IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received October 4, 2016, accepted November 1, 2016, date of publication November 16, 2016, date of current version March 8, 2017.

Digital Object Identifier 10.1109/ACCESS.2016.2628860

Soft Combination for Cooperative Spectrum Sensing in Fading Channels

HUAYAN GUO[1], (Student Member, IEEE), NIMA REISI[2], (Member, IEEE), WEI JIANG[1], (Member, IEEE), AND WU LUO[1], (Member, IEEE)

[1]State Key Laboratory of Advanced Optical Communication Systems and Networks, Peking University, Beijing 100871, China
[2]Azad University, Central Tehran Branch, Tehran, Iran

Corresponding author: W. Jiang (jiangwei@pku.edu.cn)

This work was supported by the National Natural Science Foundation of China under Grant 61171080.

ABSTRACT In this paper, we study the distributed energy-based detectors for spectrum sensing in cognitive radio networks. We assume that the sensing channel includes both small-scale and large-scale fading. The small-scale fading is modeled as Nakagami-*m* and independent for different cooperating cognitive users, while the large-scale fading is assumed to be known (or can be estimated) by the cognitive users, due to their slowly changing nature. Furthermore, we assume that the channel gains are constant in one observation interval and vary independently in different intervals. Based on the Bayesian rule, we derive the optimal energy combining rule, i.e., the average likelihood ratio (ALR) detector. We also suggest two solutions: 1) mixture of gamma (MoG)-based ALR detector and 2) generalized Gauss–Laguerre formula (GLF)-based ALR detector, to overcome the problem of the intractable integrals in the optimal rule, and we propose two novel suboptimal but practical combining rules: 1) GLF-based linear combining detector, which can be implemented by linear functions and a comparator with negligible performance degradation and 2) GLF-based weighted-energy detector applicable for the low SNR regime. The simulation results reveal that with MoG and GLF detectors, the ALR detector can be implemented almost precisely with lower complexity. Moreover, all the proposed detectors outperform the conventional ones, especially when large-scale channel gains differ for different cognitive users.

INDEX TERMS Cognitive radio, cooperative spectrum sensing, soft combining, LLR-based detector.

I. INTRODUCTION

Wireless spectrum is a vital resource in radio communications. Measurement campaigns have revealed that spectrum utilization by the licensed primary users (PU) is inefficient [1], [2]. The cognitive radio (CR) [3], [4] is a rapidly emerging technology during the last decades to overcome the problem of scarcity and inefficient utilization of the spectrum by allowing the unlicensed secondary users (SU) to opportunistically access the licensed spectrum segments, without making interference to the primary users. To avoid interference with PU systems, spectrum sensing is essential to detect the spectrum holes, i.e., the idle licensed spectrum sub-bands. Different techniques such as the matched filter detection [5]–[7], the cyclostationary detection [8]–[10], the energy detection [11]–[15], and the covariance based detection [16]–[18] are proposed for spectrum sensing, among which energy detection is the most common practical one due to its simplicity.

Due to the fading, shadowing, and the hidden node problem, the instantaneous signal-to-noise ratio (SNR) of the received signal may become too low to make the sensing result of a single SU unreliable. Cooperation among different SUs has been introduced to overcome this challenge, since the probability that all of the cooperating SUs are simultaneously in a deep shadowing or fading reduces as the number of involved SUs increases [19]–[22]. Moreover, the diversity gain provided by the multiple distributed SUs greatly improves the global detection performance and efficiency [23]–[25].

Fig. 1 depicts a classic cooperative sensing model with one PU, N SUs, and one fusion center (FC). Each SU observes the frequency band of interest through an individual (sensing) channel, processes its observation, and reports a message Y_j $(1 \leq j \leq N)$ to the FC via an error-free (reporting) channel. The FC collects all the messages, combines them, and makes the global decision about the occupancy

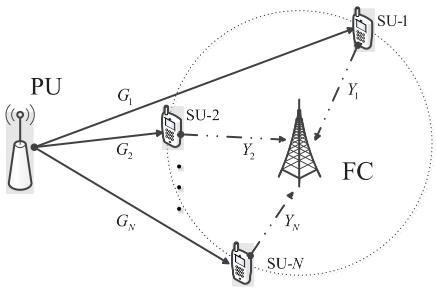


Fig. 1. Cooperative sensing model in a cognitive network with one PU, N SUs and one FC.

of the frequency band. If $\{Y_j\}$s are the local one-bit decision results, the FC combining rule is called hard combining method [26]–[29]. On the other hand, if $\{Y_j\}$s are the local test statistics (usually the local sufficient statistics or their quantized versions), the FC combining rule will be a soft combining method [30]–[33]. The hard combining methods reduce the communication cost of the reporting channel at the expense of information loss and detection performance degradation, while the soft combining methods typically lead to a better detection performance, since they have almost full information about the local observations.

It is usually argued that soft combining methods significantly increase the required bandwidth and power consumption, since more data should be transmitted. This is not necessarily true, since there may be significant overhead related to communication protocol used in transmitting the sensing results to the FC, which is still present even if only binary decisions are transmitted. Hence, the difference in the resource cost between hard and soft combining strategies may be small [3], [34]. Furthermore, for soft combining, we don't need to deal with the complex coupling relation between the local quantization rule and the fusion combining rule, which is the most important task in hard combining techniques.

In this paper, we focus on the soft combining methods and design a distributed spectrum sensing framework using different novel combining rules in the FC.

A. RELATED WORK

The distributed detection has a rich literature in the sensor network and radar research communities, e.g., [35]–[38]. However, these results are difficult to be directly applied to the CR network due to the fading effects of the wireless channels [39]–[43].

The optimal soft combining rule under a priori knowledge of the instantaneous local SNRs in observation intervals has been investigated in different papers. Such priori knowledge greatly simplifies the probability density function (PDF) of Y_j under the signal present hypothesis.

In [44], the optimal soft combining rule is derived for Gaussian primary signal. The authors also proved that the maximal ratio combining and the equal gain combining detectors are nearly optimal in low and high SNR regimes, respectively. In [45] and [46], similar conclusions are obtained for the unknown but deterministic primary signal assumption, by using the central limit theorem (CLT). A new metric, called the modified deflection coefficient, is proposed in [45], to optimize the linear combining factors, which is useful for non-Gaussian primary signal scenarios [47]–[49].

In practical scenarios, the sensing channel is subjected to multipath and fading, which causes the channel gains vary in each observation interval and makes it hard to estimate the instantaneous local SNR values. Therefore, in such scenarios, new efficient combining rules are needed.

In [50], primary signal is assumed as an unknown deterministic signal and the sensing channels are modeled as mixture of Nakagami-m and log-normal shadowing to model both small and large-scale fading. The authors assumed that only the statistical distributions of small and large-scale fading are known by the CR network. The optimal combining rule, i.e., the log-likelihood ratio (LLR) of $\{Y_j\}$s is calculated and averaged over the distribution of SNR. As a result, the optimal rule is named as the average likelihood ratio (ALR) detector. Nevertheless, the ALR involves many integrals which is hard to be implemented. The authors in [50] pointed out that, for the unknown but deterministic primary signal, the integrals caused by fading could be solved with the confluent hypergeometric function. However, the confluent hypergeometric function still needs to be solved by numerical methods [51], [52], or by a look-up table for implementation.

Usually, the large-scale fading changes slowly and the CR network can estimate it with high precision, which can simplify the design of the detector and greatly improve the detection performance. In [53], under the assumption of Gaussian primary signal and fast-fading Rayleigh-distributed sensing channel with known statistics, the optimal soft combining rule is investigated when the large-scale fading parameter is assumed to be known by the CR network. The authors have also suggested a two-point mixture of gamma (MoG) method to approximate the complex PDF of $\{Y_j\}$s, when the PU signal is present.

B. CONTRIBUTION

In this paper, we consider a distributed cognitive network and develop the optimal soft combining detector and some simplified sub-optimal detectors, when the sensing channels are subjected to both small and large scale fading.

We assume that CR users experience independent block fading, where the instantaneous SNR keeps a constant during one observation interval, but varies independently in different intervals. As in [53], we also assume that the channel gains due to the large-scale fading are available or can be estimated by the CR network.

The main contributions of our work are as follows.

- We consider the practical block-fading channel model, which is suitable when the CR observation length is comparable with the channel coherence time [12], [39], [43], [44]. Under this assumption, the solution in [53] can not be applied directly because of different channel conditions. Designing a practical combining rule with low-complexity and small performance degradation in comparison with the optimal solution for the block-fading scenario is still not fully addressed in existing literature.

- We derive the optimal fusion rule based on the Likelihood Ratio test averaged over channel statistics (ALR detector), for the block-fading scenario. Then we suggest two general solutions to solve the intractable integrals in the ALR detector. The first solution is based on the p-point MoG approximation, and the other one is based on the Generalized Gauss-Laguerre formula. Both solutions are applicable for all kinds of PU signal distributions, which is an improvement compared to [50].
- We propose a simple but efficient sub-optimal combining rule based on the Generalized Gauss-Laguerre formula solution of the ALR detector, which is implemented by linear functions and a comparator. This sub-optimal combining rule can be further simplified as an energy weighted combining method, when the SNR is low.

The rest of this paper is organized as follows. In Section II, we introduce the system model and the problem formulation. In Section III, the optimal fusion rule is derived based on the ALR detector, and two practical approximate solutions are suggested to deal with the intractable integrals in the optimal solution. In Section IV, we overview several conventional combining schemes, and propose two sub-optimal combining rules. Simulation results are presented in Section V. Finally, we draw conclusions in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. SIGNAL AND CHANNEL MODEL

We assume that the CR network consists of N cognitive users which cooperatively sense the frequency band of interest in successive observation intervals. Each interval consists of M samples which is selected according to the "bandwidth-observation time" product. The i-th received signal sample at the j-th CR user, $r_j[i]$, $1 \le j \le N$, $1 \le i \le M$ can be written as

$$r_j[i] = \begin{cases} n_j[i], & \text{under } \mathcal{H}_0 \\ \sqrt{G_j} s_j[i] + n_j[i], & \text{under } \mathcal{H}_1 \end{cases} \tag{1}$$

where $\mathcal{H}_0$ and $\mathcal{H}_1$ denote the "signal absent" and "signal present" hypotheses, respectively.

