opt} in NEP.

Fig. 3(a) and Fig. 3(b) present the iteration evolutions of the second-step efficient iterative algorithm for $\mathbf{p}^{opt}$ and the first-step bisection method for η_{tot}^{opt}, respectively, in NEP. They confirm the theoretical findings that the 'NEP' power control algorithm is insensitive to the starting value of transmit power and has rapid convergence. In addition, Fig. 3(b) shows that after several iterations, $F(\eta_{tot})$ converges to 0, which implies that the optimal η_{tot}^{opt} can be quickly found with the bisection method.

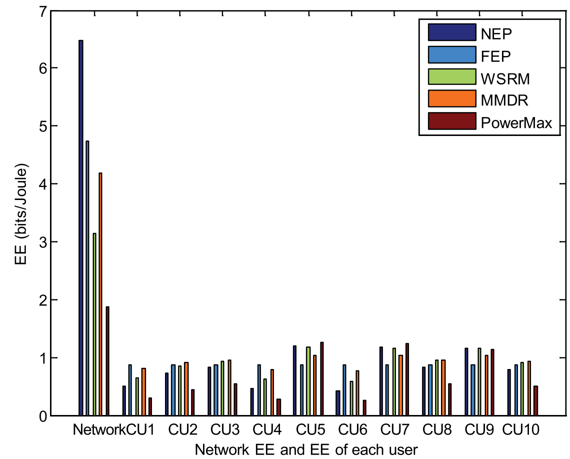


(a)

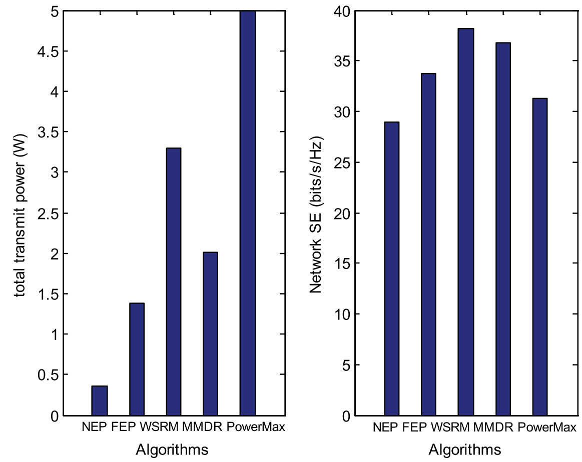


(b)

FIGURE 4. (a) Convergence evolution of the efficient iterative algorithm for p* with η_k^n = 5 bits/Joule. (b) Convergence evolution of the bisection method for η_k^* in FEP.



(a)



(b)

FIGURE 5. (a) Network EE and the EE of each CU. (b) Total transmit power consumed and Network SE under these algorithms.

Similarly, Fig. 4(a) and Fig. 4(b) depict the iteration evolutions of the second-step efficient iterative algorithm for $\mathbf{p}^*$ and the first-step bisection method for η_k^*, respectively, in FEP. They demonstrate that the 'FEP' power control algorithm converges rapidly with any feasible starting transmit power. Besides, Fig. 4(b) confirms that $\widetilde{F}(\eta_k)$ converges to 0 after several iterations. That is to say, the optimal η_k^* can be achieved very quickly.

Figure 5(a) presents the network EE and the EE of each CU under five aforementioned algorithms in the system. Specifically, 'NEP' has the highest network EE at the cost of the EE of CUs in bad channel conditions, while 'FEP' guarantees the fairness of EE among the CUs with some sacrifices in the network EE. Nevertheless, both 'NEP' and 'FEP' have preferable performance on the network EE than the other three algorithms. In order to quantitatively evaluate the fairness of these five algorithms, we introduce the Jain's fairness index (lying between 0 and 1) [34]. It is widely utilized in many resource sharing or allocation problems, for its comprehensive and highly intuitive interpretation properties. According to [34], the Jain's index of CU's EE in this system can be derived as follows:

$$J_a = \frac{\left(\sum_{k=1}^{K} \eta_k\right)^2}{K \sum_{k=1}^{K} \eta_k^2}, \tag{38}$$

where η_k denotes the EE of CU k as shown in (10). With the achievable EE of CUs by these five schemes, we can get their corresponding Jain's index. Particularly, the fairness of 'NEP' is 0.82, those of 'FEP', 'WSRM', 'MMDR' and 'PowerMax' are 1, 0.90, 0.95 and 0.72, respectively. It is worth noting that a larger J_a corresponds to better fairness performance from the perspective of Jain's index. So, 'FEP' can be considered as fair. That is to say, 'FEP' can well guarantee the fairness of EE among CUs in the system. In addition, their correspondingly total transmit power consumption and their achieved network SE are shown in Fig. 5(b). We can intuitively see that even 'NEP' has the best performance of network EE, its network SE is the lowest. While 'FEP' consumes more transmit power than 'NEP', but achieves more network SE in return.

