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Transceiver Optimization for Full-Duplex Massive MIMO AF Relaying With Direct Link

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ABSTRACT This paper focuses on the end-to-end signal-to-noise ratio (SNR) maximization for full-duplex massive multiple-input multiple-output (MIMO) amplify-and-forward (AF) relay systems in the presence of direct link. First, we rigorously prove the asymptotic optimality of the maximum-ratio combining/maximum-ratio transmission (MRC/MRT) relaying strategy by taking into account the massive MIMO setup. Then, concerning the equivalent optimization problem with respect to source beamformer, we advocate a two-tier iterative algorithm relying on bi-section search, which guarantees a globally optimal solution. As a byproduct of this approach, we show that the optimal source beamformer has an interesting generalized channel matching structure associated with both source-relay and source-destination links. In addition to the optimal design, we devise a high SNR approximation-based suboptimal scheme, which admits a closed-form solution. Simulation results verify the advantage of our full-duplex relaying designs, and also demonstrate a negligible performance gap between the proposed optimal and suboptimal methods.

INDEX TERMS Full-duplex amplify-and-forward (AF) relaying, massive multiple-input multiple-output (MIMO), direct link, signal-to-noise ratio (SNR) maximization.

I. INTRODUCTION

Wireless full-duplex relaying, which simultaneously transmits and receives signals using the same frequency resource, has emerged as a promising candidate for boosting the data rate and meanwhile expanding the coverage of future communication systems. Recently, a number of research works, e.g., [1]–[7], [12]–[14], have leveraged prominent multiple-input multiple-output (MIMO) techniques in full-duplex relays for further performance improvement.

Concretely, Riihonen *et al.* [1] advocated several efficient full-duplex relay processing strategies such as self-interference nullification based precoding. Different from this study, [2]–[7] optimized end-to-end achievable rate performance in lieu of merely dealing with self-interference, where optimization problems were mathematically formulated and addressed by methods such as gradient based search [3]–[5] and penalty-BSUM algorithm [7].[1]

[1]The transceiver optimization for half-duplex MIMO systems have been extensively investigated in literature, e.g, [8]–[11].

Alternatively, in some recent works [12]–[14], researchers proposed to apply massive MIMO technology [15], [16] to handle the inevitable self-interference in full-duplex relay systems. As revealed in these works, with a large-scale antenna array available at the relay, it is possible to mitigate the self-interference via low-complexity maximum-ratio combining/maximum-ratio transmission (MRC/MRT) or zero-forcing (ZF) strategy.

It is necessary to point out that most of the above works presumed that the source-destination link is weak enough to be neglected, which may not be suitable for characterizing certain practical scenarios. In literature, a few efforts have been devoted to transceiver optimization for multi-antenna half-duplex amplify-and-forward (AF) relaying with direct link [17]–[19]. However, we find that the extension to the more challenging full-duplex AF relaying is non-trivial and has not been investigated in prior works.

In this paper, we consider signal-to-noise ratio (SNR) maximized transceiver design for full-duplex massive MIMO AF relaying with direct link. In particular, we concern the scenario where a large-scale transmit antenna array with low transmit power is equipped at the relay. We first derive the asymptotically optimal relay receive and transmit filters in closed forms, based on which we acquire an equivalent problem with regard to source beamformer only. Then, we develop a bi-section search based iterative algorithm in order to optimally solve the problem of interest, from which we discover that the source beamformer should be the maximum eigenvector of the weighted difference of the source-relay and source-destination channels' Gram matrices. We also propose a suboptimal closed-form solution via forcing the effective channel gain of the direct link to be zero, which incurs almost no performance loss as validated via simulations.

Notations: Throughout the paper, we represent scalars, vectors and matrices by plain, bold lowercase and bold uppercase letters, respectively. $|\cdot|$ and $\|\cdot\|$ denote the absolute value and Frobenius norm, respectively. $(\cdot)^H$ represents the Hermitian operation and $\mathbb{E}\{\cdot\}$ stands for the statistical expectation operation. $\mathrm{tr}(\mathbf{A})$ refers to the trace of matrix $\mathbf{A}$. $\mathbf{A} \succeq \mathbf{0}$ means that matrix $\mathbf{A}$ is positive semidefinite. $\mathbf{I}$ denotes the identity matrix.

II. FULL-DUPLEX MIMO AF RELAY SYSTEM MODEL

We consider a dual-hop AF relay system comprised of one source, one full-duplex relay and one destination. The source has N_s antennas, the relay possesses N_r receive antennas and N_t transmit antennas, and the destination is equipped with a single antenna. A direct link exists between the source and the destination. Both source and relay adopt linear transmit or/and receive beamforming techniques for signal transmissions. In such context, the relay received signal is given by

$$\mathbf{y}_r[t] = \mathbf{H}_{sr}\mathbf{f}_s x_s[t] + \mathbf{H}_{rr}\mathbf{x}_r[t] + \mathbf{n}_r[t], \tag{1}$$

where $\mathbf{H}_{sr}$ and $\mathbf{H}_{rr}$ are the source-relay and self-interference channels, $\mathbf{f}_s$ represents the source transmit beamformer that satisfies $\|\mathbf{f}_s\|^2 \leq P_s$ with P_s being the source transmit power budget, $x_s[t]$ is the source symbol with unit variance, $\mathbf{x}_r[t]$ denotes the relay transmitted signal, and $\mathbf{n}_r[t]$ stands for the additive white Gaussian noise (AWGN) at the relay whose covariance equals $\sigma_r^2\mathbf{I}$. After the relay performs receive and transmit beamforming on $\mathbf{y}_r[t]$, we obtain the relay transmitted signal by

$$\mathbf{x}_r[t] = \mathbf{f}_r\mathbf{g}_r^H\mathbf{y}_r[t-\tau], \tag{2}$$

where $\mathbf{f}_r$ and $\mathbf{g}_r$ stand for the transmit and receive filters adopted by the relay, and τ denotes the processing delay of the relay. Then, based on (1) and (2), we are able to rewrite $\mathbf{x}_r[t]$ by the following form:

$$\begin{aligned}\mathbf{x}_r[t] = \mathbf{f}_r \sum_{j=0}^{\infty}(\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r)^j\mathbf{g}_r^H(&\mathbf{H}_{sr}\mathbf{f}_s x_s[t-(j+1)\tau] \\ &+\mathbf{n}_r[t-(j+1)\tau]).\end{aligned} \tag{3}$$