We assume both signal and noise samples are zero-mean independently and identically distributed (*i.i.d.*) circularly-symmetric complex Gaussian (CSCG) random variables with unit variance, i.e., $s_j[i] \sim \mathcal{CN}(0, 1)$ and $n_j[i] \sim \mathcal{CN}(0, 1)$. Under this assumption, the local received energy is the sufficient statistic [11], [54], so we only need to deal with the channel fading effect on different CR users when we design the fusion rule [44], [53].

We use $G_j = P_{\mathrm{L},j} h_j$ to denote the sensing channel gain between the PU and the j-th CR user, which is assumed to be constant in each observation interval. $P_{\mathrm{L},j}$ and h_j indicate the effect of the large-scale and small-scale fading, respectively. The large-scale factor $P_{\mathrm{L},j}$ describes the slow varying character of G_j due to path loss and shadowing, and the small-scale factor h_j describes the fast varying character of G_j due to multipath [55], [56]. The large-scale factor is assumed to be known or can be estimated by the CR network. On the other hand, the small-scale fading on different sensing channels are modeled as independent Nakagami-m random variables. Accordingly, the channel gain h_j follows Gamma distribution, with the following PDF:

$$f_{h_j}(u) = \frac{m^m}{\Gamma(m)} u^{m-1} e^{-mu}, \tag{2}$$

where $\Gamma(\cdot)$ is the gamma function, and $m \ge 0.5$ is the Nakagami parameter. For $m = 1$. the distribution reduces to Rayleigh fading; for $m = \infty$, there's no small-scale fading and $h_j = 1$. The Nakagami-m fading channel can also approximate many other fading models, such as Rician fading channel [57] and K-fading channel [58].

We assume that only the local average SNR $\bar{\gamma}_j = P_{\mathrm{L},j}$ could be shared between the j-th CR user and FC, while different to [44]–[46], the instantaneous SNR $\gamma_j = P_{\mathrm{L},j} h_j$ is assumed to be unavailable. Each CR user reports a soft message to the FC via a error-free channel. Then the FC makes the final decision about the occupancy of the channel and broadcasts the result to all CR users.

The conditional PDF of the γ_j given $\bar{\gamma}_j$ can be expressed as:

$$f_{\gamma_j}(\gamma | \bar{\gamma} = P_{\mathrm{L},j}) = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} e^{-\frac{m\gamma}{\bar{\gamma}}}. \tag{3}$$

One can see from (3) that γ_j also follows Gamma distribution.

B. PROBLEM FORMULATION

According to the signal and channel models, the received observation sample $r_j[i]$ follows complex Gaussian distribution:

$$r_j[i] \sim \begin{cases} \mathcal{CN}(0, 1), & \text{under } \mathcal{H}_0 \\ \mathcal{CN}(0, 1 + \gamma_j), & \text{under } \mathcal{H}_1 \end{cases} \tag{4}$$

Each CR user computes the average received power Y_j,

$$Y_j = \frac{1}{M} \sum_{i=1}^{M} \left| r_j[i] \right|^2, \tag{5}$$

and sends it via the error-free reporting channel to the FC. Since $r_j[i]$ has complex Gaussian distribution under both hypotheses, Y_j is the local sufficient statistic. According to the distributions in (4), Y_j follows a central chi-square distribution with $2M$ degrees of freedom under both hypotheses. The PDF of Y_j conditioned on $\mathcal{H}_0$ is:

$$f\left(Y_j | \mathcal{H}_0\right) = \frac{M^M}{\Gamma(M)} Y_j^{M-1} e^{-MY_j}, \tag{6}$$

and the PDF conditioned on $\mathcal{H}_1$ and γ_j is

$$f\left(Y_j|\mathcal{H}_1,\gamma_j\right)=\frac{M^M}{\left(1+\gamma_j\right)^M\Gamma(M)}Y_j^{M-1}e^{-\frac{MY_j}{1+\gamma_j}}. \tag{7}$$

The detection performance in a spectrum sensing scenario is generally indicated by two types of error probabilities: one is the false alarm (P_F), which means that the FC claims that signal is present under hypothesis $\mathcal{H}_0$:

$$P_F=\Pr\left(\text{declare } \mathcal{H}_1\,|\mathcal{H}_0\right), \tag{8}$$

the other is the missed detection (P_M), which means that the FC claims that signal is absent under hypothesis $\mathcal{H}_1$:

$$P_M=\Pr\left(\text{declare } \mathcal{H}_0\,|\mathcal{H}_1\right). \tag{9}$$

Obviously, for spectrum sensing requirements [59], both P_M and P_F should be as low as possible. A lower P_M leads to less interference to the PU, while a smaller P_F results in higher spectrum efficiency. In this paper, our aim is to minimize P_M, given the maximum tolerable P_F.

III. OPTIMAL SOFT COMBINING

A. THEORETICAL ANALYSIS FOR OPTIMAL COMBINING RULE

From the Neyman-Pearson criterion [35], the optimal combining rule for the cooperative network is the LLR test of the SUs measurement vector $(Y_1, Y_2, \ldots, Y_N)$, which can be expressed as

$$D_{OC}=\sum_{j=1}^{N}\ln\left(\frac{f\left(Y_j|\mathcal{H}_1\right)}{f\left(Y_j|\mathcal{H}_0\right)}\right)\underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}}\eta, \tag{10}$$

where D_{OC} is the decision metric, $f\left(Y_j|\mathcal{H}_i\right), i=0,1$ represents the PDF of Y_j conditioned on hypothesis $\mathcal{H}_i$, and η is the threshold determined according to the noise statistics and the constraint on the P_F.

If the instantaneous SNR values, i.e., $\{\gamma_j\}$s are known by the FC, the optimal combining rule could be derived from (6), (7) and (10) [44]:

$$D_{LR}=\sum_{j=1}^{N}\frac{\gamma_j}{1+\gamma_j}Y_j. \tag{11}$$

In [45] and [46], similar linear combining rule is obtained by applying the Gaussian approximation on the Y_j based on the CLT for the unknown deterministic primary signal model [11], where the local LLR is very complex and makes the optimal soft combining scheme very complicated.

In practical scenarios, the exact value of γ_j is not known by the j-th CR user or the FC. Consequently, the D_{LR} in (11) may not be applied directly, and we face a composite hypothesis problem with random parameters [60], [61].

To calculate the optimal fusion rule, we must compute the LLR function conditioned on the instantaneous SNR values and integrate over the distribution of the SNR. From the conditional PDF of γ_j in (3), we have the conditional PDF of Y_j given $\bar{\gamma}_j$ under $\mathcal{H}_1$:

$$\begin{aligned}f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)&=\int_0^\infty f\left(Y_j|\mathcal{H}_1,\gamma\right)f_{\gamma_j}(\gamma|\bar{\gamma}=\bar{\gamma}_j)d\gamma\\&=\int_0^\infty\left(\frac{m}{\bar{\gamma}_j}\right)^m\left(\frac{M}{1+\gamma}\right)^M\frac{Y_j^{M-1}\gamma^{m-1}}{\Gamma(M)\Gamma(m)}\\&\quad\times e^{-\frac{MY_j}{1+\gamma}-\frac{m\gamma}{\bar{\gamma}_j}}d\gamma.\end{aligned} \tag{12}$$

Applying the Bayesian rule in [60], we obtain the optimal combining rule via replacing the $f\left(Y_j|\mathcal{H}_1\right)$ in (10) with the $f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)$, which is named as the ALR detector in [50]:

$$\begin{aligned}D_{ALR}&=\sum_{j=1}^{N}\ln\frac{f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)}{f\left(Y_j|\mathcal{H}_0\right)}\\&=\sum_{j=1}^{N}\ln\frac{\int_0^\infty f\left(Y_j|\mathcal{H}_1,\gamma\right)f_{\gamma_j}(\gamma|\bar{\gamma}=\bar{\gamma}_j)d\gamma}{f\left(Y_j|\mathcal{H}_0\right)}\\&=\sum_{j=1}^{N}\ln\int_0^\infty\left(\frac{m}{\bar{\gamma}_j}\right)^m\left(\frac{1}{1+\gamma}\right)^M\frac{\gamma^{m-1}}{\Gamma(m)}e^{\frac{M\gamma Y_j}{1+\gamma}-\frac{m\gamma}{\bar{\gamma}_j}}d\gamma.\end{aligned} \tag{13}$$

One can see that due to the integral expression in the $f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)$, the D_{ALR} in (13) is hard to be implemented in practice. Although authors in [50], suggest the confluent hypergeometric function to simplify the computation of the ALR detector, it is still a hard work to solve the confluent hypergeometric function in practice. In the rest of this section, we will propose general and practical methods to realize the integrals in (13).

B. MOMENTS OF THE LOCAL STATISTICS Y_j

Most approximation methods are designed based on the fitting of the moments. In this part, we derive the k-th moment of Y_j which will be used later.

According to the signal model, the distribution of Y_j under hypothesis $\mathcal{H}_0$ only depends on the noise. Hence, Y_j follows a central chi-square distribution as shown in (6), and we can easily derive the moments of Y_j under $\mathcal{H}_0$ [62]:

Lemma 1: when the primary signal is absent, the k-th moment ($k \geq 1$) of the estimated average energy by the j-th cognitive user can be calculated according to the following expression:

$$E\left[Y_j^k|\mathcal{H}_0\right]=\frac{\Gamma(M+k)}{\Gamma(M)M^k}, \tag{14}$$

where $E[\cdot]$ denotes expectation.

