

Received October 14, 2016, accepted October 25, 2016, date of publication December 13, 2016, date of current version March 28, 2017.

Digital Object Identifier 10.1109/ACCESS.2016.2628910

Modeling and Analysis of Two-Tier HetNets With Cognitive Small Cells

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This work was supported in part by the Young Teacher Development Plan of Hunan University under Grant 2014-531107040810 and in part by the Nature Science Foundation of Hunan Province under Grant 14JJ4026.

ABSTRACT Heterogeneous networks (HetNets), which consist of traditional macro-cells overlaid with newly envisioned small cells (e.g., femtocells, picocells, microcells, and nanocells), are conceived as an appealing technology to satisfy the ever-increasing capacity requirements in future mobile networks. The cross-tier interference management is a challenging problem in conventional HetNets due to the large-scale deployment of small cells in random locations, and the lack of complete coordination. However, cognitive HetNets, where small-cell base stations are with cognitive capabilities (e.g., achieved through spectrum sensing), can efficiently overcome the posed challenge. In this paper, considering a two-tier cognitive HetNet, we utilize the statistic tool of stochastic geometry to model and analyze the coverage performance for macro-cell and small-cells over general Nakagami-m fading channels. Specifically, the exact closed-form expressions of outage probability for per-tier cell-edge users with and without cognitive interference coordination are derived, respectively. More attractively, the theoretically analytical results can be used to help to design the constraints on the configurations of small cells considering the minimum requirements of coverage performance for macro-cell and small-cell. Simulation results validate our analysis.

INDEX TERMS HetNet, small cells, stochastic geometry, cognitive interference management, coverage performance.

I. INTRODUCTION

Reliable and fast wireless data transmission is emerging as a global phenomenon and becoming a major consideration in our lives such as internet, online shopping, and social networking. According to the report provided by the International Telecommunication Union (ITU), the number of global mobile cellular subscribers is estimated to surpass 7.37 billions at the end of 2016, a number representing more than 98.6% of the world population. In the Americas, the impact of mobile cellular communications is even more significant with the current penetration rate of 111.8% [1]. Following this trend, it is predicted that there will be over 11.6 billion mobile-connected devices by the year 2020, nearly 1.5 mobile devices per capita. An important drive behind this is the worldwide rapid adoption of smartphones and mobile tablets. These devices are much more data-hungry: the data consumption of a single smartphone is equivalent to that generated by 37 featured phones; a mobile tablet can produce 2.5 times more traffic than the average smartphone. As a result, Cisco Systems predicted a staggering 53% compound annual growth rate (CAGR) for global mobile

cellular data traffic from 2015 to 2020: a 10-fold increase, and it will be grow to 30.6 exabytes per month by 2020 [2]. These statistics and predictions clearly suggest that future mobile cellular networks will face the challenge of supporting massive traffic volumes, and some experts predict that the next generation mobile cellular networks need a 1000x increase in the total data capacity demand. Conventional cellular network architectures based on large “macro” cells will be unable to meet the increased capacity needs.

Most recently, the Third Generation Partnership Project (3GPP) which is one of standards bodies about future mobile cellular network stands proposes a new network architecture: HetNet, which has been envisioned as a key solution of increasing network capacity by means of spatial reuse spectrum resources to accommodate the explosive growing mobile traffic [3]. In HetNets, there are containing network nodes with different characteristics such as transmission power and radio frequency (RF) coverage area. These network nodes can be divided into two main categories in general, one is the conventional macro-cell base station (MBS), and the other is the small-cell base

station (SBS) which is distributed among the coverage of the macro-cell. The term of small-cell base stations refers to low-power radio access nodes which have a transmission range of several tens to several hundreds of meters. The small-cells will offload users from the congested macro-cells to enhance their quality of service (QoS) and increase the overall network capacity. Small-cells include femtocells, picocells, microcells, and nanocells. A multi-tier network where small-cells are overlaid on macro-cells is generally referred to as a HetNet.

In HetNets, universal frequency reuses across coexisting network tiers is essential for spectral efficiency, and the network capacity will be highly increased [4], [5]. However, it may lead to occur serious inter-tier interference problem which will decrease the system capacity. To address this problem, cognitive radio (CR) technology has been integrated with SBSs [6]. CR-enabled SBSs can monitor and adapt to the surrounding communication environment, and the inter-tier interference is mitigated. The macro-cell system and the CR-enabled small-cell system correspond to the primary system (PUs) and the secondary system (SUs) in the CR model, respectively. There are three main cognitive interference management approaches in cognitive HetNets: power control based cognitive interference management, spectrum access based cognitive interference management, antenna schemes based cognitive interference management [7]. In this paper, we focus on spectrum access based and power based cognitive interference management approaches.

A. PREVIOUS WORKS

HetNets with cognitive small-cells (C-HetNets) has been drawing much research attention and become the research hotspot in recent years [7]–[19]. A comprehensive survey for the interference management in cognitive small-cells was presented in [7]. Interferences due to different interfering sources within cognitive small-cell networks were analyzed in [8], and then the authors introduced a stochastic dual control approach for dynamic sensing coordination with the aim of achieving efficient interference mitigation without involving global and centralized control efforts. Cheng *et al.* [9] focused on studying the role of different cognitive information on the transmission capacity, and their closed-form expressions were theoretically derived based on stochastic geometry. Elsayy and Hossain [10] investigated the user offloading and distributed channel access techniques in C-HetNets. Wildemeersch *et al.* [11] evaluated a distributed sleep-mode strategy for cognitive small-cell networks and analyzed the trade-off between traffic offloading from the macro-cell and the energy consumption of the small-cells. With the aim of maximizing the capacity of all small cells, the downlink spectrum sharing and power control problem is formulated as a mixed integer nonlinear programming problem in [12]. A energy-efficient user association algorithm in C-HetNets was proposed in [13]. The energy efficiency of C-HetNet was also investigated in [14], and an energy-efficient configured strategy for the density of

