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Multipath Estimation Based on Centered Error Entropy Criterion for Non-Gaussian Noise

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ABSTRACT With the advance of software receiver, multipath estimation becomes a key issue for high accuracy positioning systems. It is crucial for eliminating the multipath error and improving the positioning accuracy to estimate multipath parameters. The accessible multipath estimation algorithms are usually designed for Gaussian noise, and their performances degrade dramatically in non-Gaussian noise, since the mean square error criterion is adopted. To tackle the problem, a new filter based on centered error entropy criterion (CEEC) is proposed for multipath estimation. In the proposed filter, the CEEC is considered as a performance index, which is not limited to the assumption of Gaussian and linearity. According to a stochastic information gradient method, an optimal filer gain matrix is obtained by maximizing the performance function of centered error entropy. Meanwhile, a convergence analysis of the proposed filter is offered. Furthermore, a recursive estimation method based on modified Parzen windowing technique is proposed for practical implementation. The simulation results indicate that the proposed filter outperforms the filter based on minimum error entropy criterion for multipath estimation.

INDEX TERMS Multipath estimation, centered error entropy criterion (CEEC), minimum error entropy criterion (MEEC), stochastic information gradient (SIG).

I. INTRODUCTION

The positioning accuracy of GNSS is influenced by many error resources and multipath is the dominate one for high accuracy positioning systems since it is difficult to mitigate by differential techniques [1]. Multipath is the reflective replicas of the direct signals, which is produced by obstacles (buildings, hills, etc) [2]. With the development of software receiver and digital signal processing (DSP), the multipath eliminating methods based on data processing have become a hot-spot [3], [4]. The key issue of these methods is multipath estimation. Various multipath estimation methods, e.g. the maximum likelihood estimator, the extend Kalman filter (EKF) estimator, are explored to estimate multipath [5], [6]. However, most of these methods are only competent in Gaussian noise environments since they are designed with mean square error criterion (MSEC) and only the second-order statistics is taken into consideration. Although all statistics can be described by the second-order statistics for linear systems with Gaussian noise, it is not enough to describe the statistics for non-Gaussian noise using only the second-order statistics. As a result, the performances of the methods based on MSEC degrade dramatically in non-Gaussian noises. PF prototype algorithms are developed for multipath estimation in non-Gaussian noise [7]. However, the problem of sample degeneration and impoverishment limits its application. To this end, a new multipath estimation method, which can describe the higher-order statistics for non-Gaussian, needs to be proposed.

Entropy is a central quantity in information theory, which quantifies the average uncertainty involved in predicting the value of a random variable [8]. Entropy can depict not only the second-order statistics but also the higher-order statistics of a distribution and it is not limited to the assumption of Gaussian. As the entropy measures the average uncertainty contained in a random variable, its minimization makes the distribution more concentrated. Thus, minimum error entropy criterion (MEEC) is studied by many researchers [9]–[14].

Although the minimum error entropy (MEE) is a global criterion, it is shift-invariant, and a bias is needed to be set to achieve a zero-mean error. Correntropy is a similarity measure between two random variables [15]. Correntropy criterion can fix the main peak of the error PDF at the origin point. Unfortunately, it is a localized criterion.

In this paper, correntropy criterion is integrated into MEEC to form a new criterion named as centered error entropy criterion (CEEC). CEEC can fix the main peak of the error PDF at the original point and overcome the drawbacks of MEE being shift-invariant and maximum correntropy criterion (MCC) being local optimization. Then, a filter with CEEC is proposed for multipath estimation. In order to implement the proposed filter in a reasonable way, an online iterative estimation method is explored using modified Parzen windowing technique. In the proposed CEEC filter, the stochastic information gradient (SIG) method is applied to update the gain matrix. The convergence analysis is also given for the proposed filter. At last, the performance of CEEC filter for multipath estimation in non-Gaussian noise is validated by a simulation.

The remainder of this paper is organized as follows. Section II formulates the multipath estimation problem and builds the corresponding system model. Section III elaborates the CEEC filter and the gain matrix updating method. The convergence analysis and the implementation of CEEC filter are offered in Section IV. Some conclusions and the future work are given in Section V.

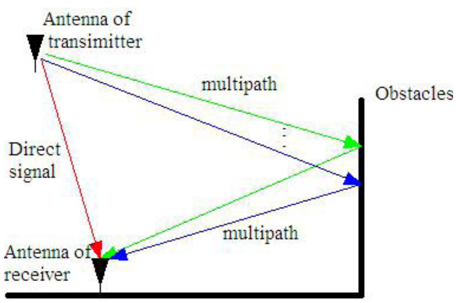

FIGURE 1. Generation of multipath.

II. PROBLEM FORMULATION

A. SIGNAL DESCRIPTION

The received signal in a multipath environment for GNSS can be modeled as an $M_0 + 1$ path model composed of a direct path signal and M_0 reflected signals plus noise, which can be depicted by Fig.1. The corresponding base-band signal can be modeled as [16].

$$r(t) = \alpha_0 c(t-\tau_0)\cos(\theta_0) + \sum_{i=1}^{M_0} \alpha_i c(t-\tau_0-\tau_i) \times \cos(\theta_0+\theta_i) + n(t) \tag{1}$$

where α_0 is the amplitude of direct signal, α_i is the amplitude of the i-th multipath, τ_0 is the time delay of direct signal, τ_i is the i-th multipath time delay relative to the direct signal, θ_0 is the direct signal phase, θ_i is the i-th multipath phase delay relative to the direct signal, and $n(t)$ is the noise. The multipath signal is normally weaker than the direct signal since some signal power is lost due to reflection, which means $\sum_{i=1}^{M_0} \alpha_i < \alpha_0$. The multipath signal arrives after the direct signal for the reason that it must travel a longer distance over the propagation path, so the multipath time delay is longer than the direct signal time delay, i.e. $\tau_i \geq 0$. Usually, the short multipath with time delay $0 \leq \tau_i < 1 + d/2$ (d is the correlator spacing between the early code and the late code) is considered. In theory there can be an infinite number of multipath signals present at any given time. In practice there is rarely more that one or two dominate multipath signals present at one time [17]. Thus, only one dominant multipath is taken into consideration for simplicity, i.e. $M_0 = 1$. Then, (1) can be simplified as

$$r(t) = \alpha_0 c(t-\tau_0)\cos(\theta_0) + \alpha_i c(t-\tau_0-\tau_i) \times \cos(\theta_0+\theta_1) + n(t) \tag{2}$$

Express the signal model in a digital form as

$$r(k) = \alpha_0 c(k-l_0)\cos(\theta_0) + \alpha_1 c(k-l_0-l_1)\cos(\theta_0+\theta_1) + n(k) \tag{3}$$

where k denotes the k-th instant, l_0 and l_1 is the digital expression of τ_0 and τ_1, respectively. Usually, the noise $n(k)$ is supposed to be Gaussian distributed, but sometimes it is not true. For example, impulsive noise is often encountered in wireless applications in many indoor and outdoor environments, which can be approximated with finite Gaussian mixture [18].