For hypothesis $\mathcal{H}_1$, the derivation of moments becomes a little complex. According to the PDF of Y_j under $\mathcal{H}_1$, i.e., $f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)$, we have:

$$\begin{aligned}E\left[Y_j^k|\mathcal{H}_1\right] &= \int_0^\infty Y_j^k f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)dY_j \\ &= \int_0^\infty\int_0^\infty Y_j^k f\left(Y_j|\mathcal{H}_1,\gamma\right)f_{\gamma_j}(\gamma|\bar{\gamma}=\bar{\gamma}_j)d\gamma dY_j \\ &= \int_0^\infty E\left[Y_j^k|\mathcal{H}_1,\gamma\right]f_{\gamma_j}(\gamma|\bar{\gamma}=\bar{\gamma}_j)d\gamma. \end{aligned}\quad (15)$$

According to $f\left(Y_j|\mathcal{H}_1,\gamma_j\right)$ in (7) and the moments of chi-square distribution [62], we have

$$E\left[Y_j^k|\mathcal{H}_1,\gamma\right] = \frac{\Gamma(M+k)}{\Gamma(M)}\left(\frac{1+\gamma}{M}\right)^k. \quad (16)$$

Substituting (16) into (15), we have

$$\begin{aligned}&E\left[Y_j^k|\mathcal{H}_1\right] \\ &= \int_0^\infty \frac{\Gamma(M+k)}{\Gamma(M)}\left(\frac{1+\gamma}{M}\right)^k f_{\gamma_j}(\gamma|\bar{\gamma}=\bar{\gamma}_j)d\gamma \\ &= \int_0^\infty \frac{\Gamma(M+k)}{\Gamma(M)M^k}\left[\sum_{i=0}^k\binom{k}{i}\gamma^i\right]f_{\gamma_j}(\gamma|\bar{\gamma}=\bar{\gamma}_j)d\gamma. \end{aligned}\quad (17)$$

Finally, according to the conditional PDF of the instantaneous SNR γ_j shown in (3), which has Gamma distribution, we obtain:

Lemma 2: when the primary signal is present, the k-th moment ($k \geq 1$) of the estimated average energy by the j-th cognitive user is calculated as:

$$E\left[Y_j^k|\mathcal{H}_1\right] = \frac{\Gamma(M+k)}{\Gamma(M)M^k}\sum_{i=0}^k\binom{k}{i}E\left[\gamma_j^i\right], \quad (18)$$

where

$$E\left[\gamma_j^i\right] = \frac{\Gamma(m+i)}{\Gamma(m)}\left(\frac{\bar{\gamma}_j}{m}\right)^i.$$

C. PRACTICAL REALIZATION OF D_{ALR}

In this part, we present two solutions to approximate the $f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)$ in (12). Then two practical combining rules are obtained by substituting the approximated expressions into D_{ALR} in (13).

1) p-POINT MoG APPROXIMATION

Our first solution is based on the two-point MoG approximation [53], [63]. In [53], this method shows high accuracy on the $f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)$ in fast-fading scenario. However, contrary to the fast-fading scenario in [53], the distribution of Y_j (i.e., $f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)$) is totally different in the block-fading scenario, so we need to check whether the MoG method still works well.

We extend the MoG approximation to p points, which is also based on the moments fitting method proposed in [63]–[65]. The $f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)$ is approximated with mixtures of p gamma distributions:

$$f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right) \approx \sum_{i=1}^p \frac{w_{i,j}}{\Gamma(k_j)\theta_{i,j}^{k_j}}Y_j^{k_j-1}e^{-\frac{Y_j}{\theta_{i,j}}}, \quad (19)$$

where the parameters k_j, $\theta_{i,j}$, and $w_{i,j}$ ($1 \leq i \leq p$) are calculated based on the first $2p$ moments of Y_j under $\mathcal{H}_1$ from *Lemma* 2 according to Appendix.

Substituting (6) and (19) into the D_{ALR} in (13), the MoG approximated ALR (MoG) detector is derived as:

$$D_{\mathrm{MoG}} = \sum_{j=1}^N\left[MY_j + (k_j - M)\ln Y_j + \ln\sum_{i=1}^p\frac{w_{i,j}}{\theta_{i,j}^{k_j}}e^{-\frac{Y_j}{\theta_{i,j}}}\right]. \quad (20)$$

The main drawback of the p-point MoG D_{MoG} in (20) is that all the parameters k_j, $\theta_{i,j}$, and $w_{i,j}$ ($1 \leq i \leq p$) should be updated when $\bar{\gamma}_j$ changes. Although in [63], some algebric simplifications were suggested to compute these parameters with numerical methods, it is still complicated, especially for larger values of p.

2) GENERALIZED GAUSS-LAGUERRE FORMULA (GLF) APPROXIMATION

To overcome the dependency of the parameters in the MoG on the average SNR values, we suggest another approximation method according to the n-th order GLF [66, eq.3.5.27]:

$$\int_0^\infty x^\beta e^{-x}f(x)dx \approx \sum_{k=1}^n w_k f(\theta_k), \quad (21)$$

where $\beta > -1$, θ_k is the k-th zero of the generalized Laguerre polynomial $L_n^\beta(x)$, which is defined as [66]–[68]

$$L_n^\beta(x) = \frac{x^{-\beta}e^x}{n!}\frac{d^n}{x^n}\left(e^{-x}x^{n+\beta}\right), \quad (22)$$

and the weight w_k is given by

$$w_k = \frac{\Gamma(n+\beta+1)\theta_k}{n!\left[(n+1)L_{n+1}^\beta(\theta_k)\right]^2}. \quad (23)$$

As presented in (12), the $f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)$ is derived by integrating $f\left(Y_j|\mathcal{H}_1,\gamma\right)$ over the PDF of the instantaneous SNR γ_j as follows:

$$\begin{aligned}f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right) &= \int_0^\infty f\left(Y_j|\mathcal{H}_1,\gamma\right)f_{\gamma_j}(\gamma|\bar{\gamma}=\bar{\gamma}_j)d\gamma \\ &= \int_0^\infty\left(\frac{m}{\bar{\gamma}_j}\right)^m\frac{\gamma^{m-1}}{\Gamma(m)}e^{-\frac{m\gamma}{\bar{\gamma}_j}}f\left(Y_j|\mathcal{H}_1,\gamma\right)d\gamma. \end{aligned}\quad (24)$$

Let $x = \frac{m\gamma}{\bar{\gamma}_j}$, then $f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)$ in (24) can be written as

$$f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right) = \int_0^\infty\frac{x^{m-1}}{\Gamma(m)}e^{-x}f\left(Y_j|\mathcal{H}_1,\frac{\bar{\gamma}_j}{m}x\right)dx. \quad (25)$$

Applying the GLF in (21) with $\beta = m-1$ results in

$$f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right) \approx \sum_{k=1}^n\frac{w_k}{\Gamma(m)}f\left(Y_j|\mathcal{H}_1,\gamma_j = \frac{\bar{\gamma}_j}{m}\theta_k\right), \quad (26)$$

where w_k and θ_k ($k = 1,\ldots,n$) are the GLF parameters defined in (22) and (23).

Substituting $f\left(Y_j|\mathcal{H}_1,\gamma\right)$ in (7) into (26), we have

$$f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)\approx\frac{M^M Y_j^{M-1}}{\Gamma(m)\Gamma(M)}\sum_{k=1}^{n}\frac{w_k}{\left(1+\frac{\bar{\gamma}_j}{m}\theta_k\right)^M}e^{-\frac{mM}{m+\bar{\gamma}_j\theta_k}Y_j}. \tag{27}$$

Actually, the GLF approximated PDF in (27) is also a kind of MoG approximation with a shape parameter M and scale parameters $\frac{1}{M}(1+\frac{\bar{\gamma}_j}{m}\theta_k)$.

Substituting the approximated PDF into the ALR in (13), the GLF approximated ALR (GLF) detector is derived as below:

$$D_{\mathrm{GLF}}=\sum_{j=1}^{N}\ln\left[\frac{1}{\Gamma(m)}\sum_{k=1}^{n}\frac{w_k}{\left(1+\frac{\bar{\gamma}_j}{m}\theta_k\right)^M}e^{\frac{M\bar{\gamma}_j\theta_k}{m+\bar{\gamma}_j\theta_k}Y_j}\right]. \tag{28}$$

Comparing with the p-point MoG approximation, all the parameters w_k and θ_k ($k=1,\ldots,n$) for the GLF approximation in (26) are independent of $\bar{\gamma}_j$. Hence, they may be calculated off-line and saved in the FC. As a result, to make the approximation error as low as possible, we can increase n with a little increasing in computational complexity of the combining rule in the FC. Furthermore, as the w_k and θ_k are not related to the distribution of Y_j, the GLF approximation in (26) is suitable for all kinds of primary signals.

Fig. 2 compares the conditional PDFs of Y_j under $\mathcal{H}_1$, i.e., $f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)$, obtained from the MoG and GLF approximations when $M=10$ and $\bar{\gamma}_j=0$ dB. An obvious mismatch exists between the PDFs from the two-point MoG approximation and the PDFs from simulation. The MoG approximation performs much better when $p=4$. On the other hand, the GLF approximation ($n=100$) always show high accuracy.

