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Finite Length Analysis of Low-Density Parity-Check Codes on Impulsive Noise Channels

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ABSTRACT Low-density parity-check (LDPC) codes with very long block lengths are well known for their powerful error correction, but it is not always desirable to employ long codes in communication systems, where latency is a serious issue, such as voice and video communication between multiple users. Finite length analyses of LDPC codes have already been presented in the literature for the additive white Gaussian noise channel, but in this paper, we consider the finite length analysis of LDPC codes for channels that exhibit impulsive noise. First, an exact uncoded bit error probability (BEP) of an impulsive noise channel, modeled as a symmetric α-stable (SαS) distribution, is derived. Then, to obtain the LDPC-coded performance, density evolution is applied to evaluate the asymptotic performance of LDPC codes on SαS channels and determine the threshold signal-to-noise ratio. Finally, we derive closed-form expressions for the BEP and block error probability of short LDPC codes on these channels, which are shown to match closely with simulated results on channels with different levels of impulsiveness, even for block lengths as low as 1000 b.

INDEX TERMS LDPC codes, impulsive noise, density evolution, finite length analysis.

I. INTRODUCTION

Short error-correcting codes can be necessary for communication systems where low latency is very important, such as real-time voice and video communications, but their performance is limited. It is well known that very long low-density parity-check (LDPC) codes approach the Shannon limit, but the performance degrades as block length decreases. An asymptotic analysis of LDPC codes is therefore not useful in this scenario since it assumes the code length is infinite and also cycle free. Hence, finite length analyses have been presented in the literature to evaluate the performance of short LDPC codes. A finite length analysis of LDPC code ensembles on the binary erasure channel (BEC) was presented in [1] using a recursive approach. In [2], the waterfall region of LDPC codes was proved to follow a scaling law over the BEC and performance was predicted accurately. However, the procedure of finding the scaling parameters on the BEC cannot be easily transferred to other channels and decoding algorithms. Recently, a waterfall region analysis based only on the threshold signal-to-noise ratio (SNR) was proposed [3]. This method estimates the block error probability (BLEP) by considering that a decoding failure is due to the actual channel quality being worse than the decoding threshold. An Extrinsic Information Transfer (EXIT) chart and Gaussian approximation (GA) combined with the BLEP were then used to obtain the BEP. This method has a low complexity and provides a good estimation of the waterfall region of finite length LDPC codes without any scaling parameters or curve fitting. In [4] and [5], Noor-A-Rahim *et al* present a similar approach, which observes the real-time channel quality and provides an improved analysis with slightly better estimation, but this requires multiple applications of density evolution (DE) during the process.

LDPC codes with short block lengths of 120 and 540 bytes have been chosen as the error-correcting codes in the G.hn/G.9960 standard for powerline channels, which are impulsive in nature. However, a comprehensive literature survey reveals that the finite length analysis of LDPC codes has only been considered on the BEC, binary symmetric channel (BSC) and binary input additive white Gaussian noise (BI-AWGN) channel. Motivated by this and the lack of published work on the finite length analysis of LDPC codes on more general memoryless channels, we present a finite length analysis of LDPC codes on impulsive noise channels.

The occurrence of impulsive noise leads to a non-Gaussian probability density function (pdf) and so the assumption that the noise has a Gaussian distribution is no longer valid. Instead, the distribution is heavy-tailed and several models have been proposed to model impulsive noise, such as the Gaussian mixture model, Middleton Class A and B noise and symmetric α-stable (SαS) distributions [6], [7]. In particular, we focus on the SαS family of distributions since they can accurately model impulsive noise present in underwater acoustic noise and atmospheric noise, as well as realistically model the statistics of radio frequency interference generated by clocks and buses in laptop and desktop computers [8] and impulsive noise in powerline communications [9]. The pdf of SαS distributions is not given in closed-form, hence sub-optimal detectors are required to reduce the complexity [10], [11]. Recently, several sub-optimal receivers combined with LDPC codes were proposed and their performance was examined [12], [13]. Moreover, good LDPC codes were designed for an OFDM-based powerline system by utilizing differential evolution [14]. However, there is still a gap between the simulated and theoretical results, especially for short length codes. Therefore, it is important to study the theoretical performance of finite length LDPC codes on impulsive noise channels.

The contributions of this paper are as follows: First we derive the exact BEP of BPSK on SαS channels for all values of α. Second, the channel capacity of SαS channels is given and the threshold of LDPC codes on such channels is determined using DE. Finally, we expand on the work of [3] to derive expressions for the BEP of short LDPC codes on several SαS impulsive noise channels for the first time. To achieve this, the obtained uncoded BEP is combined with the threshold obtained from DE to derive the BLEP and BEP for short LDPC codes on different SαS channels. The theoretical performance of short LDPC codes for several different block lengths obtained from our BLEP and BEP expressions are then compared with simulation results on different SαS channels to validate our finite length
analysis.