B. SYSTEM MODEL

The structure of signal tracking in GNSS is shown in Fig.2. After mixing with the local carrier, the correlator output vector $y_k = \left[y_k^1, y_k^2, \cdots, y_k^S\right]^T$ can be measured by correlating $r(k)$ with the local C/A code vector $c\left(k - \hat{l}_0 - d\right) = \left[c\left(k-\hat{l}_0-d_1\right), \cdots, c\left(k-\hat{l}_0-d_S\right)\right]^T$, where $d_s (s = 1, \ldots, S)$ is the correlator spacing between the i-th code and the punctual code $c\left(k-\hat{l}_0\right)$, and S is the correlator number. $c\left(k-\hat{l}_0-d_s\right)$ is the early code if $d_s < 0$, $c\left(k-\hat{l}_0-d_s\right)$ is the late code if $d_s > 0$ and $c\left(k-\hat{l}_0-d_s\right)$ is the punctual code if $d_s = 0$. The multipath parameters vector $\boldsymbol{x} = [\alpha_0, \alpha_1, \theta_0, \theta_1, l_0, l_1]^T$ can be estimated by $\boldsymbol{y}_k$. Once the multiapth parameters are estimated by an estimation algorithm, the direct signal part can be got by subtracting the multipath part from the received signal. Then, a time delay estimation of the direct signal $\hat{l}_0$ can be obtained after further processing to control the code generator, and the code generator tunes the local punctual code $c\left(k-\hat{l}_0\right)$ to synchronize the code of $r(k)$.

Without considering the influence of low pass filter in signal tracking loop, the output of s-th correlator in Fig.2 is

$$y_k^s\left(A_{0,k}, A_{1,k}, \varepsilon_k, l_{1,k}\right) = A_{0,k}R\left(\varepsilon_k\right) + A_{1,k}R\left(\varepsilon_k + l_{1,k} + d_s\right) + n_k \tag{4}$$

where $A_{0,k} = \alpha_{0,k}\cos(\theta_{0,k})$ and $A_{1,k} = \alpha_{1,k}\cos(\theta_{1,k})$ are the composite amplitude of the direct signal and the multipath at

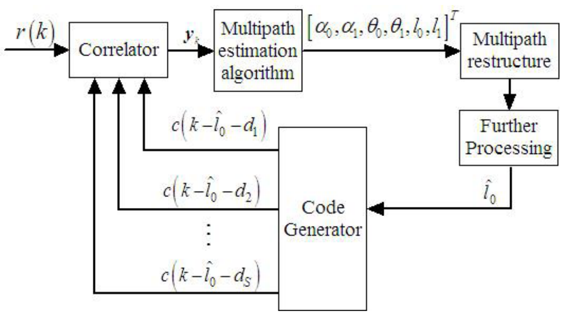

FIGURE 2. Structure of signal tracking.

time k, respectively. $R(\varepsilon)$ is the ideal autocorrelation function with $R(\varepsilon) = \begin{cases} 1-|\varepsilon|, & |\varepsilon| \le 1 \\ 0, & \text{others} \end{cases}$. $\varepsilon_k = \hat{l}_{0,k} - l_{0,k}$, $\hat{l}_{0,k}$ is the local estimation of $l_{0,k}$. $l_{0,k}$ and $l_{1,k}$ are the direct signal time delay and the relative time delay of multipath at time k, respectively. n_k denotes correlation noise at time k.

From (4), it can be seen that the parameter to be estimated at time k is reduced to $\boldsymbol{x}_k = [A_{0,k}, A_{1,k}, \varepsilon_k, l_{1,k}]^T$. Thus, the system model can be formulated as a first-order Markov process [6].

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}(\boldsymbol{x}_k) + \boldsymbol{w}_{k+1} \tag{5}$$

$$\boldsymbol{y}_k = \boldsymbol{H}(\boldsymbol{x}_k) + \boldsymbol{v}_k \tag{6}$$

where $\boldsymbol{x}_k \in \mathbf{R}^{M\times 1}$ denotes the state vectors. M equals 4 for the problem of multipath estimation in this paper. $\boldsymbol{F}(\cdot)$ is the system matrix depending on the state vector $\boldsymbol{x}_k$. $\boldsymbol{w}_k$ is assumed to be system noise with zero mean. $\boldsymbol{y}_k$ is the observing vector with $\boldsymbol{y}_k = \left[y_k^1, y_k^2, \cdots, y_k^S\right]^T$. $\boldsymbol{H}(\cdot)$ is the measurement matrix depending on $\boldsymbol{x}_k$, $\boldsymbol{v}_k$ is the measurement noise with zero mean. Then, the estimation problem can be solved by the following steps,

$$\hat{\boldsymbol{x}}_{k+1} = \boldsymbol{F}\left(\hat{\boldsymbol{x}}_k\right) + \boldsymbol{L}_{k+1}\left(\boldsymbol{y}_k - \hat{\boldsymbol{y}}_k\right) \tag{7}$$

$$\hat{\boldsymbol{y}}_k = \boldsymbol{H}\left(\hat{\boldsymbol{x}}_k\right) \tag{8}$$

where $\boldsymbol{L}_k$ is the gain matrix, which is updated at every time k. This paper aims to obtain an optimal estimation for $\boldsymbol{L}_k$ in terms of CEEC with $\boldsymbol{L}_k = \left[L_{1,k}, L_{2,k}, \cdots, L_{S,k}\right] \in \mathbf{R}^{M\times S}$, where $L_{t,k} \in \mathbf{R}^{M\times 1}$. The error e_k^s is formulated in the following form,

$$e_k^s = y_k^s - \hat{y}_k^s, \quad s = 1, 2, \ldots, S \tag{9}$$

where $\hat{\boldsymbol{y}}_k^s$ is the filter output of y_k^s.