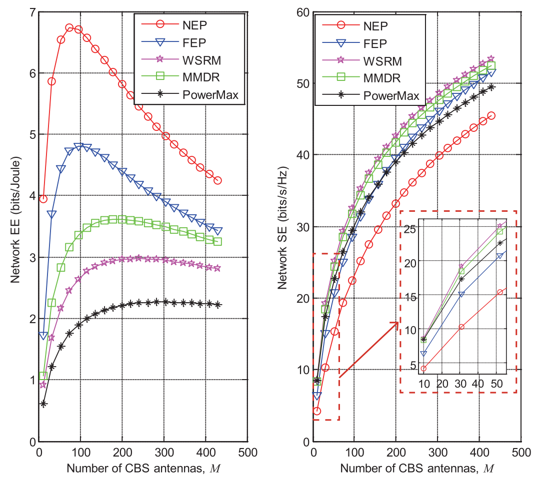
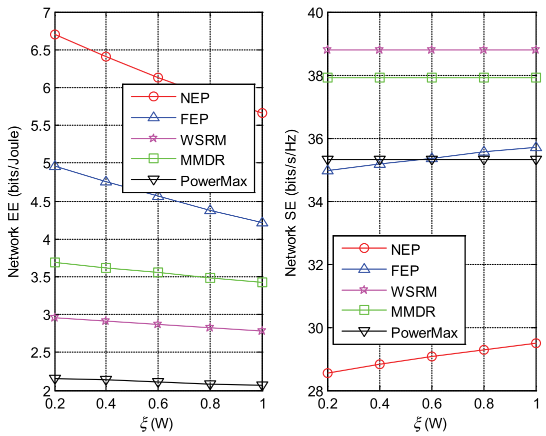
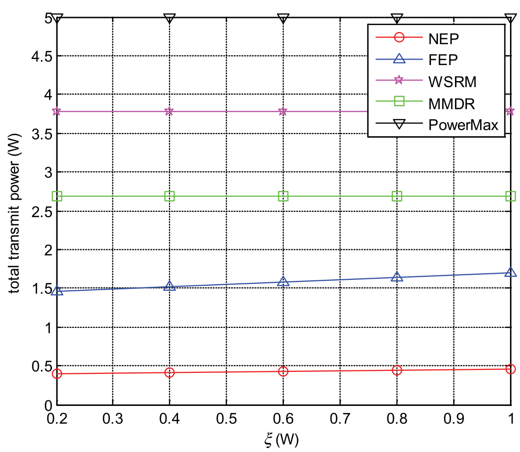


FIGURE 6. Network EE/Network SE versus different M under these algorithms.

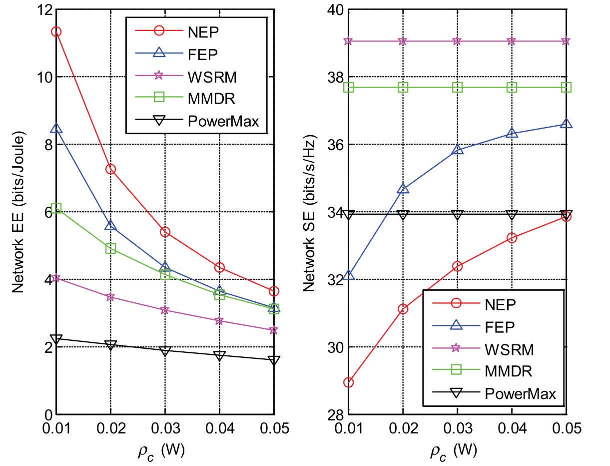


(a)

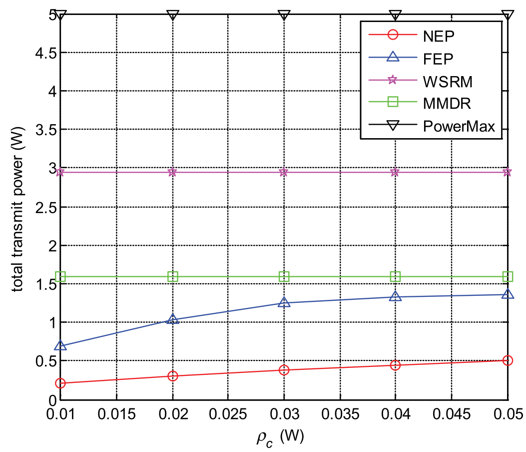


(b)

FIGURE 7. (a) Network EE/Network SE versus different static circuit power consumption ξ. (b) Total transmit power consumed versus different static circuit power consumption ξ under these algorithms.