This paper is organized as follows: Section II introduces SαS noise and defines a new expression of SNR. Section III derives the exact BEP of BPSK for SαS channels. Section IV gives the capacity of SαS channels and an asymptotic analysis of LDPC codes on these channels. The estimation of BLEP and BEP of finite length LDPC codes are presented in Section V. In Section VI, theoretical and simulation results are presented and we conclude the paper in Section VII.

II. SαS CHANNEL MODEL FOR IMPULSIVE NOISE

We consider a LDPC-coded system that generates a codeword of length N bits and is mapped to a binary phase-shift keying (BPSK) constellation to obtain the transmitted signal. The received signal is contaminated by additive impulsive noise with a SαS distribution and is defined as

$$y_j = x_j + \eta_j, \quad (1)$$

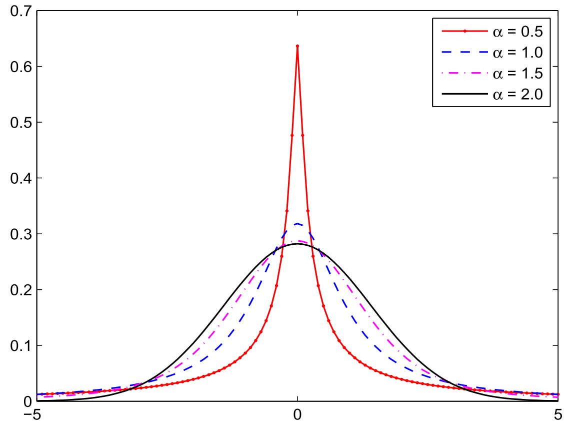

FIGURE 1. Pdfs of standard SαS distributions ($\gamma = 1$).

where y_j is the j-th symbol of the received signal, $x_j \in \{-1, +1\}$ is the BPSK signal, η_j is an additive SαS distributed noise sample and $j = 1, 2, \ldots, n$. The generation of random SαS distributed samples can be found in [10] and the pdf of a SαS random variable $x \sim S(\alpha, \gamma)$ is defined as

$$f_\alpha(x; \gamma) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-\gamma^\alpha |t|^\alpha) e^{-jtx} dt. \quad (2)$$

There are two important parameters in (2): 1) the characteristic exponent α, which has a range (0, 2] and determines the heaviness of the tails and 2) the dispersion γ^α, which is similar to the variance of a Gaussian distribution and determines the spread of the pdf. When $\alpha = 2$, the noise is Gaussian and the variance σ^2 is only defined in this case where $\sigma^2 = 2\gamma^2$. As α decreases, the tail becomes heavier which increases the likelihood of the impulses having large amplitudes. The pdf can be efficiently generated by Zolotarevs (M) parameterization [15] and the pdfs with different α's are shown in Fig. 1.

The traditional SNR cannot be defined for SαS channels since the second order moment of SαS process is infinite. Hence, we use the geometric SNR ($\mathrm{SNR_G}$) [16], which is based on zero-order statistics. First, geometric power S_0 is defined as

$$S_0 = \frac{(C_g)^{1/\alpha}\gamma}{C_g}, \quad (3)$$

where C_g is the exponential of the Euler constant and has a value of ≈ 1.78 . We can then define $\mathrm{SNR_G}$ as

$$\mathrm{SNR_G} = \frac{1}{2C_g}\left(\frac{A}{S_0}\right)^2, \quad (4)$$

where A^2 is the transmitted energy of the modulated signal and the constant $\frac{1}{2C_g}$ ensures $\mathrm{SNR_G}$ remains valid when the noise is Gaussian ($\alpha = 2$). In this paper we assume $A^2 = 1$

and the $\frac{E_b}{N_0}$ is given as

$$\frac{E_b}{N_0} = \frac{\text{SNR}_\text{G}}{2R_c} = \frac{1}{4R_c C_g^{(\frac{2}{\alpha}-1)}\gamma^2}, \tag{5}$$

where R_c is the code rate.

