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Distributed Adaptive Formation Control for Linear Swarm Systems With Time-Varying Formation and Switching Topologies

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ABSTRACT In this paper, distributed time-varying formation (TVF) control problems for general linear swarm systems with switching interaction topologies are investigated using an adaptive dynamic protocol. First, a TVF control protocol for switching interaction topologies is constructed using the states of neighboring agents. In the protocol, an adaptive controller that employs gain scheduling technique is provided to estimate the coupling weights among agents. Compared with the previous studies on formation control, the desired formation can be specified by piecewise continuously time-varying differentiable vectors, the interaction topology can be switching, and the disadvantage of requiring global information of the interaction topologies is removed in this paper. Then, an algorithm including a feasible formation condition is proposed to determine the gain matrices of the distributed adaptive formation protocol by solving a linear matrix inequality for swarm systems with switching interaction topologies. Moreover, under the designed distributed adaptive formation protocol, sufficient condition for general linear swarm systems with switching interaction topologies to achieve the given TVF is derived using the Lyapunov theory. Finally, numerical simulations are presented to demonstrate the obtained results.

INDEX TERMS Distributed adaptive protocol, time-varying formation, switching interaction topology, swarm systems, general linear dynamics.

I. INTRODUCTION

In recent years, the formation problem of swarm systems has been a significant research topic in the field of cooperative control because of its broad range of application in wide areas, such as satellite formation flying [1], [2], unmanned aerial vehicles formation [3], [4], distributed sensor networks [5], [6], mobile robots [7], [8] and so on. For a formation problem of swarm systems, the crucial task is to design distributed formation protocols based on local information, i.e., local state information of each agent and its neighbors. As a matter of fact, in robotics community, formation control problems have been studied a lot during the past years using several approaches, which are virtual structure-based approach [9], behavior-based approach [10], leader-follower based approach [11] and so on.

The main challenge faced by many studies on formation control of swarm systems is that each agent has to achieve the desired formation using local information without relying on centralized coordination [12]. A consensus-based approach was proposed for second-order swarm systems in [13], and it was proved that virtual structure, behavior, and leader-follower based approaches could be unified in the framework of consensus-based approaches. A distributed controller-observer schema for formation tracking control of a first-order swarm systems was presented in [14]. Necessary and sufficient conditions were proved to show that first-order swarm systems can achieve the rigid formation under undirected and directed topologies using the complex Laplacian approach in [15] and [16], respectively. In [17], the finite-time formation was reached by second-order swarm systems using a continuous consensus algorithm. Formation stabilization problems for second-order swarm systems were considered based on classic navigation function in [18]. It was proved in [19] that the formation errors of second-order swarm systems could converge to a small bounded region by a hybrid consensus-based approach.

It is worth emphasizing that the dynamics of each agent in swarm systems can only be described by a high-order model in some practical cases. Necessary and sufficient conditions of reaching formation for a special swarm systems were proposed in [20]. Formation stability problems for general linear high-order swarm systems were considered in [21]. Previous work [22] gave further consideration to formation stability problems for general linear high-order swarm systems with fixed and periodic switching undirected interaction topologies. Necessary and sufficient conditions for formation feasibility were presented in [23] for general linear high-order swarm systems. However, only time-invariant formation (TIF) is considered in [20]–[23], and the interaction topology in [23] can hardly change. It is more complicated and challenging to analyze and solve the formation control problems for swarm systems with switching topologies than those with fixed topologies [24]. Time-varying formation (TVF) control problems for general linear high-order swarm systems with fixed and switching interaction topologies were addressed in [25] and [26], where the above-mentioned results in [25] and [26] are not fully distributed, because the policies in [25] and [26] depend on the smallest nonzero eigenvalue of the Laplacian matrix. It is worth mentioning that the eigenvalues of the Laplacian matrix all rely on the entire interaction topology and are global information. Therefore, to address TVF control problems for high-order swarm systems, fully distributed formation protocol requiring no global information about the interaction topology precedes existing protocols. It is possible to use the adaptive based approach to design fully distributed formation protocols. The framework of fully distributed TVF control problems for high-order swarm systems with fixed interaction topology is the main focus in our existing results in [27]–[30]. Due to communication channel disconnection and reconnection among agents in practical applications, it is meaningful to study TVF control problems in a fully distributed way for high-order swarm systems with switching interaction topologies, or rather, the TVF can be achieved using distributed adaptive formation protocol without global information about the switching interaction topologies. As far as we know, TVF control problems for high-order swarm systems with adaptive gain scheduling technique and switching interaction topologies have not been comprehensively studied together.

In this paper, an adaptive based approach is applied to study the distributed TVF control problems for general linear high-order swarm systems with switching interaction topologies. Firstly, a distributed formation control protocol with an adaptive gain scheduling technique adjusting the coupling weight for each edge is constructed, where the desired formation is time-varying. Then, an algorithm including a feasible TVF condition to design the adaptive formation protocol is presented. Moreover, through applying the Lyapunov theory, it is proved that if the feasible TVF condition is satisfied, the proposed algorithm's stability can be guaranteed. Finally, numerical examples are provided to demonstrate the effectiveness of the theoretical results.

Compared with the previous studies on formation control, the novel features of this paper are threefold. Firstly, this paper proposes a fully distributed TVF control protocol which is independent of the global information about the interaction topology. However, the disadvantages of the mentioned methods in [25], [26], and [31] are that all agents share common constant formation control gains in protocols and the protocols are calculated by the minimum nonzero eigenvalue of the Laplacian matrix, which is global information. The formation control protocols in [25], [26], and [31] cannot be computed without using the global information of the interaction topology. Secondly, fully distributed formation control problems are solved for swarm systems with switching interaction topologies whereas the topologies in previous results [27]–[30] are fixed. Thirdly, the TVF problems for general linear high-order swarm systems are discussed. In [12]–[19], each agent is restricted to be low-order dynamics; and in [20]–[23], the formation is limited to be time-invariant. To address the TVF problems, the derivative of the formation information may affect the design of the formation control protocol. It is more complicated to study TVF control problems than TIF control problems. The methods in [20]–[23] cannot be directly applied to solve the problems in this paper.