small-cells was also proposed. Zhang *et al.* in [15] considered a two-tier C-HetNet, and formulated the joint uplink sub-channel and power allocation problem as a Nash bargaining game-theoretic learning problem. Taking into account of fairness and imperfect spectrum sensing, a interference-limited resource optimization was proposed for C-HetNets in [16]. Wang *et al.* [17] derived the average uplink throughput of C-HetNets with fractional power control and adaption modulation, and evaluated the optimal channel access probability of each small-cell by maximizing the system throughput. Elsayy and Hossain [18] used stochastic geometry to model and analyze the downlink outage performance of C-HetNet in a multichannel environment. On the basis of [18], Panahi and Ohtsuki [19] utilized stochastic geometry to model and analyze the downlink outage performance in C-HetNets with considering the effect of spectrum sensing errors. However, both [18] and [19] only focused on the spectrum access based interference management and the small cells operation in the open access mode.

B. OUR WORK AND CONTRIBUTIONS

Although above mentioned prior works have greatly improved our understanding on HetNets with cognitive small-cells, the problem of modeling and analysis coverage performance of macro-cell and small-cell over Nakagami-m fading channels has not been addressed considering closed access mode and multiple types of cognitive interference management strategies in C-HetNets environment. Motivated by these key observations, in this paper, considering a two-tier HetNet, we first study the outage performance at the cell edge for any typical macro-cell and small-cell user over Nakagami-m fading channels without interference coordination. Then, taking into account small cells configured cognitive capability, we further analyze the outage performance of cell edge users for macro-cell and small-cell in C-HetNets with cognitive interference coordination. Finally, considering the minimum requirement of coverage performance for macro-cell and small-cell, the constraints on the small-cell layer configurations are designed in terms of the density of small cells. Our key contributions of this paper can be summarized as follows:

- 1) We utilize the stochastic geometry theory to model the outage probability at the cell edge for any typical macro-cell user as well as small-cell user in a two-tier HetNet environment where small-cells are configured with cognitive capabilities.
- 2) We comprehensively analyze the outage performance of the two-tier HetNet with cognitive small cells under different density of small cells, transmission power, signal to interference and noise ratio (SINR) threshold requirement, and cognitive interference coordination strategies.
- 3) We propose a efficient approach to design the constraints in terms of the density of small cells on the configuration of small cells in order to satisfy the minimum coverage performance requirement.

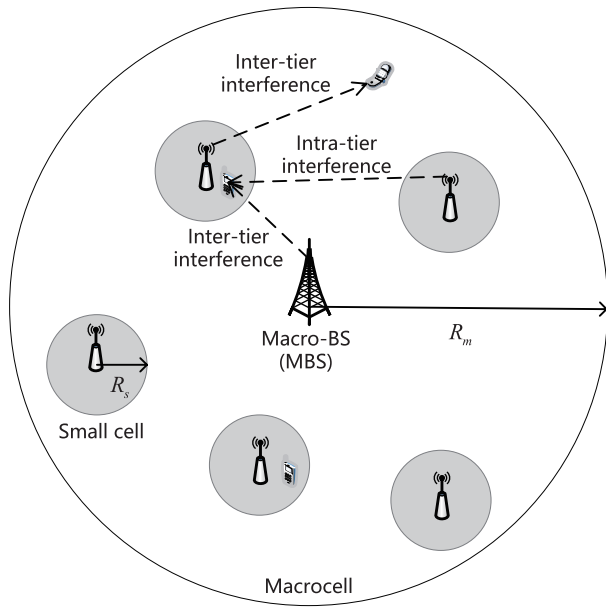


FIGURE 1. System model. A two-tier HetNets with cognitive small cells.

- 4) Both theoretical and simulation results show that the coverage performance of HetNets with opportunistic access based cognitive interference coordination is better than that with distance separation based cognitive interference coordination.

C. ORGANIZATION AND NOTATIONS

The remainder of this paper is organized as follows: Section II describes the system model of the two-tier HetNet with cognitive small cells. Section III analyzes the outage probability at cell edge for any typical macro-cell user and small-cell user with and without cognitive interference coordination. Based on the result of Section III, Section IV proposes an efficient configuration strategy for the density of small cells to satisfy the minimum coverage performance requirement. Numerical results are presented in Section V, and our conclusions are presented in Section VI.

Notation: C_K^j represents the binomial coefficient, $K!$ represents the factorial of K , and \mathbb{E} stands for the expectation. $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$, $\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1} e^{-t} dt$ and $\gamma(\alpha, x) = \int_0^x t^{\alpha-1} e^{-t} dt$ denote the gamma function [20, eq. (8.310.1)], the upper incomplete gamma function [20, eq. (8.350.2)] and the lower incomplete gamma function [20, eq. (8.350.1)], respectively. The cumulative distribution function (CDF) and the probability density function (PDF) of random variable X are expressed as $F_X(\cdot)$ and $f_X(\cdot)$, respectively.

II. SYSTEM MODEL

A. NETWORK MODEL

We consider a two-tier HetNet consisting of one macro-cell base station (MBS) and several small-cell base stations (SBSs), as illustrated in Fig. 1. Both macro-cell and small-cell are operation with orthogonal frequency division

multiple access (OFDMA). The co-channel deployment is adopted in the two-tier HetNet, which means two network tiers operate in the same spectrum simultaneously and will introduce the inter-tier inference between macro-network tier and small-network tier. The closed-access policy is also assumed, which indicates that any unregistered mobile users cannot access to small-cell networks even if they are more closed to small-cells.