III. UNCODED BIT-ERROR PROBABILITY ON SαS CHANNELS

In this section, we derive the probability of a bit error, P_b^α, for an uncoded system employing BPSK modulation on SαS channels, which will later be used to estimate the probability of a block error when a LDPC code is employed. When $\alpha = 2$, the pdf is known but the cdf is unknown in closed-form. The right tail probability function Q-function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{t^2}{2}\right) dt. \tag{6}$$

Similar to [17], we can define a tail probability for SαS distributions $Q_\alpha(x)$ as

$$Q_\alpha(x) = \int_x^\infty f_\alpha(t; 1) dt, \tag{7}$$

where $f_\alpha(t; 1)$ is the standard SαS distribution which is obtained by setting $\gamma = 1$ in (2). Hence, P_b^α for SαS channels is derived as

$$\begin{aligned} P_b^\alpha &= P(x=+1)P(e|x=+1) + P(x=-1)P(e|x=-1) \\ &= \frac{1}{2}\int_{-\infty}^0 f_\alpha(t-1;\gamma)dt + \frac{1}{2}\int_0^\infty f_\alpha(t+1;\gamma)dt \\ &= \int_1^\infty f_\alpha(u;\gamma)du, \end{aligned} \tag{8}$$

where e is a symbol error and $P(x = +1) = P(x = -1) = \frac{1}{2}$. According to the standardization of SαS random variables, if $x \sim S(\alpha, \gamma)$, then $x/\gamma \sim S(\alpha, 1)$ and the pdf should be scaled by $1/\gamma$ [15]. By using this parametrization of the SαS process, (8) can be rewritten as

$$\begin{aligned} P_b^\alpha &= \int_1^\infty \frac{1}{\gamma} f_\alpha\left(\frac{u}{\gamma}; 1\right) du \\ &= \int_{\frac{1}{\gamma}}^\infty f_\alpha(v; 1) dv \\ &= Q_\alpha\left(\frac{1}{\gamma}\right). \end{aligned} \tag{9}$$

Since the geometric SNR is defined for the whole range of α, (9) is a general expression for all SαS channels. From (5) and (9), we can obtain P_b^α in terms of E_b/N_0 as

$$P_b^\alpha = Q_\alpha\left(\frac{1}{\gamma}\right) = Q_\alpha\left(\sqrt{4R_c C_g^{(\frac{2}{\alpha}-1)}\frac{E_b}{N_0}}\right). \tag{10}$$

When $R_c = 1$, (10) represents the BEP of an uncoded BPSK system on SαS channels.

There are two special cases of SαS random variables which have a closed-form expression for the pdf: $\alpha = 1$ and $\alpha = 2$. Hence their BEP can be derived to further verify the correctness of our analysis. First we consider the case of Cauchy noise ($\alpha = 1$), where P_b^{Cauchy} is given as

$$\begin{aligned} P_b^{\text{Cauchy}} &= \int_0^\infty \frac{\gamma}{\pi} \frac{1}{(t+1)^2 + \gamma^2} dt \\ &= \int_{\frac{1}{\gamma}}^\infty \frac{1}{\pi} \frac{1}{x^2+1} dx. \end{aligned} \tag{11}$$

The Cauchy distribution of (11) has been converted to a standard pdf and P_b^{Cauchy} can be expressed in terms of $Q_\alpha(x)$ as

$$P_b^{\text{Cauchy}} = Q_\alpha\left(\frac{1}{\gamma}\right). \tag{12}$$

Now we examine the case for AWGN ($\alpha = 2$). Notice that according to the definition of the standard SαS pdf, the variance of the normal distribution is equal to two, since $\sigma^2 = 2\gamma^2$. Hence the standard SαS distribution when $\alpha = 2$ is

$$f_{\alpha=2}(t; 1) = \frac{1}{2\sqrt{\pi}} \exp(-\frac{t^2}{4}). \tag{13}$$

Then the uncoded BEP of BPSK on the AWGN channel can be expressed in terms of the Q_α-function as

$$P_b^{\text{Gauss}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q_{\alpha=2}\left(2\sqrt{\frac{E_b}{N_0}}\right). \tag{14}$$

When $\alpha = 2$, (10) reduces to (14), hence (10) is universal for all values of α. This is confirmed in Fig. 2, where P_b^α in (10) for different values of α are plotted with simulated bit-error rates, which match very closely.

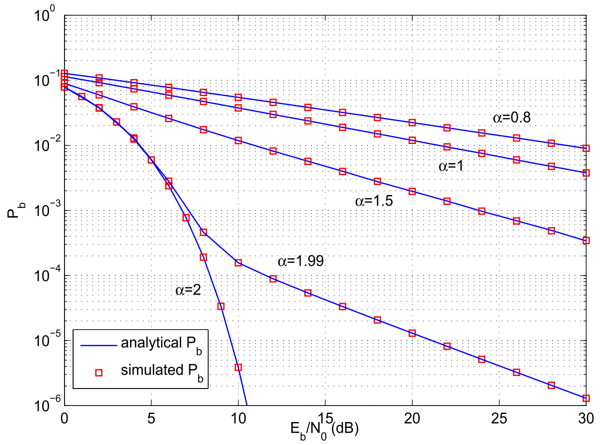

FIGURE 2. Uncoded performance of BPSK on SαS channels at $\alpha = 2, 1.99, 1.5, 1$ and 0.5, respectively.

