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An Attribute Control Chart Based on the Birnbaum-Saunders Distribution Using Repetitive Sampling

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ABSTRACT In this paper, an attribute control chart using repetitive sampling is proposed when the lifetime of a product follows the Birnbaum–Saunders distribution. The number of failures is to be monitored by designing two pairs of upper and lower control limits. The necessary measurements are derived to assess the average run length (ARL). The various tables for ARLs are presented when the scale parameter and/or the shape parameter are shifted. The efficiency of the proposed control chart is compared with an existing chart. The proposed chart is shown to be more efficient than an existing control chart in terms of ARL. A real example is given for illustration purpose.

INDEX TERMS Attribute chart, Birnbaum-Saunders distribution, average run length (ARL), control chart.

I. INTRODUCTION

Control charts have been widely used in the industry for quality monitoring. These control charts indicate the potential presence of causes of variation in a manufacturing process. A quick indication about the cause of variation on time helps to minimize the non-conforming items. A process is said to be out of control if the plotted statistic beyond the upper control limit (UCL) or lower control limit (LCL). The process is said to be in control if the plotted statistic is within the control limits. The efficiency of any control chart is assessed through the average run length (ARL). The control chart is considered more powerful if it provides quicker indication about the shifted process as compared to other control charts.

The control charts are classified into attribute and variable control charts. An attribute control chart is used for monitoring attribute data such as the number of non-conforming items. On the other hand, a variable control chart is used for continuous data such as the X-bar chart. The details about the application of attribute and variable control charts can be seen in [20], [21], [23], [24], [25], [33], [37], and [38]. Designing a variable control chart for a non-normal distribution may be difficult because the exact distribution of the associated statistic may not be known. Therefore, designing of an attribute control chart for some non-normal distribution based on a life test has attracted the attention of researchers. Particularly, a time-truncated life test is popularly employed to save the experiment time. Recently, Aslam and Jun [5] designed this type of control chart for the Weibull distribution. Aslam *et al.* [11] considered Pareto distribution of second kind.

The Birnbaum-Saunders (BS) distribution was originally developed by Birnbaum and Saunders [17] and has been widely used for analyzing positive skewed data. The BS distribution has close relation with the normal distribution. The applications of BS distribution in various fields can be seen in [11], [27], [30], and [35]. By exploring the literature, it can be seen that there are some studies on quality monitoring for BS distribution; see [29], [35]. However, there have been no works on control charts for the BS distribution using repetitive sampling.

Recently, several authors explored the use of various sampling schemes in the area of control charts to improve the sensitivity to detect a small shift in the manufacturing process. These sampling schemes such as repetitive sampling, rank set sampling, double sampling and multiple dependent state (MDS) sampling have been widely used in recent years to design control charts. Chen and Yeh [18] designed

X-bar chart using non-uniform sampling scheme. Chen [19] designed chart using variable sampling size and sampling interval (VSSI). Mehmood *et al.* [32] proposed for location using various sampling schemes. Ahmad *et al.* [4] designed a dispersion control chart for some distributions using repetitive sampling. Abid *et al.* [1] used rank set sampling in non-parametric charts. Abujiya *et al.* [2] proposed a new EWMA chart using rank set sampling. Dobbah *et al.* [22] designed a control chart using MDS sampling. Joekes *et al.* [26] designed an attribute chart using double sampling.

Repetitive sampling is considered as more efficient than single sampling. Repetitive sampling was originally developed by Sherman [36]. Later, this sampling scheme has been widely used in the area of acceptance sampling plans; see [14]. Repetitive sampling has been applied in control chart by Ahmad *et al.* [3], Aslam *et al.* [6]–[9], [12], and Azam *et al.* [15].

In this paper, the design of np control chart for the BS distribution will be considered. The structure of the proposed chart will be given when scale parameter is shifted, shape parameter is shifted and both are shifted. The efficiency of the proposed control chart will be compared with the control chart by Leiva *et al.* [28]. We show the efficiency of the proposed chart over [28] in terms of ARL. A real example is given for the illustration purpose. The paper is organized as follows: the proposed control chart is presented in Section 2. The comparative study is given in Section 3. The application of proposed chart is given in Section 4. The concluding remarks are given in last section.