The macro-cell consists of one MBS and multiple macro-user (MU). It is assumed that the serving area of macro-cell is a circle with radius R_m , and the transmit power of MBS is P_m . The macro-cell is overlaid with small-cells with radius R_s , which are spatially randomly distributed on \mathbb{R}^2 according to a homogeneous poisson point process (HPPP) $\Psi = \{l_j; j = 0, 1, 2, 3, \dots\}$ with intensity λ_s , where l_i denotes the location of the i th small-cell. One SBS and several small-cell users (SU) are composed of a small-cell, and all SBSs transmit signals with the same power P_s . The SBSs are configured with cognitive capability, thus the cognitive interference coordination can be used to mitigate the cross-tier interference. All nodes are equipped with a single antenna and operate in half-duplex mode. Assume that the radius of macro-cell is much bigger than that of small-cell, thus the distance between MBS and SU can be approximated the distance between MBS and serving SBS of SU. It is noted that we concentrate on the case where the intra-macro-cell interference is dominant in this paper, and the inter-macro-cell interference is not considered. The effects of inter-macro-cell interference will be explored in future research.

B. CHANNEL MODEL

We consider a general power-law path loss model where the signal power decays at the rate $r^{-\eta}$ with the distance r , where η is the path-loss exponent. It is assumed that both the macro-cell and small-cell tiers have the same path-loss exponent, which is denoted by α (the value is typically in the range of 2 to 4 [21]). Besides the path loss model, the flat Nakagami-m fast fading model is also considered. The channel gain between the MBS and the macro-cell edge user (E-MU), and between the SBS and the small-cell edge user (E-SU) are denoted as g_m , and g_s , whose fading severity parameters are m_1 , and m_2 , respectively. Consequently, the channel power gain between the MBS and E-MU, the SBS and E-SU are given by $g_m R_m^{-\alpha}$ and $g_s R_s^{-\alpha}$, respectively. Without any loss in generality, the background noise at all receivers are modeled as an additive white Gaussian noises (AWGN) with variance σ^2 .

C. COGNITIVE INTERFERENCE COORDINATION STRATEGIES

In this paper, we consider two main cognitive interference coordination strategies as follows:

1) OPPORTUNISTIC ACCESS BASED COGNITIVE INTERFERENCE COORDINATION (OP-BASED ICIC)

The SBS acting as a secondary user utilizes spectrum sensing to passively seek temporary frequency voids of resource

blocks (RBs) for opportunistic access. Consequently, the macro-cell network and the small-cell network are operating on orthogonal RBs, and no cross-tier interference is introduced.

2) DISTANCE SEPARATION BASED COGNITIVE INTERFERENCE COORDINATION (DS-BASED ICIC)

The SBS is able to leverage location information of MU from scheduling in macro-cell network, and it can determine the set of RBs that can be reused without causing serious cross-tier interference in its vicinity through introducing the spatial avoidance region.

III. OUTAGE PERFORMANCE ANALYSIS UNDER DIFFERENT COGNITIVE INTERFERENCE COORDINATION

The inter-tier interference seriously declines the performance of cell edge users in HetNets, and it is meaningful to exactly evaluate the outage performance of cell edge users for enhancing the performance of wireless networks. In this section, we use the statistic tool of stochastic geometry to analyze the outage performance of macro-cell user and small-cell user at cell edge under different cognitive interference coordination strategies. For convenience, some notations used in this paper are listed in Table I.

TABLE 1. Parameters notations used throughout the paper.

P_m, P_s	Transmitting power of MBS and SBS
R_m, R_s	Radius of macro-cell and small-cell
Γ_m, Γ_s	SINR threshold of macro-cell and small-cell user
γ_m, γ_s	The received SINR of macro-cell and small-cell user
P_{out}^m, P_{out}^s	Outage probability of macro-cell and small-cell user
Ψ	The set of small cells
α	Path loss exponent
σ^2	Power of background noise
d	Radius of avoid region

A. OUTAGE PERFORMANCE WITH OP-BASED ICIC

Under the case of opportunistic access based cognitive interference coordination, the macro-cell network and the small-cell network are operating on orthogonal RBs, and the downlink transmission of macro-users will not be interfered by SBSs. In other words, there is no cross-tier interference is introduced in the HetNet. Hence, the received SINR of a macro-user at cell edge can be expressed as

$$\gamma_m = \frac{P_m R_m^{-\alpha} g_m}{\sigma^2} \tag{1}$$

where g_m is the channel gain between MBS and E-MU, and it follows Gamma distribution with parameter m_1 .

According to the definition of outage, an outage event occurs while the received SINR below the target SINR Γ_m .

Therefore, the outage probability of E-MU is given by

$$P_{out}^m = \Pr \left(\frac{P_m R_m^{-\alpha} g_m}{\sigma^2} < \Gamma_m \right) = \frac{\gamma \left(m_1, \frac{m_1 \Gamma_m \sigma^2 R_m^\alpha}{P_m} \right)}{\Gamma(m_1)} \tag{2}$$

where the last equality follows from the fact that random variable (RV) g_m follows Gamma distribution with parameter m_1 . From the expression (2), it is observed that the parameters (e.g. the transmit power and the density of small-cells) do not affect the outage performance of macro-cell under OP-based ICIC.