(a)



(b)

FIGURE 8. (a) Network EE/Network SE versus different circuit power consumption per CBS antenna ρ_c. (b) Total transmit power consumed versus different circuit power consumption per CBS antenna ρ_c under these algorithms.

Figure 6 shows the evolutions of network EE and network SE of these five algorithms in terms of different CBS antennas numbers. It can be clearly observed that all their achieved network EE increases firstly and then decreases, while their network SE keeps monotonically increasing with M. In particular, 'FEP' has a relatively good SE performance, and its network SE is very close to the optimal network SE obtained by the sum rate maximization methods, i.e., 'WSRM' and 'MMDR'. As shown in Fig. 6, on the one hand, the ergodic achievable uplink rate in (3) grows as M increases, thus the network SE would be improved proportionally to M. On the other hand, the total circuit power consumption increases with M, but its growth rate is lower than that of the network SE at first. So their achieved network EE shows firstly an overall upward trend. When M is large, the growth rate of network SE is gradually slowed down with the increasing M, and the total circuit power consumption becomes dominant, consequently causing some loss of network EE. So, from the viewpoint of network EE maximization, it is more beneficial to utilize a moderate number of CBS antennas, even if there are many available CBS antennas. While from the aspect of large capacity, more CBS antennas are expected. So, to some extent, the proposed power control algorithms provide some design insights on the selection of optimal M for different specifications of EE and SE in massive MIMO CRNs.

In Fig. 7 and Fig. 8, we investigate the impacts of the static circuit power consumption ξ and the circuit power consumption per CBS antenna ρ_c on the network EE and network SE. As shown in Fig. 7(a) and Fig. 8(a), the network EE decreases with the increasing of either ξ or ρ_c, but both of them have a negligible effect on the network SE of the latter three algorithms. These results comply with the common sense. Besides, it is observed that the network SE of 'NEP' and 'FEP' goes up slightly with the increasing of ξ or ρ_c. The fact seems a bit counterintuitive at first, but it is consistent with the conclusions derived in [12] that, with the increasing of the static circuit power consumption and the circuit power consumption per CBS antenna, we can afford using more transmit power to improve the data rate before it becomes a limiting factor for EE. Note that the transmit power of 'NEP' and 'FEP' indeed increases with ξ and ρ_c, as illustrated in Fig. 7(b) and Fig. 8(b).

VII. CONCLUSION

In this paper, we investigated two energy-efficient power optimization problems, i.e., NEP and FEP, to maximize the network EE and guarantee the fairness of EE among CUs, respectively, in a massive MIMO CRN. To tackle the nonconvexity and NP-hard feature of the optimization problems, we proposed two energy-efficient power control algorithms on the basis of an interplay of fractional programming and sequential convex programming. Numerical results not only demonstrated the fast convergence and high effectiveness of these proposed algorithms, but also illustrated the impacts of CBS antennas number and the circuit power consumption on the network EE and network SE. Specifically, in practical communication systems, the network EE decreases with a too large number of CBS antennas and the increase of circuit power consumption, while the network SE keeps growing with the increasing number of CBS antennas but is slightly affected by the circuit power consumption.

APPENDIX A

Proof: For the optimal solution $\mathbf{p}^{l+1}$ of (24) in iteration l, one has

$$\begin{aligned} I^{l+1} &= f(\mathbf{p}^{l+1}) - h(\mathbf{p}^{l+1}) \\ &\geq f(\mathbf{p}^{l+1}) - [h(\mathbf{p}^{l}) + \nabla h^T(\mathbf{p}^{l})(\mathbf{p}^{l+1} - \mathbf{p}^{l})] \\ &\geq f(\mathbf{p}^{l}) - [h(\mathbf{p}^{l}) + \nabla h^T(\mathbf{p}^{l})(\mathbf{p}^{l} - \mathbf{p}^{l})] \\ &= f(\mathbf{p}^{l}) - h(\mathbf{p}^{l}) = I^{l}, \end{aligned} \tag{39}$$