We note that similar expression has also been derived in [6], [20], and [21]. Accordingly, it is readily to calculate the relay transmit power by

$$\mathbb{E}\{\|\mathbf{x}_r[t]\|^2\} = \frac{\|\mathbf{f}_r\|^2(|\mathbf{g}_r^H\mathbf{H}_{sr}\mathbf{f}_s|^2 + \sigma_r^2\|\mathbf{g}_r\|^2)}{1-|\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r|^2}, \tag{4}$$

which does not exceed a given maximum value P_r. Finally, by taking into account the signals from the relay and the source, the received signal at the destination takes the form [4], [5], [20], [22]

$$y_d[t] = \mathbf{h}_{rd}^H\mathbf{x}_r[t] + \mathbf{h}_{sd}^H\mathbf{f}_s x_s[t] + n_d[t], \tag{5}$$

where $\mathbf{h}_{rd}^H$ and $\mathbf{h}_{sd}^H$ represent the source-relay and source-destination channels, and $n_d[t]$ is the AWGN at the destination with variance σ_d^2.

From (3)–(5), we are able to obtain the destination SNR for the above illustrated system by

$$\begin{aligned}&\mathrm{SNR}_d \\ &= \frac{|\mathbf{h}_{rd}^H\mathbf{f}_r|^2|\mathbf{g}_r^H\mathbf{H}_{sr}\mathbf{f}_s|^2}{\frac{|\mathbf{h}_{rd}^H\mathbf{f}_r|^2|\mathbf{g}_r^H\mathbf{H}_{sr}\mathbf{f}_s|^2|\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r|^2}{1-|\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r|^2} + \frac{\sigma_r^2|\mathbf{h}_{rd}^H\mathbf{f}_r|^2\|\mathbf{g}_r\|^2}{1-|\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r|^2} + |\mathbf{h}_{sd}^H\mathbf{f}_s|^2 + \sigma_d^2},\end{aligned} \tag{6}$$

which is a rather complicated function of the beamforming vectors $\mathbf{f}_s$, $\mathbf{f}_r$ and $\mathbf{g}_r$.

III. SNR MAXIMIZED FULL-DUPLEX AF RELAY TRANSCEIVER UNDER MASSIVE MIMO SETUP

A. OPTIMAL RELAY BEAMFORMING WITH LARGE-SCALE RELAY TRANSMIT ARRAY AND LOW TRANSMIT POWER

According to Section II, we can readily formulate the SNR maximized transceiver optimization problem as in (7). Unfortunately, especially due to the self-interference term $|\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r|^2$, we find that it is indeed a very difficult task to address the above problem, let alone to reveal some underlying insights as the half-duplex relaying case, e.g., the optimal structure of the MIMO relay transceiver [19].

In the rest of the paper, we employ the popular massive MIMO techniques, which are known to be capable of canceling the self-interference without involving sophisticated signal processing [12]–[14]. Concretely, following [12], we assume that the relay has a large-scale transmit array and meanwhile uses a low transmit power proportional to $1/N_t$,

$$\begin{aligned}&\underset{\mathbf{f}_s,\mathbf{f}_r,\mathbf{g}_r}{\text{maximize}} \quad \frac{|\mathbf{h}_{rd}^H\mathbf{f}_r|^2|\mathbf{g}_r^H\mathbf{H}_{sr}\mathbf{f}_s|^2}{\frac{|\mathbf{h}_{rd}^H\mathbf{f}_r|^2|\mathbf{g}_r^H\mathbf{H}_{sr}\mathbf{f}_s|^2|\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r|^2}{1-|\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r|^2} + \frac{\sigma_r^2|\mathbf{h}_{rd}^H\mathbf{f}_r|^2\|\mathbf{g}_r\|^2}{1-|\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r|^2} + |\mathbf{h}_{sd}^H\mathbf{f}_s|^2 + \sigma_d^2} \\ &\text{subject to } \|\mathbf{f}_s\|^2 \leq P_s, \quad \|\mathbf{f}_r\|^2(|\mathbf{g}_r^H\mathbf{H}_{sr}\mathbf{f}_s|^2 + \sigma_r^2\|\mathbf{g}_r\|^2) \leq P_r(1-|\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r|^2).\end{aligned} \tag{7}$$

i.e., $P_r = \frac{E_r}{N_t}$ where E_r is a fixed value irrelevant of N_t. Moreover, the entries of the channels $\mathbf{h}_{rd}^H$ and $\mathbf{H}_{rr}$ are independent and identically distributed (i.i.d.) zero-mean complex Gaussian variables whose variances are γ_{rd}^2 and γ_{rr}^2, respectively. Under such circumstance, we show the asymptotically optimal AF relaying strategy in the subsequent proposition.

Proposition 1: For any given $\mathbf{f}_s$, the asymptotically optimal relay transmit and receive beamformers for a large-scale relay transmit array with a low transmit power are, respectively, given by

$$\mathbf{f}_r^* = \sqrt{\frac{E_r}{N_t^2 \gamma_{rd}^2 (\|\mathbf{H}_{sr}\mathbf{f}_s\|^2 + \sigma_r^2)}}\mathbf{h}_{rd}, \tag{8}$$

$$\mathbf{g}_r^* = \frac{\mathbf{H}_{sr}\mathbf{f}_s}{\|\mathbf{H}_{sr}\mathbf{f}_s\|}. \tag{9}$$

Proof: See Appendix A. ■

Remark 1: The proposition indicates that, when N_t is large and P_r is proportional to $1/N_t$, the classical MRC/MRT strategy is actually an asymptotically optimal solution. Compared to the existing spatial-domain self-interference suppression methods that rely on numerical algorithms [3]–[5], [7], MRC/MRT has a closed form whose implementation complexity is fairly low. Moreover, MRC/MRT is irrelevant to the self-interference channel $\mathbf{H}_{rr}$, and hence we do not need to acquire the exact value of $\mathbf{H}_{rr}$.