A. PROPOSED CONTROL CHART

Suppose that the lifetime of a product, denoted by T, follows the Birnbaum Saunders (BS) distribution. The cumulative distribution function (cdf) of the BS distribution having shape parameter b and scale parameter σ is given by

$$F(t; b, \sigma) = \Phi\left(\frac{1}{b}\xi\left(\frac{t}{\sigma}\right)\right), \quad t > 0, \tag{1}$$

where $\xi(y) = \sqrt{y} - \sqrt{1}/y$ and $\Phi(\cdot)$ is the cdf of the standard normal distribution. The mean life (μ) of the BS distribution is given by

$$\mu = \mathrm{E}[\mathrm{T}] = \sigma\left(1 + \frac{1}{2}\mathrm{b}^2\right) \tag{2}$$

Let us consider a time-truncated life test having duration t_0. Often, it is convenient to consider t_0 as a fraction of μ_0, which is the target mean for the in-control process. Let $t_0 = a\mu_0$, where a is a constant. When, for example, $\mu_0 = 1000$h and $a = 0.1$, it means that the life test duration is just 10% of the target lifetime mean. When denoting the specified shape and the scale parameter by b_0 and σ_0, respectively, the target mean will be

$$\mu_0 = \sigma_0\left(1 + \frac{1}{2}b_0^2\right)$$

The probability that a failed item occurs by time t_0, denoted by p_0, is given by

$$\begin{aligned} p_0 = P\{T \le t_0|\sigma_0, b_0\} &= \Phi\left(\frac{1}{b_0}\xi\left(\frac{t_0}{\sigma_0}\right)\right) \\ &= \Phi\left(\frac{1}{b_0}\xi\left(a\left(1 + b_0^2/2\right)\right)\right) \end{aligned} \tag{3}$$

We propose the following *np* control chart using repetitive sampling under the time truncated life test for the BS distribution.

Step 1: Select a random sample of size n at each subgroup and perform the time truncated life test with duration t_0. Count the number of failed products (D, say) by time t_0.

Step 2: Declare the process as out-of-control if $D > UCL_1$ or $D < LCL_1$. Declare the process as in-control if $LCL_2 \le D \le UCL_2$. If $UCL_1 \le D \le UCL_2$ or $LCL_1 \le D \le LCL_2$, repeat Step-1.

As seen in Step 2, there are two pairs of control limits called the outer and the inner control limits. Because D follows the binomial distribution with parameters of n (sample size) and p_0, the outer control limits for the proposed control chart are given by

$$\mathrm{UCL}_1 = \mathrm{np}_0 + \mathrm{k}_1\sqrt{\mathrm{np}_0(1 - \mathrm{p}_0)} \tag{4a}$$

$$\mathrm{LCL}_1 = \max[0, \mathrm{np}_0 - \mathrm{k}_1\sqrt{\mathrm{np}_0(1 - \mathrm{p}_0)}] \tag{4b}$$

Also, the inner control limits are given by

$$\mathrm{UCL}_2 = \mathrm{np}_0 + \mathrm{k}_2\sqrt{\mathrm{np}_0(1 - \mathrm{p}_0)} \tag{5a}$$

$$\mathrm{LCL}_2 = \mathrm{np}_0 - \mathrm{k}_2\sqrt{\mathrm{np}_0(1 - \mathrm{p}_0)} \tag{5b}$$

Here, k_1 and k_2 are control constants to be determined by considering the target in-control ARL.

In practice, p_0 is unknown, control limits are replaced using $\bar{D}$ which is the average number of failures observed from the preliminary subgroups taken from in-control process.

$$\mathrm{UCL}_1 = \bar{D} + \mathrm{k}_1\sqrt{\bar{D}(1 - \bar{D}/\mathrm{n})} \tag{6a}$$

$$\mathrm{LCL}_1 = \max[0, \bar{D} - \mathrm{k}_1\sqrt{\bar{D}(1 - \bar{D}/\mathrm{n})}] \tag{6b}$$

$$\mathrm{UCL}_2 = \bar{D} + \mathrm{k}_2\sqrt{\bar{D}(1 - \bar{D}/\mathrm{n})} \tag{7a}$$

$$\mathrm{LCL}_2 = \bar{D} - \mathrm{k}_2\sqrt{\bar{D}(1 - \bar{D}/\mathrm{n})} \tag{7b}$$

