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# Performance Analysis for Downlink Relaying Aided Non-Orthogonal Multiple Access Networks With Imperfect CSI Over Nakagami-*m* Fading

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**ABSTRACT** Non-orthogonal multiple access (NOMA) has been conceived as a breakthrough technology for the fifth generation (5G) wireless networks. With imperfect channel state information (ICSI) taken into account, we study an NOMA-based downlink amplify-and-forward (AF) relaying network under Nakagami-*m* fading in this paper. First, we investigate the system outage behavior, and close-form expressions for the exact and tight lower bounds of the outage probability are attained, respectively. By further evaluating the outage probability at the high SNR region, it is observed that an error floor exists in the outage probability due to the presence of ICSI. Finally, numerical results are presented to demonstrate the validity of our analysis and show the advantages of NOMA over conventional orthogonal multiple access. Moreover, simulation results verify that the optimal relay location for NOMA should be close to the source node.

**INDEX TERMS** Relaying networks, non-orthogonal multiple access (NOMA), outage probability, Nakagami-*m* fading, imperfect channel state information (ICSI).

## I. INTRODUCTION

Radio access is the key technology for wireless communications [1]. With the aim to deal with high traffic volume and optimize spectral efficiency, a novel access technology, non-orthogonal multiple access (NOMA), is widely considered to be a candidate multiple access for the fifth generation (5G) [2], [3]. At the transmitter, NOMA superposes the signals of multiple users by splitting them in the power domain. At the receiver side, successive interference cancellation (SIC) is utilized to separate multiplexed users' signals [4]. In addition to spectrum utilization, user fairness is also a key feature of NOMA. Unlike conventional water-filling power allocation, NOMA allocates less power to users with better channel conditions and more power to users with worse channel conditions, in order to realize an improved trade-off between system throughput and user fairness. As a result, multiple users are served at the same time, frequency and spreading codes but different power level, which leads to an increase in spectral efficiency and user fairness is guaranteed as well. The author in [5] studied the ergodic capacity maximization problem for multiple-input-multiple-output (MIMO) NOMA networks and an optimal power allocation scheme was then proposed. In [6], the outage behavior and the ergodic sum rate for a cellular downlink NOMA networks was analyzed.

In order to improve the system capacity and enhance the reliability of NOMA, cooperative NOMA (C-NOMA) networks have attracted some attentions [7]–[10]. The author in [7] studied the achievable average rate for C-NOMA networks. In [8], NOMA was combined with multiple-antenna cooperative network to enhance the system performance, and the system outage behavior was then analyzed. In [9], NOMA was applied into coordinated directed and relay transmission to improve the spectral efficiency. In [10], a novel detection scheme for the C-NOMA network was proposed and a suboptimal and practical power allocation strategy was also provided.

Most of the previous literatures about C-NOMA are analyzed over Rayleigh fading with the assumption that perfect channel state information (CSI) is known. However, by reason of the channel estimation errors, this assumption is too idealistic in practice. Moreover, Nakagami-*m* distribution models empirical data better than Rayleigh distribution, and it is useful for a wider class of fading environments,


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which comprises the Rayleigh fading ($m = 1$) as a special case [11].

Motivated by these, in this paper, we consider a downlink C-NOMA network with imperfect CSI (ICSI) under Nakagami-$m$ fading channels. The main contributions of this paper are summarized as follows:

1) We combine NOMA with relaying networks to improve spectral efficiency of the network, and the outage performance for the network is analyzed.
2) Exact expressions for the outage probability are obtained in closed-form, which is the theoretical basis to optimize and guide the actual communication system design. However, the obtained expressions are very complex and we can hardly perceive any insight from this formulation. Thus, a simple lower bound for the outage probability is attained and the asymptotic outage behavior is studied in the high-SNR regime to grasp the inward nature of the outage behavior.
3) Finally, simulation results corroborate the accuracy of our analytical results and the advantage of NOMA. Moreover, an error floor (EF) appears in the outage probability due to the presence of ICSI even though the system SNR is very high.

Throughout this paper, $\Pr[\cdot]$ symbolizes probability; $f_X(\cdot)$ and $F_X(\cdot)$ denote the probability density function (PDF) and the cumulative distribution function (CDF) of a random variable $X$, respectively; $E[\cdot]$ represents the expectation operator.

## II. SYSTEM MODEL

Consider a downlink relaying aided NOMA network as shown in Fig. 1, in which a source node $S$ serves $N$ users $D = \{D_1, D_2, \cdots, D_N\}$, simultaneously through an amplify-and-forward (AF) relay $R$. All nodes are single-antenna devices and operate in a half-duplex mode. The direct links between the source node and the users are unavailable which is common in the scenarios where two sources are located far away from each other or within heavily shadowed areas. We assume that all users are clustered very close such that a homogeneous network topology is considered in our paper. [1] The channels associated with each link exhibit Nakagami-$m$ fading and additive white Gaussian noise (AWGN). The channel coefficient pertaining to the link $S \to R$ is denoted by $g_{SR}$ with fading parameter $m_1$ and $E\left(|g_{SR}|^2\right) = \Omega_{SR}$, and the channel coefficient between $R$ and $D_k$ is denoted by $g_{RD_k}$ with fading parameter $m_2$ and $E\left(\left|g_{RD_k}\right|^2\right) = \Omega_{RD}$, $k = 1, 2, \cdots, N$. Letting $d_{SR}$ denote the distance between $S$ and $R$ and $d_{RD}$ denote the distance between $R$ and $D$, we have $\Omega_{SR} = {d_{SR}}^{-\alpha}$ and $\Omega_{RD} = {d_{RD}}^{-\alpha}$, where $\alpha$ is the path loss exponent.