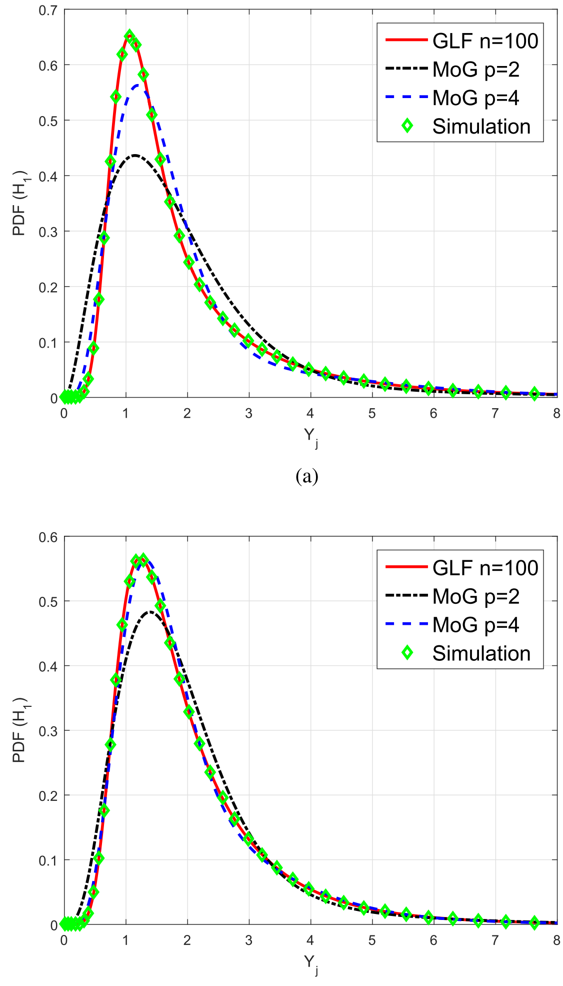


Fig. 2. **Comparison among the PDFs of Y_j under $\mathcal{H}_1$ obtained by the suggested MoG and GLF approximations and the PDFs obtained by simulation, for different m with $M=10$ samples and $\bar{\gamma}_j=0$ dB. (a) $m=0.5$. (b) $m=1$.**

IV. SIMPLIFIED BUT SUBOPTIMAL SOFT COMBINING

In practical applications, the CR network should make a tradeoff between the detection performance and the implementation complexity. For instance, for a single CR user scenario, the optimal detector is the local LLR test [11]–[15], but the most widely used detector is the energy detector for its low-complexity and good performance. This motivates us to investigate a simple-structure suboptimal combining rule with low performance degradation compared to the optimal one, i.e., the ALR detector, for the multiple CR users scenarios.

A. CONVENTIONAL COMBINING RULES

In the existing literature, there are several well-known conventional combining rules. These rules will be compared with our proposed suboptimal rules in section V.

1) EQUAL GAIN COMBINING (EGC)

The FC simply sums the received energies $\{Y_j\}$s and compares the result to a constant threshold, which is computed according to the constraint on the P_{F}:

$$D_{\mathrm{EGC}}=\sum_{j=1}^{N}Y_j\underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}}\eta_{\mathrm{EGC}}. \tag{29}$$

2) MAXIMAL RATIO COMBINING (MRC)

If the average SNRs $\{\bar{\gamma}_j\}$ of cooperating CR users are different, a weighted combination may work better:

$$D_{\mathrm{MRC}}=\frac{1}{\sum_{j=1}^{N}\bar{\gamma}_j}\sum_{j=1}^{N}\bar{\gamma}_jY_j\underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}}\eta_{\mathrm{MRC}}. \tag{30}$$

The weighting factors in D_{MRC} are chosen based on a very simple principle: a high-SNR SU should be assigned a larger weighting coefficient to enhance its contribution to the global decision, while the weight of a low-SNR SU should be decreased in order to limit its contribution in the final decision.

Remark that the MRC in (30) is different from the one defined in [44]. Since in this paper we assume that the instantaneous SNRs are unavailable, we use the average SNRs as the weighting factors instead.

3) MODIFIED DEFLECTION COEFFICIENT (MDC) BASED LINEAR COMBINING

The deflection coefficient (DC) is a widely-used metric for detection performance [15], [60], [69]:

$$d^2 = \frac{(E[D|\mathcal{H}_1] - E[D|\mathcal{H}_0])^2}{Var[D|\mathcal{H}_0]}, \tag{31}$$

where D is the decision variable in the fusion rule. In [45] and [33], a modified DC is suggested, which is defined as

$$d_{\text{M}}^2 = \frac{(E[D|\mathcal{H}_1] - E[D|\mathcal{H}_0])^2}{Var[D|\mathcal{H}_1]}. \tag{32}$$

An optimal linear combining rule is proposed in [45], as follows:

$$D_{\text{MDC}} = \sum_{j=1}^{N} w_{j,\text{MDC}} Y_j \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_{\text{MDC}}, \tag{33}$$

where $w_{j,\text{MDC}}$ is the weighting factor, which is derived such that the MDC will be maximized [45]:

$$\begin{aligned} w_{j,\text{MDC}} &= \frac{E\left[Y_j|\mathcal{H}_1\right] - E\left[Y_j|\mathcal{H}_0\right]}{Var\left[Y_j|\mathcal{H}_1\right]} \\ &= \frac{mM\bar{\gamma}_j}{m + 2m\bar{\gamma}_j + (M+m+1)\,\bar{\gamma}_j^{\,2}}. \end{aligned} \tag{34}$$

The D_{MDC} shows good detection performance in the scenarios where the instantaneous SNRs $\{\gamma_j\}$s are known [45].

4) GENERALIZED LIKELIHOOD RATIO (GLR) TEST

In this paper, as we discussed, we assume that the instantaneous SNR γ_j is a random variable. When the FC doesn't have a priori information about γ_j, the GLR can be applied as the decision variable [60]:

$$D_{\text{GLR}} = \sum_{j=1}^{N} \ln \frac{f\left(Y_j|\mathcal{H}_1, \hat{\gamma}_{j,\text{ML}}\right)}{f\left(Y_j|\mathcal{H}_0\right)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_{\text{GLR}}, \tag{35}$$

where $\hat{\gamma}_{j,\text{ML}} = Y_j - 1$ is the maximum likelihood estimation (MLE) of γ_j [70], and we have

$$f\left(Y_j|\mathcal{H}_1, \hat{\gamma}_{j,\text{ML}}\right) = \frac{M^M}{\Gamma(M)} Y_j^{-1} e^{-M}. \tag{36}$$

By discarding irrelevant constant terms, the D_{GLR} finally becomes:

$$D_{\text{GLR}} = \sum_{j=1}^{N} \left(Y_j - \ln Y_j\right). \tag{37}$$

B. NOVEL SUBOPTIMAL LINEAR COMBINING RULE

Although the conventional combining rules presented above have simpler structure comparing to the D_{ALR} in (13), the D_{MoG} in (20) and the D_{GLF} in (28), they do not fully utilize the priori information about the primary signal and channel gains. Hence, they could not guarantee good performance in all the cases.

In this subsection, we propose a novel suboptimal combining rule according to the GLF approximated ALR detector (D_{GLF}) in (28). Considering that our design is based on the ALR detector (the optimal combining rule), we expect that the proposed combining rule will perform better than the conventional rules.

As we know, the optimal combining rule D_{ALR} is the sum of the local LLRs. According to the GLF approximation, we denote the local LLR for the j-th CR user as a function of Y_j:

$$\begin{aligned} G(Y_j;\bar{\gamma}_j) &= \ln \frac{f\left(Y_j|\mathcal{H}_1,\bar{\gamma}_j\right)}{f\left(Y_j|\mathcal{H}_0\right)} \\ &= \ln\left[\sum_{k=1}^{n} g\left(Y_j;k,\bar{\gamma}_j\right)\right], \end{aligned} \tag{38}$$

where

$$g\left(Y_j;k,\bar{\gamma}_j\right) = \frac{w_k}{\Gamma(m)\left(1+\frac{\bar{\gamma}_j}{m}\theta_k\right)^M} e^{\frac{M\bar{\gamma}_j\theta_k}{m+\bar{\gamma}_j\theta_k} Y_j}. \tag{39}$$

Then the D_{GLF} in (28) can be expressed as

$$D_{\text{GLF}} = \sum_{j=1}^{N} G(Y_j;\bar{\gamma}_j). \tag{40}$$

The LLR function $G(Y_j;\bar{\gamma}_j)$ in (38) is a log-sum-exp function [71]–[73]. We modify it with a log-max-exp function:

$$\overline{G}(Y_j;\bar{\gamma}_j) = \ln\left[\max_{1\le k\le n} g\left(Y_j;k,\bar{\gamma}_j\right)\right], \tag{41}$$

which means that we choose the maximum terms among the n exponential functions $g\left(Y_j;k,\bar{\gamma}_j\right)$ $(1 \le k \le n)$ as the combining variable corresponding to the j-th CR user.

The $\overline{G}(Y_j;\bar{\gamma}_j)$ in (41) can be further simplified as the maximum among a group of linear functions, since the logarithm function $\ln(\cdot)$ is strictly increasing:

$$\begin{aligned} \overline{G}(Y_j;\bar{\gamma}_j) &= \max_{1\le k\le n} \ln\left[g\left(Y_j;k,\bar{\gamma}_j\right)\right] \\ &= \max_{1\le k\le n} \left(a_{j,k}Y_j + b_{j,k}\right), \end{aligned} \tag{42}$$

where for the k-th linear function, the slope is

$$a_{j,k} = \frac{M\bar{\gamma}_j\theta_k}{m+\bar{\gamma}_j\theta_k}, \tag{43}$$

and the intercept is

$$b_{j,k} = \ln\left[\frac{w_k}{\Gamma(m)\left(1+\frac{\bar{\gamma}_j}{m}\theta_k\right)^M}\right]. \tag{44}$$

The slope $a_{j,k}$ in (43) is non-negative. Therefore, $\overline{G}(Y_j; \bar{\gamma}_j)$ is the maximum of increasing linear functions, and it is convex.