where the first inequality holds since $h(\mathbf{p})$ is concave and for any given $\mathbf{p}$, $h(\mathbf{p}) \leq h(\mathbf{p}^l) + \nabla h^T(\mathbf{p}^l)(\mathbf{p} - \mathbf{p}^l)$ [31], while the second inequality holds since $\mathbf{p}^{l+1}$ is the optimal solution of (24) for obtaining the largest objective value. Hence, the objective value of (24) is improved after each iteration. Furthermore, the objective value of (24) is upper bounded due to the transmit power constraints. Hence, the iterative algorithm is guaranteed to converge. ■

APPENDIX B

Proof: Let $m(\mathbf{p}) = f(\mathbf{p}) - [h(\mathbf{p}^l) + \nabla h^T(\mathbf{p}^l)(\mathbf{p} - \mathbf{p}^l)]$. We note that $m(\mathbf{p})$ is differentiable and strictly concave on the constraint convex set of (24) [31]. As a result, the obtained optimal transmit power converges to an accumulation point [35]. Here, we assume $\mathbf{p}^l = \mathbf{p}^{l+1}$ in the limit, and $\mathbf{p}^{l+1} = \arg\max_{\mathbf{p}\in\{C1,C2',C3\}}\{f(\mathbf{p}) - [h(\mathbf{p}^l) + \nabla h^T(\mathbf{p}^l)(\mathbf{p} - \mathbf{p}^l)]\}$. According to the optimality condition [35],

$$\nabla m^T(\mathbf{p}^l)(\mathbf{p} - \mathbf{p}^l) = \nabla m^T(\mathbf{p}^{l+1})(\mathbf{p} - \mathbf{p}^{l+1}) \leq 0, \tag{40}$$

and the vector satisfying the optimality condition is referred to as a stationary point. So, $\mathbf{p}^l$ is the stationary point of (24), i.e., (16). ■

APPENDIX C

(1) The efficient iterative algorithm always converges.

Proof: Let $\mathbf{p}^{l+1}$ be the optimal solution of (37) in iteration l. One has

$$\begin{aligned} J^{l+1} &= \min_k[\widetilde{f}_k(\mathbf{p}^{l+1}) - \widetilde{h}_k(\mathbf{p}^{l+1})] \\ &\geq \min_k[\widetilde{f}_k(\mathbf{p}^{l+1}) - \widetilde{h}_k(\mathbf{p}^{l}) - \nabla\widetilde{h}_k^T(\mathbf{p}^{l})(\mathbf{p}^{l+1} - \mathbf{p}^{l})] \\ &\geq \min_k[\widetilde{f}_k(\mathbf{p}^{l}) - \widetilde{h}_k(\mathbf{p}^{l}) - \nabla\widetilde{h}_k^T(\mathbf{p}^{l})(\mathbf{p}^{l} - \mathbf{p}^{l})] \\ &= \min_k[\widetilde{f}_k(\mathbf{p}^{l}) - \widetilde{h}_k(\mathbf{p}^{l})] = J^{l}, \end{aligned} \tag{41}$$

where the first inequality holds since $\widetilde{h}_k(\mathbf{p}) \leq \widetilde{h}_k(\mathbf{p}^l) + \nabla\widetilde{h}_k^T(\mathbf{p}^l)(\mathbf{p} - \mathbf{p}^l)$ [31], the second inequality holds since the optimal solution $\mathbf{p}^{l+1}$ always maximizes the objective value of (37). Hence, the objective value of (37) increases after each iteration. Furthermore, due to the transmit power constraints, this objective value is upper bounded. Therefore, the algorithm converges after finite iterations. ■

(2) The obtained optimal transmit power converges to a stationary point of (37), i.e., (29), with any feasible initial value.