III. THE FILTER BASED ON CENTERED ERROR ENTROPY CRITERION

Based on the systems of model (7) and (8), the CEEC filter is proposed for multipath estimation in this section. Before proceeding the MEEC and MCC are reviewed.

A. MINIMUM ERROR ENTROPY CRITERION (MEEC)

The MEE estimation aims to minimize the entropy of the estimation error, and hence decreases the uncertainty in estimation. The Renyi's entropy is adopted due to its easy calculation. Assume a random variable $\boldsymbol{e}$ with PDF $f_{\boldsymbol{E}}(\boldsymbol{e})$, the Renyi's entropy is defined by [19]

$$H(\boldsymbol{e}) = -\log \int f_{\boldsymbol{E}}^2(\boldsymbol{e}) d\boldsymbol{e} \tag{10}$$

The kernel density estimation (KDE) is focused on because of its wide applicability and its relationship with Renyi's entropy [14]. Given a set of i.i.d. data $\{\boldsymbol{e}_i\}_{i=1}^N$ drawn from the distribution, the KDE of the PDF is

$$\hat{f}_{\boldsymbol{E}}(\boldsymbol{e}) = \frac{1}{N}\sum_{i=1}^{N} G_{\sum}(\boldsymbol{e} - \boldsymbol{e}_i) \tag{11}$$

$G_{\Sigma}(\boldsymbol{e} - \boldsymbol{e}_i)$ is the Gaussian function with the following expression,

$$G_{\sum}(\boldsymbol{e} - \boldsymbol{e}_i) = \frac{1}{\sqrt{2\pi\left(\det \sum\right)}} \cdot \exp\left(-\frac{1}{2}(\boldsymbol{e} - \boldsymbol{e}_i)^T \sum^{-1} (\boldsymbol{e} - \boldsymbol{e}_i)\right) \tag{12}$$

where N is the number of the data points and $\boldsymbol{\Sigma}$ is the kernel parameter. In this paper, $\boldsymbol{\Sigma}$ is assumed to be a diagonal matrix with the s- th diagonal element being the variance δ_s^2 for e_s in $\boldsymbol{e}$, $s = 1, 2, \ldots, S$. The kernel parameter is a free parameter that must be chosen by the user. In this paper, the Σ is set experimentally.

Therefore, using KDE, the Renyi's quadratic entropy can be formulated as following,

$$\begin{aligned} H_2(\boldsymbol{e}) &= -\log \int \left(\frac{1}{N}\sum_{i=1}^{N} G_{\sum}(\boldsymbol{e} - \boldsymbol{e}_i)\right)^2 d\boldsymbol{e} \\ &= -\log \frac{1}{N^2} \int \left(\sum_{i=1}^{N}\sum_{j=1}^{N} G_{\sum}(\boldsymbol{e} - \boldsymbol{e}_i) G_{\sum}\left(\boldsymbol{e} - \boldsymbol{e}_j\right)\right) d\boldsymbol{e} \\ &= -\log \frac{1}{N^2} \left(\sum_{i=1}^{N}\sum_{j=1}^{N} \int G_{\sum}(\boldsymbol{e} - \boldsymbol{e}_i)\, G_{\sum}\left(\boldsymbol{e} - \boldsymbol{e}_j\right) d\boldsymbol{e}\right) \\ &= -\log \frac{1}{N^2} \left(\sum_{i=1}^{N}\sum_{j=1}^{N} G_{\sqrt{2}\sum}\left(\boldsymbol{e}_i - \boldsymbol{e}_j\right)\right) \\ &= -\log \frac{1}{N^2} \left(\sum_{i=1}^{N}\sum_{j=1}^{N} G_{\sum_2}\left(\boldsymbol{e}_i - \boldsymbol{e}_j\right)\right) \end{aligned} \tag{13}$$

where

$$V(\boldsymbol{e}) = \frac{1}{N^2}\left(\sum_{i=1}^{N}\sum_{j=1}^{N} G_{\sum_2}\left(\boldsymbol{e}_i - \boldsymbol{e}_j\right)\right) \tag{14}$$

$V(\boldsymbol{e})$ is called the information potential (IP) of variable $\boldsymbol{e}$ and $\boldsymbol{\Sigma}_2 = \sqrt{2}\boldsymbol{\Sigma}$. Thus, the minimizing of the Renyi's entropy $H_2(\boldsymbol{e})$ is equivalent to maximize the IP $V(\boldsymbol{e})$ due to the monotonic increasing property of the $\log(\cdot)$ function. In order to decrease the calculating complexity, the Parzen windowing technique and the instantaneous IP at time k is used as the performance index, i.e.

$$J_1(\boldsymbol{e}) = \frac{1}{W}\sum_{i=k-W+1}^{k} G_{\sum_2}(\boldsymbol{e}_k - \boldsymbol{e}_i) \tag{15}$$

where W is the length of the Parzen window. J_1 in (15) is used as the performance index for MEEC in this paper.

The problem with MEEC is to determine the location of error PDF because the criterion is shift-invariant. A solution to this problem is to bias the system output to the desired signal mean to make the error mean equal to zero. However, the desired mean cannot be known for the problem of multipath estimation described in this paper.

B. MAXIMUM CORRENTROPY CRITERION (MCC)

Correntropy is closely related to Renyi's quadratic entropy. With Gaussian kernel, correntropy is a localized similarity measure between two random variables. Correntropy is a robust adaptation criterion in presence of non-Gaussian impulsive noise [20].

The objective of state estimation is to optimize a performance index function (or cost function) in such a way that the observation output $\boldsymbol{y}_k$ resembles the filter output $\hat{\boldsymbol{y}}_k$ as closely as possible. Under MCC, the cost function that we want to maximize is the correntropy between the measurement output and the model output, i.e.,

$$J_2 = E\left[G_{\sum_1}(\boldsymbol{e}_k)\right] \tag{16}$$

$\boldsymbol{\Sigma}_1$ is the kernel parameter of J_2.

In practical applications, one often uses the following empirical correntropy as the performance index,

$$J_2 = \frac{1}{W}\sum_{i=k-W+1}^{k} G_{\sum_1}(\boldsymbol{e}_k) \tag{17}$$

where W has the same meaning as that in (15). J_2 in (17) is used as the performance index for MCC in this paper.

The problem of MCC is it is a local criterion because it only cares about the local part of error PDF falling within the kernel bandwidth. So the kernel size has to be chosen carefully.