For practical insights, obtaining the perfect CSI of wireless network is extremely difficult due to channel estimation errors. Thus, the channel coefficient can be modeled as $g_i = \hat{g}_i + e_i, i = \{SR, RD_k\}$, where $\hat{g}_i$ represents the estimated channel coefficient and $e_i \sim \mathcal{CN}\left(0, {\sigma_{e_i}}^2\right)$ denotes the channel estimation error which can be approximated as a Gaussian random variable [12]. In addition, $\hat{\Omega}_i = \Omega_i - {\sigma_{e_i}}^2$ can be attained by assuming that $\hat{g}_i$ is statistically independent of $e_i$. Letting $\rho_i = {\sigma_{e_i}}^2/\Omega_i$ denote the relative channel estimation error, we have ${\sigma_{e_i}}^2 = \rho_i {d_i}^{-\alpha}$ and $\hat{\Omega}_i = (1-\rho_i)\,{d_i}^{-\alpha}$. Without loss of generality, we assume the estimated channel gain between the relay node and all users are ordered as $\left|\hat{g}_{RD_1}\right|^2 \le \left|\hat{g}_{RD_2}\right|^2 \le \cdots \le \left|\hat{g}_{RD_N}\right|^2$.

In this paper, AF protocol is used for the NOMA downlink cooperative network. Two separate phases are involved to accomplish the data transmission. In the first phase, the source node sends unit-power symbol $x_S$ to the relay node. According to the principle of NOMA, $x_S = \sum_{i=1}^{N} \sqrt{a_i P_S} x_i$ denotes the superposition of $N$ independent user-dedicated symbols, where $P_S$ is the transmit power at $S$, $x_i$ represents the transmit symbol of user $i$, and $a_i$ is the power allocation factor of user $i$ with $a_1 \ge a_2 \ge \cdots \ge a_N$ and $\sum_{i=1}^{N} a_i = 1$. The received signal at $R$ is given by

$$y_R = \left(\hat{g}_{SR} + e_{SR}\right) \sum_{i=1}^{N} \sqrt{a_i P_S} x_i + w_R \tag{1}$$

in which $w_R \sim \mathcal{CN}(0, {\sigma_R}^2)$ denotes the AWGN at $R$. In the second phase, $R$ broadcasts the received signal to $D$ after amplifying it with an amplifying coefficient $G = 1\Big/\sqrt{\left(P_S\left|\hat{g}_{SR}\right|^2 + P_S \rho_{SR} {d_{SR}}^{-\alpha} + {\sigma_R}^2\right)}$. The received signal at the $n$th user can be formulated as

$$\begin{aligned} y_{RD_n} &= \sqrt{P_R} G \left(\hat{g}_{RD_n} + e_{RD_n}\right)\left(\hat{g}_{SR} + e_{SR}\right) \sum_{i=1}^{N} \sqrt{a_i P_S} x_i \\ &\quad + \sqrt{P_R} G \left(\hat{g}_{RD_n} + e_{RD_n}\right) w_R + w_{D_n} \end{aligned} \tag{2}$$

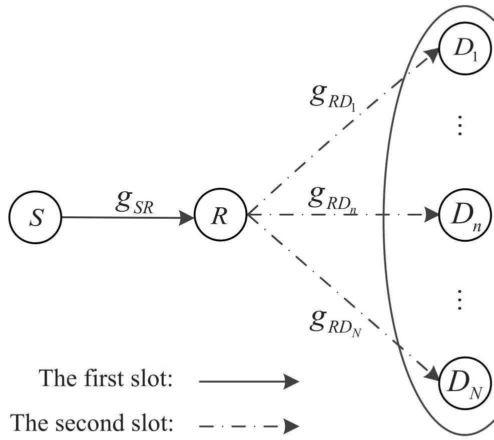


**FIGURE 1.** Downlink relaying aided NOMA networks.

[1] Although the subsequent analysis focuses on i.i.d. Nakagami-m fading scenario for the links $R \to D_n$, the presented analysis can be extended to the independent but not necessarily identically distributed Rayleigh fading scenario in a straightforward manner.

where $w_{D_n} \sim \mathcal{CN}(0, {\sigma_{D_n}}^2)$ represents the AWGN at $D_n$, and $P_R$ denotes the transmit power at $R$.

When decoding at each user, SIC will be performed at each user to separate superimposed symbols and mitigate the inter-user interference. NOMA allocates less transmit power to users with better channel conditions and more transmit power to users with worse channel conditions in order to achieve a balanced trade-off between system throughput and user fairness. Thus, the optimal order for SIC is in the order of the increasing channel gain, where users with better channel conditions need to decode the signals for the others before decoding their own. Therefore, $D_n$ should detect firstly the signals of $D_j$, $j < n$, before decoding its own signal. For mathematical tractability, in the following, we set $P_S = P_R = P$, ${\sigma_R}^2 = {\sigma_{D_n}}^2 = \sigma^2$, and define $\gamma \triangleq \frac{P}{\sigma^2}$ as the average system SNR. Then the SINR for the $n$th user to decode the $j$th user's signal $x_j, j \le n$, can be expressed as

$$\Gamma_{RD_{j\to n}} = \frac{a_j\gamma^2|\hat{g}_{SR}|^2|\hat{g}_{RD_n}|^2}{\tilde{a}_j\gamma^2|\hat{g}_{SR}|^2|\hat{g}_{RD_n}|^2 + \theta_1\gamma|\hat{g}_{SR}|^2 + \theta_2\gamma|\hat{g}_{RD_n}|^2 + \theta_3} \tag{3}$$