The outline of this paper is shown as follows. In Section 2, mathematical preliminaries required in this paper are reviewed. The problem formulation is given in Section 3. An algorithm to design the distributed formation protocol with switching interaction topologies are investigated and stability analysis of the algorithm is proposed in Section 4. Simulation examples are presented for illustration in Section 5. Finally, Section 6 concludes the whole work.

Notation: In this paper, I_n is the $n \times n$ identity matrix and $\otimes$ denotes the Kronecker product. Let 0 denote zero matrices of appropriate size with zero vectors and zero number as special cases. Let $\mathbf{1}$ be a column vector of appropriate size with one as its elements. The superscript T means transpose for real matrices and $\mathcal{I}_n$ is the index set $\{1, 2, \ldots, n\}$. The 2-norm of a vector x is denoted by $\|x\|$.

II. PRELIMINARIES

A weighted graph is used to describe the interaction topology between the N agents. A graph $\mathcal{G}$ is a 3-tuple $(\mathcal{V}, \mathcal{E}, \mathcal{W})$, where $\mathcal{V} = \{v_p : p \in \mathcal{I}_N\}$ is the node set, $\mathcal{E} = \{(v_p, v_q) : v_p, v_q \in \mathcal{V}\}$ is the edge set and the nonnegative weighted adjacency matrix is $\mathcal{W} = [w_{pq}] \in \mathbb{R}^{N\times N}$. Let $e_{pq} = (v_p, v_q)(p \neq q)$ denote an edge in graph $\mathcal{G}$. For the weighted adjacency matrix $\mathcal{W}$, if and only if $e_{pq} \in \mathcal{E}$, $w_{qp} > 0$; otherwise, $w_{qp} = 0$. Moreover, $w_{pp} = 0, \forall p \in \mathcal{V}$. The neighbor set of node p is denoted as $\mathcal{N}_p = \{q \in \mathcal{V} | e_{qp} \in \mathcal{E}\}$. An undirected graph $\mathcal{G}$ is defined such that $e_{qp} \in \mathcal{E} \Leftrightarrow e_{pq} \in \mathcal{E}$. Each edge in the graph $\mathcal{G}$ is bidirectional and $w_{qp} = w_{pq}$. The graph $\mathcal{G}$ is connected if there is a bidirectional path between each node pair. Let $\mathcal{L} = [\ell_{pq}] \in \mathbb{R}^{N\times N}$ be the Laplacian matrix of the graph $\mathcal{G}$, which is defined by

$$\ell_{pq} = \begin{cases} \sum_{k=1, k\neq p}^{N} w_{pk}, & p = q, \\ -w_{pq}, & p \neq q. \end{cases}$$

We have the following lemma.

Lemma 1 [32]: The Laplacian matrix $\mathcal{L}$ of the graph $\mathcal{G}$ is with the following properties: i) $\mathcal{L}$ has at least one zero eigenvalues and $\mathbf{1}_N$ is the associated eigenvector, namely $\mathcal{L}\mathbf{1}_N = 0$ ii) For a connected graph $\mathcal{G}$, 0 is a simple eigenvalue of $\mathcal{L}$ and all the other $N-1$ eigenvalues are real and positive.

The interaction topology of the general linear swarm system is assumed to be switching. Let $[t_p, t_{p+1})(p \in \mathbb{N})$ be an infinite sequence of uniformly bounded non-overlapping time interval, where $t_1 = 0$, $0 < \Delta_{t0} \leq t_{p+1} - t_p \leq \Delta_{t1}$ and $\mathbb{N}$ is the set of natural numbers. The interaction topology changes at the switching sequence t_p and remains fixed during the time interval Δ_{t0} which is also known as the dwell time. A switching signal is denoted by $\delta(t) : \mathbb{R}_{\geq 0} \to \mathcal{I}_{G_n}$, where $\mathcal{I}_{G_n} \in \mathbb{N}$ is the graph index set associated with the element in G_n. It means that $\mathcal{G}_p = G_n$ when $p = \delta(t)$. The possible interaction topologies set G_n is finite. The interaction topology of the general linear swarm system and the associated Laplacian matrix at $\delta(t)$ are respectively denoted by $\mathcal{G}_{\delta(t)}$ and $\mathcal{L}_{\delta(t)}$. The neighbor set of agent p at $\delta(t)$ is expressed as $\mathcal{N}_p^{\delta(t)}$.

III. PROBLEM DESCRIPTION

Consider a general linear swarm system with N agents. The dynamics of each agent is given by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \tag{1}$$

where $i = 1, 2, \ldots, N$, $x_i(t) \in \mathbb{R}^r$ are the states, $u_i(t) \in \mathbb{R}^n$ are the control inputs, $A \in \mathbb{R}^{r\times r}$ and $B \in \mathbb{R}^{r\times n}$ are system matrices. The switching interaction topology of the swarm system can be treated as a graph $\mathcal{G}_{\delta(t)}$, where each node in $\mathcal{G}_{\delta(t)}$ represents an agent in the swarm system. Define $w_{ij}(t)$ as the (i, j)-th element of the nonnegative weighted adjacency matrix associated with $\mathcal{G}_{\delta(t)}$. For $i, j \in \mathcal{I}_N$, the communication channel between agent i and j is denoted by the edge e_{ij} associated with the communication strength $w_{ij}(t)$.

Definition 1: A TVF is described by $h(t) = \left[h_1^T(t), h_2^T(t), \ldots, h_N^T(t)\right]^T \in \mathbb{R}^{rN}$ with $h_i(t)$ $(i \in \mathcal{I}_N)$ piecewise continuously differentiable. For any given bounded initial states, the TVF specified by $h(t)$ is said to be achieved by swarm system (1) if

$$\lim_{t\to\infty} ((x_i(t) - x_j(t)) - (h_i(t) - h_j(t))) = 0 \; (i, j \in \mathcal{I}_N). \tag{2}$$

Remark 1: Definition 1 implies that the TVF problem degenerates into the consensus problem when $h(t) \equiv 0$. Swarm system (1) achieves TIF if $h(t)$ equals to a constant vector in (2). Therefore, consensus problems and TIF problems for linear high-order swarm system with switching interaction topologies can be solved by applying the results in this paper.

Assumption 1: All possible communication graphs in G_n are connected.