B. IN-CONTROL ARL

The probability that the process is declared to be out of control at each subgroup when the process is actually in control is given as follows

$$\begin{aligned} \mathrm{P}^0_{\mathrm{out},1} &= \mathrm{P}(\mathrm{D} > UCL_1) + \mathrm{P}(\mathrm{D} < LCL_1) \\ &= \sum\nolimits_{\mathrm{d}=\lfloor \mathrm{UCL}_1\rfloor+1}^{\mathrm{n}} \binom{\mathrm{n}}{\mathrm{d}} \mathrm{p}_0^{\mathrm{d}}(1 - \mathrm{p}_0)^{\mathrm{n}-\mathrm{d}} \\ &\quad + \sum\nolimits_{\mathrm{d}=0}^{\lceil \mathrm{LCL}_1\rceil} \binom{\mathrm{n}}{\mathrm{d}} \mathrm{p}_0^{\mathrm{d}}(1 - \mathrm{p}_0)^{\mathrm{n}-\mathrm{d}} \end{aligned} \tag{8}$$

TABLE 1. The values of ARLs when $\beta = 0.31$, $n = 10$.

f	k_1=3.3596; k_2=2.3683;a=0.9225									
	g									
	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
	ARLs									
1	370.10	347.83	320.37	286.50	245.00	195.19	138.02	78.23	27.21	1.85
0.9	179.02	152.04	124.05	95.84	68.56	43.75	23.33	9.21	2.33	1.01
0.8	19.63	14.03	9.46	5.96	3.49	1.98	1.24	1.02	1.00	1.00
0.7	3.21	2.30	1.67	1.29	1.09	1.02	1.00	1.00	1.00	1.00
0.6	1.22	1.10	1.03	1.01	1.00	1.00	1.00	1.00	1.00	1.00
0.5	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

The probability of repetition for in control process is given by

$$\mathrm{P}^0_{rep} = \mathrm{P}\left(UCL_2 < D < UCL_1\right) + \mathrm{P}\left(LCL_1 < D < LCL_2\right)$$
$$= \sum\nolimits_{d=\lfloor LCL_1\rfloor+1}^{\lfloor LCL_2\rfloor} \binom{n}{d} p_0^d \left(1-p_0\right)^{n-d}$$
$$+ \sum\nolimits_{d=\lfloor UCL_2\rfloor+1}^{\lfloor UCL_1\rfloor} \binom{n}{d} p_0^d \left(1-p_0\right)^{n-d} \quad (9)$$

Therefore, the probability that the process is declared to be out-of-control when the process is in control under repetitive sampling is given by

$$\mathrm{P}^0_{out} = \frac{\mathrm{P}^0_{out,1}}{1-\mathrm{P}^0_{rep}} \quad (10)$$

Hence the in-control ARL is given by

$$\mathrm{ARL}_0 = \frac{1}{\mathrm{P}^0_{out}} \quad (11)$$

C. OUT OF CONTROL ARL

It is assumed that due to some extraneous factors, both scale parameter σ and shape parameter b are shifted to $\sigma_1 = f\sigma_0$ and $b_1 = gb_0$. Note that f=1 indicates that the scale parameter remains unchanged and that g=1 means that the shape parameter remains unchanged. The probability that an item fails (p_1) by time t_0 is obtained by

$$p_1 = P\{T \le t_0|\sigma_1, b_1\} = \Phi\left(\frac{1}{b_1}\xi\left(\frac{t_0}{\sigma_1}\right)\right)$$
$$= \Phi\left(\frac{1}{gb_0}\xi\left(\frac{a(1+\frac{1}{2}b_0^2)}{f}\right)\right) \quad (12)$$

The probability of declaring out-of-control $\left(\mathrm{P}^1_{out,1}\right)$ process at each subgroup when the process is shifted is given by

$$\mathrm{P}^1_{out,1} = \sum\nolimits_{d=\lfloor UCL_1\rfloor+1}^{n} \binom{n}{d} p_1^d \left(1-p_1\right)^{n-d}$$
$$+ \sum\nolimits_{d=0}^{\lceil LCL_1\rceil} \binom{n}{d} p_1^d \left(1-p_1\right)^{n-d} \quad (13)$$