where $\theta_1 = \gamma\rho_{RD}{d_{RD}}^{-\alpha} + 1$, $\theta_2 = \gamma\rho_{SR}{d_{SR}}^{-\alpha} + 1$, $\theta_3 = \gamma^2\rho_{SR}\rho_{RD}{d_{SR}}^{-\alpha}{d_{RD}}^{-\alpha} + \gamma\left(\rho_{SR}{d_{SR}}^{-\alpha} + \rho_{RD}{d_{RD}}^{-\alpha}\right) + 1$, $\tilde{a}_j = \sum_{i=j+1}^{N} a_i$ for $j < N$, and $\tilde{a}_j = 0$ for $j = N$. If $x_j$ is decoded correctly, i.e., $\Gamma_{RD_{j\to n}} \ge \gamma_{thj}$, where $\gamma_{thj}$ denotes the targeted SINR for $D_j$, then $x_j$ will be cancelled from the $n$th user's observation. This SIC will be implemented until $n$ users' messages are all decoded, where the SINR for the $n$th user to decode its own signal can be given by

$$\Gamma_{RD_n} = \frac{a_n\gamma^2|\hat{g}_{SR}|^2|\hat{g}_{RD_n}|^2}{\tilde{a}_n\gamma^2|\hat{g}_{SR}|^2|\hat{g}_{RD_n}|^2 + \theta_1\gamma|\hat{g}_{SR}|^2 + \theta_2\gamma|\hat{g}_{RD_n}|^2 + \theta_3} \tag{4}$$

For $D_N$, all the other users' signals should be detected first, and the SNR for the $N$th user to decode its own signal can be given by

$$\Gamma_{RD_N} = \frac{a_N\gamma^2\left|\hat{g}_{SR}\right|^2\left|\hat{g}_{RD_N}\right|^2}{\theta_1\gamma\left|\hat{g}_{SR}\right|^2 + \theta_2\gamma\left|\hat{g}_{RD_N}\right|^2 + \theta_3} \tag{5}$$

## III. OUTAGE PERFORMANCE ANALYSIS

In this section, we investigated the outage behavior for the NOMA downlink cooperative network with ICSI. To this end, exact expressions for the outage probability is studied first. In order to reduce the computation complexity and better understand the behavior of the network, a tight lower bound for the outage probability is derived, while the diversity order for the network can be easily achieved by approximating the lower bound in the high-SNR regime.

### A. EXACT OUTAGE PROBABILITY

An outage event occurs at the $n$th user when the $n$th user fails to decode its own signal or the signal of the $j$th user, $1 \le j \le n$. The event that the $n$th user can correctly decode the $j$th user's signal can be expressed as

$$\begin{aligned} E_{n,j} &= \left\{\frac{a_j\gamma^2|\hat{g}_{SR}|^2|\hat{g}_{RD_n}|^2}{\tilde{a}_j\gamma^2|\hat{g}_{SR}|^2|\hat{g}_{RD_n}|^2 + \theta_1\gamma|\hat{g}_{SR}|^2 + \theta_2\gamma|\hat{g}_{RD_n}|^2 + \theta_3} \ge \gamma_{thj}\right\} \\ &= \left\{\left[\left(a_j - \gamma_{thj}\tilde{a}_j\right)\gamma^2\left|\hat{g}_{RD_n}\right|^2 - \theta_1\gamma\gamma_{thj}\right]\left|\hat{g}_{SR}\right|^2 \ge \gamma_{thj}\left(\theta_3 + \theta_2\gamma\left|\hat{g}_{RD_n}\right|^2\right)\right\} \\ &\underset{(a)}{=} \left\{\left|\hat{g}_{RD_n}\right|^2 \ge \frac{\gamma_{thj}\theta_1}{\left(a_j - \gamma_{thj}\tilde{a}_j\right)\gamma} \triangleq \lambda_j, \left|\hat{g}_{SR}\right|^2 \ge \frac{\lambda_j\left(\theta_3 + \theta_2\gamma\left|\hat{g}_{RD_n}\right|^2\right)}{\theta_1\gamma\left(\left|\hat{g}_{RD_n}\right|^2 - \lambda_j\right)}\right\} \end{aligned} \tag{6}$$

where step (a) relies on $a_j > \gamma_{thj}\tilde{a}_j$. If the condition is not satisfied, the $n$th user can never decode the $j$th user's signal successfully regardless of the system SNR. Therefore, the outage probability for the $n$th user is given by

$$\begin{aligned} \mathcal{P}_{out}^n &= 1 - \Pr\left(E_{n,1} \cap \cdots \cap E_{n,n}\right) \\ &= 1 - \Pr\left(\left|\hat{g}_{RD_n}\right|^2 \ge \lambda_n^*, \left|\hat{g}_{SR}\right|^2 \ge \frac{\lambda_n^*\left(\theta_3 + \theta_2\gamma\left|\hat{g}_{RD_n}\right|^2\right)}{\theta_1\gamma\left(\left|\hat{g}_{RD_n}\right|^2 - \lambda_n^*\right)}\right) \end{aligned} \tag{7}$$

where $\lambda_n^* = \max_{1\le j\le n} \lambda_j$.