By substituting $(\mathbf{f}_r^*, \mathbf{g}_r^*)$ in Proposition 1 into problem (7) and employing the fact that $(\mathbf{g}_r^*)^H \mathbf{H}_{rr}\mathbf{f}_r^* \to 0$ and $\frac{\|\mathbf{h}_{rd}\|^2}{N_t} \to \gamma_{rd}^2$ hold when $N_t \to \infty$ (refer to Appendix A), we acquire the following problem with only one optimization variable $\mathbf{f}_s$:

$$\begin{aligned} \underset{\mathbf{f}_s}{\text{maximize}} \quad & \frac{E_r \gamma_{rd}^2 \|\mathbf{H}_{sr}\mathbf{f}_s\|^2}{(\|\mathbf{H}_{sr}\mathbf{f}_s\|^2 + \sigma_r^2)(|\mathbf{h}_{sd}^H\mathbf{f}_s|^2 + \sigma_d^2) + E_r\gamma_{rd}^2\sigma_r^2} \\ \text{subject to} \quad & \|\mathbf{f}_s\|^2 \le P_s. \end{aligned} \tag{10}$$

We need to highlight that, the above problem has a quite different objective from the one under the conventional half-duplex relaying [17], [19]. In fact, we cannot apply the algorithm developed in [17], and [19] to address problem (10) straightforwardly. Moreover, as we will see later, the optimal structure of the source beamformer revealed in [19, Th. 1] does not hold any more. In what follows, we first develop a bi-section search based approach to achieve its globally optimal solution in a semi-closed form. Subsequently, we derive a suboptimal but closed-form solution which performs very close to the optimal method.

B. SEMI-CLOSED FORM SOLUTION TO OPTIMAL SOURCE BEAMFORMER

The denominator of the objective in (10) contains a cumbersome quartic term $\|\mathbf{H}_{sr}\mathbf{f}_s\|^2|\mathbf{h}_{sd}^H\mathbf{f}_s|^2$, making it difficult to find the optimal solution directly. To obtain a tractable reformulation, we introduce an auxiliary variable t and equivalently rewrite problem (10) as

$$\begin{aligned} \underset{\mathbf{f}_s, t}{\text{maximize}} \quad & \frac{E_r \gamma_{rd}^2 \|\mathbf{H}_{sr}\mathbf{f}_s\|^2}{(\|\mathbf{H}_{sr}\mathbf{f}_s\|^2 + \sigma_r^2)(t + \sigma_d^2) + E_r\gamma_{rd}^2\sigma_r^2} \\ \text{subject to} \quad & |\mathbf{h}_{sd}^H\mathbf{f}_s|^2 \le t, \quad \|\mathbf{f}_s\|^2 \le P_s. \end{aligned} \tag{11}$$

It is clear that the objective function is monotonically increasing with the term $\|\mathbf{H}_{sr}\mathbf{f}_s\|^2$. Therefore, for any fixed t, problem (11) amounts to

$$\begin{aligned} \underset{\mathbf{f}_s}{\text{maximize}} \quad & \|\mathbf{H}_{sr}\mathbf{f}_s\|^2 \\ \text{subject to} \quad & |\mathbf{h}_{sd}^H\mathbf{f}_s|^2 \le t, \quad \|\mathbf{f}_s\|^2 \le P_s. \end{aligned} \tag{12}$$

This is a non-convex quadratically constrained quadratic program (QCQP) with two quadratic constraints, which can be transformed into an equivalent semidefinite program (SDP) via semidefinite relaxation (SDR) [23]. Nonetheless, solving an SDP requires a high computational cost especially when the problem size is large. Moreover, the SDP based solution can hardly provide a deep insight into the essence of the optimal solution. Therefore, instead of applying the standard SDR tool, we resort to an alternative approach that yields a semi-closed form solution as below.

Theorem 1: The optimal $\mathbf{f}_s^*$ to problem (12) has a semi-closed form expression as

$$\mathbf{f}_s^* = \sqrt{P_s}\text{max\_eig}(\mathbf{H}_{sr}^H\mathbf{H}_{sr} - \alpha^*\mathbf{h}_{sd}\mathbf{h}_{sd}^H), \tag{13}$$

where max_eig$(\cdot)$ represents the normalized maximum eigenvector that corresponds to the maximum eigenvalue of the input matrix, and α^* is calculated by

$$\alpha^* = \begin{cases} 0 & P_s|\mathbf{h}_{sd}^H\text{max\_eig}(\mathbf{H}_{sr}^H\mathbf{H}_{sr})|^2 < t \le P_s\|\mathbf{h}_{sd}\|^2 \\ \tilde{\alpha}^* & 0 \le t \le P_s|\mathbf{h}_{sd}^H\text{max\_eig}(\mathbf{H}_{sr}^H\mathbf{H}_{sr})|^2, \end{cases} \tag{14}$$

in which $\tilde{\alpha}^*$ is the nonnegative root of the equation $P_s|\mathbf{h}_{sd}^H\text{max\_eig}(\mathbf{H}_{sr}^H\mathbf{H}_{sr} - \alpha\mathbf{h}_{sd}\mathbf{h}_{sd}^H)|^2 = t$ and can be found via bisection search.

Proof: See Appendix B. ■

Theorem 1 indicates that the optimal source transmit strategy is to use all available power to transmit along the maximum eigenvector of the weighted difference of Gram matrices $\mathbf{H}_{sr}^H\mathbf{H}_{sr}$ and $\mathbf{h}_{sd}\mathbf{h}_{sd}^H$. We would like to note that, since $\mathbf{H}_{sr}^H\mathbf{H}_{sr} - \alpha^*\mathbf{h}_{sd}\mathbf{h}_{sd}^H$ is not necessarily positive semidefinite, the maximum eigenvector here differs from the commonly known dominant eigenvector corresponding to the eigenvalue with maximum absolute value.