Proof: From (1), we can conclude that the obtained optimal transmit power converges in finite iterations. Here we assume $\mathbf{p}^l = \mathbf{p}^{l+1}$ in the limit, and $\mathbf{p}^{l+1} = \arg\max_{\mathbf{p}\in\{C1,C2',C3\}}\{\min_k[\widetilde{f}_k(\mathbf{p}) - \widetilde{h}_k(\mathbf{p}^l) - \nabla\widetilde{h}_k^T(\mathbf{p}^l)(\mathbf{p} - \mathbf{p}^l)]\}$. Let $n(\mathbf{p}) = \widetilde{f}_k(\mathbf{p}) - [\widetilde{h}_k(\mathbf{p}^l) + \nabla\widetilde{h}_k^T(\mathbf{p}^l)(\mathbf{p} - \mathbf{p}^l)]$. According to the optimality condition [35],

$$\min_k[\nabla n^T(\mathbf{p}^l)(\mathbf{p} - \mathbf{p}^l)] = \min_k[\nabla n^T(\mathbf{p}^{l+1})(\mathbf{p} - \mathbf{p}^{l+1})] \leq 0, \tag{42}$$

and the optimal vector satisfying the optimality condition is referred to as a stationary point. So, $\mathbf{p}^l$ is the stationary point of (37), i.e., (29). ■

REFERENCES

[1] Ericsson White Paper, "More than 50 billion connected devices," Ericsson, Stockholm, Sweden, Tech. Rep. 284 23-3149 Uen, Feb. 2011.