In what follows, we focus on the calculation of $\mathcal{P}_{out}^n$. First, $\mathcal{P}_{out}^n$ can be rewritten as

$$\mathcal{P}_{out}^n = \underbrace{\int_0^{\lambda_n^*} f_{|\hat{g}_{RD_n}|^2}(y)dy}_{I_1} + \underbrace{\int_{\lambda_n^*}^{\infty} f_{|\hat{g}_{RD_n}|^2}(y)F_{|\hat{g}_{SR}|^2}\left(\frac{\lambda_n^*\left(\theta_3 + \theta_2\gamma y\right)}{\theta_1\gamma\left(y - \lambda_n^*\right)}\right)dy}_{I_2} \tag{8}$$

Next, the PDF and CDF of the gamma random variable $\left|\hat{g}_{SR}\right|^2$ with parameter $m_1$ and $\hat{\Omega}_{SR}$ can be expressed as

$$f_{|\hat{g}_{SR}|^2}(y) = \frac{{m_1}^{m_1}y^{m_1-1}{d_{SR}}^{\alpha m_1}}{\left(1-\rho_{SR}\right)^{m_1}\Gamma\left(m_1\right)}e^{-\frac{m_1 y {d_{SR}}^{\alpha}}{1-\rho_{SR}}} \tag{9}$$

$$F_{|\hat{g}_{SR}|^2}(y) = \frac{\gamma\left(m_1, ym_1{d_{SR}}^{\alpha}/\left(1-\rho_{SR}\right)\right)}{\Gamma\left(m_1\right)} \tag{10}$$

where $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1}e^{-x}dx$ denotes the Gamma function and $\gamma(\alpha, x) = \int_0^{x} x^{\alpha-1}e^{-x}dx$ denotes the incomplete Gamma function [13, eq.(8.310.1), eq.(8.350.1)]. In addition, $\Gamma(\alpha) =$

$(\alpha-1)!$ and $\gamma(\alpha,x) = (\alpha-1)!\left[1-e^{-x}\sum_{m=0}^{\alpha-1}\frac{x^m}{m!}\right]$ when $\alpha$ takes integer values greater than one [13, eq. (8.339.1), eq. (8.352.6)]. In this paper, we constrain the fading parameter $m_j, j = 1, 2$, to take integer values. Then, we can rewrite $F_{|\hat{g}_{SR}|^2}(y)$ as

$$F_{|\hat{g}_{SR}|^2}(y) = 1 - e^{-\frac{y m_1 d_{SR}^{\alpha}}{1-\rho_{SR}}}\sum_{s=0}^{m_1-1}\frac{1}{s!}\left(\frac{y m_1 d_{SR}^{\alpha}}{1-\rho_{SR}}\right)^s \tag{11}$$

Similarly, the PDF and CDF of the unordered gamma random variable $\left|\tilde{\hat{g}}_{RD_n}\right|^2$ can be expressed as

$$f_{\left|\tilde{\hat{g}}_{RD_n}\right|^2}(y) = \frac{{m_2}^{m_2} y^{m_2-1} {d_{RD}}^{\alpha m_2}}{(1-\rho_{RD})^{m_2}(m_2-1)!}e^{-\frac{m_2 {d_{RD}}^{\alpha}}{1-\rho_{RD}}} \tag{12}$$

$$F_{\left|\tilde{\hat{g}}_{RD_n}\right|^2}(y) = 1 - e^{-\frac{y m_2 {d_{RD}}^{\alpha}}{1-\rho_{RD}}}\sum_{r=0}^{m_2-1}\frac{1}{r!}\left(\frac{y m_2 {d_{RD}}^{\alpha}}{1-\rho_{RD}}\right)^r \tag{13}$$

Using order statistics, the PDF of the ordered channel gain $\left|\hat{g}_{RD_n}\right|^2$ can be written as [14]

$$\begin{aligned} &f_{|\hat{g}_{RD_n}|^2}(y) \\ &= \Theta_n f_{\left|\tilde{\hat{g}}_{RD_n}\right|^2}(y)\left[F_{\left|\tilde{\hat{g}}_{RD_n}\right|^2}(y)\right]^{n-1}\left[1-F_{\left|\tilde{\hat{g}}_{RD_n}\right|^2}(y)\right]^{N-n} \\ &= \Theta_n \sum_{k=0}^{N-n}(-1)^k\binom{N-n}{k} f_{\left|\tilde{\hat{g}}_{RD_n}\right|^2}(y)\left[F_{\left|\tilde{\hat{g}}_{RD_n}\right|^2}(y)\right]^{n+k-1} \end{aligned} \tag{14}$$

where we define $\Theta_n \triangleq \frac{N!}{(N-n)!(n-1)!}$. The corresponding CDF of $\left|\hat{g}_{RD_n}\right|^2$ is given by

$$F_{|\hat{g}_{RD_n}|^2}(y) = \Theta_n \sum_{k=0}^{N-n}\frac{(-1)^k}{n+k}\binom{N-n}{k}\left[F_{\left|\tilde{\hat{g}}_{RD_n}\right|^2}(y)\right]^{n+k} \tag{15}$$

Now, we concentrate on the analysis of $I_2$. By plugging the CDF of $\left|\hat{g}_{SR}\right|^2$ and the PDF of $\left|\hat{g}_{RD_n}\right|^2$ into $I_2$ and after some algebraic manipulations, (16) can be attained shown at the bottom of this page. In order to facilitate the subsequent analysis, a more convenient expression of $I_3$ can be obtained as Lemma 1.