The probability of repetition for the shifted process is given by

$$\mathrm{P}^1_{rep} = \sum\nolimits_{d=\lfloor LCL_1\rfloor+1}^{\lfloor LCL_2\rfloor} \binom{n}{d} p_1^d \left(1-p_1\right)^{n-d}$$
$$+ \sum\nolimits_{d=\lfloor UCL_2\rfloor+1}^{\lfloor UCL_1\rfloor} \binom{n}{d} p_1^d \left(1-p_1\right)^{n-d} \quad (14)$$

The probability of declaring out-of-control process for the shifted process under repetitive sampling is given by

$$\mathrm{P}^1_{out} = \frac{\mathrm{P}^1_{out,1}}{1-\mathrm{P}^1_{rep}} \quad (15)$$

Therefore, the out-of-control ARL for the shifted process is given by

$$\mathrm{ARL}_1 = \frac{1}{\mathrm{P}^1_{out}} \quad (16)$$

The simulation is used to determine the values of values of ARL_0 and ARL_1. Let r_0 be the specified ARL_0. The values of control coefficients k_1 and k_2 are chosen such that $\mathrm{ARL}_0 \ge r_0$. Then, the Eq. (16) is used to find ARL_1 for various shifts. We wrote R codes to determine the parameters of the proposed control chart. The codes are available from author upon request.

The values of ARLs are determined for various specified parameters. Tables 1-3 are presented when specified r_0. is 370, $b = 0.31$ and $n = 10$, 20 and 30. Tables 4-5 are presented when specified r_0 is 370, $b = 1$ and $n = 20$ and 30. The trend in ARLs is also presented in Figure 1 when specified r_0 is 370, $b = 1$ and $n = 10$.

The following algorithm is used to assess ARL_1. Let b_0 be specified value of b.

1. Specify control chart parameters r_0, b_0, n and (f, g)
2. Determine the values of control chart coefficients k_1 and k_2 such that $\mathrm{ARL}_0 \ge r_0$

TABLE 2. The values of ARLs when $\beta = 0.31$, $n = 20$.

f	k_1=2.9527; k_2=1.5404;a=0.9070									
	g									
	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
	ARLs									
1	370.08	351.96	314.95	257.12	184.15	109.85	50.09	14.59	2.18	1.00
0.9	24.84	20.72	16.59	12.57	8.80	5.53	3.03	1.54	1.03	1.00
0.8	1.71	1.41	1.20	1.08	1.02	1.00	1.00	1.00	1.00	1.00
0.7	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE 3. The values of ARLs when $\beta = 0.31$, $n = 30$.

f	k_1=2.8770; k_2=2.2200;a=0.9997									
	g									
	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
	ARLs									
1	370.15	350.26	308.07	243.76	167.36	95.54	42.30	12.61	2.21	1.00
0.9	13.92	8.69	5.12	2.89	1.69	1.16	1.02	1.00	1.00	1.00
0.8	1.43	1.18	1.06	1.01	1.00	1.00	1.00	1.00	1.00	1.00
0.7	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

3. Use selected k_1 and k_2 to determine ARL_1 for various c

From Tables 1-5 and Figure 1, we note the following trend in control chart parameters.

1. The values of ARL_1 increases as n increases.
2. The values of ARL_1 decreases as b increases.
3. For fixed value of f, ARL_1 decreases as g decreases.
4. For fixed value of g, ARL_1 decreases as f decreases.

It is noted that ARL values are larger when both parameters are shifted than when shape parameter is shifted or scale parameter is shifted.

D. COMPARATIVE STUDY

1) ARL COMPARISON TO THE CHART BY LEIVA ET AL. (2015)

Leiva *et al.* [28] proposed an attribute control chart for the BS distribution using single sampling. In their chart, a decision about the state of process is taken on the basis of information from a single sample. The Leiva *et al.* [28] chart is easy to use in practice but it requires a larger ARL to make the decision when the process is out-of-control.