The downlink transmission of small-cell users is also not interfered by MBS due to there is not existing cross-tier interference. However, the intra-interference among those small-cells will occur because that they using the same spectrum simultaneously and there is not interference coordination among them. Hence, the received SINR of a small-cell user at small-cell edge can be written as

$$\gamma_s = \frac{P_s R_s^{-\alpha} g_s}{\sigma^2 + I_{s,s}} \tag{3}$$

where $I_{s,s} = \sum_{l_j \in \Psi \setminus \{l_0\}} P_s d_{j,s}^{-\alpha} g_{j,s}$ denotes the intra-interference from surrounding SBSs. $g_{j,s}$ and $d_{j,s}$ are the channel gain and the distance between the SBS at l_j and a typical SBS at the origin, respectively. The fast fading gain $g_{j,s}$ follows Gamma distribution with parameters m_3 . As a result, for the given target SINR Γ_s , the outage probability of the E-SU is given by

$$P_{out}^s = \Pr \left(\frac{P_s R_s^{-\alpha} g_s}{\sigma^2 + I_{s,s}} < \Gamma_s \right) = \Pr \left(g_s < \frac{\Gamma_s R_s^\alpha (\sigma^2 + I_{s,s})}{P_s} \right) = \mathbb{E}_{I_{s,s}} \left[\frac{\gamma \left(m_2, \frac{m_2 \Gamma_s R_s^\alpha (\sigma^2 + I_{s,s})}{P_s} \right)}{\Gamma(m_2)} \right] \tag{4}$$

where the last equality follows from the fact that RV g_s follows Gamma distribution with parameter m_2 , and \mathbb{E} denotes calculating the expectation.

Theorem 1: The outage probability of the small-cell user at cell boundary under the case of opportunistic access based cognitive interference coordination is given by

$$P_{out}^s = 1 - e^{-s\sigma^2} \sum_{i=0}^{m_2-1} \sum_{j=0}^i \left[\frac{C_i^j (\sigma^2)^{-i-j} s^i (-1)^j d^j \exp \left(M_\alpha s^{\frac{2}{\alpha}} \right)}{i! d^j} \right] \tag{5}$$

where $s = \frac{m_2 \Gamma_s R_s^\alpha}{P_s}$, $M_\alpha = \frac{-\pi \lambda_s \Gamma \left(1 - \frac{2}{\alpha} \right) \Gamma \left(\frac{2}{\alpha} + m_3 \right) (P_s)^{\frac{2}{\alpha}}}{(m_3)^{\frac{2}{\alpha}} \Gamma(m_3)}$, and $\frac{d^n F(s)}{ds^n}$ denotes the n th order derivative of function $F(s)$.

Proof: See Appendix A. ■

B. OUTAGE PERFORMANCE WITHOUT INTERFERENCE COORDINATION

While there is not interference coordination in the HetNet, the macro-cell network and the small-cell network are operating on the same spectrum simultaneously. Although it can improve the spectrum utilization efficiency, it will introduce serious cross-tier interference. Taking into account the downlink transmission for a E-MU, the received SINR is given by

$$\gamma_m = \frac{P_m R_m^{-\alpha} g_m}{\sigma^2 + I_{s,m}} \tag{6}$$

where $I_{s,m} = \sum_{l_j \in \Psi} P_s d_{j,m}^{-\alpha} g_{j,m}$ is the cross-tier interference caused by surrounding SBSs. $g_{j,m}$ and $d_{j,m}$ are the channel gain and the distance between the SBS at l_j and MBS, respectively. The fast fading gain $g_{j,m}$ follows Gamma distribution with parameters m_4 .

Similarly with (2), for a given target SINR threshold Γ_m , the outage probability can be calculated as

$$\begin{aligned} P_{out}^m &= \Pr\left(\frac{P_m R_m^{-\alpha} g_m}{\sigma^2 + I_{s,m}} \leq \Gamma_m\right) \\ &= \Pr\left(g_m \leq \frac{\Gamma_m R_m^\alpha (\sigma^2 + I_{s,m})}{P_m}\right) \\ &= \mathbb{E}_{I_{s,m}} \left[\frac{\gamma\left(m_1, \frac{m_1 \Gamma_m R_m^\alpha (\sigma^2 + I_{s,m})}{P_m}\right)}{\Gamma(m_1)} \right] \end{aligned} \tag{7}$$

Theorem 2: The outage probability of the macro-cell user at the cell boundary without interference coordination is given by

$$P_{out}^m = 1 - e^{-s\sigma^2} \sum_{i=0}^{m_1-1} \sum_{j=0}^i \left[\frac{C_i^j (\sigma^2)^{i-j} s^i (-1)^j d^j \exp(\hat{M}_\alpha s^{\frac{2}{\alpha}})}{i!} \frac{d^j}{ds^j} \right] \tag{8}$$

where $s = \frac{m_1 \Gamma_m R_m^\alpha}{P_m}$ and $\hat{M}_\alpha = \frac{-\pi \lambda_s \Gamma(1-\frac{2}{\alpha}) \Gamma(\frac{2}{\alpha} + m_4) (P_m)^{\frac{2}{\alpha}}}{(m_4)^{\frac{2}{\alpha}} \Gamma(m_4)}$.

Proof: With the similar method utilized in Appendix I, and with the replacement of $m_2 \rightarrow m_1, m_3 \rightarrow m_4, \Gamma_s \rightarrow \Gamma_m, R_s \rightarrow R_m$ and $P_s \rightarrow P_m$, then the expression (8) can be got. ■

For small-cell users, its downlink transmission is also interfered by the cross-tier interference from MBS excepting for the intra-tier interference from other surrounding SBSs. Therefore, the received SINR at E-SU can be written as

$$\gamma_s = \frac{P_s R_s^{-\alpha} g_s}{\sigma^2 + I_{m,s} + I_{s,s}} \tag{9}$$

where $I_{m,s} = P_m d_{m,s}^{-\alpha} g_{m,s}$ denotes the inter-tier interference from MBS, $g_{m,s}$ and $d_{m,s}$ are the channel gain and the distance between the MBS and E-SU, respectively. The fast fading gain $g_{m,s}$ follows Gamma distribution with parameters m_5 .