Let $h(t)$ be a desired TVF. Consider the following distributed adaptive TVF control protocol to each agent in switching interaction topologies as

$$\begin{cases} u_i(t) = F_1x_i(t) + F_2 \sum_{j=\mathcal{N}_i^{\delta(t)}}^{N} c_{ij}(t)w_{ij}(t)(x_{ij}(t) - h_{ij}(t)), \\ \dot{c}_{ij}(t) = \varepsilon_{ij}w_{ij}(t)(x_{ij}(t) - h_{ij}(t))^T \Phi(x_{ij}(t) - h_{ij}(t)), \end{cases} \tag{3}$$

where $x_{ij}(t) = x_i(t) - x_j(t)$, $h_{ij}(t) = h_i(t) - h_j(t)$. The time-varying coupling weight for the edge e_{ij} is denoted by $c_{ij}(t)$ with $c_{ij}(0) = c_{ji}(0)$, $\varepsilon_{ij} = \varepsilon_{ji}$ are given positive constants, $F_1, F_2 \in \mathbb{R}^{n\times r}$ and $\Phi \in \mathbb{R}^{r\times r}$ are feedback gain matrices.

Remark 2: Protocol (3) provides a general framework for adaptive TVF protocols. The role of F_1 in the adaptive formation protocol (3) is to expand the set of feasible TVF $h(t)$. If $F_1 = 0$, protocol (3) only uses neighboring relative states. F_2 and $c_{ij}(t)$ are used to drive the swarm system to achieve the desired formation under switching interaction topologies. $c_{ij}(t)(i \in \mathcal{I}_N, j = \mathcal{N}_i^{\delta(t)})$ are adaptive gains applied to adjust the interaction strength among agents. The object of using adaptive scheme for $c_{ij}(t)$ is to remove calculating the minimum of the smallest nonzero eigenvalues of Laplacian matrix $\mathcal{L}_{\delta(t)}$, which is global information. Protocol (3) has generality. If $F_1 = 0$, $\dot{c}_{ij}(t) = 0$ and $c_{ij}(0) = 1$, the protocol (3) in [23] becomes a special case of protocol (3) in this section. Compared with the distributed adaptive TVF control protocols in [28] and [29], protocol (3) in this section can be applied to deal with the distributed adaptive TVF control problems for swarm system with switching interaction topologies. Protocols in [28] and [29] can be considered as special cases of protocol (3) in this section. The problem discussed in this paper has more generality than those in [28] and [29].

Under protocol (3), swarm system (1) can be rewritten as follows

$$\begin{cases} \dot{x}_i(t) = (A+BF_1)x_i(t) + BF_2 \sum_{j=\mathcal{N}_i^{\delta(t)}}^{N} c_{ij}(t)w_{ij}(t)(x_{ij}(t) - h_{ij}(t)), \\ \dot{c}_{ij}(t) = \varepsilon_{ij}w_{ij}(t)(x_{ij}(t) - h_{ij}(t))^T \Phi(x_{ij}(t) - h_{ij}(t)), \end{cases} \tag{4}$$

where $i = 1, 2, \ldots, N$. The main focus of this paper is how to determine the parameters in distributed adaptive formation control protocol (3) such that the general linear high-order swarm system (1) with switching interaction topologies can achieve the desired TVF.

IV. MAIN RESULTS

In this section, firstly an algorithm is presented to design distributed adaptive TVF control protocol (3). Sufficient condition for swarm system (1) with switching interaction topologies using the proposed algorithm to achieve the TVF is derived.

The following procedure determines the control parameters of protocol (3) with two steps.

Algorithm 1: The control parameters of the distributed adaptive formation control protocol (3) applied in the swarm system (1) can be determined in the following procedure.

Step 1): In an attempt to design F_1, test the following feasible TVF condition (5) for all $i, j \in \mathcal{I}_N$. If there exists F_1 satisfying condition (5), continue; else the TVF specified by $h(t)$ is not feasible for the swarm system (1) under protocol (3) and the algorithm stops.

$$\lim_{t\to\infty}((A+BF_1)h_{ij}(t)-\dot{h}_{ij}(t))=0. \tag{5}$$

Step 2): Calculate a symmetric positive definite matrix Q using the following linear matrix inequality (LMI).

$$Q(A+BF_1)^T+(A+BF_1)Q-2BB^T<0. \tag{6}$$

Then F_2 and Φ can be given by $F_2 = -B^TQ^{-1}$ and $\Phi = Q^{-1}BB^TQ^{-1}$. It is verified in [33] that there is a Q if (A, B) is stabilizable.

Remark 3: The feasible formation condition (5) shows that not all possible formations can be realized by swarm system (1). Formation feasible condition is also considered in [23], [25], and [27]. In condition (5), the performance of F_1 is to expand the time-varying feasible formation set. Noting that F_1 has no effects on swarm system (1) achieving TVF. If $F_1 = 0$, the feasible formation set is restricted to $\lim_{t\to\infty}(Ah_{ij}(t)-\dot{h}_{ij}(t)) = 0$. TVF control problems for heterogeneous linear swarm systems with switching interaction topologies have been studied in [36] and [37]. The given TVF in [36] and [37] is subjected to $\dot{h}_i(t) = A_0h_i(t)(i = 1, 2, \ldots, N)$. Since identical dynamics are considered in this paper, the feasible TVF condition (5) is general. The feasible formation condition (5) degenerates into the feasible condition for TIF in [23] if $F_1 = 0$ and $\dot{h}_i(t) \equiv 0$.

Based on Algorithm 1, we can obtain the main results of this section.

Theorem 1: Suppose that Assumption 1 holds. For any arbitrary switching interaction topology $\mathcal{G}_{\delta(t)}$ in G_n, if (A, B) is stabilizable and the feasible formation condition (5) holds, swarm system (1) can achieve the TVF under the distributed adaptive control protocol (3) designed in Algorithm 1 without global information about the interaction topologies.