TABLE 4. The values of ARLs when $\beta = 1, n = 20$.

	k_1=3.0153; k_2=1.5768;a=0.7633									
	g									
	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
f	ARLs									
1	370.06	374.51	365.51	333.81	272.38	186.32	97.80	33.43	5.07	1.01
0.9	241.33	191.29	139.88	92.33	53.16	25.20	8.95	2.32	1.03	1.00
0.8	79.35	54.05	33.80	18.90	9.14	3.77	1.56	1.04	1.00	1.00
0.7	22.16	13.68	7.76	4.05	2.07	1.24	1.02	1.00	1.00	1.00
0.6	5.88	3.54	2.13	1.40	1.10	1.01	1.00	1.00	1.00	1.00
0.5	1.86	1.37	1.13	1.03	1.00	1.00	1.00	1.00	1.00	1.00
0.4	1.09	1.03	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE 5. The values of ARLs when $\beta = 1, n = 30$.

	k_1=2.9247; k_2=1.5909;a=0.8335									
	g									
	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
f	ARLs									
1	370.26	368.51	314.70	217.12	117.27	47.91	13.45	2.48	1.02	1.00
0.9	193.59	126.23	72.67	36.10	14.86	4.91	1.61	1.02	1.00	1.00
0.8	46.47	25.98	12.99	5.76	2.41	1.26	1.02	1.00	1.00	1.00
0.7	9.93	5.23	2.68	1.51	1.10	1.01	1.00	1.00	1.00	1.00
0.6	2.41	1.55	1.17	1.03	1.00	1.00	1.00	1.00	1.00	1.00
0.5	1.15	1.04	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.4	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

In this section, we compare the efficiency of the proposed chart over an existing control chart proposed by Leiva *et al.* [28]. For repetitive sampling, the sample size required at a subgroup may increase. But, the increase in the sample size is marginal because the probability of repetition is quite small. The average sample size for the proposed chart is given as

$$\text{average sample size} = \frac{n}{1 - \mathrm{P}^0_{\text{rep}}} \quad (17)$$

We compare both control charts in terms of ARLs for the same values of subgroup size and other specified parameters, which are given in Table 6.

From Tables 6, it can be seen that for all specified parameters, the proposed control chart is always more powerful to detect a shift in process as compared to the existing chart. For example, when $g = 0.7$ and $f = 0.7$, $n = 20$ and $b = 1$, the value of ARL from the proposed chart is 4 while it is 12 from the chart by Leiva *et al.* [28].

2) SIMULATION STUDY

In this section, we will show that the proposed chart has ability to detect a shift in the manufacturing process earlier than the existing control chart by Leiva *et al.* [28]. It is assumed that due to some extraneous factors, both scale parameter σ and shape parameter b are shifted to $\sigma_1 = f\sigma_0$ and $b_1 = gb_0$.

TABLE 6. The ARL comparisons of Proposed Chart and Existing Chart.

		b=0.31, n=20		b=1, n=20	
f	g	Existing	Proposed	Existing	Proposed
1.0	1.0	370	370	370.09	370.06
	0.9	366.77	351.96	368.01	367.51
	0.8	361.46	314.95	366.53	365.51
	0.7	353.96	257.12	352.79	333.81
	0.6	342.94	184.15	341.43	272.38
0.9	1.0	38.85	24.84	284.98	241.33
	0.9	29.83	20.72	267.63	191.90
	0.8	21.82	16.59	246.30	139.88
	0.7	15.02	12.57	220.07	92.33
	0.6	9.58	8.80	188.09	53.16
0.8	1.0	4.11	1.71	103.64	79.35
	0.9	3.06	1.41	85.66	54.05
	0.8	2.26	1.20	67.85	33.80
	0.7	1.68	1.08	50.78	18.90
	0.6	1.30	1.02	35.18	9.14
0.7	1.0	1.34	1.01	33.91	22.16
	0.9	1.18	1.0	25.83	13.68
	0.8	1.07	1.0	18.75	7.76
	0.7	1.02	1.0	12.82	4.05
	0.6	1.0	1.0	8.14	2.07
0.6	1.0	1.03	1.0	11.62	5.80
	0.9	1.0	1.0	8.51	3.54
	0.8	1.0	1.0	6.00	2.13
	0.7	1.0	1.0	4.07	1.40
	0.6	1.0	1.0	2.67	1.10

For in control process let us assume that $b_0 = 1$, $\sigma_0 = 1.5$ for a BS distribution. , For a shifted process, we assume that f=0.9, g=0.9 so that $\sigma_1 = 0.9 * 1.5 = 1.35$ and $b_1 = 0.9$. For this simulation, first 20 subgroups are generated from in- control process and the next 10 subgroups are generated from the shifted process. The sample size of a subgroup is 30 (n=30). The test time constant is chosen as a=0.99975. The four control limits of proposed control chart are shown

TABLE 7. The plotting statistic for coupon data.