Hence, the outage probability of E-SU due to the SINR falling below the reception threshold Γ_s is given by

$$\begin{aligned} P_{out}^s &= \Pr\left(\frac{P_s R_s^{-\alpha} g_s}{\sigma^2 + I_{m,s} + I_{s,s}} \leq \Gamma_s\right) \\ &= \Pr\left(g_s \leq \frac{\Gamma_s R_s^\alpha (\sigma^2 + I_{m,s} + I_{s,s})}{P_s}\right) \\ &= \mathbb{E}_{I_{m,s}, I_{s,s}} \left[\frac{\gamma\left(m_2, \frac{m_2 \Gamma_s R_s^\alpha (\sigma^2 + I_{m,s} + I_{s,s})}{P_s}\right)}{\Gamma(m_2)} \right] \end{aligned} \tag{10}$$

In order to get the exact closed-form expression of P_{out}^s , we firstly derive the Laplace transform of inter-tier interference $I_{m,s}$. Considering that the radius of macro-cell is much bigger than that of small-cell, thus the distance between MBS and E-SU can be approximated as the distance between MBS and the corresponding serving SBS of E-SU. Furthermore, SBSs are randomly distributed in the area according to a HPPP. Therefore, the PDF of the distance between MBS and E-SU can be approximated as

$$f_{d_{m,s}}(r) = \frac{2r}{R_m^2} \tag{11}$$

Then, the Laplace transform of $I_{m,s}$ can be calculated as (12), as shown at the bottom of the next page, where $a = \frac{m_5 R_m^\alpha}{s P_m}, b = \frac{2}{\alpha} + m_5 - 1$, and $F_1(\cdot; \cdot; \cdot)$ denotes the Gaussian hyper-geometric function.

Theorem 3: The outage probability of the small-cell user at the cell boundary without interference coordination is given by

$$\begin{aligned} P_{out}^s &= 1 - e^{-s\sigma^2} \sum_{i=0}^{m_2-1} \sum_{j=0}^i \left[\frac{C_i^j (\sigma^2)^{i-j} s^i (-1)^j d^j (L_{I_{m,s}}(s) L_{I_{s,s}}(s))}{i!} \frac{d^j}{ds^j} \right] \end{aligned} \tag{13}$$

where $s = \frac{m_1 \Gamma_s R_s^\alpha}{P_s}$ and $M_\alpha = \frac{-\pi \lambda_s \Gamma(1-\frac{2}{\alpha}) \Gamma(\frac{2}{\alpha} + m_4) (P_s)^{\frac{2}{\alpha}}}{(m_4)^{\frac{2}{\alpha}} \Gamma(m_4)}$.

Proof: According the characteristics of Laplace transform, we have

$$L_{I_{m,s} + I_{s,s}}(s) = L_{I_{m,s}}(s) L_{I_{s,s}}(s) \tag{14}$$

Then, replacing the parameter $I_{s,s}$ with the parameter $I_{m,s} + I_{s,s}$ in (5), and using the expression (14), the result can be obtained. ■

C. OUTAGE PERFORMANCE WITH DS-BASED ICIC

As a result of the closed-access strategy, the cross-tier interference is serious for without interference coordination especially when macro-cell users are close to SBSs. One of available solutions is introducing the avoidance region around the macro-cell user, which means that within this region the SBSs cannot reuse the spectrum originally allocated to macro-cell users.

Under this case, the received SINR at E-MU is given by

$$\gamma_m = \frac{P_m R_m^{-\alpha} g_m}{\sigma^2 + I'_{s,m}} \quad (15)$$

where $I'_{s,m} = \sum_{l_j \in \Psi \setminus B(0,d)} P_s d_{j,m}^{-\alpha} g_{j,m}$ denotes the inter-tier interference from SBSs. $B(0, d)$ denotes the avoidance region which is a circle with radius d .

Theorem 4: The outage probability of the macro-cell user at the cell boundary under the case of distance separation based cognitive interference coordination is given by

$$P_{out}^m = 1 - e^{-s\sigma^2} \sum_{i=0}^{m_2-1} \sum_{j=0}^i \left[\frac{C_i^j (\sigma^2)^{i-j} s^i (-1)^j d^j (\exp(-T(\alpha, s)))}{i! d^j} \right] \quad (16)$$

where $s = \frac{m_1 \Gamma_m P_m^\alpha}{P_m}$, and

$$T(\alpha, s) = \pi \lambda_s d^2 \left[1 - \left(\frac{m_4}{m_4 + \frac{s P_s}{d^\alpha}} \right)^{m_4} F_1 \left(1, m_4; 1 - \frac{2}{\alpha}; \frac{s P_s}{s P_s + m_4 d^\alpha} \right) \right] \quad (17)$$

Proof: See Appendix B. ■

For the E-SU, the outage probability under the case of distance separation based cognitive interference coordination is the same with that under the case of without interference coordination.

IV. CONSTRAINTS ON THE CONFIGURATIONS OF SMALL CELLS

With the theoretically derived expressions of outage probability for macro-cell and small-cell user at cell edge, we can suggest on the system design without exhaustive simulations. In this section, the constraints on the configurations of density of small cells are investigated.

To guarantee the coverage performance of HetNets, there is an outage constraint at macro-cell and small-cell edge user with maximum outage probability. Assume the tolerable maximum outage probability of macro-cell and small-cell users at cell edge are ε_m and ε_s . As can be observed from the derived expression of outage probability (e.g. (5), (8), (13), (16)), the outage probability is a function of λ_s . Hence,

we can calculate $P_{out}^m(\lambda_s)$ and $P_{out}^s(\lambda_s)$ until $P_{out}^m(\lambda_s) \leq \varepsilon_m$ and $P_{out}^s(\lambda_s) \leq \varepsilon_s$ are satisfied. The process of find the proper λ_s is given below.