[2] "The 1000x data challenge," Qualcomm, San Diego, CA, USA, pp.1-38, Nov. 2013. [Online]. Available: http://www.qualcomm.com/1000x
[3] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.
[4] H. Park and T. Hwang, "Energy-efficient power control of cognitive femto users for 5G communications," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 4, pp. 772–785, Apr. 2016.
[5] J. Zhang, F.-C. Zheng, X.-Q. Gao, and H.-B. Zhu, "Sensing-energy efficiency tradeoff for cognitive radio networks," *IET Commun.*, vol. 8, no. 18, pp. 3414–3423, 2014.
[6] S. Buzzi and D. Saturnino, "A game-theoretic approach to energy-efficient power control and receiver design in cognitive CDMA wireless networks," *IEEE J. Sel. Topics Signal Process.*, vol. 5, no. 1, pp. 137–150, Feb. 2011.
[7] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.
[8] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, "Energy and spectral efficiency of very large multiuser MIMO systems," *IEEE Trans. Commun.*, vol. 61, no. 4, pp. 1436–1449, Apr. 2013.
[9] H. Q. Ngo, M. Matthaiou, and E. G. Larsson, "Massive MIMO with optimal power and training duration allocation," *IEEE Wireless Commun. Lett.*, vol. 3, no. 6, pp. 605–608, Dec. 2014.
[10] E. Björnson, J. Hoydis, M. Kountouris, and M. Debbah, "Massive MIMO systems with non-ideal hardware: Energy efficiency, estimation, and capacity limits," *IEEE Trans. Inf. Theory*, vol. 60, no. 11, pp. 7112–7139, Nov. 2014.
[11] H. Yang and T. L. Marzetta, "Total energy efficiency of cellular large scale antenna system multiple access mobile networks," in *Proc. IEEE GreenCom*, Oct. 2013, pp. 27–32.
[12] E. Björnson, L. Sanguinetti, J. Hoydis, and M. Debbah, "Optimal design of energy-efficient multi-user MIMO systems: Is massive MIMO the answer?" *IEEE Trans. Wireless Commun.*, vol. 14, no. 6, pp. 3059–3075, Jun. 2015.
[13] M. Cui, B.-J. Hu, X. Li, and H. Chen, "A novel power control algorithm for massive MIMO cognitive radio systems based on game theory," in *Proc. IEEE VTC-Spring*, May 2015, pp. 1–5.
[14] L. Wang, H. Q. Ngo, M. Elkashlan, T. Q. Duong, and K.-K. Wong, "Massive MIMO in spectrum sharing networks: Achievable rate and power efficiency," *IEEE Syst. J.*, to be published.
[15] C. C. Zarakovitis and Q. Ni, "Maximizing energy efficiency in multiuser multicarrier broadband wireless systems: Convex relaxation and global optimization techniques," *IEEE Trans. Veh. Technol.*, vol. 65, no. 7, pp. 5275–5286, Jul. 2016.
[16] Z. Xu, C. Yang, G. Y. Li, S. Zhang, Y. Chen, and S. Xu, "Energy-efficient configuration of spatial and frequency resources in MIMO-OFDMA systems," *IEEE Trans. Commun.*, vol. 61, no. 2, pp. 564–575, Feb. 2013.
[17] Y. Hao, Q. Ni, H. Li, and S. Hou, "Energy and spectral efficiency tradeoff with user association and power coordination in massive MIMO enabled HetNets," *IEEE Commun. Lett.*, vol. 20, no. 10, pp. 2091–2094, Oct. 2016.
[18] M. Lauridsen, L. Noël, T. B. Sørensen, and P. Mogensen, "An empirical LTE smartphone power model with a view to energy efficiency evolution," *Intel Technol. J.*, vol. 61, no. 2, pp. 567–575, Apr. 2013.
[19] Y. Hao, Z. Song, S. Hou, and H. Li, "Energy- and spectral-efficiency tradeoff in massive MIMO systems with inter-user interference," in *Proc. IEEE PIMRC*, Aug./Sep. 2015, pp. 553–557.
[20] W. Liu, A. Zappone, C. Yang, and E. Jorswieck, "Global EE optimization of massive MIMO systems," in *Proc. IEEE SPAWC*, Jun. 2015, pp. 221–225.
[21] X. Zhang and H. Li, "Energy efficiency optimization for MIMO cognitive radio network," in *Proc. IEEE ICC*, Jun. 2015, pp. 7713–7718.
[22] X. Chen and C. Yuen, "Efficient resource allocation in a rateless-coded MU-MIMO cognitive radio network with QoS provisioning and limited feedback," *IEEE Trans. Veh. Technol.*, vol. 62, no. 1, pp. 395–399, Jan. 2013.
[23] A. Zappone, L. Sanguinetti, G. Bacci, E. Jorswieck, and M. Debbah, "Energy-efficient power control: A look at 5G wireless technologies," *IEEE Trans. Signal Process.*, vol. 64, no. 7, pp. 1668–1683, Apr. 2016.
[24] A. Zappone and E. Jorswieck, "Energy efficiency in wireless networks via fractional programming theory," *Found. Trends Commun. Inf. Theory*, vol. 11, nos. 3–4, pp. 185–396, Jun. 2015.
[25] C. Isheden, Z. Chong, E. Jorswieck, and G. Fettweis, "Framework for link-level energy efficiency optimization with informed transmitter," *IEEE Trans. Wireless Commun.*, vol. 11, no. 8, pp. 2946–2957, Aug. 2012.
[26] W. Dinkelbach, "On nonlinear fractional programming," *Manage. Sci.*, vol. 13, no. 7, pp. 492–498, Mar. 1967.
[27] B. R. Marks and G. P. Wright, "A general inner approximation algorithm for nonconvex mathematical programs," *Oper. Res.*, vol. 26, no. 4, pp. 681–683, Aug. 1978.
[28] A. Zappone, E. Björnson, L. Sanguinetti, and E. A. Jorswieck, "Achieving global optimality for energy efficiency maximization in wireless networks," *IEEE Trans. Signal Process.*, to be published.
[29] Y. Li, M. Sheng, X. Wang, Y. Zhang, and J. Wen, "Max–min energy-efficient power allocation in interference-limited wireless networks," *IEEE Trans. Veh. Technol.*, vol. 64, no. 9, pp. 4321–4326, Sep. 2015.
[30] H. H. Kha, H. D. Tuan, and H. H. Nguyen, "Fast global optimal power allocation in wireless networks by local D.C. programming," *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, pp. 510–515, Feb. 2012.
[31] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
[32] Y. Li, M. Sheng, C. Yang, and X. Wang, "Energy efficiency and spectral efficiency tradeoff in interference-limited wireless networks," *IEEE Commun. Lett.*, vol. 17, no. 10, pp. 1924–1927, Oct. 2013.
[33] M. Udell and S. Boyd, *Maximizing a Sum of Sigmoids*. May 2014, pp. 1–25. [Online]. Available: http://stanford.edu/~boyd/papers/max_sum_sigmoids.html
[34] R. Jain, D. Chiu, and W. Hawe, "A quantitative measure of fairness and discrimination for resource allocation in shared systems," Digit. Equip. Corp., Maynard, MA, USA, Tech. Rep. DEC-TR-301, Sep. 1984, pp. 1–37.
[35] D. P. Bertsekas and D. P. Bertsekas, *Nonlinear Programming*, 2nd ed. Belmont, MA, USA: Athena Scientific, Sep. 1999.
[36] M. Cui, B.-J. Hu, H. Chen, and X. Li, "Max-min fair power control algorithm in massive MIMO cognitive radio networks," in *Proc. IEEE WCSP*, Oct. 2016, pp. 1–5.

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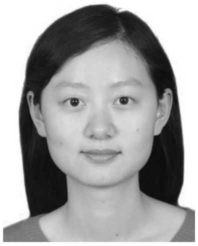

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