*Lemma 1:* The convenient expression of $I_3$ can be expressed as

$$I_3 = \sum_{i=0}^{n+k-1}\bigcup_i (-1)^i \Xi_{1,i}\Xi_{2,i} y^{\bar{i}} e^{-y m_2 i {d_{RD}}^{\alpha}/1-\rho_{RD}} \tag{17}$$

in which $\bigcup_i \triangleq \sum_{i_1=0}^{i}\sum_{i_2=0}^{i-i_1}\cdots\sum_{i_{m_2-1}=0}^{i-i_1-\cdots i_{m_2-2}}$, $\Xi_{1,i} = \binom{n+k-1}{i} \times \binom{i}{i_1}\binom{i-i_1}{i_2}\cdots\binom{i-i_1-\cdots i_{m_2-2}}{i_{m_2-1}}$, $\Xi_{2,i} = \prod_{r=0}^{m_2-2}\left(\frac{{m_2}^r {d_{RD}}^{\alpha r}}{r!(1-\rho_{RD})^r}\right)^{i_{r+1}} \times \left(\frac{{m_2}^{m_2-1}{d_{RD}}^{\alpha(m_2-1)}}{(m_2-1)!(1-\rho_{RD})^{m_2-1}}\right)^{i-i_1-\cdots i_{m_2-1}}$, and $\bar{i} = (m_2-1)\times(i-i_1)-(m_2-2)\,i_2-(m_2-3)\,i_3-\cdots i_{m_2-1}$.

*Proof:* See Appendix A.

By substituting (17) into (16) and making the change of variates $z = y - \lambda_n^*$, we have

$$\begin{aligned} I_2 = 1 - I_1 - &\frac{\Theta_n {m_2}^{m_2} {d_{RD}}^{\alpha m_2}}{(1-\rho_{RD})^{m_2}(m_2-1)!} \\ &\times \sum_{k=0}^{N-n}\sum_{i=0}^{n+k-1}\sum_{s=0}^{m_1-1}\sum_{s_1=0}^{s}\bigcup_i(-1)^i \\ &\times \Xi_{1,i}\Xi_{2,i}(-1)^k\binom{N-n}{k}\binom{s}{s_1}\frac{{\mu_1}^{s_1}{\mu_2}^{s-s_1}}{s!} \\ &\times e^{-\mu_2-\frac{m_2\lambda_n^* {d_{RD}}^{\alpha}(i+1)}{1-\rho_{RD}}} \\ &\times \underbrace{\int_0^\infty z^{-s_1}\left(z+\lambda_n^*\right)^{m_2+\bar{i}-1} e^{-\frac{m_2 z {d_{RD}}^{\alpha}(i+1)}{1-\rho_{RD}}} e^{-\mu_1/z} dz}_{I_4} \end{aligned} \tag{17}$$

where $\mu_1 \triangleq \frac{\lambda_n^* m_1 {d_{SR}}^{\alpha}(\theta_3+\theta_2\gamma\lambda_n^*)}{\theta_1\gamma(1-\rho_{SR})}$, and $\mu_2 \triangleq \frac{\lambda_n^* m_1\theta_2 {d_{SR}}^{\alpha}}{\theta_1(1-\rho_{SR})}$. Applying the binomial theorem and with the aid of [13, eq.(3.471.9)], $I_4$ can be computed as

$$\begin{aligned} I_4 = 2&\sum_{s_2=0}^{m_2+\bar{i}-1}\binom{m_2+\bar{i}-1}{s_2}\left(\frac{\mu_1(1-\rho_{RD})}{m_2 {d_{RD}}^{\alpha}(i+1)}\right)^{\frac{s_2-s_1+1}{2}} \\ &\times {\lambda_n^*}^{m_2+\bar{i}-1-s_2} K_{s_2-s_1+1}\left(2\sqrt{\frac{\mu_1 m_2 {d_{RD}}^{\alpha}(i+1)}{(1-\rho_{RD})}}\right) \end{aligned}$$

$$\begin{aligned} I_2 &= \Theta_n \sum_{k=0}^{N-n}(-1)^k\binom{N-n}{k}\int_{\lambda_n^*}^{\infty} f_{\left|\tilde{\hat{g}}_{RD_n}\right|^2}(y)\left[F_{\left|\tilde{\hat{g}}_{RD_n}\right|^2}(y)\right]^{n+k-1}\left(1-e^{-\frac{\lambda_n^*(\theta_3+\theta_2\gamma y)m_1 d_{SR}}{\theta_1\gamma(1-\rho_{SR})(y-\lambda_n^*)}}\sum_{s=0}^{m_1-1}\left[\frac{\lambda_n^*(\theta_3+\theta_2\gamma y)\,d_{SR}}{\theta_1\gamma(1-\rho_{SR})\left(y-\lambda_n^*\right)}\right]^s\frac{1}{s!}\right)dy \\ &= 1 - I_1 - \Theta_n \sum_{k=0}^{N-n}(-1)^k\binom{N-n}{k}\int_{\lambda_n^*}^{\infty} f_{\left|\tilde{\hat{g}}_{RD_n}\right|^2}(y)\underbrace{\left[F_{\left|\tilde{\hat{g}}_{RD_n}\right|^2}(y)\right]^{n+k-1}}_{I_3} \\ &\quad \times e^{-\frac{\lambda_n^*(\theta_3+\theta_2\gamma y)m_1 d_{SR}}{\theta_1\gamma(1-\rho_{SR})(y-\lambda_n^*)}}\left(\sum_{s=0}^{m_1-1}\left[\frac{\lambda_n^*(\theta_3+\theta_2\gamma y)\,d_{SR}}{\theta_1\gamma(1-\rho_{SR})\left(y-\lambda_n^*\right)}\right]^s\frac{1}{s!}\right)dy \end{aligned} \tag{16}$$