In Fig. 3, $G(Y_j; \bar{\gamma}_j)$ and $\overline{G}(Y_j; \bar{\gamma}_j)$ are compared for different values of observation samples M and Nakagami parameter m with $n = 100$ and $\bar{\gamma}_j = 0$ dB. We can see that $\overline{G}(Y_j; \bar{\gamma}_j)$ could always keep nearly the same slope of $G(Y_j; \bar{\gamma}_j)$ for all the values of Y_j. As a result, we can conclude that $\overline{G}(Y_j; \bar{\gamma}_j)$ contains the main information about $G(Y_j; \bar{\gamma}_j)$, and it is possible to build a well-performed combining rule based on $\overline{G}(Y_j; \bar{\gamma}_j)$.

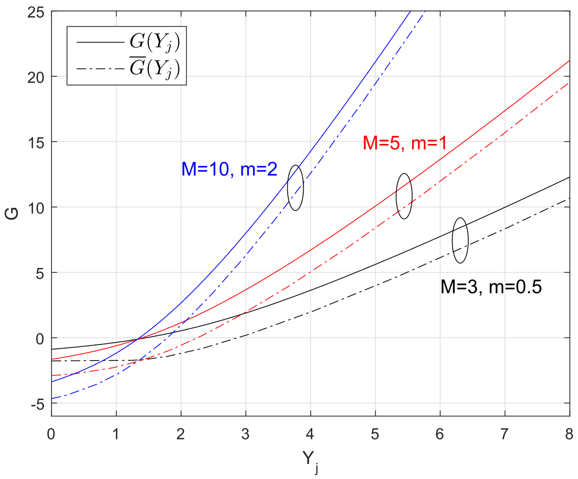


Fig. 3. **Comparison between $G(Y_j; \bar{\gamma}_j)$ in (38) and $\overline{G}(Y_j; \bar{\gamma}_j)$ in (42) for different values of observation samples M and Nakagami parameter m when $\bar{\gamma}_j = 0$ dB and GLF approximation order $n = 100$.**

Our proposed combining rule, which is named as the GLF-based linear combining (GLFL) detector, can be constructed as follows:

$$D_{\text{GLFL}} = \sum_{j=1}^{N} \Big[\max_{1\le k\le n} \left(a_{j,k} Y_j + b_{j,k}\right) \Big] \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_{\text{GLFL}}, \tag{45}$$

where the $a_{j,k}$ and $b_{j,k}$ have been defined in (43) and (44), respectively. Comparing with the D_{GLF} in (28) and the D_{MoG} in (20), there is no exponential or logarithm function in the D_{GLFL}, and it can be implemented by linear functions and a comparator.

We denote the index of the maximum element in (42) as follows

$$k_j^*(Y_j) = \arg\max_k \ln \left[g\left(Y_j; k, \bar{\gamma}_j\right) \right]. \tag{46}$$

Fig. 4 depicts the $k_j^*(Y_j)$ associated with the $\overline{G}(Y_j; \bar{\gamma}_j)$ in Fig. 3. The $k_j^*(Y_j)$ is a non-decreasing step function due to the limited value of n. Thus the $\overline{G}(Y_j; \bar{\gamma}_j)$ is a piecewise linear function, and the slope of the tangent line at $Y_j = y_0$ is $a_{j,k_j^*(y_0)}$.

According to the IEEE 802.22 standard, in CR networks, CR users should reliably detect primary signals, on the basis of the requirements on both P_{F} and P_{M}, in low SNR regime due to possible shadowing [14], [59]. When $\bar{\gamma}_j \approx 0$, we have $E\left[Y_j|\mathcal{H}_1\right] \approx E\left[Y_j|\mathcal{H}_0\right]$, and Y_j distributes around $E\left[Y_j|\mathcal{H}_0\right]$ with probability nearly unity.

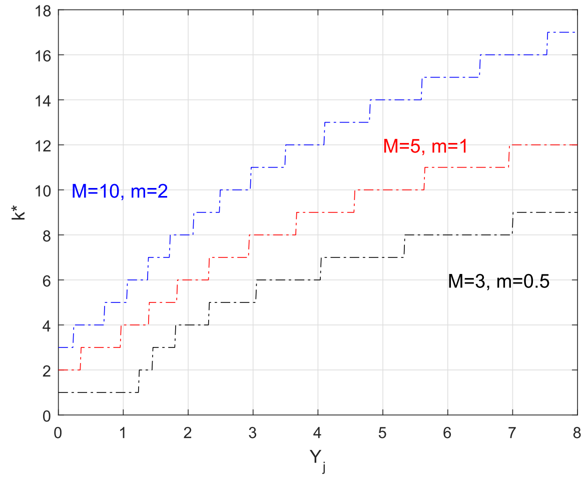


Fig. 4. **The index $k_j^*(Y_j)$ of the maximum component of $G(Y_j; \bar{\gamma}_j)$ corresponding to the $\overline{G}(Y_j; \bar{\gamma}_j)$ curves shown in Fig. 3.**

If the CR network is designed for a low-SNR sensing scenario, the D_{GLFL} in (45) can be further simplified. We suggest a novel energy weighted combining method as below by discarding irrelevant constant terms, and we name it as the GLF-based weighted-energy (GLFW) detector:

$$D_{\text{GLFW}} = \sum_{j=1}^{N} a_{j,\text{GLFW}} Y_j \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_{\text{GLFW}}, \tag{47}$$

where the weighting factor $a_{j,\text{GLFW}}$ is the slope of the tangent function for $Y_j = E\left[Y_j|\mathcal{H}_0\right]$:

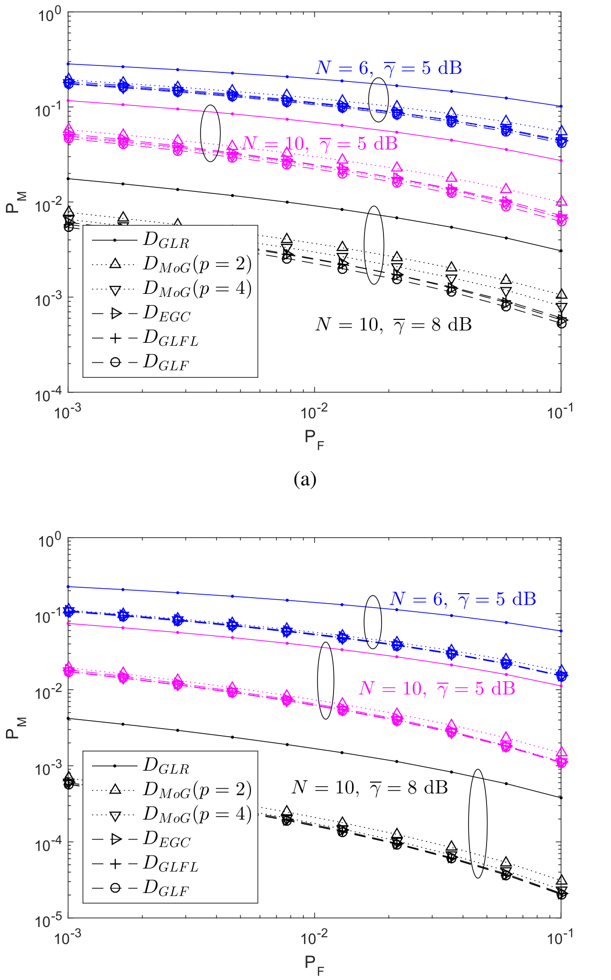
$$a_{j,\text{GLFW}} = \frac{M \bar{\gamma}_j \theta_{k_j^*(Y_j = E[Y_j|\mathcal{H}_0])}}{m + \bar{\gamma}_j \theta_{k_j^*(Y_j = E[Y_j|\mathcal{H}_0])}}. \tag{48}$$

V. SIMULATION RESULTS

In this section, to evaluate the performance of the proposed detectors, we plot the complementary Receiver Operating Characteristic (ROC) curve, which is the missed detection P_{M} in terms of false alarm P_{F}.

First, we consider the special case of equal large-scale fading for all the CR users, which causes all the CR nodes experience same average SNRs $\bar{\gamma}$. In this case, the D_{MRC} detector in (30), D_{MDC} detector in (33), and D_{GLFW} detector (47) all reduce to the EGC detector. The performance of D_{GLR}, D_{EGC}, D_{MoG}, D_{GLFL} and D_{GLF} are depicted in Fig. 5 for different values of $\bar{\gamma}$ and N with observation samples equal to $M = 2$.

According to Fig. 5, the D_{GLF} ($n = 100$) always has the best performance, and the D_{GLFL} has very close performance to the D_{GLF}. For the D_{MoG}, there is a small performance degradation for the case of the MoG approximation points $p = 2$ especially when $m = 0.5$ and $\bar{\gamma}$ is high. Nevertheless, the degradation is alleviated when we increase the MoG approximation points to $p = 4$, which can be inferred from the comparison of PDFs in Fig. 2. In fact, when $\bar{\gamma}$ approaches to 0, Y_j converges to the Gamma distribution



(a)

(b)

Fig. 5. **Performance comparison among the D_{GLR}, D_{EGC}, D_{MoG}, D_{GLFL} and D_{GLF}, when all the sensing channels have same average SNRs and $M = 2$. (a) $m = 0.5$. (b) $m = 1$.**

under hypothesis $\mathcal{H}_1$, so $p = 2$ is enough in low SNR case. However, when $\bar{\gamma}$ is high, the distribution of Y_j under $\mathcal{H}_1$ is mainly determined by the signal component, and it is very complex as shown in (12), so we need a larger p for a better approximation. We set $p = 4$ for the D_{MoG} in the rest simulations, as the additional computation complexity is not too much to increase p from 2 to 4.

As we see, for the case of equal average SNRs, the D_{EGC} performs well. However, the D_{GLR} has very poor performance. Since the D_{GLR} in (37) doesn't utilize the available information about $\bar{\gamma}_j$, it's performance will be degraded more in the scenarios where CR users have different values of average SNRs. Hence, we will not consider the D_{GLR} in the rest of the simulations.