C. THE FILTER BASED ON CEEC USING STOCHASTIC INFORMATION GRADIENT

Since MEE has the property of shift-invariant, and estimation results obtained by the algorithm with MEEC may not converge to the true value. Fortunately, MCC is a localized similarity measure between the observation output $\boldsymbol{y}_k$ and the filter output $\hat{\boldsymbol{y}}_k$. Under MCC, a concave cost function can be constructed by choosing a large enough kernel parameter according to the results of [8], and a unique optimal solution can be expected. Hence, a natural consideration is a global optimal solution can be fixed by combining MCC and MEEC. The combination of MCC and MEEC is called CEEC.

Under CEEC, the performance index can be expressed as

$$J_k(\boldsymbol{e}) = \lambda\left[\frac{1}{W}\sum_{i=k-W+1}^{k} G_{\sum_1}(\boldsymbol{e}_i)\right] + (1-\lambda)\left[\frac{1}{W}\sum_{i=k-W+1}^{k} G_{\sum_2}(\boldsymbol{e}_k - \boldsymbol{e}_i)\right] \tag{18}$$

where λ is a weighting constant between 0 and 1. Specially, when $\lambda = 0$, J_k reduces to a MEEC; when $\lambda = 1$, it is MCC.

According to the property of multidimensional Gaussian probability density function, if $e_1, e_2, \ldots, e_s, \ldots, e_S$ are independent of each other, one can obtain

$$\begin{aligned} f_k(\boldsymbol{e}) &= \frac{1}{W}\sum_{i=k-W+1}^{k} G_{\sum_1}(\boldsymbol{e}_i) \\ &= \frac{1}{W}\sum_{i=k-W+1}^{k}\prod_{s=1}^{S}\kappa_{\delta_{s,1}}\left(e_i^s\right) \end{aligned} \tag{19}$$

where $\boldsymbol{\kappa}_\delta(e) = 1/\sqrt{2\pi}\delta \cdot \exp\left(e^2/2\delta^2\right)$ is a Gaussian kernel function, e_i^s is the s-th element of $\boldsymbol{e}_i$, $\delta_{s,1}^2$ is the s-th diagonal element of $\boldsymbol{\Sigma}_1$. Therefore, we only need to maximize

$$J_k(\boldsymbol{e}) = \lambda\left[\frac{1}{W}\sum_{i=k-W+1}^{k}\prod_{s=1}^{S}\boldsymbol{\kappa}_{\delta_{s,1}}\left(e_i^s\right)\right] + (1-\lambda)\left[\frac{1}{W}\sum_{i=k-W+1}^{k}\prod_{s=1}^{S}\boldsymbol{\kappa}_{\delta_{s,2}}\left(e_k^s - e_i^s\right)\right] \tag{20}$$

where $\delta_{s,2}^2$ is the s-th diagonal element of Σ_2.

In order to update gain matrix $\boldsymbol{L}_k$ at every time k adaptively, the stochastic information gradient (SIG) method is used, and $\boldsymbol{L}_k$ can be updated as

$$L_{s,k+1} = L_{s,k} + \eta\cdot\frac{\partial J_k(\boldsymbol{e})}{\partial L_{s,k}} \tag{21}$$

where $L_{s,k} \in \mathbf{R}^{M\times 1}$, η is learning rate for adaptation.

$$\begin{aligned} \frac{\partial J_k(\boldsymbol{e})}{\partial L_{s,k}} &= -\lambda\cdot\frac{1}{W}\sum_{i=k-W+1}^{k}\left\{\left[\prod_{s=1}^{S}\kappa_{\delta_{s,1}}\left(e_i^s\right)\right]'\cdot\left[\boldsymbol{H}'_{:,s}(\hat{\boldsymbol{x}}_i)\cdot\frac{\partial\hat{\boldsymbol{x}}_i}{\partial L_{s,i}}\right]^T\right\} \\ &\quad + (1-\lambda)\cdot\frac{1}{W}\sum_{i=k-W+1}^{k}\left\{\left[\prod_{s=1}^{S}\kappa_{\delta_{s,2}}\left(e_k^s - e_i^s\right)\right]' \right. \\ &\quad \left.\cdot\left[\boldsymbol{H}'_{:,s}(\hat{\boldsymbol{x}}_i)\cdot\frac{\partial\hat{\boldsymbol{x}}_i}{\partial L_{s,i}} - \boldsymbol{H}'_{:,s}(\hat{\boldsymbol{x}}_k)\cdot\frac{\partial\hat{\boldsymbol{x}}_k}{\partial L_{s,k}}\right]^T\right\} \end{aligned} \tag{22}$$

with

$$\begin{aligned} \left[\prod_{s=1}^{S}\kappa_{\delta_{s,1}}\left(e_i^s\right)\right]' &= -\prod_{s=1}^{S}\kappa_{\delta_{s,1}}\left(e_i^s\right)\cdot\left[\sum_{s=1}^{S}e_i^s/\left(\delta_{s,1}^2\right)\right] \\ \left[\prod_{s=1}^{S}\kappa_{\delta_{s,2}}\left(e_k^s - e_i^s\right)\right]' &= -\prod_{s=1}^{S}\kappa_{\delta_{s,2}}\left(e_k^s - e_i^s\right) \\ &\quad\cdot\left[\sum_{s=1}^{S}\left(e_k^s - e_i^s\right)/\left(\delta_{s,2}^2\right)\right] \end{aligned} \tag{23}$$

where $\frac{\partial \hat{x}_k}{\partial L_{s,k}}$ can be obtained by the chain rule,

$$\frac{\partial \hat{\boldsymbol{x}}_k}{\partial L_{s,k}} = \left[\boldsymbol{F}'\left(\hat{\boldsymbol{x}}_{k-1}\right) - \boldsymbol{L}_{k-1} \cdot \boldsymbol{H}'_{:,s}\left(\hat{\boldsymbol{x}}_{k-1}\right)\right] \cdot \frac{\partial \hat{\boldsymbol{x}}_{k-1}}{\partial L_{s,k-1}} + \left(y^s_{k-1} - \hat{y}^s_{k-1}\right) \cdot \mathbf{I}_{N\times N} \quad (24)$$

with $\mathbf{I}_{N\times N}$ being identity matrix, and

$$\boldsymbol{F}'\left(\hat{\boldsymbol{x}}_{k-1}\right) = \frac{\partial \boldsymbol{F}\left(\hat{\boldsymbol{x}}_{k-1}\right)}{\partial \hat{\boldsymbol{x}}_{k-1}}, \quad \boldsymbol{H}'\left(\hat{\boldsymbol{x}}_{k-1}\right) = \frac{\partial \boldsymbol{H}\left(\hat{\boldsymbol{x}}_{k-1}\right)}{\partial \hat{\boldsymbol{x}}_{k-1}} \quad (25)$$

$\boldsymbol{H}'_{:,s}\left(\hat{\boldsymbol{x}}_{k-1}\right)$ is the s-th column of $\boldsymbol{H}'\left(\hat{\boldsymbol{x}}_{k-1}\right)$.