$$\mathcal{P}_{out}^{n} = 1 - \frac{2\Theta_n m_2^{m_2} {d_{RD}}^{\alpha m_2}}{(1-\rho_{RD})^{m_2}(m_2-1)!} \sum_{k=0}^{N-n}\sum_{i=0}^{n+k-1}\sum_{s=0}^{m_1-1}\sum_{s_1=0}^{s}\sum_{s_2=0}^{m_2+\bar{i}-1} \bigcup_{i} (-1)^{k+i}\Xi_{1,i}\Xi_{2,i}\binom{N-n}{k}\binom{s}{s_1}\binom{m_2+\bar{i}-1}{s_2}$$
$$\times \frac{{\mu_1}^{s_1}{\mu_2}^{s-s_1}{\lambda_n^*}^{m_2+\bar{i}-1-s_2}}{s!} e^{-\mu_2-\frac{m_2\lambda_n^* d_{RD}{}^{\alpha}(i+1)}{1-\rho_{RD}}} \left(\frac{\mu_1(1-\rho_{RD})}{m_2 d_{RD}{}^{\alpha}(i+1)}\right)^{\frac{s_2-s_1+1}{2}} K_{s_2-s_1+1}\left(2\sqrt{\frac{\mu_1 m_2 d_{RD}{}^{\alpha}(i+1)}{(1-\rho_{RD})\Omega_{RD}}}\right) \quad (20)$$

(18)

where $K_v(\cdot)$ denotes the $v$th-order modified Bessel function of the second kind.

By combining (8), (18) and (19), a closed-form expression for the outage probability of the $n$th user can be expressed as (20), shown at the top of this page.

### B. LOWER BOUND FOR THE OUTAGE PROBABILITY

Although the exact expressions for the outage probability have been obtained, it is of great difficulty to get further insights. Therefore, a simpler bound for the outage probability should be attained.

To begin with, we rewrite (7) as

$$\begin{aligned}\mathcal{P}_{out}^{n} &= 1-\Pr\left(\frac{\theta_1\gamma|\hat{g}_{SR}|^2|\hat{g}_{RD_n}|^2}{\theta_1\gamma|\hat{g}_{SR}|^2+\theta_2\gamma|\hat{g}_{RD_n}|^2+\theta_3} > \lambda_n^*\right)\\ &= 1-\Pr\left(\frac{\frac{\theta_1\theta_2\gamma^2}{{\theta_3}^2}|\hat{g}_{SR}|^2|\hat{g}_{RD_n}|^2}{\frac{\theta_1\gamma}{\theta_3}|\hat{g}_{SR}|^2+\frac{\theta_2\gamma}{\theta_3}|\hat{g}_{RD_n}|^2+1} > \frac{\lambda_n^*\theta_2\gamma}{\theta_3}\right)\end{aligned} \quad (21)$$

Using the inequality $xy/(x+y+1) \le \min(x,y)$ [15], $\mathcal{P}_{out}^{n}$ can be lower-bounded as follows

$$\begin{aligned}\mathcal{P}_{out,LB}^{n} &= 1-\Pr\left(\min\left(\theta_1|\hat{g}_{SR}|^2, \theta_2|\hat{g}_{RD_n}|^2\right) > \lambda_n^*\theta_2\right)\\ &= F_{|\hat{g}_{SR}|^2}\left(\frac{\lambda_n^*\theta_2}{\theta_1}\right)+F_{|\hat{g}_{RD_n}|^2}\left(\lambda_n^*\right) - F_{|\hat{g}_{SR}|^2}\left(\frac{\lambda_n^*\theta_2}{\theta_1}\right)\\ &\quad\times F_{|\hat{g}_{RD_n}|^2}\left(\lambda_n^*\right)\end{aligned} \quad (22)$$

By plugging (11) and (15) into (22), the lower bound for $\mathcal{P}_{out}^{n}$ can be addressed.

By defining $\Lambda_n^* = \max_{1\le j\le n}\left[\gamma_{thj}/\left(a_j-\gamma_{thj}\tilde{a}_j\right)\right]$, at high SNR region ($\gamma \to \infty$) it yields

$$\lambda_n^* = \frac{\Lambda_n^*\theta_1}{\gamma} = \Lambda_n^*\left(\rho_{RD}d_{RD}{}^{-\alpha}+\frac{1}{\gamma}\right) \simeq \Lambda_n^*\rho_{RD}d_{RD}{}^{-\alpha} \quad (23)$$

$$\mu_2 = \frac{\Lambda_n^* m_1 d_{SR}{}^{\alpha}}{1-\rho_{SR}}\left(\rho_{SR}d_{SR}{}^{-\alpha}+\frac{1}{\gamma}\right) \simeq \frac{\Lambda_n^* m_1\rho_{SR}}{1-\rho_{SR}} \quad (24)$$

By applying the approximations, we have

$$F_{|\hat{g}_{SR}|^2}\left(\frac{\lambda_n^*\theta_2}{\theta_1}\right) \simeq 1-e^{-\frac{\Lambda_n^* m_1\rho_{SR}}{1-\rho_{SR}}}\sum_{s=0}^{m_1-1}\frac{1}{s!}\left(\frac{\Lambda_n^* m_1\rho_{SR}}{1-\rho_{SR}}\right)^{s} \quad (25)$$

$$F_{|\hat{g}_{RD_n}|^2}(\lambda_n^*) \simeq \Theta_n\sum_{k=0}^{N-n}\frac{(-1)^k}{n+k}\binom{N-n}{k}\left[1-e^{-\frac{\Lambda_n^*\rho_{RD}m_2}{1-\rho_{RD}}}\times\sum_{r=0}^{m_2-1}\frac{1}{r!}\left(\frac{\Lambda_n^*\rho_{RD}m_2}{1-\rho_{RD}}\right)^{r}\right]^{n+k} \quad (26)$$

Note that $F_{|\hat{g}_{SR}|^2}\left(\frac{\lambda_n^*\theta_2}{\theta_1}\right)$ and $F_{|\hat{g}_{RD_n}|^2}(\lambda_n^*)$ approach a fixed value which is independent of $d_{SR}$ and $d_{RD}$ when $\gamma \to \infty$. Thus the outage probability maintains a fixed non-zero value due to the presence of channel estimation errors even if $\gamma$ is sufficiently large. This fixed value is named as EF, which indicates that the system diversity order equals to zero at high SNR. Thus, the level of EF for $\mathcal{P}_{out}^{n}$ is independent of $d_{SR}$ and $d_{RD}$, and the EF can be attained by substituting (25) and (26) into (22).

## IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, numerical examples are presented to validate the outage performance of the downlink C-NOMA network under Nakagami-$m$ fading channels with ICSI. Moreover, C-NOMA is compared with conventional cooperative orthogonal multiple access (C-OMA) networks, where a random user is scheduled from all users in a transmission period. The targeted SNR for C-OMA satisfies $\frac{1}{2}\sum_{j=1}^{N}\log\left(1+\gamma_{thj}\right) = \frac{1}{2}\log(1+\gamma_{th})$. We consider that the base station, the relay node, and mobile users are located on a straight line. Without loss of generality, we assume the distance between $S$ and $D$ is normalized to unity. In the following simulations, we set the relative channel estimation error $\rho_{SR} = \rho_{RD} = \rho$, $N = 2$, $a_1 = 3/4$, $a_2 = 1/4$, $\gamma_{th1} = 2$, $\gamma_{th2} = 2.5$, $\alpha = 3$, and different fading parameters $\{m_1, m_2\} : \{1, 1\}, \{2, 2\}$.

In Fig. 2, the outage probability versus system SNR is presented in different fading parameters. First of all, the exact analytical results and simulation results are in excellent agreement, and the lower bounds of the outage probability in (22) are shown to be tight bounds in the medium-and high-SNR regimes. Moreover, as the system SNR increases, the outage probability decreases and then reaches a fixed level (EF) by reason of the errors in the channel estimation, which leads the diversity order turns to zero. Another important observation is that the outage probability for User 2 of NOMA outperforms C-OMA. Note that the outage performance for User 1 is slightly worse than C-OMA, but NOMA can offer better spectral efficiency and user fairness since multiple users are served simultaneously.

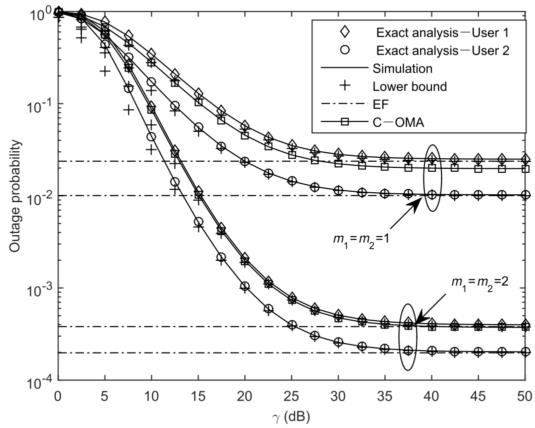


**FIGURE 2.** Outage probability vs. system SNR ($d_{SR} = 0.5$ and $\rho = 0.001$).

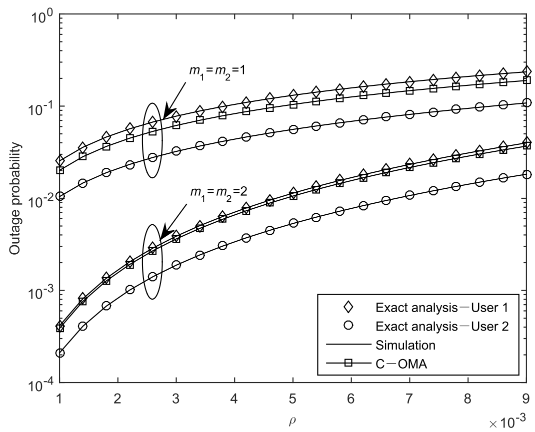


**FIGURE 3.** Outage probability vs. $\rho$ ($d_{SR} = 0.5$ and $\gamma = 50$ dB).

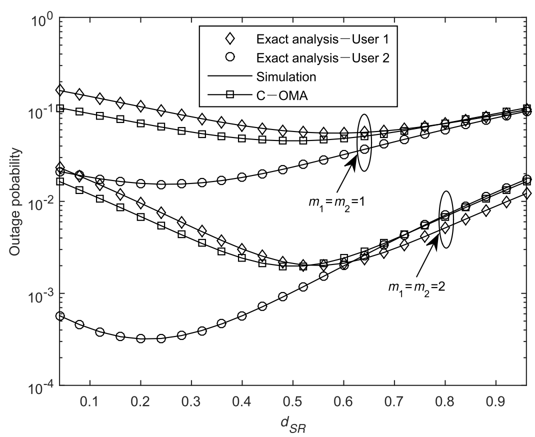


**FIGURE 4.** Outage probability vs. $d_{SR}$ ( $\gamma = 20$ dB and $\rho = 0.001$).

Fig. 3 illustrates the impact of the relative channel estimation error on the outage performance. It can be observed that the outage performance decreases with $\rho$ due to the EF effect, and User 2 of NOMA provides more performance gain over C-OMA regardless of the value of $\rho$.