IV. ASYMPTOTIC PERFORMANCE OF LDPC CODES ON SαS CHANNELS

A. THE CAPACITY OF SαS CHANNELS

Channel capacity is a fundamental upper bound on the rate at which information can be reliably transmitted. For the AWGN channel, the channel capacity has been well studied in the literature [18]. For binary memoryless symmetric channels (BMSC), the capacity can be evaluated as a function of the pdf of log-likelihood ratios (LLRs) [18]. As a type of BMSC, the capacity of SαS channels can be expressed as

$$C_\alpha = 1 - E\left\{\log_2\left(1+e^{-L}\right)\right\}, \tag{15}$$

where $L = \ln \frac{P(x=+1|y)}{P(x=-1|y)}$ is the channel LLR. The expectation operator in (15) can be replaced by a time average. Hence the capacity of SαS channels can be obtained as

$$C_\alpha = 1 - \lim_{N\to\infty}\left\{\frac{1}{N}\sum_{n=1}^{N}\log_2\left(1+e^{-x_nL_n}\right)\right\}, \tag{16}$$

where x_n is the modulated signal. This capacity limit can be measured by a large number, N, of LLR values and it will be used as a benchmark for the coded performance in Section VI.

B. DENSITY EVOLUTION OF LDPC CODES ON SαS CHANNELS

The asymptotic performance of an ensemble of LDPC codes can be accurately predicted from several methods, namely, DE, EXIT charts and GA. However, only DE can be employed to analyze the iterative behavior of the decoder when the channel is impulsive, since it has a non-Gaussian distribution. In this subsection, we will show how to apply DE to SαS channels.

First we characterize LDPC ensembles by degree distributions. A (d_v, d_c) regular LDPC code is defined by all variable nodes having degree d_v and all check nodes having degree d_c. Naturally, irregular LDPC codes are defined by non-uniform variable node degrees and check node degrees. The edge degree distributions are expressed as:

$$\lambda(x) = \sum_{i=2}^{d_v}\lambda_i x^{i-1}, \; \rho(x) = \sum_{i=2}^{d_c}\rho_i x^{i-1}, \tag{17}$$

where d_v and d_c are the maximum variable node degree and check node degree respectively and λ_i or ρ_i is the fraction of edges that are connected to variable or check nodes of degree i.

It is known that DE assumes the channel output is symmetric and we can prove the symmetry property for SαS channels, as shown below:

$$\begin{aligned} P(y|x=1) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp(-\gamma^\alpha|t|^\alpha)e^{-jt(y-1)}dt \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp(-\gamma^\alpha|t|^\alpha)e^{-jt[-(y-1)]}dt \\ &= P(-y|x=-1). \end{aligned} \tag{18}$$

DE tracks the pdf of LLRs during the iterative Sum-Product decoding (SPD) process. The initial LLR of a variable node is given as

$$\omega^{(0)} = \ln\frac{P(x=+1|y)}{P(x=-1|y)} = \ln\frac{f_\alpha(y-1;\gamma)}{f_\alpha(y+1;\gamma)}. \tag{19}$$

It is difficult to evaluate the pdf of (19) analytically, except when $\alpha = 2$, hence a numerical and histogram method is used to find the pdf of (19). Then the DE of a check node is expressed as

$$p_\phi^{(l)} = \Lambda^{-1}\left[\sum_{i=2}^{d_c}\rho_i\left(\Lambda\left[p_\omega^{(l-1)}\right]\right)^{\otimes(i-1)}\right], \tag{20}$$

where $p_\phi^{(l)}$ is the pdf of each check node output $\phi_i^{(l)}$ and $p_\omega^{(l)}$ is the pdf of each variable node output $\omega_j^{(l)}$ at the l-th iteration. The symbol $\otimes$ represents convolution, Λ and Λ^{-1} are the changes of density due to the transformations $g(\cdot)$ and $g^{-1}(\cdot)$ respectively, where $g(y) = (\text{sign}(y), \ln\coth(|y/2|))$. The DE of a variable node is

$$p_\omega^{(l)} = p_\omega^{(0)} \otimes \sum_{i=2}^{d_v}\lambda_i\left(p_\phi^{(l)}\right)^{\otimes(i-1)}. \tag{21}$$

The summations in the variable node update of the SPD become convolution operations in (21). Let us assume that the all-zero codeword ($x = +1$) is transmitted. During this two-stage iterative algorithm, the fraction of incorrect messages for the l-th iteration can be denoted as

$$P_e^{(l)} = \int_{-\infty}^{0} p_\omega^{(l)}(x)dx. \tag{22}$$

We note that $P_e^{(l)}$ will be used to derive an expression for the estimated BEP for finite length LDPC codes on SαS impulsive noise channels in the next section.