Subgroup No.	*d*	Subgroup No.	*d*
1	9	16	9
2	9	17	11
3	8	18	7
4	9	19	10
5	8	20	11
6	6	21	9
7	12	22	11
8	10	23	5
9	10	24	12
10	10	25	10
11	8	26	13
12	8	27	7
13	6	28	7
14	10	29	10
15	8	30	3

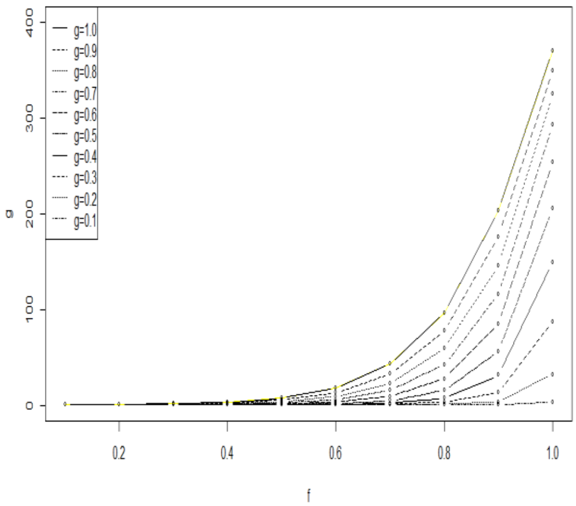

FIGURE 1. ARLs when b=1, n=10.

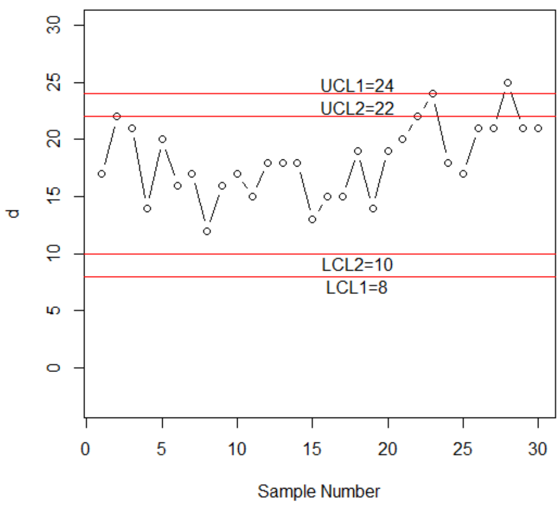

FIGURE 2. The proposed control chart for simulated data.

in Figure 2 along with the number of defectives from each of 30 subgroups.

From Figure 2, it can be seen that the proposed control chart detects the shift at 28th subgroup or 8th subgroup after the actual shift.

Figure 3 shows the control chart by Leiva *et al.* [28] for simulated data. It can be seen that no value of D is beyond the control limits. It shows that the process is declared as in control even for the shifted process. Hence, we may say that

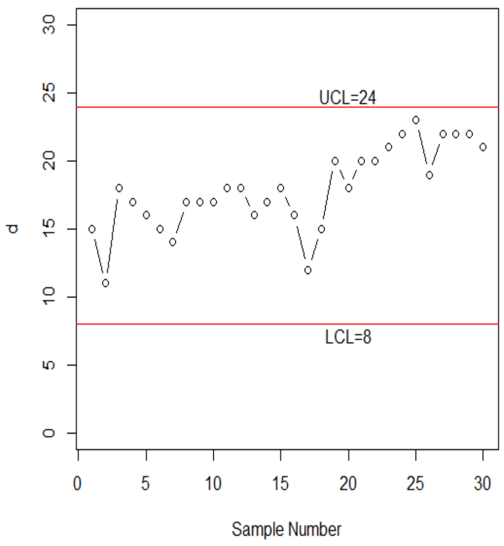

FIGURE 3. The control chart by Leiva *et al.* [28] for simulated data.