Fig. 4 and Fig. 5 compare the outage probability of the considered cooperative NOMA networks with C-OMA networks versus relay location for $\gamma = 20$ and 50 dB, respectively. In Fig. 4, it is shown that the best relay location for user 2

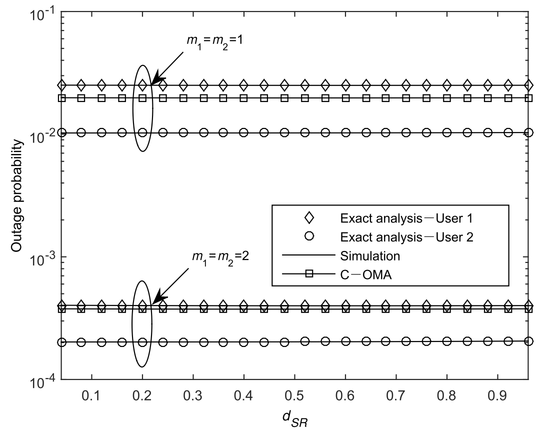


**FIGURE 5.** Outage probability vs. $d_{SR}$ ( $\gamma = 50$ dB and $\rho = 0.001$).

is nearer to the source node than User 1. This is because less transmit power is allocated to User 2, the best relay location for User 2 should be nearer to the source node so as to obtain high received SNR at the relay node. Moreover, User 2 of C-NOMA outperforms C-OMA and the outage performance gap is decrease as the relay node gets close to the users, hence the relay location for C-NOMA networks should be close to the source node. In Fig. 5, the outage probability remains unchanged with the relay location. This is because at high SNR region, the outage probability approaches the EF which is independent of $d_{SR}$ and $d_{RD}$.

## V. CONCLUSION

In this paper, with ICSI taken into account, the outage performance for downlink C-NOMA networks over Nakagami-$m$ fading channels is investigated. Expressions for the exact and lower bound of the outage probability are obtained in closed-form, which is the theoretical basis to provide valuable guidelines in the actual communication system design. Simulation results demonstrate that C-NOMA provides significant performance gain over the conventional C-OMA in terms of outage performance. It is observed that an EF appears in the outage probability due to the channel estimation errors. In addition, it is proved that the best relay location for C-NOMA networks should be close to the source node.

## APPENDIX
## PROOF OF LEMMA 1

First of all, $I_3$ can be reexpressed as

$$I_3 = \left[1 - e^{-\frac{y m_2 d_{RD}{}^{\alpha}}{1-\rho_{RD}}} \sum_{r=0}^{m_2-1} \frac{1}{r!}\left(\frac{y m_2 d_{RD}{}^{\alpha}}{1-\rho_{RD}}\right)^r\right]^{n+k-1}$$
$$= \sum_{i=0}^{n+k-1} (-1)^i \binom{n+k-1}{i} e^{-\frac{y m_2 i d_{RD}{}^{\alpha}}{1-\rho_{RD}}}$$
$$\times \underbrace{\left[\sum_{r=0}^{m_2-1} \frac{1}{r!}\left(\frac{y m_2 d_{RD}{}^{\alpha}}{1-\rho_{RD}}\right)^r\right]^i}_{I_{3,1}} \quad (27)$$

Denote $A_r = \frac{1}{r!}\left(\frac{m_2 d_{RD}^{\alpha}}{1-\rho_{RD}}\right)^r$. By successive binomial expansion on $I_{3,1}$, we have

$$\begin{aligned} I_{3,1} &= \left(\sum_{r=0}^{m_2-1} A_r y^r\right)^i \\ &= \sum_{i_1=0}^{i}\binom{i}{i_1} {A_0}^{i_1}\left(\sum_{r=1}^{m_2-1} A_r y^r\right)^{i-i_1} \\ &= \sum_{i_1=0}^{i}\sum_{i_2=0}^{i-i_1}\binom{i}{i_1}\binom{i-i_1}{i_2} {A_0}^{i_1}(A_1 y)^{i_2}\left(\sum_{r=2}^{m_2-1} A_r y^r\right)^{i-i_1-i_2} \\ &= \sum_{i_1=0}^{i}\sum_{i_2=0}^{i-i_1}\cdots\sum_{i_{m_2-1}=0}^{i-i_1-\cdots i_{m_2-2}}\binom{i}{i_1}\binom{i-i_1}{i_2} \\ &\quad \cdots\binom{i-i_1-\cdots i_{m_2-2}}{i_{m_2-1}} \\ &\quad \times\left(A_{m_2-1}y^{m_2-1}\right)^{i-i_1-\cdots i_{m_2-1}}\prod_{r=0}^{m_2-2}\left(A_r y^r\right)^{i_{r+1}} \end{aligned} \tag{28}$$

Next, by substituting (28) into (27) and then denoting $\bigcup_i \triangleq \sum_{i_1=0}^{i}\sum_{i_2=0}^{i-i_1}\cdots\sum_{i_{m_2-1}=0}^{i-i_1-\cdots i_{m_2-2}}$, $\Xi_{1,i} = \binom{n+k-1}{i}\times\binom{i}{i_1}\binom{i-i_1}{i_2}\cdots\binom{i-i_1-\cdots i_{m_2-2}}{i_{m_2-1}}$, $\Xi_{2,i} = \prod_{r=0}^{m_2-2}\left(\frac{{m_2}^r d_{RD}^{\alpha r}}{r!(1-\rho_{RD})^r}\right)^{i_{r+1}} \times \left(\frac{{m_2}^{m_2-1} d_{RD}^{\alpha(m_2-1)}}{(m_2-1)!(1-\rho_{RD})^{m_2-1}}\right)^{i-i_1-\cdots i_{m_2-1}}$, and $\bar{i} = (m_2-1)\times(i-i_1)-(m_2-2)\,i_2-(m_2-3)\,i_3-\cdots i_{m_2-1}$, we can easily attain (17), which completes the proof.

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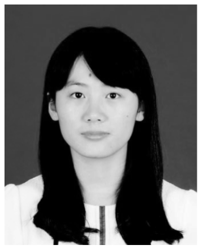

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