For a given noise parameter γ, this two-stage iterative algorithm is performed until the error probability either tends to zero or remains at a fixed value. Hence, the threshold γ_{th} of a specific LDPC ensemble on SαS channels is defined as

$$\gamma_{\text{th}} = \sup\left\{\gamma : \lim_{l\to\infty}\int_{-\infty}^{0} p_\omega^{(l)}(x)dx = 0\right\}, \tag{23}$$

When $\gamma < \gamma_{\text{th}}$, the decision error converges to zero as the number of iterations tends to infinity and when $\gamma > \gamma_{\text{th}}$, the error will be bounded away from zero.

The obtained threshold γ_{th} can be used to predict the asymptotic performance of LDPC codes since DE assumes the block length is infinite and cycle-free. In the results section, the corresponding SNR of the threshold will be included in figures to indicate the start of the waterfall region.

V. BLOCK AND BIT-ERROR PROBABILITY OF FINITE LENGTH LDPC CODES ON SαS CHANNELS

A. ESTIMATING THE BLOCK-ERROR PROBABILITY

1) OBSERVED BIT-ERROR RATE ON SαS CHANNELS

The observed bit-error rate (BER) P_{obs}^α is defined as the BER of any received word of length N [3]. We assume the all-zero codeword $\boldsymbol{c}$ is transmitted, hence an error occurs when $L(y_j) = \frac{P(c_j=0|y_j)}{P(c_j=1|y_j)}$ is negative. To find the pdf of P_{obs}^α, we take N samples from the LLR distribution, where each bit has a probability P_b^α of being incorrect. Then, the probability mass function (pmf) of P_{obs}^{α} is given as

$$f_{P_{\text{obs}}^{\alpha}}(N, P_b^{\alpha}) = \binom{N}{K}(P_b^{\alpha})^K(1 - P_b^{\alpha})^{N-K}, \tag{24}$$

where $K = NP_{\text{obs}}^{\alpha}$ is the average number of errors in a codeword of length N, which follows a binomial distribution $B(N, P_b^{\alpha})$. When $N \to \infty$, the pdf of P_{obs}^{α} can be approximated by a normal distribution $\mathcal{N}(P_b^{\alpha}, P_b^{\alpha}(1 - P_b^{\alpha})/N)$.

2) BLOCK ERROR PROBABILITY

We employ a threshold method to estimate the BLEP $P_{\text{B}}^{\alpha}(N, \lambda, \rho)$ for LDPC codes of block length N on SαS impulsive noise channels. The threshold of a specific ensemble of LDPC codes is defined as the maximum channel parameter where the probability of a bit error from the sum-product decoder converges to zero. Once the threshold γ_{th} is obtained, we can use the threshold method to estimate $P_{\text{B}}^{\alpha}(N, \lambda, \rho)$.

To find $P_{\text{B}}^{\alpha}(N, \lambda, \rho)$ of a specific ensemble of LDPC codes, the probability that the observed channel behaves worse than the decoding threshold is calculated using P_{obs}^{α} defined earlier. First, we calculate the corresponding BEP P_{th} of the SαS impulsive noise channel at the threshold γ_{th} using (10)

$$P_{\text{th}} = Q_{\alpha}\left(\sqrt{4R_c C_g^{(\frac{2}{\alpha}-1)}\left(\frac{E_b}{N_0}\right)_{\text{th}}}\right), \tag{25}$$

where $\left(\frac{E_b}{N_0}\right)_{\text{th}}$ is the threshold SNR defined by γ_{th}. Then we calculate $f_{P_{\text{obs}}^{\alpha}}(N, P_{\text{obs}}^{\alpha})$ according to the block length N of the codeword and the probability that $P_{\text{obs}}^{\alpha} > P_{\text{th}}$. Hence, the estimated BLEP is

$$P_{\text{B}}^{\alpha}(N, \lambda, \rho) = \int_{P_{\text{th}}}^{1} f_{P_{\text{obs}}^{\alpha}}(N, x)dx, \tag{26}$$