For the rest of the simulations, we consider a CR network consisting of $N = 15$ users. All CR users are uniformly-located from $d = 50$m to $d = 200$m from the PU, so they have different path-losses. Similar to [53], the path-losses are obtained from the simplified model $P_{\mathrm{L},j}(d) = P_0(\frac{d_0}{d})^{\rho}$, where $d_0 = 50$m is the reference distance from PU, P_0 is the large-scale factor at d_0, and ρ is the path-loss exponent. Here, the outdoor environment of an urban microcell is investigated where $\rho = 3$. We set Nakagami parameter $m = 1$, i.e., Rayleigh fading is considered. The average SNRs are different for different users due to different geographical positions. We define the global average SNR as $\bar{\gamma}_{\mathrm{G}} = \frac{1}{N}\sum_{j=1}^{N}\bar{\gamma}_j$.

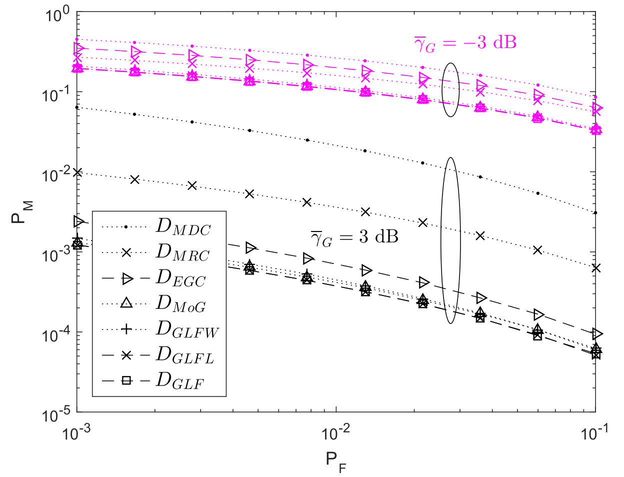


Fig. 6. **Complementary ROC curves of all presented combining rules except the D_{GLR}, with $M = 6$ and $N = 15$ for different $\bar{\gamma}_{\mathrm{G}}$.**

In Fig. 6, we depict the complementary ROC curves of the conventional combining rules D_{EGC}, D_{MRC} and D_{MDC}, the proposed approximated solutions of the ALR detector, i.e., D_{MoG} and D_{GLF}, and the proposed sub-optimal combining rules, i.e., D_{GLFL} and D_{GLFW}, with $M = 6$ observation samples for different values of $\bar{\gamma}_{\mathrm{G}}$. From these curves, we can conclude that the proposed combining rules always outperform the D_{EGC}, and the D_{GLF} always shows the best performance. The D_{GLFL} and D_{MoG} ($p = 4$) have very close performance compared to the D_{GLF}. The energy weighted combining method D_{GLFW} has a slight performance degradation, but it still outperforms the D_{EGC}. On the other hand, the D_{MDC} performs poorly in all cases. The performance of D_{MRC} is slightly better than the D_{EGC} when $\bar{\gamma}_{\mathrm{G}} = -3$dB, but it degrades when $\bar{\gamma}_{\mathrm{G}}$ increases to 3dB.

In Fig. 7, we fix the $\bar{\gamma}_{\mathrm{G}} = -3$ dB to verify the detection performance in low SNR cases. We compare all the combining rules presented in Fig. 6 for observation samples equal to $M = 6$ and $M = 30$. The D_{MDC} has poor performance as before, and the D_{MRC} preforms worse than the D_{EGC} when $M = 30$. Hence, we conclude that the D_{MDC} and D_{MRC} are not appropriate to be applied for the block-fading scenarios. On the other hand, the D_{GLF} still has the best performance as expected. As another result, one can see that, increasing the observation samples, can improve the detection performance. The MoG approximation based rule D_{MoG} and the proposed linear combining rule D_{GLFL} have similar performances to the D_{GLF}. The energy weighted combining method D_{GLFW} still has a little performance degradation. We conjecture that this is because of the inner performance gap between the energy

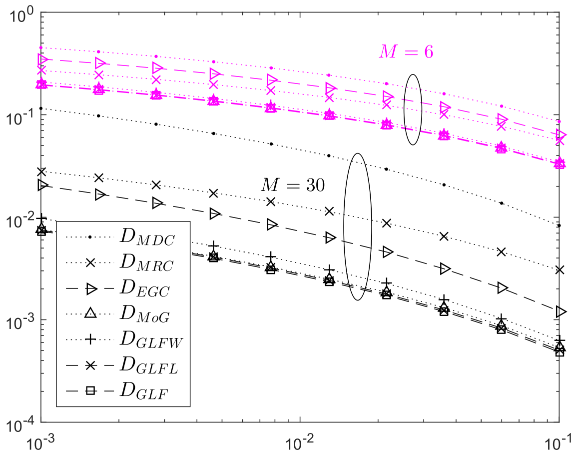


Fig. 7. **Complementary ROC curves of all presented combining rules except the D_{GLR}, with $\bar{\gamma}_{\text{G}} = -3$ dB and $N = 15$ for different M.**

weighted combining method and the optimal ALR detector. Nevertheless, this is acceptable, since the combining structure of D_{GLFW} is very simple, and there is still a significant improvement comparing to the D_{EGC}.

TABLE 1. **The values of P_{M} when $P_{\text{F}} = 0.1$ in Fig. 6 and Fig. 7. (a) $\bar{\gamma}_{\text{G}} = -3$ dB and $M = 6$. (b) $\bar{\gamma}_{\text{G}} = 3$ dB and $M = 6$. (c) $\bar{\gamma}_{\text{G}} = -3$ dB and $M = 30$.**

(a) $\bar{\gamma}_{\text{G}} = -3$ dB and $M = 6$

	MDC	MRC	EGC	MoG	GLFW	GLFL	GLF
P_{M} $\times 10^2$	8.58	5.61	6.36	3.34	3.49	3.31	3.28

(b) $\bar{\gamma}_{\text{G}} = 3$ dB and $M = 6$

	MDC	MRC	EGC	MoG	GLFW	GLFL	GLF
P_{M} $\times 10^4$	30.8	6.36	0.94	0.60	0.62	0.54	0.52

(c) $\bar{\gamma}_{\text{G}} = -3$ dB and $M = 30$

	MDC	MRC	EGC	MoG	GLFW	GLFL	GLF
P_{M} $\times 10^3$	8.30	3.04	1.21	0.54	0.63	0.50	0.48

As a case in point, in TABLE 1, we fix the false alarm probability at $P_{\text{F}} = 0.1$ and list the values of P_{M} in Fig. 6 and Fig. 7. We can see that the GLF detector always has the lowest missed detection probability. Besides, the MoG detector ($p = 4$) is also well-performed. The proposed GLFL detector has a little lower P_{M} than the MoG detector ($p = 4$), although it has a simpler structure for implementation. We should mention that, if we increase the p, the MoG may become better. But considering the extremely limited performance gains and the computation complexity, we didn't try it in the simulations. Moreover, we can see that the energy weighted combining method, i.e., the GLFW detector outperforms the conventional EGC detector, and has a slight degradation comparing to the GLF, MoG and GLFL detectors. For instance, when $\bar{\gamma}_{\text{G}} = -3$ dB and $M = 30$, the missed detection probability of GLFW detector is 6.3×10^{-4}, which is much lower than the 1.21×10^{-3} of EGC detector. Finally, the P_{M} of the MDC detector is much higher than the one of the EGC detector in all the cases. The MRC detector has competitive performance with the EGC detector when $\bar{\gamma}_{\text{G}} = -3$ dB and $M = 6$, but its missed detection probability decreases too slowly comparing to the EGC detector when the average SNR $\bar{\gamma}_{\text{G}}$ or sample length M increases.

VI. CONCLUSION

We investigate the soft combining methods for cooperative spectrum sensing over block-fading channels under the assumption that the instantaneous SNR is unknown for the CR network. The optimal fusion rule D_{ALR} is derived based on the Bayesian rule. To implement the D_{ALR} which has many intractable integrals, we suggest two practical solutions D_{MoG} and D_{GLFL} via MoG approximation and GLF approximation, respectively. Based on the GLF approximated ALR (D_{GLFL}), we propose a sub-optimal but well-performed combining rule D_{GLFL}, which can be implemented by linear functions and a comparator. Furthermore, when the SNR is low, we simplify the D_{GLFL} as an energy weighted combining method D_{GLFW}, which has a more practical structure but suffers from a little performance degradation. Simulation results show that all the proposed detectors provide better sensing performance than the conventional ones, especially when the CR users have different large-scale fading characters.

APPENDIX
PARAMETERS FOR THE *p*-POINT MoG APPROXIMATION

The p-point MoG approximation is firstly proposed in [63] to approximate the distribution of the weighted sum of *i.i.d.* chi-square distributions. We briefly summarize the moments fitting method proposed in [63] to determine the parameters: k_j, $\theta_{i,j}$, and $w_{i,j}$ $(1 \le i \le p)$.

Firstly, the r-th pseudo-moment $\delta_r(\lambda)$ is defined as

$$\delta_r(\lambda) = \frac{E\left[Y_j^r|\mathcal{H}_1\right]}{(1+\lambda)(1+2\lambda)\cdots(1+(r+1)\lambda)}. \tag{A.1}$$

Then the p-th pseudo-moment matrix $\Delta_p(\lambda)$ is given

$$\Delta_p(\lambda) = \begin{bmatrix} 1 & \delta_1(\lambda) & \delta_2(\lambda) & \cdots & \delta_p(\lambda) \\ \delta_1(\lambda) & \delta_2(\lambda) & \delta_3(\lambda) & \cdots & \delta_{p+1}(\lambda) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \delta_p(\lambda) & \delta_{p+1}(\lambda) & \delta_{p+2}(\lambda) & \cdots & \delta_{2p}(\lambda) \end{bmatrix}. \tag{A.2}$$

Let $\tilde{\lambda}_p$ be the smallest nonnegative root, if it exists, of $\det[\Delta_p(\lambda)]$. The common shaping parameter is $k_j = 1/\tilde{\lambda}_p$.