Once $\boldsymbol{L}_k$ is obtained, the estimation result $\hat{\boldsymbol{x}}_k$ can be got by (7).

IV. THE CONVERGENCE ANALYSIS AND IMPLEMENTATION OF CEEC FILTER

A. CONVERGENCE ANALYSIS

Since the filter gain $\boldsymbol{L}_k$ is influenced by the learning rate η according to (21), the choice of η should be analyzed to ensure the convergence of CEEC filter. After such analysis, an acceptable range of learning rate which results in decrease of the squared sum of output error would be given. Under known errors $\boldsymbol{e}_{k-1}$, the following conditional expectation needs to be satisfied for convergence [21],

$$D_k = \mathrm{E}\left[\boldsymbol{e}^T_k \boldsymbol{e}_k \,|\boldsymbol{e}_{k-1}\right] - \boldsymbol{e}^T_{k-1}\boldsymbol{e}_{k-1} < 0 \quad (26)$$

Using the approximation $\boldsymbol{H}\left(\boldsymbol{x}_k\right) \approx \boldsymbol{H}'\left(\boldsymbol{x}_k\right)\boldsymbol{x}_k = \boldsymbol{C}_k\boldsymbol{x}_k$ and (8), one can obtain,

$$\begin{aligned}\boldsymbol{e}^{\mathrm{T}}_k\boldsymbol{e}_k &\approx \left[\boldsymbol{y}^T_k - \hat{\boldsymbol{x}}^T_k\boldsymbol{C}^T_k\right] \times \left[\boldsymbol{y}_k - \boldsymbol{C}_k\hat{\boldsymbol{x}}_k\right] \\ &= \boldsymbol{y}^T_k\boldsymbol{y}_k - \boldsymbol{y}^T_k\boldsymbol{C}_k\hat{\boldsymbol{x}}_k - \hat{\boldsymbol{x}}^T_k\boldsymbol{C}^T_k\boldsymbol{y}_k + \hat{\boldsymbol{x}}^T_k\boldsymbol{C}^T_k\boldsymbol{C}_k\hat{\boldsymbol{x}}_k \quad (27)\end{aligned}$$

where $\boldsymbol{C}_k$ is determined by $\hat{\boldsymbol{x}}_k$. According to (7), $\boldsymbol{C}_k$ can be determined by $\hat{\boldsymbol{x}}_{k-1}$ and $\boldsymbol{e}_{k-1}$.

Using the approximation $\boldsymbol{F}\left(\hat{\boldsymbol{x}}_k\right) \approx \boldsymbol{F}'\left(\hat{\boldsymbol{x}}_k\right)\hat{\boldsymbol{x}}_k = \boldsymbol{A}_k\hat{\boldsymbol{x}}_k$, together with (7) and the update law of learning rate expressed by (21), the following formula can be got by substituting (27) into (26),

$$\begin{aligned}D_k &= \mathrm{E}\{\boldsymbol{y}^T_k\boldsymbol{y}_k - \boldsymbol{y}^T_k\boldsymbol{C}_k\left[\boldsymbol{A}_{k-1}\hat{\boldsymbol{x}}_{k-1} + \left(\boldsymbol{L}_{k-1} + \eta\frac{\partial \boldsymbol{J}_{k-1}}{\partial \boldsymbol{L}_{k-1}}\right)\boldsymbol{e}_{k-1}\right] \\ &\quad - \left[\boldsymbol{A}_{k-1}\hat{\boldsymbol{x}}_{k-1} + \left(\boldsymbol{L}_{k-1} + \eta\frac{\partial \boldsymbol{J}_{k-1}}{\partial \boldsymbol{L}_{k-1}}\right)\boldsymbol{e}_{k-1}\right]^T \boldsymbol{C}^T_k\boldsymbol{y}_k \\ &\quad + \left[\boldsymbol{A}_{k-1}\hat{\boldsymbol{x}}_{k-1} + \left(\boldsymbol{L}_{k-1} + \eta\frac{\partial \boldsymbol{J}_{k-1}}{\partial \boldsymbol{L}_{k-1}}\right)\boldsymbol{e}_{k-1}\right]^T \boldsymbol{C}^T_k\boldsymbol{C}_k \\ &\quad \cdot \left[\boldsymbol{A}_{k-1}\hat{\boldsymbol{x}}_{k-1} + \left(\boldsymbol{L}_{k-1} + \eta\frac{\partial \boldsymbol{J}_{k-1}}{\partial \boldsymbol{L}_{k-1}}\right)\boldsymbol{e}_{k-1}\right]\} - \boldsymbol{e}^T_{k-1}\boldsymbol{e}_{k-1} \\ &< 0 \quad (28)\end{aligned}$$

After some algebraic calculation, (28) can be reformulated as

$$D_k = a\eta^2 - 2b\eta + c < 0 \quad (29)$$

with

$$a = \boldsymbol{e}^T_{k-1}\frac{\partial \boldsymbol{J}^T_{k-1}}{\boldsymbol{L}^T_{k-1}}\boldsymbol{C}^{\mathrm{T}}_k\boldsymbol{C}_k\frac{\partial \boldsymbol{J}_{k-1}}{\boldsymbol{L}_{k-1}}\boldsymbol{e}_{k-1} \quad (30)$$

and

$$\begin{aligned}b &= \boldsymbol{y}^T_k\boldsymbol{C}_k\frac{\partial \boldsymbol{J}_{k-1}}{\boldsymbol{L}_{k-1}}\boldsymbol{e}_{k-1} - \hat{\boldsymbol{x}}^T_{k-1}\boldsymbol{A}^T_{k-1}\boldsymbol{C}^T_k\boldsymbol{C}_k\frac{\partial \boldsymbol{J}_{k-1}}{\boldsymbol{L}_{k-1}}\boldsymbol{e}_{k-1} \\ &\quad - \boldsymbol{e}^T_{k-1}\boldsymbol{L}^T_{k-1}\frac{\partial \boldsymbol{J}_{k-1}}{\boldsymbol{L}_{k-1}}\boldsymbol{e}_{k-1} \quad (31)\end{aligned}$$