where (26) calculates the probability of a block error for an ensemble of LDPC codes with block length N and degree distributions $\lambda(x)$ and $\rho(x)$. When N is large, $f_{P_{\text{obs}}^{\alpha}}(N, P_{\text{obs}}^{\alpha})$ can be approximated as a normal distribution. Hence the BLEP can be expressed as

$$P_{\text{B}}^{\alpha}(N, \lambda, \rho) = Q\left(\frac{P_{\text{th}} - \mu_{P_{\text{obs}}^{\alpha}}}{\sigma_{P_{\text{obs}}^{\alpha}}}\right), \tag{27}$$

where $\mu_{P_{\text{obs}}^{\alpha}} = P_0^{\alpha}$ and $\sigma_{P_{\text{obs}}^{\alpha}} = P_0^{\alpha}(1 - P_0^{\alpha})/N$.

B. ESTIMATING THE BIT ERROR PROBABILITY

The coded BEP can be derived from the BLEP by observing that the error rate when the decoder fails does not change significantly for channel parameters worse than the threshold. Consequently, there is an error probability of $P_e^{(l)}$ when the decoder fails at the l-th iteration, as defined in (22). Since each block has a probability $P_{\text{B}}^{\alpha}(N, \lambda, \rho)$ of an error occurring, the BEP is given as

$$P_{\text{b}}^{\alpha}(N, \lambda, \rho) = P_e^{(l_{\max})} P_{\text{B}}^{\alpha}(N, \lambda, \rho), \tag{28}$$

where $l_{\max}$ is the maximum number of iterations when DE is performed. We model the observed BER as a random

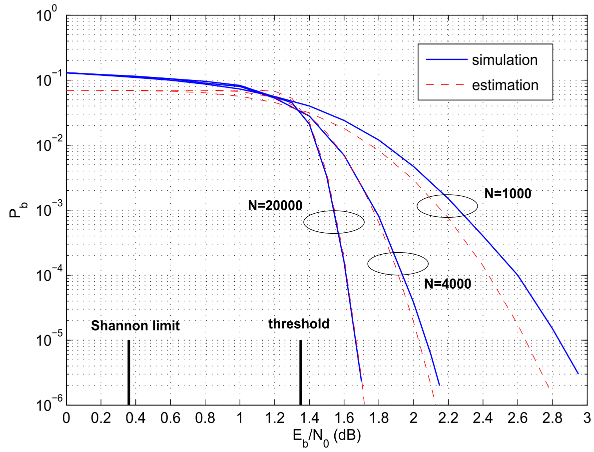

FIGURE 3. BEP comparison of regular (3, 6) LDPC codes showing estimated and simulation results with different block lengths on SαS channels when $\alpha = 1.9$.

variable with a binomial distribution as in [3]. However, this paper extends the estimation of the BLEP of LDPC codes on AWGN channels to more general non-Gaussian SαS channels by using our derived uncoded BEP and threshold dispersion for SαS channels.

VI. RESULTS AND DISCUSSION

The accuracy of the estimated performance for finite length LDPC codes is now investigated by comparing theoretical bit-error probabilities with bit-error rates obtained by simulations of rate 1/2 regular and irregular LDPC codes of different block lengths ($N = 1000, 4000, 20000$) at different values of α ($\alpha = 0.8, 1, 1.5, 1.9$). The decoding algorithm is the sum-product algorithm and the maximum number of decoder iterations is set to 100. For regular codes, we fix the column weights and row weights to 3 and 6, respectively. For irregular codes, the degree distributions are selected to be $\lambda(x) = 0.30013x + 0.28395x^2 + 0.41592x^7$, $\rho(x) = 0.22919x^5 + 0.77081x^6$ and $\lambda(x) = 0.4x^2 + 0.4x^5 + 0.2x^8$, $\rho(x) = x^8$. The first degree distribution pair is chosen from [19] which is the optimized code with maximum variable node degree of 8. The second degree distribution pair is chosen from [3]. In our simulations, short and medium length codes ($N \leq 4000$ bits) are constructed using a progressive edge-growth (PEG) algorithm [20], which maximizes the local girth to reduce the effects of cycles. For long LDPC codes, we use Mackay's random construction since the complexity of the PEG algorithm at large block lengths is very high.

In Figs. 3 - 7, the capacity and thresholds of each LDPC ensemble for SαS channels are given. For regular LDPC codes and different values of α, as shown in Figs. 3 - 5, the gap between the estimated and simulated performance becomes smaller as the block length increases. When $N = 1000$, the gap between the estimated and simulated performance is around 0.2 dB and reduces to about 0.1 dB when N increases to 4000. When $N = 20000$, the estimated and simulated performance are almost identical. We also observe that these differences in performance are independent of α.