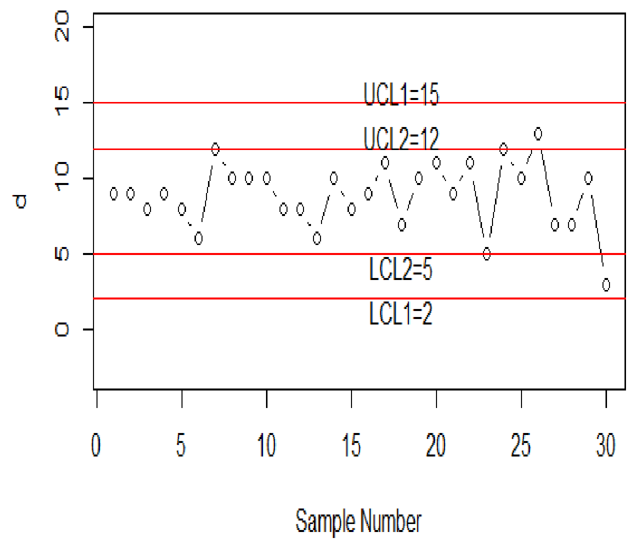

FIGURE 4. The proposed control chart for coupon data.

the proposed control chart performs better than the chart by Leiva *et al.* [28].

E. APPLICATION OF PROPOSED CHART

In this example, coupon data is used to illustrate the application of the proposed control chart. The similar coupon data was also reported in [17]. The plotting statistics are shown in Table 7, which will be used to apply the proposed control chart. The data is known to follow the BS distribution with shape parameter $b = 0.31$. For this example, it is assumed that $a = 0.9070$. The control coefficients of the proposed chart are obtained by $k_1 = 2.9527$, $k_2 = 1.5404$, while n=20 and $p_0 = 0.44$. The calculated control limits are given in Figure 4.

From Figure 4, it can be seen that some subgroups fall in the repetitive areas. The 7-th subgroup first falls in the repetitive area, but we do not make a decision at this time and proceed to the next (8-th) subgroup. We observe 10 failures at the 8-th subgroup, so it is declared as in-control.

The same values of d are also plotted on Leiva *et al.* [28] chart. From Figure 5, it can be noted that all points are within the control limits which indicates that process is in-control state and there is no shift in the process.

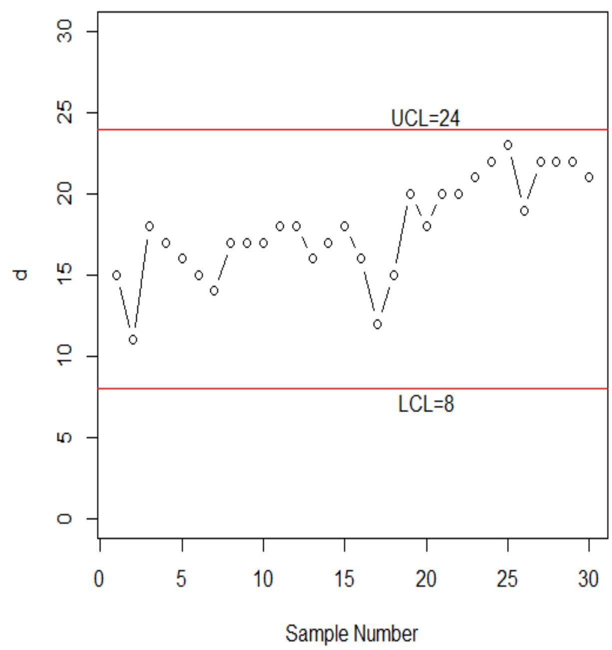

FIGURE 5. The control chart by Leiva *et al.* [28] for real data.

II. CONCLUSION AND RECOMMENDATIONS

An attribute control chart for a BS distribution is proposed using repetitive sampling. The structure of the proposed control chart is presented when the shape and/or the scale parameters are shifted. Some tables are given for practical use for each case. The efficiency of the proposed chart is discussed over an existing control chart by Leiva *et al.* [28] based on ARL comparison and by using simulation study. A real industrial data is used to explain the application of the proposed control chart. The proposed control can be used in industry where the particular characteristics may be modeled by the BS distribution such as in the area of environment assessment and earth sciences [28], health monitoring assessment [13], fatigue life measures of aluminum coupons [31]. The proposed control using EWMA statistic can be considered as future research. The proposed chart can be extended using some other statistical distributions such as a gamma distribution.