Proof: Let $\phi_i(t) = x_i(t) - h_i(t)(i = 1, 2, \ldots, N)$. It follows from swarm system (4) that

$$\begin{cases}\dot{\phi}_i(t)=(A+BF_1)\phi_i(t)+(A+BF_1)h_i(t)-\dot{h}_i(t)\\ \qquad +BF_2\sum\limits_{j=\mathcal{N}_i^{\delta(t)}}^{N}c_{ij}(t)w_{ij}(t)(\phi_i(t)-\phi_j(t)),\\ \dot{c}_{ij}(t)=\varepsilon_{ij}w_{ij}(t)(\phi_i(t)-\phi_j(t))^T\Phi(\phi_i(t)-\phi_j(t)).\end{cases} \tag{7}$$

Let $\varsigma_i(t) = \phi_i(t) - \frac{1}{N}\sum_{j=1}^{N}\phi_j(t)$, $\varsigma(t) = [\varsigma_1^T(t), \varsigma_2^T(t), \ldots, \varsigma_N^T(t)]^T$ and $\phi(t) = [\phi_1^T(t), \phi_2^T(t), \ldots, \phi_N^T(t)]^T$. Then $\varsigma(t) = (\Pi \otimes I_r)\phi(t)$, where $\Pi = I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T$. Π has a simple eigenvalue 0 associated with the right eigenvector $\mathbf{1}$. Since $\varepsilon_{ij} = \varepsilon_{ji}$ and $c_{ij}(0) = c_{ji}(0)$, it follows from protocol (3) that $c_{ij}(t) = c_{ji}(t)$ for $\forall t \geq 0$. According to the definition of $\varsigma(t)$, it can be obtained that

$$\begin{cases}\dot{\varsigma}_i(t)=\dot{\phi}_i(t)-\frac{1}{N}\sum\limits_{j=1}^{N}\dot{\phi}_j(t),\\ \dot{c}_{ij}(t)=\varepsilon_{ij}w_{ij}(t)(\varsigma_i(t)-\varsigma_j(t))^T\Phi(\varsigma_i(t)-\varsigma_j(t)).\end{cases} \tag{8}$$

Substituting (7) into (8), one has

$$\begin{cases}\dot{\varsigma}_i(t)=(A+BF_1)\varsigma_i(t)\\ \qquad +\dfrac{1}{N}\sum\limits_{j=1}^{N}[(A+BF_1)h_{ij}(t)-\dot{h}_{ij}(t)]\\ \qquad +BF_2\sum\limits_{j=1}^{N}c_{ij}(t)w_{ij}(t)(\varsigma_i(t)-\varsigma_j(t)),\\ \dot{c}_{ij}(t)=\varepsilon_{ij}w_{ij}(t)(\varsigma_i(t)-\varsigma_j(t))^T\Phi(\varsigma_i(t)-\varsigma_j(t)).\end{cases} \tag{9}$$

Choose the following Lyapunov functional candidate

$$V(t)=\frac{1}{2}\sum_{i=1}^{N}\varsigma_i^T(t)Q^{-1}\varsigma_i(t)+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{(c_{ij}(t)-\beta)^2}{4\varepsilon_{ij}}, \tag{10}$$

where β is a positive constant to be determined later. The time derivative of $V(t)$ is given by

$$\dot{V}(t)=\sum_{i=1}^{N}\varsigma_i^T(t)Q^{-1}\dot{\varsigma}_i(t)+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{(c_{ij}(t)-\beta)}{2\varepsilon_{ij}}\dot{c}_{ij}(t). \tag{11}$$

Substituting (9) into (11) yields

$$\dot{V}(t)=\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)+\dot{V}_4(t), \tag{12}$$

where

$$\dot{V}_1(t)=\sum_{i=1}^{N}\varsigma_i^T(t)Q^{-1}(A+BF_1)\varsigma_i(t),$$

$$\dot{V}_2(t)=\sum_{i=1}^{N}\sum_{j=1}^{N}\varsigma_i^T(t)Q^{-1}BF_2c_{ij}(t)w_{ij}(t)(\varsigma_i(t)-\varsigma_j(t)),$$

$$\dot{V}_3(t)=\sum_{i=1}^{N}\sum_{j=1}^{N}\varsigma_i^T(t)Q^{-1}\frac{1}{N}[(A+BF_1)h_{ij}(t)-\dot{h}_{ij}(t)],$$

$$\dot{V}_4(t)=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\times(c_{ij}(t)-\beta)w_{ij}(t)(\varsigma_i(t)-\varsigma_j(t))^T\Phi(\varsigma_i(t)-\varsigma_j(t)).$$

Then, it can be derived that

$$\dot{V}_4(t)=\sum_{i=1}^{N}\sum_{j=1}^{N}\left(c_{ij}(t)-\beta\right)w_{ij}(t)\varsigma_i^T(t)\Phi(\varsigma_i(t)-\varsigma_j(t)). \tag{13}$$

According to Algorithm 1, substitute F_2 and Φ into $\dot{V}_2(t)$ and $\dot{V}_4(t)$ respectively, one has

$$\dot{V}_2(t) = -\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}(t)w_{ij}(t)\varsigma_i^T(t)Q^{-1}BB^TQ^{-1}(\varsigma_i(t)-\varsigma_j(t)), \tag{14}$$

and

$$\dot{V}_4(t) = \sum_{i=1}^{N}\sum_{j=1}^{N}\left(c_{ij}(t)-\beta\right)w_{ij}(t)\varsigma_i^T(t)Q^{-1}BB^TQ^{-1}(\varsigma_i(t)-\varsigma_j(t)). \tag{15}$$

$\dot{V}(t)$ can be written as

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_3(t) - \dot{V}_5(t), \tag{16}$$

where $\dot{V}_5(t) = \beta\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}(t)\varsigma_i^T(t)Q^{-1}BB^TQ^{-1}(\varsigma_i(t) - \varsigma_j(t))$. Let $\tilde{\varsigma}_i(t) = Q^{-1}\varsigma_i(t)$, then one gets

$$\dot{V}_1(t) = \frac{1}{2}\sum_{i=1}^{N}\tilde{\varsigma}_i^T(t)((A+BF_1)Q + Q(A+BF_1)^T)\tilde{\varsigma}_i(t), \tag{17}$$

$$\dot{V}_3(t) = \sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{\varsigma}_i^T(t)\frac{1}{N}[(A+BF_1)h_{ij}(t) - \dot{h}_{ij}(t)], \tag{18}$$

and

$$\dot{V}_5(t) = \beta\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}(t)\tilde{\varsigma}_i^T(t)BB^T\tilde{\varsigma}_i(t). \tag{19}$$