Based upon the second part of the proof for Theorem 1 and the fact that the constraint $|\mathbf{h}_{sd}^H\mathbf{f}_s|^2 \le t$ of problem (11) is active at the optimal point, it does not lose optimality to assume $t \in [0, P_s|\mathbf{h}_{sd}^H\text{max\_eig}(\mathbf{H}_{sr}^H\mathbf{H}_{sr})|^2]$. Thereby, problem (11) becomes

$$\begin{aligned} \underset{t}{\text{maximize}} \quad & \frac{E_r\gamma_{rd}^2 f(t)}{(f(t) + \sigma_r^2)(t + \sigma_d^2) + E_r\gamma_{rd}^2\sigma_r^2} \\ \text{subject to} \quad & 0 \le t \le P_s|\mathbf{h}_{sd}^H\text{max\_eig}(\mathbf{H}_{sr}^H\mathbf{H}_{sr})|^2, \end{aligned} \tag{15}$$

where $f(t)$ denotes the optimal objective value of problem (12). The major intricacy of solving this problem lies in the implicit form of the function $f(t)$ which renders a closed-form solution almost impossible. Nonetheless, after performing an in-depth analysis, we discover that the optimal solution to t can be found via a simple bi-section search owing to the particular properties of $f(t)$ and problem (15) elaborated as follows.

Proposition 2: $f(t)$ is a concave function and problem (15) is a quasiconvex problem.

Proof: See Appendix C. ■

In summary, the proposed algorithm that optimally solves problem (11) consists of two tiers. In the inner tier, we apply Theorem 1 to calculate the optimal $\mathbf{f}_s^*$ with t given. In the outer tier, we determine the optimal solution to t by performing a bi-section search for problem (15) thanks to Proposition 2.

C. CLOSED-FORM SUBOPTIMAL SOURCE BEAMFORMER SOLUTION

The above developed algorithm requires an iterative search process, which can lead to a high computational cost in practice. To reduce the implementation complexity, we now develop a suboptimal but closed-form solution to the source beamformer $\mathbf{f}_s$.

Before presenting the low complexity method, let us first investigate a special case when the noise variance at the relay σ_r^2 is very small. Under such circumstance, since $\sigma_r^2 \to 0$, we can approximate the denominator of the objective in (10) as $\|\mathbf{H}_{sr}\mathbf{f}_s\|^2(|\mathbf{h}_{sd}^H\mathbf{f}_s|^2 + \sigma_d^2)$ by neglecting the terms that include σ_r^2. Accordingly, when $\sigma_r^2 \to 0$, problem (10) is simplified as

$$\begin{aligned} &\underset{\mathbf{f}_s}{\text{minimize}} \ |\mathbf{h}_{sd}^H\mathbf{f}_s|^2 + \sigma_d^2 \\ &\text{subject to } \|\mathbf{f}_s\|^2 \le P_s. \end{aligned} \tag{16}$$

It is readily to see that we must have $\mathbf{h}_{sd}^H\mathbf{f}_s = 0$ at optimality. This fact implies that, when the relay SNR is high, the source should convey information only on the source-relay-destination link so as to maximize the end-to-end SNR. Motivated by this interesting phenomenon, we propose to fix the structure of $\mathbf{f}_s$ such that $\mathbf{h}_{sd}^H\mathbf{f}_s = 0$ is fulfilled. To be more specific, we let $\mathbf{f}_s = \left(\mathbf{I} - \frac{\mathbf{h}_{sd}\mathbf{h}_{sd}^H}{\|\mathbf{h}_{sd}\|^2}\right)\mathbf{v} \triangleq \mathbf{P}_{\mathbf{h}_{sd}}\mathbf{v}$ and plug it into problem (10) (without forcing $\sigma_r^2 \to 0$), which gives

$$\begin{aligned} &\underset{\mathbf{v}}{\text{maximize}} \ \mathbf{v}^H\mathbf{P}_{\mathbf{h}_{sd}}\mathbf{H}_{sr}^H\mathbf{H}_{sr}\mathbf{P}_{\mathbf{h}_{sd}}\mathbf{v} \\ &\text{subject to } \mathbf{v}^H\mathbf{P}_{\mathbf{h}_{sd}}\mathbf{v} \le P_s. \end{aligned} \tag{17}$$

Similar to problem (12), this is also a non-convex QCQP problem with, however, only one quadratic constraint. We are able to obtain its optimal solution in closed form as below.

Proposition 3: The optimal solution to problem (17) is

$$\mathbf{v}^* = d_{v^*}\text{max\_eig}\left(\mathbf{P}_{\mathbf{h}_{sd}}\mathbf{H}_{sr}^H\mathbf{H}_{sr}\mathbf{P}_{\mathbf{h}_{sd}}\right) \triangleq d_{v^*}\tilde{\mathbf{v}}^*, \tag{18}$$

where $d_{v^*} = \sqrt{P_s/((\tilde{\mathbf{v}}^*)^H\mathbf{P}_{\mathbf{h}_{sd}}\tilde{\mathbf{v}}^*)}$.

Proof: See Appendix D. ■

Compared to the optimal approach proposed in Section III-B, the above solution only involves one eigenvalue decomposition and hence has much lower complexity. Moreover, it will be verified via simulations that this method achieves almost the same performance as the optimal one.

IV. SIMULATION RESULTS

We conduct simulations to evaluate the rate performance of the following methods:

- The optimal full-duplex relay transceiver with ideal self-interference cancelation ('FD Ideal SIC')
- The proposed optimal full-duplex relay transceiver ('FD Optimal')
- The proposed suboptimal full-duplex relay transceiver ('FD Suboptimal')
- The eigenmode transmission based full-duplex relay transceiver ('FD Eigenmode')
- The optimal half-duplex relay transceiver [19] ('HD Optimal').