Define the polynomial:

$$S_p(\tilde{\lambda}_p, t) = \det \begin{bmatrix} 1 & \delta_1(\tilde{\lambda}_p) & \cdots & \delta_{p-1}(\tilde{\lambda}_p) & 1 \\ \delta_1(\tilde{\lambda}_p) & \delta_2(\tilde{\lambda}_p) & \cdots & \delta_p(\tilde{\lambda}_p) & t \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \delta_p(\tilde{\lambda}_p) & \delta_{p+1}(\tilde{\lambda}_p) & \cdots & \delta_{2p-1}(\tilde{\lambda}_p) & t^p \end{bmatrix}. \tag{A.3}$$

The support points $\tilde{t}_1, \ldots, \tilde{t}_p$ are the p roots of $S_p(\tilde{\lambda}_p, t) = 0$ [64], [65], and then we have $\theta_{i,j} = \tilde{t}_i\tilde{\lambda}_p$.

The weights $w_{i,j}$ can be obtained by solving the following linear equation

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ \tilde{t}_1 & \tilde{t}_2 & \cdots & \tilde{t}_p \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{t}_1^{p-1} & \tilde{t}_2^{p-1} & \cdots & \tilde{t}_p^{p-1} \end{bmatrix} \begin{bmatrix} w_{1,j} \\ w_{2,j} \\ \vdots \\ w_{p,j} \end{bmatrix} = \begin{bmatrix} 1 \\ \delta_1(\tilde{\lambda}_p) \\ \vdots \\ \delta_{p-1}(\tilde{\lambda}_p) \end{bmatrix}. \tag{A.4}$$

The matrix on the left is a Vandermonde matrix, and the linear function in (A.4) could be solved with the Bjork-Pereyra algorithm [74]–[76].

REFERENCES

[1] R. Tandra, S. M. Mishra, and A. Sahai, "What is a spectrum hole and what does it take to recognize one?" *Proc. IEEE*, vol. 97, no. 5, pp. 824–848, May 2009.

[2] A. Ghasemi and E. S. Sousa, "Spectrum sensing in cognitive radio networks: Requirements, challenges and design trade-offs," *IEEE Commun. Mag.*, vol. 46, no. 4, pp. 32–39, Apr. 2008.

[3] J. Lunden, V. Koivunen, and H. V. Poor, "Spectrum exploration and exploitation for cognitive radio: Recent advances," *IEEE Signal Process. Mag.*, vol. 32, no. 3, pp. 123–140, May 2015.

[4] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Commun. Surveys Tut.*, vol. 11, no. 1, pp. 116–130, 1st Quart. 2009.

[5] G. B. Giannakis and M. K. Tsatsanis, "Signal detection and classification using matched filtering and higher order statistics," *IEEE Trans. Acoust., Speech Signal Process.*, vol. 38, no. 7, pp. 1284–1296, Jul. 1990.

[6] S. Kapoor, S. Rao, and G. Singh, "Opportunistic spectrum sensing by employing matched filter in cognitive radio network," in *Proc. Int. Conf. CSNT*, 2011, pp. 580–583.

[7] X. Zhang, R. Chai, and F. Gao, "Matched filter based spectrum sensing and power level detection for cognitive radio network," in *Proc. IEEE GlobalSIP*, Dec. 2014, pp. 1267–1270.

[8] Z. Yonghong and L. Ying-Chang, "Robustness of the cyclostationary detection to cyclic frequency mismatch," in *Proc. 21st Annu. IEEE Int. Symp. Pers. Indoor Mobile Radio Commun.*, Sep. 2010, pp. 2704–2709.

[9] A. Tkachenko, D. Cabric, and R. W. Brodersen, "Cyclostationary feature detector experiments using reconfigurable BEE2," in *Proc. IEEE Int. Symp. New Frontiers Dyn. Spectr. Access Netw.*, Apr. 2007, pp. 216–219.

[10] J. Riba, J. Font-Segura, J. Villares, and G. Vazquez, "Frequency-domain GLR detection of a second-order cyclostationary signal over fading channels," *IEEE Trans. Signal Process.*, vol. 62, no. 8, pp. 1899–1912, Apr. 2014.

[11] H. Guo, W. Jiang, and W. Luo, "A modified energy detector for random signals in Gaussian noise," *IEEE Commun. Lett.*, vol. 19, no. 8, pp. 1358–1361, Aug. 2015.

[12] F. F. Digham, M.-S. Alouini, and M. K. Simon, "On the energy detection of unknown signals over fading channels," *IEEE Trans. Commun.*, vol. 55, no. 1, pp. 21–24, Jan. 2007.

[13] A. A. Alkheir and H. T. Mouftah, "An improved energy detector using outdated channel state information," *IEEE Commun. Lett.*, vol. 19, no. 7, pp. 1237–1240, Jul. 2015.

[14] S. Atapattu, C. Tellambura, H. Jiang, and N. Rajatheva, "Unified analysis of low-SNR energy detection and threshold selection," *IEEE Trans. Veh. Technol.*, vol. 64, no. 11, pp. 5006–5019, Nov. 2015.

[15] I. Sobron, P. S. R. Diniz, W. A. Martins, and M. Velez, "Energy detection technique for adaptive spectrum sensing," *IEEE Trans. Commun.*, vol. 63, no. 3, pp. 617–627, Mar. 2015.

[16] M. Naraghi-Pour and T. Ikuma, "Autocorrelation-based spectrum sensing for cognitive radios," *IEEE Trans. Veh. Technol.*, vol. 59, no. 2, pp. 718–733, Feb. 2010.

[17] W. Han, C. Huang, J. Li, Z. Li, and S. Cui, "Correlation-based spectrum sensing with oversampling in cognitive radio," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 5, pp. 788–802, May 2015.

[18] Y. Zeng and Y. C. Liang, "Spectrum-sensing algorithms for cognitive radio based on statistical covariances," *IEEE Trans. Veh. Technol.*, vol. 58, no. 4, pp. 1804–1815, May 2009.

[19] A. Ghasemi and E. S. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *Proc. IEEE Int. Symp. New Frontiers Dyn. Spectr. Access Netw. (DySPAN)*, Nov. 2005, pp. 131–136.

[20] J. Unnikrishnan and V. V. Veeravalli, "Cooperative sensing for primary detection in cognitive radio," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 18–27, Feb. 2008.

[21] Z. Quan, S. Cui, H. V. Poor, and A. H. Sayed, "Collaborative wideband sensing for cognitive radios," *IEEE Signal Process. Mag.*, vol. 25, no. 6, pp. 60–73, Nov. 2008.

[22] S. J. Zahabi, A. A. Tadaion, and S. Aissa, "Neyman-pearson cooperative spectrum sensing for cognitive radio networks with fine quantization at local sensors," *IEEE Trans. Commun.*, vol. 60, no. 6, pp. 1511–1522, Jun. 2012.

[23] K. B. Letaief and W. Zhang, "Cooperative communications for cognitive radio networks," *Proc. IEEE*, vol. 97, no. 5, pp. 878–893, May 2009.

[24] R. Deng, J. Chen, C. Yuen, P. Cheng, and Y. Sun, "Energy-efficient cooperative spectrum sensing by optimal scheduling in sensor-aided cognitive radio networks," *IEEE Trans. Veh. Technol.*, vol. 61, no. 2, pp. 716–725, Feb. 2012.

[25] Y. Liu, S. Xie, R. Yu, Y. Zhang, and C. Yuen, "An efficient MAC protocol with selective grouping and cooperative sensing in cognitive radio networks," *IEEE Trans. Veh. Technol.*, vol. 62, no. 8, pp. 3928–3941, Oct. 2013.

[26] J. Shen, S. Liu, L. Zeng, G. Xie, J. Gao, and Y. Liu, "Optimisation of cooperative spectrum sensing in cognitive radio network," *IET Commun.*, vol. 3, no. 7, pp. 1170–1178, Jul. 2009.

[27] Q. Wang, D. W. Yue, and F. C. M. Lau, "Performance of cooperative spectrum sensing over fading channels with low signal-to-noise ratio," *IET Commun.*, vol. 6, no. 13, pp. 1988–1999, Sep. 2012.

[28] Y. Zhou, Z. Zhou, and B. Li, "Sensing nodes selection and data fusion in cooperative spectrum sensing," *IET Commun.*, vol. 8, no. 13, pp. 2308–2314, Sep. 2014.

[29] E. C. Y. Peh, Y.-C. Liang, Y. L. Guan, and Y. Zeng, "Cooperative spectrum sensing in cognitive radio networks with weighted decision fusion schemes," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3838–3847, Dec. 2010.

[30] W. Han, J. Li, Z. Li, J. Si, and Y. Zhang, "Efficient soft decision fusion rule in cooperative spectrum sensing," *IEEE Trans. Signal Process.*, vol. 61, no. 8, pp. 1931–1943, Apr. 2013.

[31] M. B. Ghorbel, H. Nam, and M. S. Alouini, "Soft cooperative spectrum sensing performance under imperfect and non identical reporting channels," *IEEE Commun. Lett.*, vol. 19, no. 2, pp. 227–230, Feb. 2015.

[32] W. Ejaz, G. Hattab, N. Cherif, M. Ibnkahla, F. Abdelkefi, and M. Siala, "Cooperative spectrum sensing with heterogeneous devices: Hard combining versus soft combining," *IEEE Syst. J.*, to be published.

[33] J. A. Maya, L. R. Vega, and C. G. Galarza, "A locally optimal soft linear-quadratic scheme for CR systems in shadowing environments," *IEEE Wireless Commun. Lett.*, vol. 5, no. 3, pp. 296–299, Jun. 2016.