-Step 1: Assign an initial value to λ_s .

-Step 2: Calculate the outage probability $P_{out}^m(\lambda_s)$ and $P_{out}^s(\lambda_s)$ with derived expressions.

-Step 3: If $P_{out}^m(\lambda_s) \leq \varepsilon_m$ and $P_{out}^s(\lambda_s) \leq \varepsilon_s$, stop calculation, and record the current value of λ_s . Else set $\lambda_s = \lambda_s \rho_\lambda$ and return to Step 2, where $\rho_\lambda < 1$ denotes the step coefficient of λ_s .

With the above mentioned algorithm, we can determine the configuration of maximum density of small cells under different cognitive interference coordination for satisfying the requirement of coverage performance.

V. NUMERICAL RESULTS

In this section, considering a typical type of small cells: femtocell, numerical results are presented to evaluate theoretical analysis by Monte-Carlo simulations. Meanwhile, we discuss the impact of related system parameters on the coverage performance of macro-cell and small-cell under different cognitive interference coordination strategies. The simulation parameters are given in Table II.

TABLE 2. Simulation parameters.

	Macro-cell	Femto-cell
Transmit power	$P_m = 43dBm$,	$P_s = 20dBm$
Radius of cell	$R_m = 300m$,	$R_s = 15m$
SINR Threshold	$\Gamma_m = 1$,	$\Gamma_s = 1$
Path loss exponent	$\alpha = 4$	
Power of noise	$\sigma^2 = -110dBm$	
Radius of avoidance region	$d = 50, 100, 150m$	

The coverage performance of HetNet without interference coordination is illustrated in Fig. 2. Specifically, Fig. 2 plots the outage probability of macro-cell and femtocell (i.e. small-cell) user at cell edge with different density of femtocells. The analytical curves for outage probability are obtained from (8) and (13), which are matched well to the Monte Carlo simulations. From this figure, we can observe that the coverage performance of macro-cell and femtocell decreases with the increasing of density of femtocells. The reason is that

$$L_{I_{m,s}}(s) = E_{I_{m,s}} [e^{-sI_{m,s}}] = E_{g_{m,s}, d_{m,s}} [e^{-sP_m g_{m,s} d_{m,s}^{-\alpha}}] = \int_0^{R_m} \int_0^\infty e^{-sP_m xy^{-\alpha}} \frac{(m_5)^{m_5} x^{m_5-1} e^{-m_5 x}}{\Gamma(m_5)} \frac{2y}{R_m^2} dx dy$$

$$= \frac{2}{\alpha R_m^2} \left(\frac{s P_m}{m_5} \right)^{\frac{2}{\alpha}} \left\{ \prod_{i=1}^{m_5-1} \left[\frac{(m_5-i)(b+1-i) a^{b-m_5+2}}{(b-m_5+2)(a+1)} F_1 \left(1, 1; b-m_5+3; \frac{a}{a+1} \right) \right] - \sum_{i=1}^{m_5-2} \left[\frac{a^{b-i} \prod_{j=0}^i (m_5-1-i) \prod_{l=0}^{i-1} (b-l)}{(1+a)^{m-1-i}} \right] \right\} \quad (12)$$

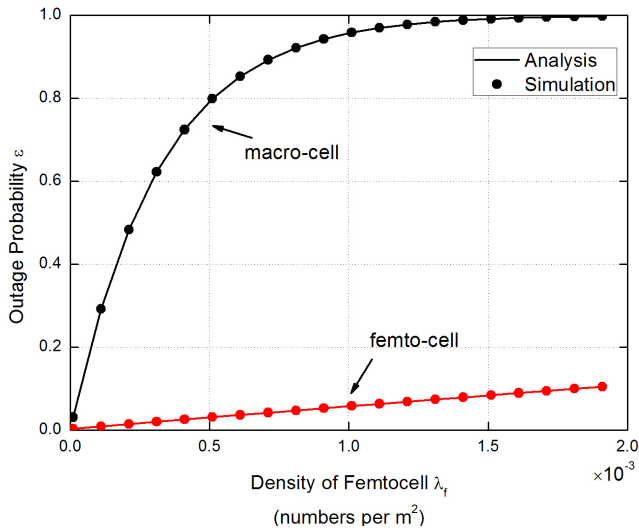


FIGURE 2. Relationship between outage performance and the density of femtocells without interference coordination.

both inter-tier interference and intra-tier interference increase with the increasing of the density of femtocells. From Fig. 2 we can also find that the effect of the density of femtocells on the coverage performance of macro-cell system is bigger than that of femtocell system.

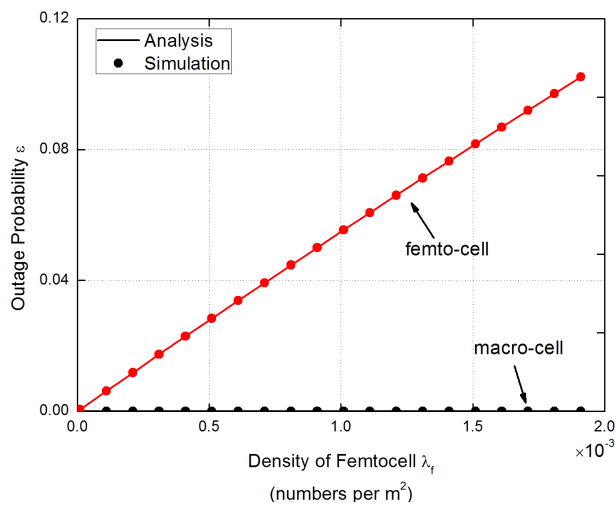


FIGURE 3. Relationship between outage performance and the density of femtocells with opportunistic access based cognitive interference coordination.