APPENDIX

R codes for Simulation Study

```
library(VGAM)

SSC <- function(lamda,f,g,n,b){
d1<-c();d2<-c();
options(digits = 2)
a=0.99975
k1=2.8770; k2=2.2200

        x=a*(1+b^2/2)
        fx= sqrt(x)-sqrt(1/x)
        p0=pnorm(1/b*fx)

u=lamda*(1+b^2/2)
usl=a*u
print(cbind(p0,usl))
UCL1 = as.integer(n*p0 + k1*sqrt(n*p0*(1-p0)));
        LCL1 = as.integer(n*p0 - k1*sqrt(n*p0*(1
        -p0)))
UCL2 = as.integer(n*p0 + k2*sqrt(n*p0*(1-p0)));
        LCL2 = as.integer(n*p0 - k2*sqrt(n*p0*(1
        -p0)))
        for(i in 1:20){
          r1=rbisa(n, lamda, b)
          print(r1)
          w1=which(r1<usl)
          d1[i]=length(w1)
          }
        for(i in 1:10){
            r2=rbisa(n, g*lamda, f*b)
           w2=which(r2<usl)
          print(r2)
          d2[i]=length(w2)
          }
d=append(d1,d2)
print(d)
phat<-d
l=length(d)
x=c(1:l)
Observations=x
plot(Observations, phat, type="b",col=1, ylim = c(5,
          30), xlab="Sample Number", ylab="d",
          main="")
abline(h=LCL1, col=2); text(18, LCL1-0.8,
          "LCL1=8", col = "black")
abline(h=LCL2, col=2); text(18, LCL2-0.8,
          "LCL2=10", col = "black")
abline(h=UCL2, col=2); text(18, UCL2+0.8,
          "UCL2=22", col = "black")
abline(h=UCL1, col=2); text(18, UCL1+0.8,
          "UCL1=24", col = "black")

w=which(d>UCL1 | d<LCL1)
print(cbind(d))
print(w)
print(cbind(LCL1,LCL2,UCL2,UCL1))
}
SSC(1.5,0.9,0.9,30,1)
```

R codes for Real Example

```
library(VGAM)

SSC <- function(lamda,f,g,n,b){
d1<-c();d2<-c();
options(digits = 2)
k1=2.9527; k2=1.5404;a=0.9070
        x=a*(1+b^2/2)
        fx= sqrt(x)-sqrt(1/x)
        p0=pnorm(1/b*fx)
u=lamda*(1+b^2/2)
usl=a*u
print(cbind(p0,usl))
UCL1 = as.integer(n*p0 + k1*sqrt(n*p0*(1-p0)));
        LCL1 = as.integer(n*p0 - k1*sqrt(n*p0*(1
        -p0)))
UCL2 = as.integer(n*p0 + k2*sqrt(n*p0*(1-p0)));
        LCL2 = as.integer(n*p0 - k2*sqrt(n*p0*(1
        -p0)))
        for(i in 1:20){
        r1=rbisa(n, lamda, b)
        print(r1)
        w1=which(r1<usl)
        d1[i]=length(w1)
        }
        for(i in 1:10){
        r2=rbisa(n, g*lamda, f*b)
        w2=which(r2<usl)
        print(r2)
        d2[i]=length(w2)
        }
d=append(d1,d2)
print(d)
phat<-d
l=length(d)
x=c(1:l)
Observations=x
plot(Observations, phat, type="b",col=1, ylim = c(-
        3, 20), xlab="Sample Number", ylab="d",
        main="")
abline(h=LCL1, col=2); text(18, LCL1-0.8,
        "LCL1=2", col = "black")
abline(h=LCL2, col=2); text(18, LCL2-0.8,
        "LCL2=5", col = "black")
abline(h=UCL2, col=2); text(18, UCL2+0.8,
        "UCL2=12", col = "black")
abline(h=UCL1, col=2); text(18, UCL1+0.8,
        "UCL1=15", col = "black")

w=which(d>UCL1 | d<LCL1)
print(cbind(d))
print(w)
```

```
print(cbind(LCL1,LCL2,UCL2,UCL1))
}
SSC(2,1,1,20,0.31)
```

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