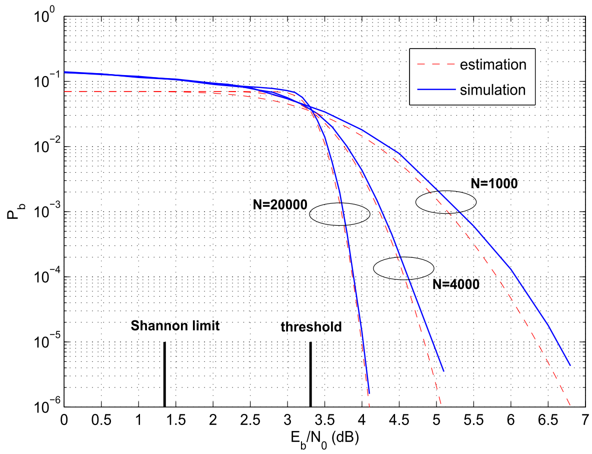

FIGURE 4. **BEP comparison of regular (3, 6) LDPC codes showing estimated and simulation results with different block lengths on SαS channels when $\alpha = 1$.**

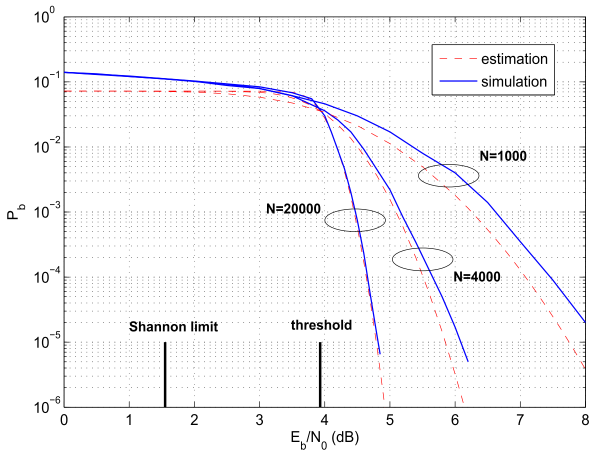

FIGURE 5. **BEP comparison of regular (3, 6) LDPC codes showing estimated and simulation results with different block lengths on SαS channels when $\alpha = 0.8$.**

Our estimation method is also shown to match closely with the simulation results for irregular LDPC codes. As shown in Fig. 6, the actual performance is accurately predicted by our analytically derived P_B^α and P_b^α in (27) and (28) with only a 0.15 dB difference at a bit-error rate of 10^{-5}, while the gap to the threshold is 1.04 dB. In Fig. 7, the performance of optimized LDPC codes is presented when $\alpha = 1$. It is shown that the gaps between the estimated and simulated performance for different block lengths are similar to the results for the regular LDPC codes, with both sets of results becoming almost identical when $N = 20000$ bits. Compared with Fig. 4, we note that the performance of this optimized code is about 1 dB better than the regular (3, 6) LDPC codes with the same block lengths.

We observe that the gap between the estimated and simulated results is greater at shorter block lengths. There are two reasons for this result: First, the threshold γ_{th} and its corresponding P_{th} obtained from DE assume the LDPC code is cycle-free. However, short cycles cannot be avoided for

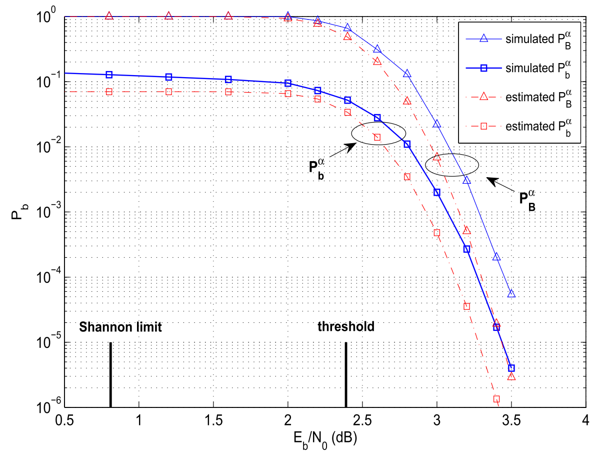

FIGURE 6. **Block and bit error probability of irregular LDPC codes with degree distribution $\lambda(x) = 0.4x^2 + 0.4x^5 + 0.2x^8$, $\rho(x) = x^8$ showing estimated and simulation results with $N = 4000$ on SαS channels when $\alpha = 1.5$.**