Let $\tilde{\varsigma}(t) = [\tilde{\varsigma}_1^T(t), \tilde{\varsigma}_2^T(t), \ldots, \tilde{\varsigma}_N^T(t)]^T$. From (17), (18) and (19), it holds that

$$\begin{aligned}\dot{V}(t) = \frac{1}{2}\tilde{\varsigma}^T(t)[I_N \otimes (Q(A+BF_1)^T &+ (A+BF_1)Q - 2\beta\mathcal{L}_{\delta(t)} \otimes BB^T)]\tilde{\varsigma}(t)\\ &+ \tilde{\varsigma}^T(t)[\Pi \otimes (A+BF_1)]h(t) - \tilde{\varsigma}^T(t)[\Pi \otimes I_r]\dot{h}(t),\end{aligned} \tag{20}$$

where $\mathcal{L}_{\delta(t)}$ is the Laplacian matrix associated with topology $\mathcal{G}_{\delta(t)}$. Since $\mathcal{G}_{\delta(t)}$ is connected and $\Pi\mathbf{1} = 0$, $(\mathbf{1}^T \otimes I)\tilde{\varsigma}(t) = 0$, $\tilde{\varsigma}^T(t)(\mathcal{L}_{\delta(t)} \otimes I)\tilde{\varsigma}(t) \geq \lambda_2^{\min}\tilde{\varsigma}^T(t)\tilde{\varsigma}(t)$ is obtained, where $\lambda_2^{\min}$ denotes the minimum of the smallest nonzero eigenvalues of $\mathcal{L}_{\delta(t)}$ for all $\mathcal{G}_{\delta(t)} \in G_n$.

Therefore, one has

$$\begin{aligned}\dot{V}(t) \leq \frac{1}{2}\tilde{\varsigma}^T(t)[I_N \otimes (Q(A+BF_1)^T &+ (A+BF_1)Q - 2\beta\lambda_2^{\min}BB^T)]\tilde{\varsigma}(t)\\ &+ \tilde{\varsigma}^T(t)[\Pi \otimes (A+BF_1)]h(t) - \tilde{\varsigma}^T(t)[\Pi \otimes I_r]\dot{h}(t).\end{aligned} \tag{21}$$

Choose sufficiently large β such that $\beta\lambda_2^{\min} \geq 1$ and we have

$$\begin{aligned}Q(A+BF_1)^T &+ (A+BF_1)Q - 2\beta\lambda_2^{\min}BB^T\\ &\leq Q(A+BF_1)^T + (A+BF_1)Q - 2BB^T\\ &< 0.\end{aligned} \tag{22}$$

Let

$$\zeta(t) = \frac{1}{2}\tilde{\varsigma}^T(t)[I_N \otimes (Q(A+BF_1)^T + (A+BF_1)Q - 2\beta\lambda_2^{\min}BB^T)]\tilde{\varsigma}(t),$$

and

$$\xi(t) = \tilde{\varsigma}^T(t)[\Pi \otimes (A+BF_1)]h(t) - \tilde{\varsigma}^T(t)[\Pi \otimes I_r]\dot{h}(t).$$

It is obtained that $\zeta(t) \leq 0$. Since appropriate F_1 is chosen and $h(t)$ satisfies the feasible formation condition (5), we have $\lim_{t\to\infty}\xi(t) = 0$; so there exists a finite time t', which satisfies $\zeta(t') + \xi(t') \leq 0$. Then we can obtain $\lim_{t\in[t',\infty]}\dot{V}(t) \leq 0$. By LaSalle-Yoshizawa theorem [38], we have $\lim_{t\in[t',\infty]}\tilde{\varsigma}(t) = 0$, implying that $\lim_{t\in[t',\infty]}\varsigma(t) = 0$, which means that $\lim_{t\in[t',\infty]}((x_i(t) - x_j(t)) - (h_i(t) - h_j(t))) = 0 (i, j \in \mathcal{I}_N)$. Accordingly, using distributed adaptive formation control protocol (3), swarm system (1) can achieve the TVF specified by $h(t)$; that is, the fully distributed TVF control problem for high-order linear swarm system with switching interaction topologies is solved. ■

Remark 4: Compared with previous results for swarm system with switching interaction topologies achieving TVF, Theorem 1 in this paper indicates that protocol (3) can be designed without requiring global information about the interaction topologies. However, existing results in [25], [27], [34], and [35] all suffer from the same limitation that TVF control protocols in [25], [27], [34], and [35] rely on the minimum nonzero eigenvalue of the Laplacian matrix. The minimum nonzero eigenvalue of the Laplacian matrix is global information because it is calculated by the entire interaction topology. Those approaches for achieving TVF in [25], [27], [34], and [35] by swarm system with switching interaction topologies are not fully distributed only according to the local information of each agent and its neighbors.

Remark 5: Noting that directed interaction topology [39] should be considered in many practical cases. It is of interest to further study adaptive TVF control problems for the case with directed interaction topology. Interesting future topics will also focus on extending the results to the case with time delay and output feedback.

V. NUMERICAL SIMULATIONS

In this section, two numerical examples are presented to validate the theoretical result obtained in the previous section. Firstly, a three-order swarm system consists of ten agents with switching interaction topologies is considered in the first example to achieve the TVF using distributed adaptive formation control protocol (3). In the second example, the theoretical result is applied to solve multi-vehicle systems cooperative reconnaissance and detection problems.

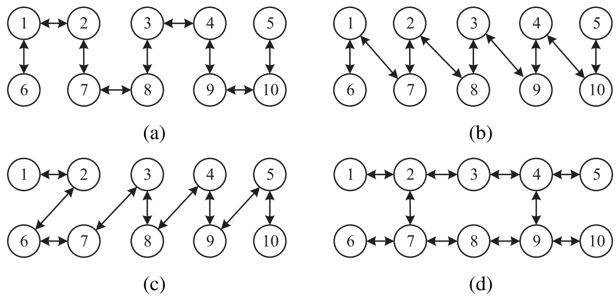

FIGURE 1. Switched interaction topologies set for the third-order swarm system. (a) G_1. (b) G_2. (c) G_3. (d) G_4.