The benchmark method 'FD Ideal SIC' is achieved by replacing the constant γ_{rd}^2 in 'FD Optimal' with $\frac{\|\mathbf{h}_{rd}\|^2}{N_t}$, which is optimal for any finite N_t due to the absence of self-interference. For 'FD Eigenmode', the source beamformer only matches the source-relay channel $\mathbf{H}_{sr}$ and is given by $\sqrt{P_s}\text{max\_eig}(\mathbf{H}_{sr}^H\mathbf{H}_{sr})$, while the relay receive and transmit beamformers are the same as 'FD Optimal' and 'FD Suboptimal'. Note that, 'FD Optimal', 'FD Suboptimal' and 'FD Eigenmode' all adopt the asymptotically optimal $\mathbf{f}_r^*$ and $\mathbf{g}_r^*$ in Proposition 1, which may violate the relay power constraint for finite N_t. To deal with this issue, we scale $\mathbf{f}_r^*$ such that the power constraint is satisfied. During the simulations, we set $N_s = N_r = 4$ and $P_s = E_r = 1$. The entries of channels $\mathbf{H}_{sr}$, $\mathbf{h}_{rd}^H$, $\mathbf{H}_{rr}$ and $\mathbf{h}_{sd}$ are generated with i.i.d. zero-mean complex Gaussian variables with variances being 0 dB, 0 dB, γ_{rr} dB and γ_{sd} dB.

The achievable rates of the above listed methods are compared in Fig. 1. From the figure, we find that the proposed two methods attain almost the same rate and actually perform very close to the benchmark scheme 'FD Ideal SIC'. Compared to 'FD Eigenmode' that neglects direct link, both proposed schemes achieve remarkable gains. This is due to the fact that the direct link in the considered full-duplex relaying protocol is regarded as an interfering link and will severely degrade the achievable rate if the source beamformer is not properly designed. Moreover, the proposed methods clearly outperform 'HD Optimal' [19] as exemplified in the figure.

In Fig. 2, we show the achievable rate versus self-interference level γ_{rr}. It can be observed that, when γ_{rr} increases, the performance of 'FD Optimal', 'FD Suboptimal' and 'FD Eigenmode' degrades gradually, but they still perform better than 'HD Optimal'. Here we would like to remark that, although the MRC/MRT relaying strategy used in the three full-duplex transmission schemes can eliminate self-interference when N_t approaches infinity (see Appendix A), there still exists non-zero self-interference

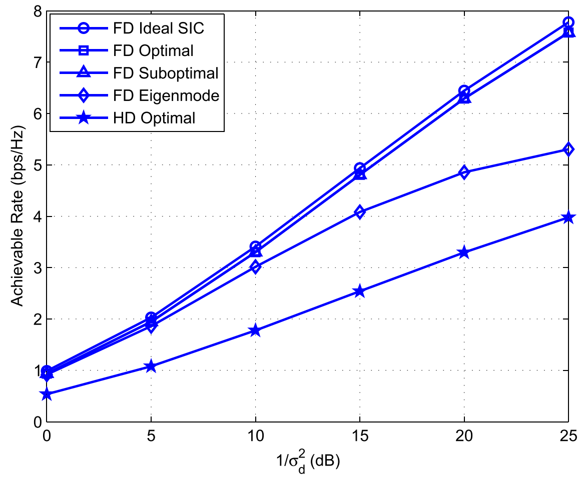

FIGURE 1. **Achievable rate versus $1/\sigma_d^2$ for various transmission schemes ($N_t = 32$, $\sigma_r^2 = -20$ dB, $\gamma_{sd} = -15$ dB, $\gamma_{rr} = -10$ dB).**

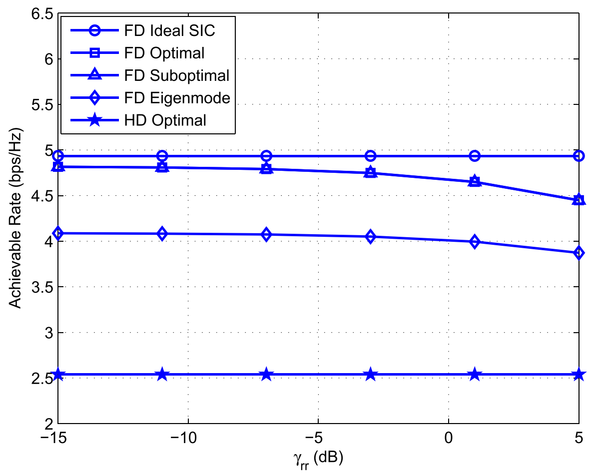

FIGURE 2. **Achievable rate versus γ_{rr} for various transmission schemes ($N_t = 32$, $\sigma_r^2 = -20$ dB, $\sigma_d^2 = -15$ dB, $\gamma_{sd} = -15$ dB).**

for finite N_t, which results in performance degradation especially when γ_{rr} is large.

The rate performance versus direct link gain γ_{sd} is depicted in Fig. 3, where we find that the proposed optimal and suboptimal schemes achieve evident rate gains over 'FD Eigenmode' when γ_{sd} is large, since the latter method does not take into account the direct link. It is also interesting to observe that, unlike the half-duplex counterpart, the performance of the proposed full-duplex relaying designs is not sensitive to the value of γ_{sd}.

We investigate the relationship between the achievable rate and the number of relay transmit antennas in Fig. 4, from which we can see that the rate of each method is enhanced with the increase in N_t. Furthermore, the gap between the proposed full-duplex schemes and the benchmark method with ideal self-interference cancelation becomes more and more negligible when N_t gets larger, which verifies the asymptotical optimality property shown in Proposition 1.

We compare the achievable rate of the proposed optimal and suboptimal transmission schemes in Fig. 5. It can be

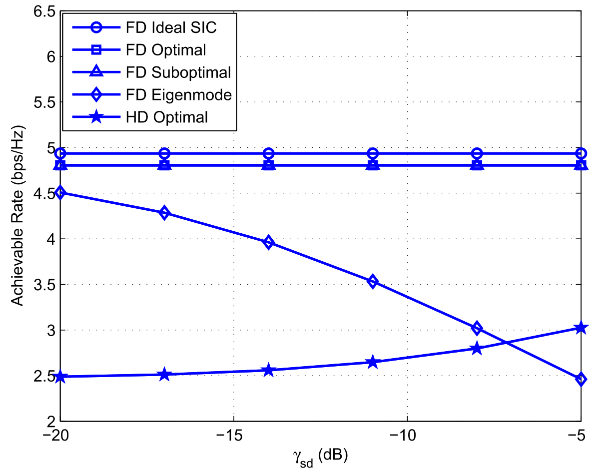

FIGURE 3. **Achievable rate versus γ_{sd} for various transmission schemes ($N_t = 32$, $\sigma_r^2 = -20$ dB, $\sigma_d^2 = -15$ dB, $\gamma_{rr} = -10$ dB).**