and also

$$\begin{aligned}c &= \boldsymbol{y}^T_k\boldsymbol{y}_k - 2\boldsymbol{y}^T_k\boldsymbol{C}_k\boldsymbol{A}_{k-1}\hat{\boldsymbol{x}}_{k-1} - 2\boldsymbol{y}^T_k\boldsymbol{C}_k\boldsymbol{L}_{k-1}\boldsymbol{e}_{k-1} \\ &\quad + \hat{\boldsymbol{x}}^T_{k-1}\boldsymbol{A}^T_{k-1}\boldsymbol{C}^T_k\boldsymbol{C}_k\boldsymbol{A}_{k-1}\hat{\boldsymbol{x}}_{k-1} \\ &\quad + 2\hat{\boldsymbol{x}}^T_{k-1}\boldsymbol{A}^T_{k-1}\boldsymbol{C}^T_k\boldsymbol{C}_k\boldsymbol{L}_{k-1}\boldsymbol{e}_{k-1} \\ &\quad + \boldsymbol{e}^T_{k-1}\boldsymbol{L}^T_{k-1}\boldsymbol{C}^T_k\boldsymbol{C}_k\boldsymbol{L}_{k-1}\boldsymbol{e}_{k-1} \quad (32)\end{aligned}$$

Solving (32) for η, the constant η_1 and η_2 can be determined as:

$$\eta_1 = \frac{-b - \sqrt{b^2 - ac}}{a} \quad (33)$$

$$\eta_2 = \frac{-b + \sqrt{b^2 - ac}}{a} \quad (34)$$

Thus, in order to ensure the convergence of CEEC filter, the learning rate η should be chosen so that

$$\eta_1 < \eta < \eta_2. \quad (35)$$

Other parameters are chosen by empirical method. For example, the kernel size $\sigma_{s,1}$ of MCC function is set large enough to ensure the MCC function concave. The kernel size $\sigma_{s,2}$ of MEE function needs to be set properly to guarantee MEE function capturing higher-order statistics property.

B. IMPLEMENTATION OF CEEC FILTER FOR MULTIPATH ESTIMATION

For the multipath estimation problem, only one measurement output vector is obtained at one instant and there are not enough samples used for implementing the CEEC filter in the way of data-driven at the initial iteration stage. To apply the CEEC filter in a practical way for multipath estimation, an online iterative method using modified Parzen windowing technique is proposed in this section.

In (18), W error samples $\boldsymbol{e}_i$ ($i = k - W + 1, k - W + 2, \ldots, k$) are needed to compute the performance function $J_k(\boldsymbol{e})$ at time k. However, there are not enough samples can be used to calculate $J_k(\boldsymbol{e})$ if $k \leq W$. To tackle this problem, the following formula is used for (18).

$$\begin{aligned}J_k\left(\boldsymbol{e}\right) &= \lambda\left[\frac{1}{t}\sum_{i=k-t+1}^{k} G_{\sum_1}\left(\boldsymbol{e}_i\right)\right] \\ &\quad + (1-\lambda)\left[\frac{1}{t}\sum_{i=k-t+1}^{k} G_{\sum_2}\left(\boldsymbol{e}_k - \boldsymbol{e}_i\right)\right] \quad (36)\end{aligned}$$

where $t = \begin{cases} k & k \leq W \\ W & k > W \end{cases}$.

According to (36), the CEEC filter can be implemented in the way shown in Fig.3.

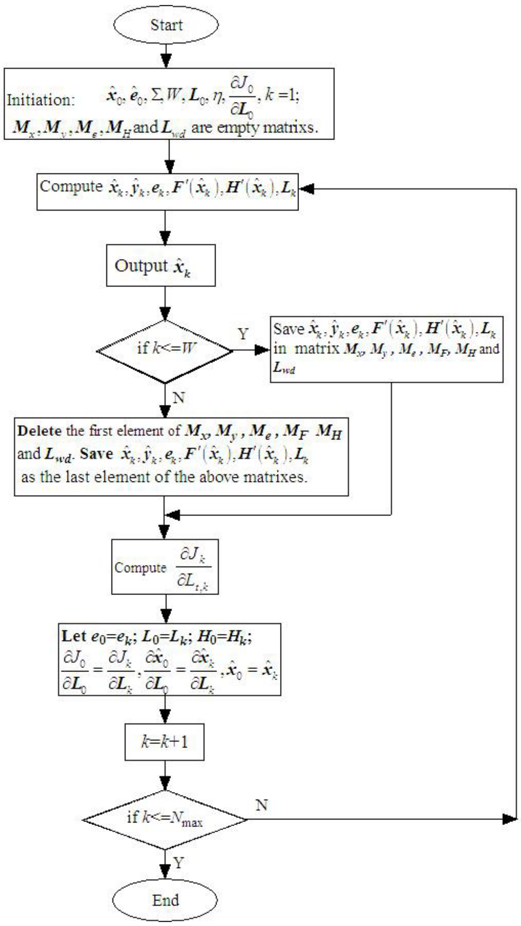

FIGURE 3. The implementation of CEEC filter.

This iterative algorithm is different from other iterative algorithms mentioned in [6], [21], [22], and [23]. For instance, the iterative algorithm based on EKF mentioned in [6] use only one sample (or sample vector) to perform filter in one iteration. It is not suitable for multipath estimation to reduce the randomness of estimation error because enough samples are needed to capture the error PDF's stochastic property in non-Gasussian noise. The iterative algorithm in [21] can be performed only when W samples are collected, which leads to a waiting time at the initial stage and it is not suitable for positioning systems due to its high real-time need. The iterative algorithms in [22] and [23] are adopt to reduce the order of filter gain for continuous-time system, which is not suitable for the estimation problem of discrete system described in this paper.

Remark: k samples can be used to calculate J_k even if $k \leq W$ at the initial estimation stage and only the newest W samples are utilized for calculating J_k if $k > W$. In this way, one does not need to wait to collect all the needed samples for calculating J_k at the initial stage and the calculation complexity can be controlled by limiting the sample number as iteration goes on. The most important is that the error randomness can be reduced under the criterion of CEEC.