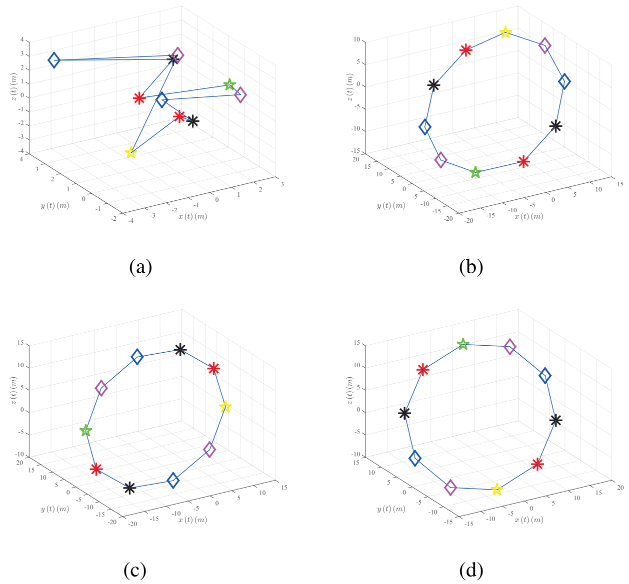

FIGURE 2. State trajectory snapshots of the ten agents. (a) $t = 0$s. (b) $t = 20$s. (c) $t = 35$s. (d) $t = 50$s.

Example 1: Consider a general linear third-order swarm system comprising ten agents. Each agent has the dynamics as

$$A = \begin{bmatrix} 1 & -1 & -2 \\ 5 & 0 & 2 \\ -2 & -0.1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The state of each agent is with $x_i(t) = [x_i^1(t), x_i^2(t), x_i^3(t)]^T$ $(i = 1, 2, \ldots, 10)$. As shown in Fig. 1, let $\mathcal{G}_{\delta(t)}$ switch every 5s among G_1, G_2, G_3 and G_4 randomly. The swarm system is asked to reach and keep a periodic time-varying decagon formation which is specified by

$$h_i(t) = \begin{bmatrix} r\sin(\omega t + \frac{(i-1)\pi}{5}) + r\cos(\omega t + \frac{(i-1)\pi}{5}) \\ r\sin(\omega t + \frac{(i-1)\pi}{5}) - r\cos(\omega t + \frac{(i-1)\pi}{5}) \\ r\sin(\omega t + \frac{(i-1)\pi}{5}) \end{bmatrix},$$

where $i = 1, 2, \ldots, 10$, $r = 10m$ and $\omega = 2rad/s$. Choose $c_{ij}(0) = c_{ji}(0) = 0$ and initialize each agents state as $x_i^k(0) = i(\kappa - 0.5)$ $(i = 1, 2, \ldots, 10; k = 1, 2, 3)$, where κ is a random value between 0 and 1. Let $\varepsilon_{ij} = \varepsilon_{ji} = 0.01$ $(i, j = 1, 2, \ldots, 10)$ in protocol (3). The gain matrices in protocol (3) can be computed in accordance with

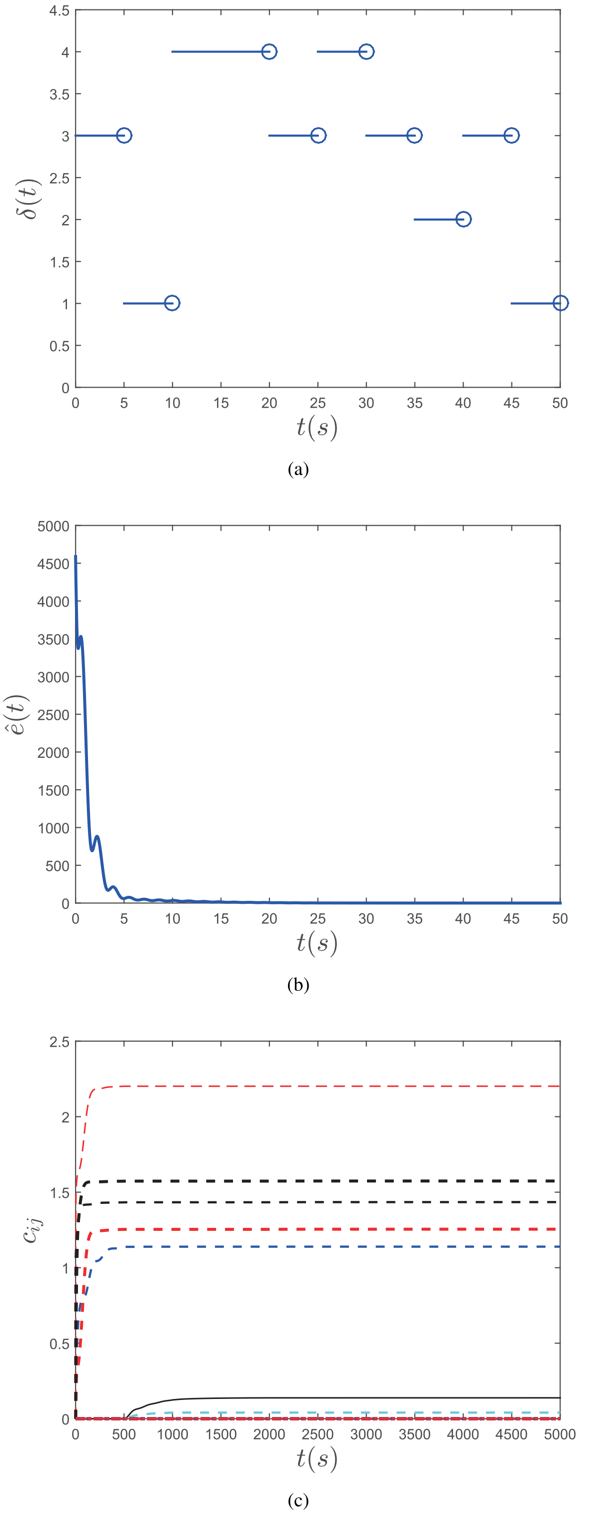

FIGURE 3. (a) The switching signal of the three-order swarm system; (b) The formation error of the three-order swarm system; (c) The coupling weights of the three-order swarm system.

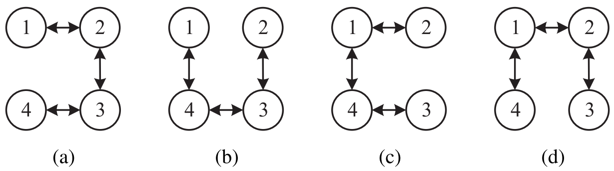

FIGURE 4. Switched interaction topologies set of multi-vehicle system. (a) G_1. (b) G_2. (c) G_3. (d) G_4.