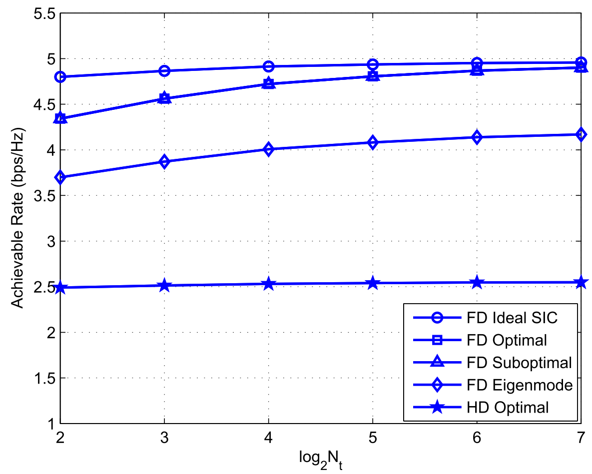

FIGURE 4. **Achievable rate versus $\log_2 N_t$ for various transmission schemes ($\sigma_r^2 = -20$ dB, $\sigma_d^2 = -15$ dB, $\gamma_{rr} = -10$ dB, $\gamma_{sd} = -15$ dB).**

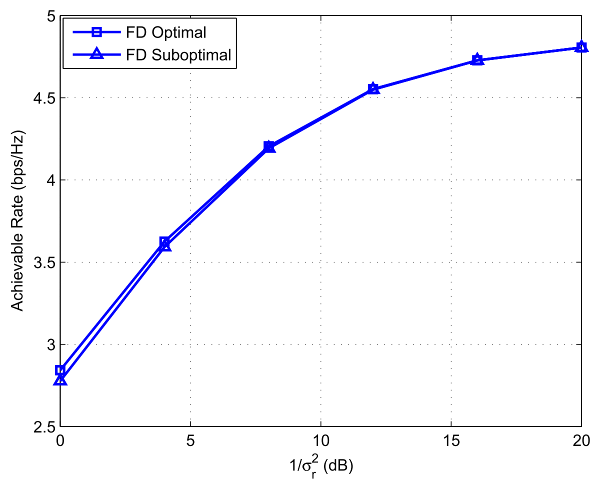

FIGURE 5. **Achievable rate versus $1/\sigma_r^2$ for various transmission schemes ($N_t = 32$, $\sigma_d^2 = -15$ dB, $\gamma_{rr} = -10$ dB, $\gamma_{sd} = -15$ dB).**

found that the suboptimal scheme performs quite close to the optimal one even when σ_r^2 is large, thus validating the effectiveness of the simplification adopted in Section III-C.

V. CONCLUSIONS

We maximized the end-to-end SNR of a full-duplex multi-antenna AF relay system with direct link, which uses a large-scale transmit antenna array with low transmit power at the relay. We first proved that MRC/MRT is an asymptotically optimal relaying strategy. An iterative algorithm was then devised to obtain an optimal source beamformer, which has a semi-closed form that generalizes the conventional channel matching structure. In addition, we also established a suboptimal source beamformer solution in closed form while causing only a negligible performance loss compared to the optimal design.

APPENDIX A
PROOF OF PROPOSITION 1

First, without loss of generality, let us assume that $\|\mathbf{g}_r\| = 1$ at optimality. It is then clear to see that the SNR objective in problem (7) is monotonically increasing with the terms $|\mathbf{h}_{rd}^H\mathbf{f}_r|$ and $|\mathbf{g}_r^H\mathbf{H}_{sr}\mathbf{f}_s|$, and meanwhile decreasing with $|\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r|$. Therefore, the asymptotically optimal solution to $\mathbf{f}_r$ and $\mathbf{g}_r$ must maximize $|\mathbf{h}_{rd}^H\mathbf{f}_r|$ and $|\mathbf{g}_r^H\mathbf{H}_{sr}\mathbf{f}_s|$ and minimize $|\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r|$ when $N_t \to \infty$.

We set $\mathbf{f}_r^* = c_{f_r}\mathbf{h}_{rd}$ and $\mathbf{g}_r^* = \frac{\mathbf{H}_{sr}\mathbf{f}_s}{\|\mathbf{H}_{sr}\mathbf{f}_s\|}$, which maximize $|\mathbf{h}_{rd}^H\mathbf{f}_r|$ and $|\mathbf{g}_r^H\mathbf{H}_{sr}\mathbf{f}_s|$, respectively. Then, let us set the coefficient $c_{f_r} = \sqrt{\frac{E_r}{N_t^2\gamma_{rd}^2(\|\mathbf{H}_{sr}\mathbf{f}_s\|^2+\sigma_r^2)}}$, resulting in $(\mathbf{g}_r^*)^H\mathbf{H}_{rr}\mathbf{f}_r^* = \sqrt{\frac{E_r}{N_t^2\gamma_{rd}^2(\|\mathbf{H}_{sr}\mathbf{f}_s\|^2+\sigma_r^2)}}\frac{(\mathbf{H}_{sr}\mathbf{f}_s)^H}{\|\mathbf{H}_{sr}\mathbf{f}_s\|}\mathbf{H}_{rr}\mathbf{h}_{rd}$. Recall that the entries of $\mathbf{h}_{rd}^H$ and $\mathbf{H}_{rr}$ are i.i.d. zero-mean complex Gaussian variables with variances γ_{rd}^2 and γ_{rr}^2, respectively. Hence, when $N_t \to \infty$, $\frac{\mathbf{H}_{rr}\mathbf{h}_{rd}}{N_t} \to \mathbf{0}$ and accordingly $(\mathbf{g}_r^*)^H\mathbf{H}_{rr}\mathbf{f}_r^* \to 0$, meaning that $(\mathbf{f}_r^*, \mathbf{g}_r^*)$ minimizes $\mathbf{g}_r^H\mathbf{H}_{rr}\mathbf{f}_r$. Furthermore, since $\frac{\|\mathbf{h}_{rd}\|^2}{N_t} \to \gamma_{rd}^2$, we can show that $(\mathbf{f}_r^*, \mathbf{g}_r^*)$ also fulfills the relay power constraint when $N_t \to \infty$. Thereby, $(\mathbf{f}_r^*, \mathbf{g}_r^*)$ is asymptotically optimal.