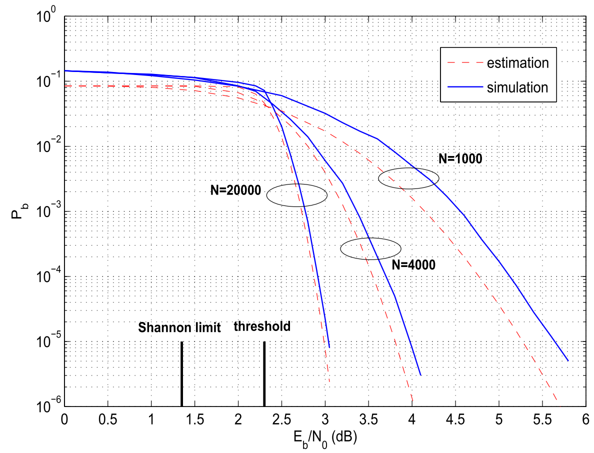

FIGURE 7. **BEP comparison of irregular LDPC codes with degree distribution $\lambda(x) = 0.30013x + 0.28395x^2 + 0.41592x^7$, $\rho(x) = 0.22919x^5 + 0.77081x^6$ showing estimated and simulation results with different block lengths on SαS channels when $\alpha = 1$.**

short LDPC codes. Hence, the effect of cycles on short block length LDPC codes is more serious and this degrades performance [20]. For long LDPC codes, the prediction becomes more accurate since the concentration theorem states that the average behavior of individual codes concentrates around its expected behavior as the block length grows and this average behavior converges to the cycle-free case [21]. Second, the pdf of P_{obs}^α is not well approximated by a Gaussian distribution when N is small, which means (27) and (28) become less accurate. To numerically evaluate the accuracy of the Gaussian approximation, the Kullback-Leibler (KL) divergence is employed to calculate the difference between the two pdfs. KL divergence is defined as $D_{\text{KL}}(P||Q) = \sum_i P(i) \log \frac{P(i)}{Q(i)}$, where P is the true pdf and Q is an approximation of P. In our case, P is the binomial pdf $B(N, P_b^\alpha)$ and Q is the normal distribution $\mathcal{N}(P_b^\alpha, P_b^\alpha(1 - P_b^\alpha)/N)$. We note that the accuracy of this approximation generally improves as

N increases and P_b^α is not near to 0 or 1. In order to investigate the validity of Gaussian approximation, as an example, we choose the smallest $P_b^\alpha = 0.0802$ which can be calculated from (10) at $E_b/N_0 = 3$ dB which is the largest SNR in Fig. 3. Knowing the value of block length N and P_b^α, the pdf of P_{obs}^α can be determined. Therefore, the KL divergence between the pdf of P_{obs}^α and Gaussian distribution is obtained as $8.02 \times 10^{-4}, 1.99 \times 10^{-4}, 3.98 \times 10^{-5}$ for $N = 1000, 4000, 20000$, respectively. This indicates that the approximation becomes more accurate as the block length increases and we also observe that the Gaussian approximation is very accurate even for short length LDPC codes ($N = 1000$), since the KL divergence is very small.

VII. CONCLUSION

In this paper, we have performed a finite length analysis of regular and irregular LDPC codes to derive the block and bit-error probabilities on additive impulsive noise channels with SαS pdfs. First, a general expression of the exact BEP of BPSK on SαS channels is derived. Then DE has been employed to obtain the decoding threshold of infinitely long LDPC codes since the simpler Gaussian approximation method was not feasible due to the non-Gaussian nature of the channel. This was used to derive the block and bit error probabilities of long and short LDPC codes. At long block lengths ($N = 20000$ bits), the estimated BEPs are almost identical to the simulated bit-error rates for different values of α, but it has been observed that the gap between theoretical and simulation results increases as the block length decreases. The reasons for this are the effect of short cycles in the Tanner graph and the Gaussian approximation of the observed error probabilities becoming weaker as the block length is reduced, although the gap was still only around 0.2 dB when the block length is as low as 1000 bits. Hence, we have shown that for a given degree distribution pair our method can be used to obtain accurate estimates of the block and bit error probabilities of finite length LDPC codes on SαS additive impulsive noise channels. Furthermore, our analysis implies that for a given uncoded BEP and threshold, the prediction of the actual performance for short LDPC codes could be accomplished on more general memoryless channels.

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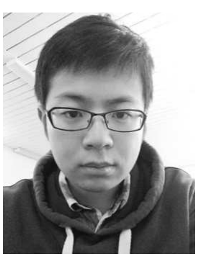

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