Algorithm 1 as

$$F_1 = \begin{bmatrix} -6 & -3 & 4 \\ 3 & -0.9 & 0 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} -3.1321 & -4.2783 & 7.8239 \\ 7.9246 & 7.8239 & -18.3374 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 72.6103 & 75.4020 & -169.8230 \\ 75.4020 & 79.5176 & -176.9433 \\ -169.8230 & -176.9433 & 397.4737 \end{bmatrix}.$$

It should be pointed out that F_1, F_2, and Φ are designed without using the global information of the interaction topologies.

The state snapshots of the ten agents at $t = 0s$, $t = 20s$, $t = 35s$ and $t = 50s$ are illustrated in Fig. 2. Figs. 2(a) and (b) show that the TVF is reached. It can be seen from Figs. 2(b), (c) and (d) that the formation keeps rotation. The switching signal of the swarm system is depicted in Fig. 3(a). Let $\tilde{e}_i(t) = x_{i1}(t) - h_{i1}(t)(i = 2, 3, \ldots, N)$ and $\tilde{e}(t) = [\tilde{e}_2^T(t), \tilde{e}_3^T(t), \ldots, \tilde{e}_N^T(t)]^T$. Define $\hat{e}(t) = \tilde{e}^T(t)\tilde{e}(t)$ as the formation error of the swarm system. Fig. 3(b) shows that the formation error converges to zero. From Fig. 3(c), the coupling weights $c_{ij}(t)$ are clearly bounded. Therefore, using distributed protocol (3), the desired TVF is achieved by the swarm system (1) with switching interaction topologies.

Example 2: Consider using a multi-vehicle system, which is equipped with different specific sensors, to maximally explore an unknown region. Firstly, the multi-vehicle system should reach a formation. Secondly, the formation should be kept rotation to guarantee that each direction can be detected by different specific sensors. Due to communication constraints and link variations, the interaction topology of the multi-vehicle system may change. Without loss of generality, the switching interaction topologies are considered in this application.

Suppose that a team of four vehicles forms the multi-vehicle system. Each vehicle has dynamics with

$$\begin{cases} \dot{P}_i^x(t) = V_i(t)\cos(\theta_i(t)), \\ \dot{P}_i^y(t) = V_i(t)\sin(\theta_i(t)), \\ \dot{\theta}_i(t) = \omega_i(t), \end{cases}$$

where $i = 1, 2, 3, 4$, $P_i^x(t) \in \mathbb{R}$ and $P_i^y(t) \in \mathbb{R}$ denote the position onto the X-Y plane for the ith vehicle. $\omega_i(t) \in \mathbb{R}$, $V_i(t) \in \mathbb{R}$ and $\theta_i(t) \in \mathbb{R}$ denote the angular velocity, linear velocity and heading angle of the ith vehicle respectively. The multi-vehicle system can be transformed as a linearized system described by (1) using the method in [40] with

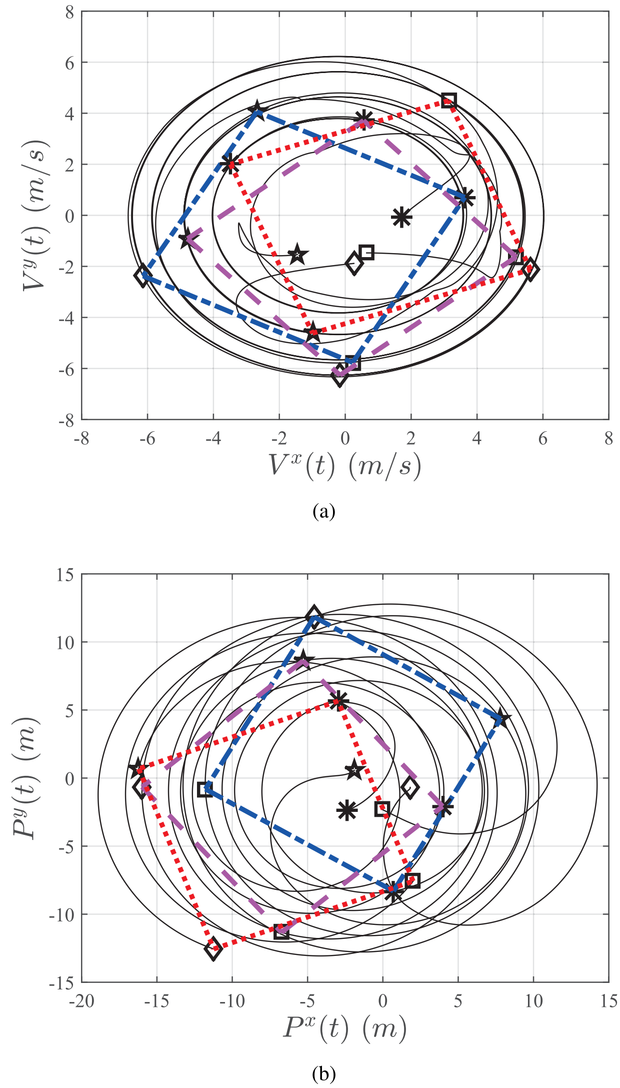

FIGURE 5. (a) Position trajectories of the multi-vehicle system; (b) Velocity trajectories of the multi-vehicle system.