APPENDIX B
PROOF OF THEOREM 1

According to [24, Th. 2.4], problem (12) admits an optimal solution $\mathbf{f}_s^*$ if and only if there exist $\alpha^* \geq 0$ and $\beta^* \geq 0$ such that the following optimality conditions hold

$$(\mathbf{H}_{sr}^H\mathbf{H}_{sr} - \alpha^*\mathbf{h}_{sd}\mathbf{h}_{sd}^H)\mathbf{f}_s^* = \beta^*\mathbf{f}_s^*, \tag{19a}$$
$$\alpha^*(|\mathbf{h}_{sd}^H\mathbf{f}_s^*|^2 - t) = 0, \tag{19b}$$
$$\beta^*(\|\mathbf{f}_s^*\|^2 - P_s) = 0, \tag{19c}$$
$$\beta^*\mathbf{I} + \alpha^*\mathbf{h}_{sd}\mathbf{h}_{sd}^H - \mathbf{H}_{sr}^H\mathbf{H}_{sr} \succeq \mathbf{0}, \tag{19d}$$
$$|\mathbf{h}_{sd}^H\mathbf{f}_s^*|^2 \leq t, \quad \|\mathbf{f}_s^*\|^2 \leq P_s. \tag{19e}$$

It can be inferred from (19a) that β^* is an eigenvalue of matrix $\mathbf{\Psi} \triangleq \mathbf{H}_{sr}^H\mathbf{H}_{sr} - \alpha^*\mathbf{h}_{sd}\mathbf{h}_{sd}^H$ with $\mathbf{f}_s^*$ being the corresponding eigenvector. Moreover, by defining $\mathbf{z} = \text{max\_eig}(\mathbf{\Psi})$ and invoking (19e), we obtain $\mathbf{z}^H(\beta^*\mathbf{I}-\mathbf{\Psi})\mathbf{z} = \beta^*-\lambda_{\max}(\mathbf{\Psi}) \geq 0$, where $\lambda_{\max}(\mathbf{\Psi})$ denotes the maximum eigenvalue of $\mathbf{\Psi}$. Consequently, we conclude that $\beta^* = \lambda_{\max}(\mathbf{\Psi})$ and $\mathbf{f}_s^*$ is a scaled version of $\mathbf{z}$. To verify the expression of $\mathbf{f}_s^*$ in (13), we need to further show that the constraint $\|\mathbf{f}_s\|^2 \leq P_s$ must be active at optimality. Suppose that $\|\mathbf{f}_s^*\|^2 < P_s$. Then, $\beta^* = 0$ holds from (19c). Based on the fact that β^* is the largest eigenvalue of $\mathbf{\Psi}$, $\mathbf{\Psi}$ should be negative semidefinite. However, this cannot be true because we can always construct a vector $\mathbf{v} = \left(\mathbf{I} - \frac{\mathbf{h}_{sd}\mathbf{h}_{sd}^H}{\|\mathbf{h}_{sd}\|^2}\right)\mathbf{w} \neq \mathbf{0}$ such that $\mathbf{v}^H\mathbf{\Psi}\mathbf{v} = \mathbf{w}^H\left(\mathbf{I} - \frac{\mathbf{h}_{sd}\mathbf{h}_{sd}^H}{\|\mathbf{h}_{sd}\|^2}\right)\mathbf{\Psi}\left(\mathbf{I} - \frac{\mathbf{h}_{sd}\mathbf{h}_{sd}^H}{\|\mathbf{h}_{sd}\|^2}\right)\mathbf{w} > 0$.[2] Therefore, $\|\mathbf{f}_s^*\|^2 = P_s$ and (13) holds.

We now determine the value for α^*. Define $\rho(\alpha) = P_s|\mathbf{h}_{sd}^H\text{max\_eig}(\mathbf{H}_{sr}^H\mathbf{H}_{sr} - \alpha\mathbf{h}_{sd}\mathbf{h}_{sd}^H)|^2$ and let $\theta(\alpha)$ denote the maximum eigenvalue of $\mathbf{H}_{sr}^H\mathbf{H}_{sr} - \alpha\mathbf{h}_{sd}\mathbf{h}_{sd}^H$. Then, it follows from [25] that $\frac{d\theta(\alpha)}{d\alpha} = -|\mathbf{h}_{sd}^H\text{max\_eig}(\mathbf{H}_{sr}^H\mathbf{H}_{sr} - \alpha\mathbf{h}_{sd}\mathbf{h}_{sd}^H)|^2 = -\frac{\rho(\alpha)}{P_s}$ and accordingly $\frac{d^2\theta(\alpha)}{d^2\alpha} = -\frac{1}{P_s}\frac{d\rho(\alpha)}{d\alpha}$. Since $\lambda_{\max}(\mathbf{X})$ is convex for any Hermitian matrix $\mathbf{X}$ [26], $\frac{d^2\theta(\alpha)}{d^2\alpha} \geq 0$ must hold and hence $\frac{d\rho(\alpha)}{d\alpha} \leq 0$. Consider the case where $t > P_s|\mathbf{h}_{sd}^H\text{max\_eig}(\mathbf{H}_{sr}^H\mathbf{H}_{sr})|^2 = \rho(0)$. It is deduced from (13) and the monotonically decreasing property of $\rho(\alpha)$ that $|\mathbf{h}_{sd}^H\mathbf{f}_s^*|^2 = \rho(\alpha^*) \leq \rho(0) < t$. Thus, based on (19b), we have $\alpha^* = 0$. For the case where $t \leq \rho(0)$, let us assume that $|\mathbf{h}_{sd}^H\mathbf{f}_s^*|^2 = \rho(\alpha^*) < t$. Then, (19b) indicates that $\alpha^* = 0$ which contradicts the condition $t \leq \rho(0)$. Hence, $\rho(\alpha^*) = t$ and we can find α^* via bisection method owing to the decreasing monotonicity of $\rho(\alpha)$.