Fig. 3 shows the coverage performance of macro-cell and femtocell under the case of opportunistic access based cognitive interference coordination. In this figure, the curves marked with 'macro-cell' and 'femto-cell' stand for the outage probability for macro-cell and femtocell, respectively. From the figure, it is clearly observed that the coverage performance of macro-cell keep unchanged with the increase of λ_s , which is different from that under the case of without interference coordination. The reason is that there is not inter-tier interference, but the intra-interference among femtocells is still existing.

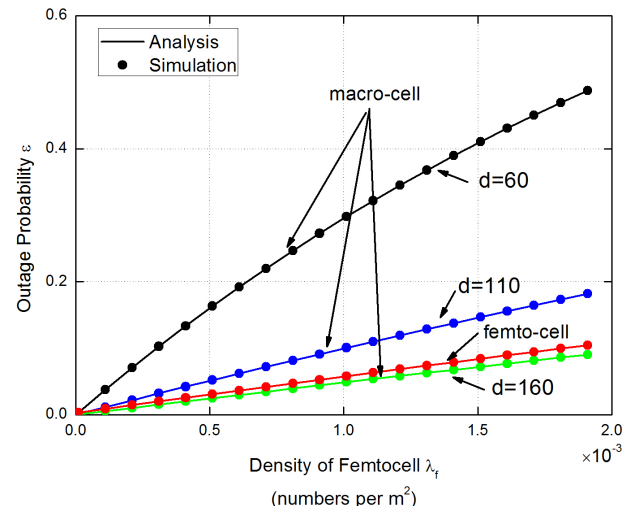


FIGURE 4. Relationship between outage performance and the density of femtocells with distance separation based cognitive interference coordination.

Fig. 4 illustrated the relationship between the coverage performance and the density of femtocells with different radius of avoidance region. From Fig. 4, it can be found that the coverage performance of marco-cell system improves with the d , but it does not affect the coverage performance of femto-cell system. It is because that the distance separation based cognitive interference coordination can reduce the inter-tier interference but not for intra-tier interference.

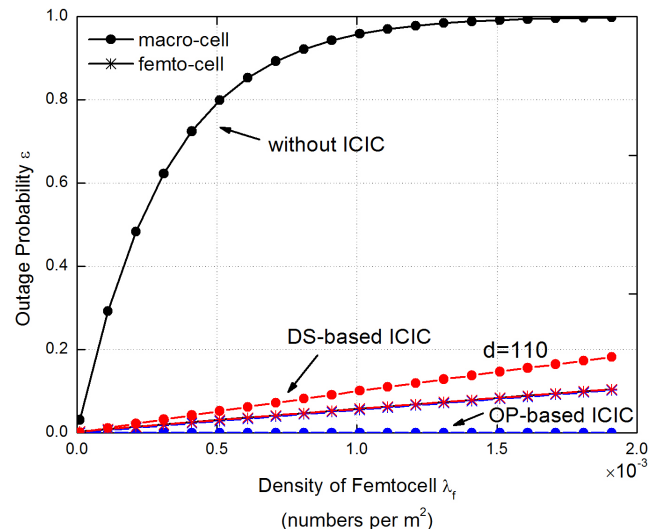


FIGURE 5. Relationship between outage performance and the density of femtocells with different cognitive interference coordination strategies.

The comparison of the three cognitive interference coordination strategies is depicted in Fig. 5. For a certain density λ_s of femtocells, the coverage performance is different for three cognitive interference coordination strategies. It can be seen that the coverage performance of OP-based ICIC is the best, and that of the case without interference coordination is the worst. When the avoidance distance is large enough, the coverage performance of DS-based ICIC is very close to

that of OP-based ICIC, thus the distance separation based cognitive interference coordination strategy is recommended to be used.

VI. CONCLUSION

This paper analyzes and compares the coverage performance of a two-tier HetNet with cognitive small cells for different cognitive interference coordination strategies (e.g. opportunistic access based and distance separation based cognitive interference coordination). The relationship between coverage performance and density of small-cells for the two-tier network is evaluated. Furthermore, an algorithm is proposed to design the constraints on the configurations of small cells considering the minimum requirements of coverage performance for macro-cell and small-cell. The theoretical analysis is validated by simulations. The results show that the coverage performance of the two-tier networks with opportunistic access based cognitive interference coordination is better than that with distance separation based cognitive interference coordination. This work provides essential understanding for successful deployment of cognitive heterogeneous networks.

**APPENDIX A
PROOF OF LEMMA 1**

We have following expansion for an lower incomplete gamma function [20, eq. (8.352.6)]

$$\gamma(n, x) = \Gamma(n) \left(1 - e^{-x} \sum_{i=0}^{n-1} \frac{x^i}{i!} \right) \quad (18)$$

Hence, substituting (18) into (4), and replacing $\frac{m_2 \Gamma_s R_s^\alpha}{P_s}$ with s , then the outage probability P_{out}^s can be transformed into

$$\begin{aligned} P_{out}^s &= \mathbb{E}_{I_{s,s}} \left[1 - e^{-s(\sigma^2 + I_{s,s})} \sum_{i=0}^{m_2-1} \frac{(s(\sigma^2 + I_{s,s}))^i}{i!} \right] \\ &= 1 - e^{-s\sigma^2} \sum_{i=0}^{m_2-1} \sum_{j=0}^i \left\{ \frac{C_i^j (\sigma^2)^{i-j} s^i}{i!} E_{I_{s,s}} \left[(I_{s,s})^j e^{-sI_{s,s}} \right] \right\} \\ &= 1 - e^{-s\sigma^2} \sum_{i=0}^{m_2-1} \sum_{j=0}^i \left[\frac{C_i^j (\sigma^2)^{i-j} s^i}{i!} L_{f_{I_{s,s}}(x)}(I_{s,s})^j(s) \right] \end{aligned} \quad (19)$$

where $s = \frac{m_2 \Gamma_s R_s^\alpha}{P_s}$, and $f_{I_{s,s}}(x)$ is the PDF of $I_{s,s}$.