$x_i(t) = [P_i^x(t), V_i^x(t), P_i^y(t), V_i^y(t)]^T$, $u_i(t) = [u_i^x(t), u_i^y(t)]^T$, where $V_i^x(t) \in \mathbb{R}$ (resp. $u_i^x(t) \in \mathbb{R}$) and $V_i^y(t) \in \mathbb{R}$ (resp. $u_i^y(t) \in \mathbb{R}$) are the linear velocity (resp. control input) along the X and Y axes, respectively. The system matrices A and B are obtained as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The desired TVF is presented by

$$h_i(t) = \begin{bmatrix} r\cos(\omega t + \frac{(i-1)\pi}{2}) \\ -r\omega\sin(\omega t + \frac{(i-1)\pi}{2}) \\ r\sin(\omega t + \frac{(i-1)\pi}{2}) \\ r\omega\cos(\omega t + \frac{(i-1)\pi}{2}) \end{bmatrix} \quad (i = 1, 2, 3, 4),$$

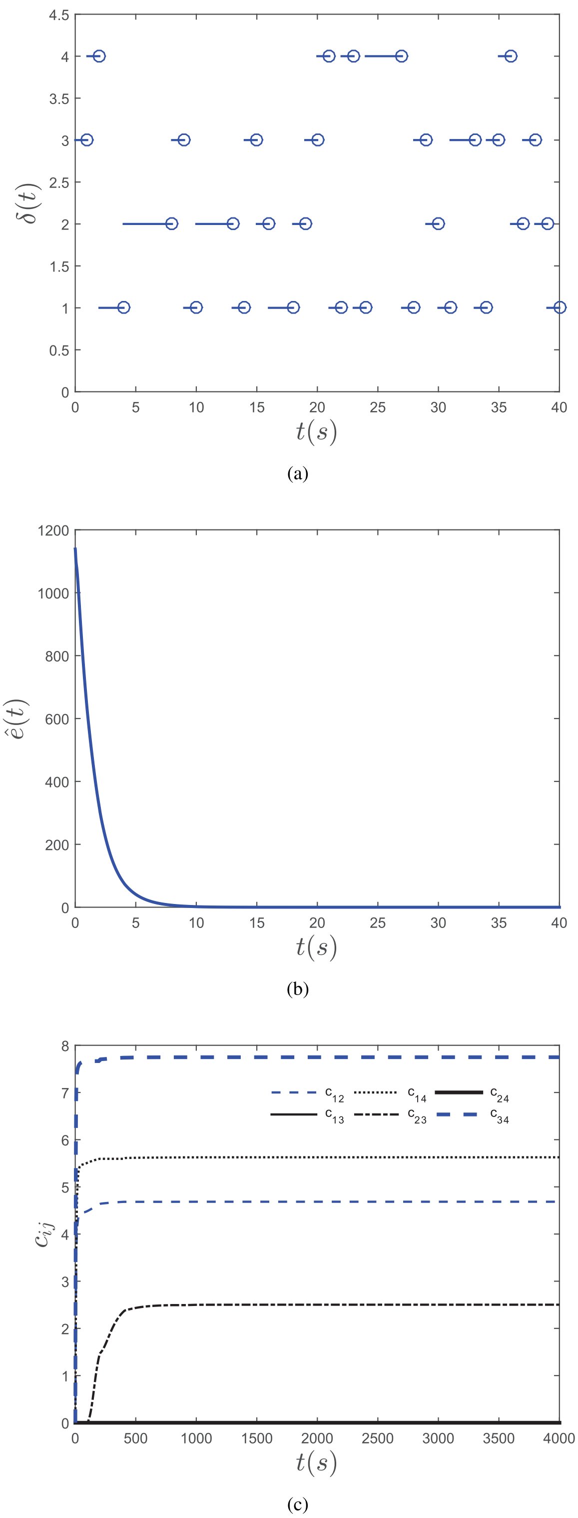

FIGURE 6. (a) The switching signal of the multi-vehicle system; (b) The formation error of the multi-vehicle system; (c) The coupling weights of the multi-vehicle system.

where $r = 10m$ and $\omega = 0.5rad/s$. Choose $c_{ij}(0) = c_{ji}(0) = 0$ and initialize the multi-vehicle system by $5(\kappa - 0.5)$. Let $\varepsilon_{ij} = \varepsilon_{ji} = 1 (i, j = 1, 2, 3, 4)$. The switching interaction topologies set is depicted in Fig. 4. Satisfying the feasible formation condition (5) and using Algorithm 1, all gain matrices are obtained as

$$F_1 = \begin{bmatrix} 0 & 0.02 & 0.01 & -0.5 \\ 0.01 & 0.2 & -0.15 & -0.02 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} -0.5636 & -1.7363 & -0.1948 & -0.0862 \\ 0.1636 & -0.0862 & -0.4466 & -1.550 \end{bmatrix},$$
$$\Phi = \begin{bmatrix} 0.3444 & 0.9644 & 0.0367 & -0.2053 \\ 0.9644 & 3.0221 & 0.3767 & 0.2833 \\ 0.0367 & 0.3767 & 0.2374 & 0.7099 \\ -0.2053 & 0.2833 & 0.7099 & 2.4161 \end{bmatrix}.$$

Solid blue lines in Figs. 5(a) and (b) respectively show the velocity and position trajectories of the multi-vehicle system from $t = 0s$ to $t = 40s$. Using the star, asterisk, square and diamond to represent the states of each vehicle, the TVF of the multi-vehicle system under switching interaction topologies is illustrated at $t = 10s$ using bold dash-dotted lines, at $t = 25s$ using bold dashed lines and at $t = 40s$ using bold dotted lines. From Fig. 5, we can observe that both the velocities and positions of the multi-vehicle system reach parallel rectangle formations and the parallel rectangles keep rotation. In Fig. 6, the switching signal, formation error, and coupling weights are displayed respectively. Fig. 6(a) shows that the interaction topology $\mathcal{G}_{\delta(t)}$ of the multi-vehicle system switch every 1s among G_1, G_2, G_3 and G_4 randomly. From Fig. 6(b), the formation error of the multi-vehicle system converges to zero which means that the TVF is achieved. In Fig. 6(c), the coupling weights is bounded when the TVF is achieved. Therefore the multi-vehicle system under switching interaction topologies achieves the given TVF based on an adaptive based approach.

VI. CONCLUSIONS

Distributed TVF control problems for general linear swarm systems with switching interaction topologies were investigated using an adaptive based approach in this paper. A fully distributed formation protocol was proposed based on the local states of neighboring agents. An algorithm was presented to determine the gain matrices of the protocol. The solvability of the algorithm can be guaranteed if the dynamics of each agent is stabilizable. A feasible TVF condition was given in the algorithm, and the stability of the proposed algorithm was proved by constructing the Lyapunov function. It was shown that general linear swarm systems with switching interaction topologies could achieve the TVF under the distributed adaptive formation control protocol if the feasible TVF condition were satisfied. The limitation of using global information about the interaction topologies was removed. On the basis of this result, it is of interest to further study adaptive TVF control problems for the case with directed interaction topology. Interesting future topics will also focus on extending the results to the case with time delay and output feedback.

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