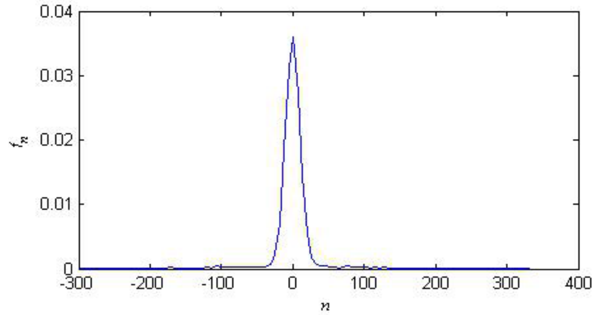

FIGURE 4. PDF of $n(k)$.

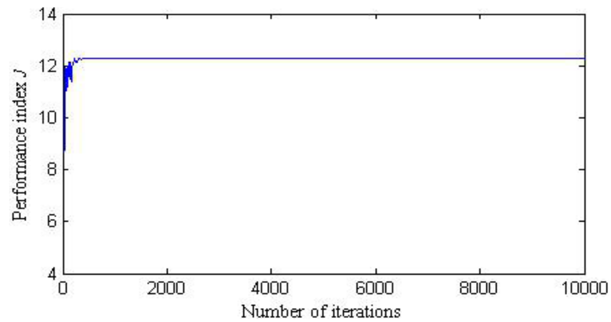

FIGURE 5. The performance index of CEEC.

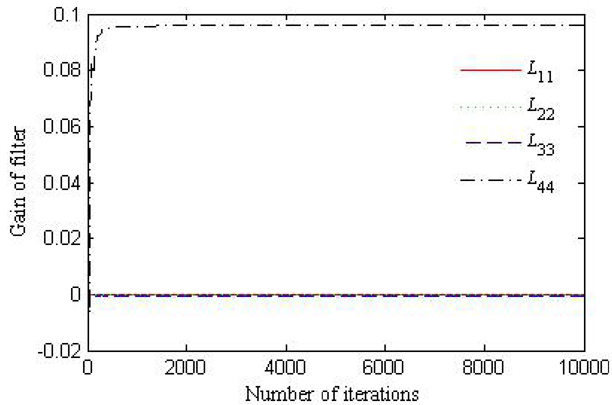

FIGURE 6. Gain of the CEEC estimator.

V. SIMULATION RESULTS

Without loss of generality, a C/A signal of GPS is simulated assuming a scenario composed of a direct signal and a multipath signal. Set $A_0 = 0.8$, $A_1 = 0.4$, $l_0 = 10T_c$, $l_1 = 0.5T_c$, $T_s = T_c/10$, where T_c is the C/A code chip duration with 1/1023 ms, 1023 is the number of C/A code chip in a period, T_s is the sampling interval. The local estimation of l_0 is $\hat{l}_0 = 10.4T_c$, so $\varepsilon = \hat{l}_0 - l_0 = 0.4T_c$. Our goal is to estimate A_0, A_1, ε, τ_1. In this simulation, Four correlation outputs are used to construct a observation vector $\boldsymbol{y}_k$, i.e. $S = 4$, and the correlation space vector is set as $\boldsymbol{d} = [d_1, d_2, d_3, d_4] = [-0.5, -0.3, 0.3, 0.5]T_c$. Assume the multipath parameter being unchanged during the observating period. Then, the system matrix $\boldsymbol{F}_k$ is set as an $M \times M$ identity matrix.

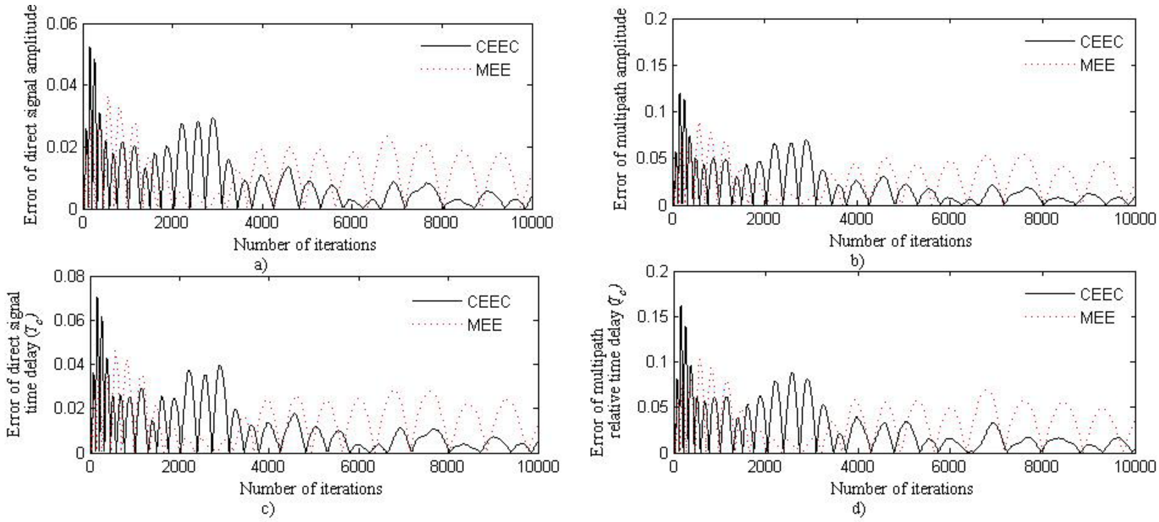

FIGURE 7. The multipath estimation results under different criterion.

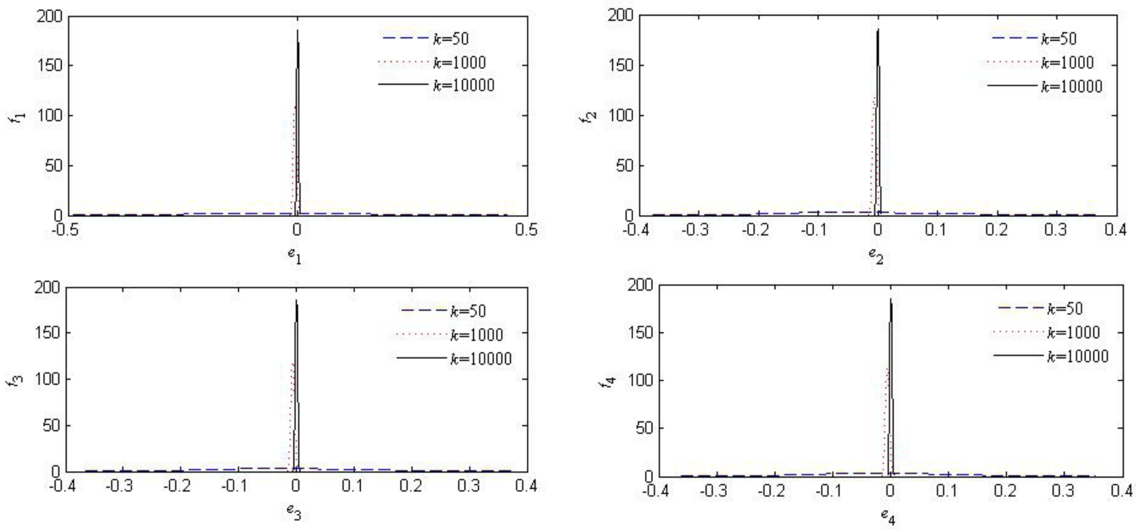

FIGURE 8. The error PDF at different instant for CEEC filter.