From (19), it is observed that we have to calculate the Laplace transform of $f_{I_{s,s}}(x) (I_{s,s})^j$. Fortunately, it is known that the Laplace transform of $f_{I_{s,s}}(x) (I_{s,s})^j$ is $(-1)^n \frac{d^n L_{I_{s,s}}(s)}{ds^n}$ [22]. Hence, utilizing this result, the expression of P_{out}^s can be transformed into

$$P_{out}^s = 1 - e^{-s\sigma^2} \sum_{i=0}^{m_2-1} \sum_{j=0}^i \left[\frac{C_i^j (\sigma^2)^{i-j} s^i}{i!} (-1)^j \frac{d^j L_{I_{s,s}}(s)}{ds^j} \right] \quad (20)$$

where $s = \frac{m_2 \Gamma_s R_s^\alpha}{P_s}$.

The Laplace transform of the intra-tier interference $I_{s,s}$ is calculated as

$$\begin{aligned} L_{I_{s,s}}(s) &= \mathbb{E}_{I_{s,s}} [\exp(-sI_{s,s})] \\ &= \mathbb{E}_{\Psi, g_{j,s}} [\exp(-s \sum_{l_j \in \Psi \setminus \{l_0\}} P_s g_{j,s} d_{j,s}^{-\alpha})] \\ &= \mathbb{E}_{\Psi} \left[\prod_{l_j \in \Psi \setminus \{l_0\}} \mathbb{E}_{g_{j,s}} [\exp(-s P_s g_{j,s} d_{j,s}^{-\alpha})] \right] \\ &= \exp \left\{ -\mathbb{E}_{g_{j,s}} \left[2\pi \lambda \int_0^\infty (1 - \exp(-s P_s g_{j,s} r^{-\alpha})) r dr \right] \right\} \end{aligned} \quad (21)$$

where the last equality follows from the definition of the probability generating function (PGFL) of HPPP [23].

Then, using the result obtained in [24] and with some simplification, $L_{I_{s,s}}(s)$ is obtained as follows

$$L_{I_{s,s}}(s) = \exp \left(\frac{-\pi \lambda \Gamma \left(1 - \frac{2}{\alpha} \right) \Gamma \left(\frac{2}{\alpha} + m_3 \right) (P_s)^{\frac{2}{\alpha}}}{(m_3)^{\frac{2}{\alpha}} \Gamma(m_3)} s^{\frac{2}{\alpha}} \right) \quad (22)$$

Finally, substituting (22) into (20), the exact closed-form expression of outage probability for E-SU can be obtained.

**APPENDIX B
PROOF OF LEMMA 2**

With the similar method utilized in Appendix A, the outage probability of E-MU can be expressed as

$$P_{out}^m = 1 - e^{-s\sigma^2} \sum_{i=0}^{m_2-1} \sum_{j=0}^i \left[\frac{C_i^j (\sigma^2)^{i-j} s^i (-1)^j d^j (L'_{I'_{s,m}}(s))}{i! ds^j} \right] \quad (23)$$

where $s = \frac{m_1 \Gamma_m R_m^\alpha}{P_m}$.

The Laplace transform of $I'_{s,m}$ can be evaluated as

$$\begin{aligned} L'_{I'_{s,m}}(s) &= \mathbb{E}_{\Psi, g_{j,m}} [\exp(-s \sum_{l_j \in \Psi \setminus B(0,d)} P_s d_{j,m}^{-\alpha} g_{j,m})] \\ &= \exp \left(-\mathbb{E}_{g_{j,m}} \left[2\pi \lambda \int_d^\infty (1 - \exp(-s P_s g_{j,m} r^{-\alpha})) r dr \right] \right) \\ &= \exp \left(-2\pi \lambda \left(\frac{1}{2} d^2 + \frac{(s P_s)^{\frac{2}{\alpha}}}{\alpha} \right) \right) \\ &\quad \times \underbrace{\mathbb{E}_{g_{j,m}} \left[(g_{j,m})^{\frac{2}{\alpha}} \gamma \left(-\frac{2}{\alpha}, \frac{s P_s g_{j,m}}{d^\alpha} \right) \right]}_A \end{aligned} \quad (24)$$

Taking account of the fading gain $g_{j,m}$ following Gamma distribution for Nakagami-m fading channel and with the

expansion expression (42) in appendix A, the expectation part A in (48) can be calculated as follows

$$\begin{aligned}
 A &= \frac{(m_4)^{m_4}}{\Gamma(m_4)} \int_0^\infty t^{\frac{2}{\alpha} + m_4 - 1} e^{-m_4 t} \gamma\left(-\frac{2}{\alpha}, \frac{sP_s}{d^\alpha} t\right) dt \\
 &= -\frac{\alpha}{2} \left(\frac{m_4}{m_4 + \frac{sP_s}{d^\alpha}}\right)^{m_4} \left(\frac{sP_s}{d^\alpha}\right)^{-\frac{2}{\alpha}} \\
 &\quad \times F_1\left(1, m_4; 1 - \frac{2}{\alpha}; \frac{sP_s}{sP_s + m_4 d^\alpha}\right) \quad (25)
 \end{aligned}$$

where $F_1(\cdot, \cdot; \cdot; \cdot)$ is the Gaussian hyper-geometric function.

Finally, substituting (25) and (24) into (23), the result can be obtained.

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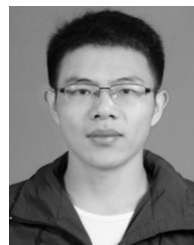
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