[34] S. Chaudhari, J. Lunden, V. Koivunen, and H. V. Poor, "Cooperative sensing with imperfect reporting channels: Hard decisions or soft decisions?" *IEEE Trans. Signal Process.*, vol. 60, no. 1, pp. 18–28, Jan. 2012.

[35] P. K. Varshney, *Distributed Detection and Data Fusion*. New York, NY, USA: Springer-Verlag, 1997.

[36] R. Viswanathan and P. K. Varshney, "Distributed detection with multiple sensors part I. Fundamentals," *Proc. IEEE*, vol. 85, no. 1, pp. 54–63, Jan. 1997.

[37] R. S. Blum, S. A. Kassam, and H. V. Poor, "Distributed detection with multiple sensors II. Advanced topics," *Proc. IEEE*, vol. 85, no. 1, pp. 64–79, Jan. 1997.

[38] S. Appadwedula, V. V. Veeravalli, and D. L. Jones, "Decentralized detection with censoring sensors," *IEEE Trans. Signal Process.*, vol. 56, no. 4, pp. 1362–1373, Apr. 2008.

[39] S. P. Herath, N. Rajatheva, and C. Tellambura, "Energy detection of unknown signals in fading and diversity reception," *IEEE Trans. Commun.*, vol. 59, no. 9, pp. 2443–2453, Sep. 2011.

[40] A. Rao and M. S. Alouini, "Performance of cooperative spectrum sensing over non-identical fading environments," *IEEE Trans. Commun.*, vol. 59, no. 12, pp. 3249–3253, Dec. 2011.

[41] Y. F. Sharkasi, D. McLernon, and M. Ghogho, "Cooperative spectrum sensing over frequency-selective Nakagami-m fading channels," in *Proc. IEEE SSPD*, Sep. 2012, pp. 1–5.
[42] S. Nallagonda, S. K. Bandari, S. D. Roy, and S. Kundu, "Performance of cooperative spectrum sensing with soft data fusion schemes in fading channels," in *Proc. IEEE INDICON*, Dec. 2013, pp. 1–6.
[43] E. Adebola and A. Annamalai, "Unified analysis of energy detectors with diversity reception in generalised fading channels," *IET Commun.*, vol. 8, no. 17, pp. 3095–3104, 2014.
[44] J. Ma, G. Zhao, and Y. Li, "Soft combination and detection for cooperative spectrum sensing in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4502–4507, Nov. 2008.
[45] Z. Quan, S. Cui, and A. H. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 28–40, Feb. 2008.
[46] V. G. Chavali and C. R. C. M. Da Silva, "Collaborative spectrum sensing based on a new SNR estimation and energy combining method," *IEEE Trans. Veh. Technol.*, vol. 60, no. 8, pp. 4024–4029, Oct. 2011.
[47] A. S. Cacciapuoti, M. Caleffi, and L. Paura, "Widely linear cooperative spectrum sensing for cognitive radio networks," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*, Dec. 2010, pp. 1–5.
[48] D. Sun, T. Song, M. Wu, J. Hu, J. Guo, and B. Gu, "Optimal sensing time of soft decision cooperative spectrum sensing in cognitive radio networks," in *Proc. IEEE WCNC*, Apr. 2013, pp. 4124–4128.
[49] X. Li, W. Li, and Y. Hei, "Joint spectrum sensing and user selection strategy for cognitive radio networks," in *Proc. IEEE WCSP*, Oct. 2012, pp. 1–6.
[50] N. Reisi, S. Gazor, and M. Ahmadian, "Distributed cooperative spectrum sensing in mixture of large and small scale fading channels," *IEEE Trans. Wireless Commun.*, vol. 12, no. 11, pp. 5406–5412, Nov. 2013.
[51] J. Abad and J. Sesma, "A new expansion of the confluent hypergeometric function in terms of modified Bessel functions," *J. Comput. Appl. Math.*, vol. 78, no. 1, pp. 97–101, 1997.
[52] J. L. López and P. J. Pagola, "The confluent hypergeometric functions M(a,b;z) and U(a,b;z) for large b and z," *J. Comput. Appl. Math.*, vol. 233, no. 6, pp. 1570–1576, 2010.
[53] C. Wonseok, S. Moon-Gun, A. Jaeha, and I. Gi-Hong, "Soft combining for cooperative spectrum sensing over fast-fading channels," *IEEE Commun. Lett.*, vol. 18, no. 2, pp. 193–196, Feb. 2014.
[54] W. Peterson, T. Birdsall, and W. Fox, "The theory of signal detectability," *Trans. IRE Prof. Group Inf. Theory*, vol. 4, no. 4, pp. 171–212, Sep. 1954.
[55] D. Tse and P. Viswanath, *Fundamentals of Wireless Communications*. Cambridge, U.K.: Cambridge Univ. Press, 2005.
[56] S. M. Mishra, A. Sahai, and R. W. Brodersen, "Cooperative sensing among cognitive radios," in *Proc. IEEE Int. Conf. Commun.*, vol. 4. Jun. 2006, pp. 1658–1663.
[57] A. Goldsmith, *Wireless Communications*. Cambridge, U.K.: Cambridge Univ. Press, 2005.
[58] G. V. Weinberg and V. G. Glenny, "Optimal Rayleigh approximation of the K-distribution via the Kullback-Leibler divergence," *IEEE Signal Process. Lett.*, vol. 23, no. 8, pp. 1067–1070, Aug. 2016.
[59] *Wireless Rans: Spectrum Sensing Requirements Summary*, IEEE Standard P802.22, Mar. 2008. [Online]. Available: http://www.ieee802.org/22/Meeting_documents/2006_July/
[60] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*. Upper Saddle River, NJ, USA: Prentice-Hall, 1998.
[61] S. Zarrin and L. T. Joon, "Cooperative spectrum sensing in cognitive radios with incomplete likelihood functions," *IEEE Trans. Signal Process.*, vol. 58, no. 6, pp. 3272–3281, Jun. 2010.
[62] K. Krishnamoorthy, *Handbook of Statistical Distributions With Applications*. Boca Raton, FL, USA: CRC Press, 2006.
[63] B. G. Lindsay, R. S. Pilla, and P. Basak, "Moment-based approximations of distributions using mixtures: Theory and applications," *Ann. Inst. Statist. Math.*, vol. 52, no. 2, pp. 215–230, 2000.
[64] B. G. Lindsay, "On the determinants of moment matrices," *Ann. Statist.*, vol. 17, no. 2, pp. 711–721, 1989.
[65] B. G. Lindsay, "Moment matrices: Applications in mixtures," *Ann. Math. Statist.*, vol. 17, no. 2, pp. 722–740, 1989.
[66] F. W. J. Olver, D. W. Lozier, R. F. Boisvert, and C. W. Clark, *NIST Handbook of Mathematical Functions*. Cambridge, U.K.: Cambridge Univ. Press, 2010.
[67] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products*. San Diego, CA, USA: Academic, 2007.
[68] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables*. New York, NY, USA: Dover, 1970.
[69] B. Picinbono, "On deflection as a performance criterion in detection," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 31, no. 3, pp. 1072–1081, Jul. 1995.
[70] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Upper Saddle River, NJ, USA: Prentice-Hall, 1998.
[71] Y. Chen and D. Y. Gao, "Global solutions to nonconvex optimization of 4th-order polynomial and log-sum-exp functions," *J. Global Optim.*, vol. 64, no. 3, pp. 417–431, 2016.
[72] K. Hara, K. Inoue, and K. Urahama, "A differentiable approximation approach to contrast-aware image fusion," *IEEE Signal Process. Lett.*, vol. 21, no. 6, pp. 742–745, Jun. 2014.
[73] M. Chen, S. C. Liew, Z. Shao, and C. Kai, "Markov approximation for combinatorial network optimization," *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6301–6327, Oct. 2013.
[74] A. Bjorck and V. Pereyra, "Solution of vandermonde systems of equations," *Math. Comput.*, vol. 24, no. 112, pp. 893–903, 1970.
[75] G. Galimberti and V. Pereyra, "Numerical differentiation and the solution of multidimensional vandermonde systems," *Math. Comput.*, vol. 24, no. 110, pp. 357–364, 1970.
[76] N. J. Higham, "Stability analysis of algorithms for solving confluent vandermonde-like systems," *SIAM J. Matrix Anal. Appl.*, vol. 11, no. 1, pp. 23–41, 1990.

HUAYAN GUO (S'16) received the B.S. degree in electronic information science and technology from the Beijing University of Posts and Telecommunications, Beijing, China, in 2012. He is currently pursuing the Ph.D. degree with the State Key Laboratory of Advanced Optical Communication Systems and Networks, School of Electronics Engineering and Computer Science, Peking University, Beijing. His research interests include spectrum sensing and satellite communications.

NIMA REISI (S'09–M'16) received the B.Sc. degree from the Isfahan University of Technology and the M.Sc. and Ph.D. degrees from the K. N. Toosi University of Technology, all in electrical engineering. Since 2015, he has been with Azad University-Central Tehran Branch, Tehran, Iran, as a Faculty Member. He has also been a Visiting Researcher with Queen's University, Canada, for one year. His main research interests include cognitive radio networks, massive MIMO, interference alignment, and cooperative communications.

WEI JIANG (M'04) received the B.S. and Ph.D. degrees in electronics engineering from Peking University in 1997 and 2003, respectively. He is currently an Associate Professor with the School of Electronics Engineering and Computer Science, Peking University. His research interests include communication algorithms, multiple antenna technologies, and satellite communications.

WU LUO (M'09) received the B.S., M.S., and Ph.D. degrees from Peking University in 1991, 1998, and 2006, respectively, all in electronics engineering. He is currently a Professor with the School of Electronics Engineering and Computer Science, Peking University. His research interests include wireless and satellite communication networks, channel estimation, interference cancellation and suppression, MIMO networks, and channel coding techniques.