In non-Gaussian noise environments, the impulse noise is considered. The PDF of impulse noise can be modeled by a Gaussian mixture PDF. In the receiving signal $r(k)$, the PDF of $n(k)$ is modeled as $f_n = \lambda_1 N(\mu_1, \sigma_1) + \lambda_2 N(\mu_2, \sigma_2)$, where $N(\mu, \sigma)$ is the Gaussian distribution with mean μ and variance σ. In this simulation, we set $\lambda_1 = 0.9$, $\mu_1 = 0$, $\sigma_1 = 10$, $\lambda_2 = 0.1$, $\mu_2 = 0$, $\sigma_2 = 100$. The PDF of $n(k)$ is shown in Fig.4. $\boldsymbol{x}_0$ is set as the true value, i.e. $\boldsymbol{x}_0 = [A_1 A_0 \varepsilon l_1]^T = [0.8\ 0.4\ 0.4\ 0.5]^T$, $\boldsymbol{e}_0 = [0\ 0\ 0\ 0]^T$.

In this simulation, $\boldsymbol{e}_k = [(k-1)\boldsymbol{e}_{k-1} + \boldsymbol{e}_k]/k$ is used to improve performance since the multipath parameters are unchanged during observating period. The performance index is given in Fig.5. It can be seen that the maximum of $\boldsymbol{J}$ is achieved after about 300 iterations. The change trend of the gain $\boldsymbol{L}_k$ is shown in Fig.6. For simplicity, only the diagonal elements of $\boldsymbol{L}_k$ are given in this figure.

The multipath estimation results are shown in Fig.7, where $\lambda = 0$, $\delta_{s,2}^2 = 0.38$, $W = 32$, $\eta = 0.00001$ for MEE. $\lambda = 0.32$, $\delta_{s,1}^2 = 0.15$, $\delta_{s,2}^2 = 0.38$, $W = 32$, $\eta = 0.00005$ for CEEC. In this figure, the absolute error is considered. It indicates the CEEC filter has better performance than MEE filter for multipath estimation. This is due to the fact that CEEC is a balance criterion between MCC and MEE. CEEC can fix the main peak of the error PDF at the original point and overcome the drawbacks of MEE being shift-invariant and maximum correntropy criterion (MCC) being local optimization.

The error PDF of multipath parameters for CEEC was shown in Fig.8. It is shown that the shape of the PDF of estimation error turns to be narrower and sharper over the

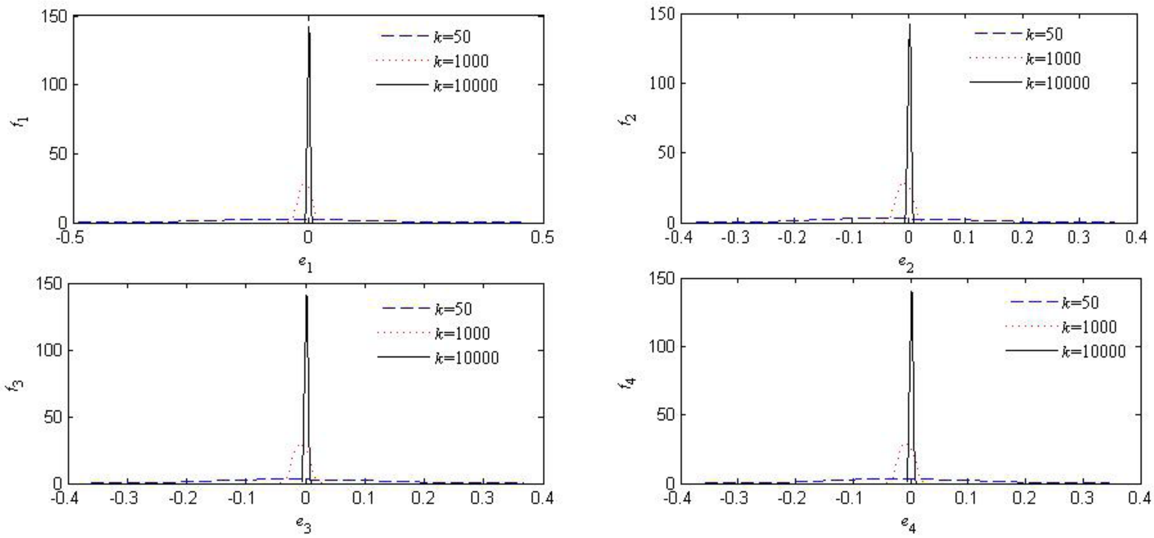

FIGURE 9. **The error PDF at different instant for MEEC filter.**

iteration process, which means the error becomes smaller. The error PDF of multipath parameters for MEEC was shown in Fig.9. It can be seen that the error PDF of MEEC filter is wider than that of CEEC filter at the same instant, which means the CEEC filter has a better performance than MEEC filter.

VI. CONCLUSIONS

In this paper, the multipath estimation problem is converted into a state estimation problem. The CEEC instead of MSEC is used as the performance index for filter design to describe the high order statistics of error PDF for multipath estimation in non-Gaussian noise. The multipath parameters are estimated by the proposed CEEC filter in an iterative way. The comparison of CEEC filter and MEEC filter shows the former has a better performance for multipath estimation in terms of estimation accuracy.

Compared with the previous work, the main contributions of this paper are two folds: (1) The CEEC filter instead of MSEC filter are proposed for multipath estimation in non-Gaussion noise; (2) An online iterative estimation method based on modified Parzen windowing technique is presented for practical implementation. However, the CEEC estimator is sensitive to the initial state, initial gain matrix. These problems should be focused on in the future work.

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