APPENDIX C
PROOF OF PROPOSITION 2

Let us perform SDR on the QCQP problem in (12), which, according to [23], yields an equivalent SDP problem as

$$\begin{aligned}&\underset{\mathbf{Q}_s\succeq 0}{\text{minimize}} \quad -\text{tr}(\mathbf{H}_{sr}^H\mathbf{H}_{sr}\mathbf{Q}_s)\\ &\text{subject to } t - \text{tr}(\mathbf{h}_{sd}\mathbf{h}_{sd}^H\mathbf{Q}_s) \geq 0, \quad P_s - \text{tr}(\mathbf{Q}_s) \geq 0.\end{aligned} \tag{20}$$

Then, by invoking [27, Corollary 2.1], we arrive at the conclusion that the optimal objective value of the above problem is a convex function of the parameter t. Therefore, the optimal objective value of problem (12), i.e., $f(t)$, is concave.

To show the quasiconvexity of problem (15), we rewrite it as

$$\begin{aligned}&\underset{t}{\text{minimize}} \quad \frac{(f(t)+\sigma_r^2)(t+\sigma_d^2) + E_r\gamma_{rd}^2\sigma_r^2}{f(t)}\\ &\text{subject to } 0 \leq t \leq P_s|\mathbf{h}_{sd}^H\text{max\_eig}(\mathbf{H}_{sr}^H\mathbf{H}_{sr})|^2.\end{aligned} \tag{21}$$

Denote the above objective by $g(t)$. Then, we calculate the first-order and second-order derivatives of $g(t)$, which are given by $g'(t) = -\frac{(\sigma_r^2 t+\sigma_r^2\sigma_d^2+E_r\gamma_{rd}^2\sigma_r^2)f'(t)}{f^2(t)} + \frac{\sigma_r^2}{f(t)} + 1$ and $g''(t) = \frac{2(\sigma_r^2 t+\sigma_r^2\sigma_d^2+E_r\gamma_{rd}^2\sigma_r^2)(f'(t))^2-2\sigma_r^2 f(t)f'(t)}{f^3(t)} - \frac{(\sigma_r^2 t+\sigma_r^2\sigma_d^2+E_r\gamma_{rd}^2\sigma_r^2)f''(t)}{f^2(t)}$, respectively. Assuming that $g'(t_0) = 0$, we accordingly have

$$(\sigma_r^2 t + \sigma_r^2\sigma_d^2 + E_r\gamma_{rd}^2\sigma_r^2)f'(t_0) = f^2(t_0) + \sigma_r^2 f(t_0). \tag{22}$$

[2] We implicitly assume that $\mathbf{h}_{sd}^H$ is linearly independent of at least one row of $\mathbf{H}_{sr}$, which is a reasonable assumption for random wireless channels.

Furthermore, by substituting (22) into $g''(t_0)$, we obtain

$$g''(t_0) = \frac{2f'(t_0)}{f(t_0)} - \frac{(\sigma_r^2 t + \sigma_r^2 \sigma_d^2 + E_r \gamma_{rd}^2 \sigma_r^2) f''(t_0)}{f^2(t_0)}. \quad (23)$$

Since $f(t_0) \geq 0$, it follows from (22) that $f'(t_0) \geq 0$. Moreover, based on the fact that $f(t)$ is concave, we deduce that $f''(t_0) \leq 0$ holds. Consequently, we know that $g''(t_0) \geq 0$ from (23). According to the second-order conditions for quasiconvexity [26, Sec. 3.4.3], $g(t)$ is a quasiconvex function and thus problem (21) (or problem (15)) is also quasiconvex.

APPENDIX D
PROOF OF PROPOSITION 3

It can be readily confirmed via contradiction that the constraint must be activated at optimality. Then, according to [28], we arrive at the conclusion that the optimal $\mathbf{v}^*$ is the dominant generalized eigenvector of matrix pencil $\left(\mathbf{P}_{\mathbf{h}_{sd}} \mathbf{H}_{sr}^H \mathbf{H}_{sr} \mathbf{P}_{\mathbf{h}_{sd}}, \mathbf{P}_{\mathbf{h}_{sd}}\right)$. Now we are ready to verify (18). Suppose that $\mathbf{v}_i$ and μ_i are the i-th eigenvector and eigenvalue of matrix $\mathbf{P}_{\mathbf{h}_{sd}} \mathbf{H}_{sr}^H \mathbf{H}_{sr} \mathbf{P}_{\mathbf{h}_{sd}}$, i.e., $\mathbf{P}_{\mathbf{h}_{sd}} \mathbf{H}_{sr}^H \mathbf{H}_{sr} \mathbf{P}_{\mathbf{h}_{sd}} \mathbf{v}_i = \mu_i \mathbf{v}_i$. Then, we have $\mathbf{P}_{\mathbf{h}_{sd}} \mathbf{P}_{\mathbf{h}_{sd}} \mathbf{H}_{sr}^H \mathbf{H}_{sr} \mathbf{P}_{\mathbf{h}_{sd}} \mathbf{v}_i \overset{(a)}{=} \mathbf{P}_{\mathbf{h}_{sd}} \mathbf{H}_{sr}^H \mathbf{H}_{sr} \mathbf{P}_{\mathbf{h}_{sd}} \mathbf{v}_i = \mu_i \mathbf{P}_{\mathbf{h}_{sd}} \mathbf{v}_i$, where (a) is due to the idempotent property of the projection matrix $\mathbf{P}_{\mathbf{h}_{sd}}$. Hence, $\mathbf{v}_i$ and μ_i are also the i-th generalized eigenvector and eigenvalue of matrix pencil $(\mathbf{P}_{\mathbf{h}_{sd}} \mathbf{H}_{sr}^H \mathbf{H}_{sr} \mathbf{P}_{\mathbf{h}_{sd}}, \mathbf{P}_{\mathbf{h}_{sd}})$. Therefore, by further taking into account the equality $(\mathbf{v}^*)^H \mathbf{P}_{\mathbf{h}_{sd}} \mathbf{v}^* = P_s$, we obtain (18). Note that we cannot simply apply Rayleigh quotient [28] to achieve (18) since the constraint of problem (17) is not $\|\mathbf{v}\|^2 \leq P_s$.

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