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# Facial Emotion Recognition Based on Biorthogonal Wavelet Entropy, Fuzzy Support Vector Machine, and Stratified Cross Validation

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**ABSTRACT** Emotion recognition represents the position and motion of facial muscles. It contributes significantly in many fields. Current approaches have not obtained good results. This paper aimed to propose a new emotion recognition system based on facial expression images. We enrolled 20 subjects and let each subject pose seven different emotions: happy, sadness, surprise, anger, disgust, fear, and neutral. Afterward, we employed biorthogonal wavelet entropy to extract multiscale features, and used fuzzy multiclass support vector machine to be the classifier. The stratified cross validation was employed as a strict validation model. The statistical analysis showed our method achieved an overall accuracy of  $96.77 \pm 0.10\%$ . Besides, our method is superior to three state-of-the-art methods. In all, this proposed method is efficient.

**INDEX TERMS** Facial emotion recognition, facial expression, biorthogonal wavelet entropy, support vector machine, fuzzy logic.

## I. INTRODUCTION

Emotion recognition (ER) [1] studies how to identify human emotions, typically, the sources are facial expressions, and we call it as facial emotion recognition (FER) [2]. Academically, the facial expression denotes motions and positions of facial muscles beneath skin of the face. Those movements are a form of nonverbal communication, and convey the emotional status to an observer. The research of FER benefits a massive of fields, for example, the in-car conversational interface [3], EEG signal [4], the theatre performance [5], speech emotion recognition [6], bipolar disorder [7], adolescents with disabilities [8], depressive symptom [9], Parkinson's disease [10], etc.

There are various studies using various methods to solve the FER problems. Drume and Jalal [11] presented

a two-level classification approach, i.e., principal component analysis (PCA) at level 1 was boosted by support vector machine (SVM) at level 2. Vivek and Guddeti [12] combined cat swarm optimization (CSO), genetic algorithm (GA), and particle swarm optimization (PSO). This hybrid bio-inspired algorithm was in conjunction with SVM. Ali *et al.* [13] used radon transform (RT), higher-order spectral (HOS), and SVM. Takehara *et al.* [14] proposed to use a small-world network model. Alhussein [15] used multi-scale Weber local descriptor (MS-WLD) and SVM.

Nevertheless, the performances of above methods are not satisfying. Besides, their method cannot handle outliers and noises. In this study, our contribution is to propose a new FER system, which uses biorthogonal wavelet entropy as the feature detector. It combines the advantages of both

biorthogonal wavelet transform and Shannon entropy. Furthermore, a powerful variant of SVM, fuzzy SVM, was introduced to improve the performance of conventional SVM.

The structure of the remainder is organized as follows: Section II gives the materials used in this study, and presents the preprocessing method. Section III provides the methodology on feature extraction. Section IV describes the classification method. Section V contains the results and discussions. Finally, Section VI concludes the paper.

## II. SUBJECTS

We enrolled 20 subjects with ages from twenty to thirty-five by online advertisements. Each subject was requested to lay seven facial expressions (happy, sadness, surprise, anger, disgust, fear, and neutral). We photographed each facial expression of each subject ten times by Canon digital camera, and finally picked up five photos approved by three experienced psychologists. In the final, we have 700 images in total.

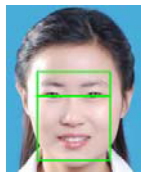


FIGURE 1. Face detection from a female face model.

Face detection method was used based on the face model shown in Figure 1. Suppose the distance between two centers of eyes be  $D$ , we have the geometric face model rules [16] as follows: (i) Width of the mouse is  $D$ . (ii) Width of nose is  $0.8D$ . (iii) Vertical distance between eyes and center of mouth is  $D$ . (iv) Vertical distance between eyes and center of nostril is  $0.6D$ . (v) Vertical distance between eyes and eyebrows is  $0.4D$ . To cover the whole face, we extend  $0.4D$  in both upper and lower margin, and  $0.4D$  in both left and right margins. Finally, we get a photo image with size of width of  $1.8D$  and height of  $2.2D$ . Figure 2 shows the seven emotional expressions of both a male and a female faces.

## III. FEATURE EXTRACTION

### A. DISCRETE WAVELET TRANSFORM

Signal processing has gone through three important generations as shown in Figure 3. For the first generation, Fourier transform (FT) decomposes the signal into its frequencies (as the music chord). Then, the second generation analyze local sections of signal, and the famous technique is short time FT (ST-FT). In the third generation, the wavelet transform (WT) used a variable-size window to analyze the signal in multi-scales [17]. In the past decade, the discrete WT (DWT) has been proven as one of the most efficient signal processing techniques, and applied in various fields, such as fault detection [18], JPEG 2000 [19], dendrite spine recognition [20], voltage signal feature extraction [21], QRS detection [22], etc.

Suppose we have a continuous signal  $x(t)$ , we then sample it at initial time  $t_0$  with interval  $\Delta t$ , and thus we get the

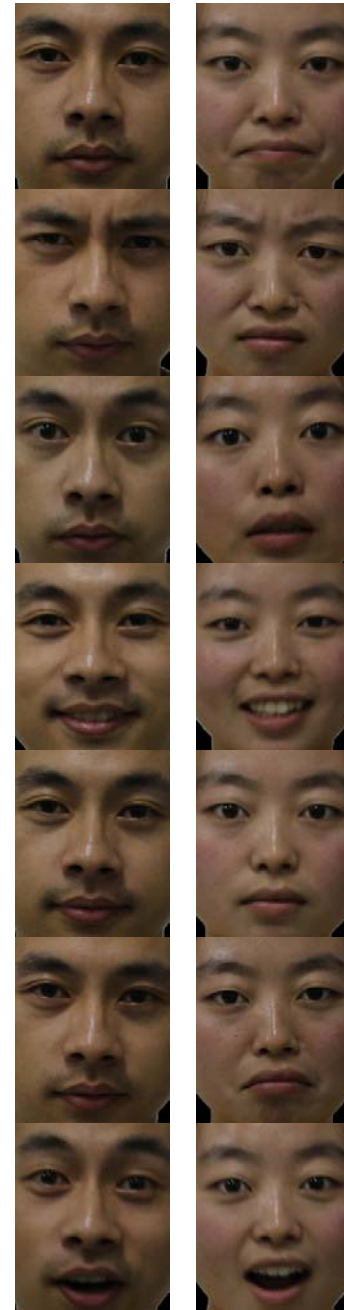


FIGURE 2. Samples of the dataset (The first column gives the seven emotion expressions of a male face, and the second column gives the seven emotion expressions of a female face).

discrete version of the continuous signal as

$$x(n) = x(t_0 + n\Delta t) \tag{1}$$

where  $n$  is in the range of integer set  $(0, 1, \dots, N - 2, N - 1)$ . To get the DWT results, we need to pass  $x(n)$  through a series of both low-pass filter  $l$  and high-pass filter  $k$ , and then downsampled by 2:

$$C_l(n) = \sum_{m=-\infty}^{+\infty} x(m) \times l(2n - m) \tag{2}$$

$$C_k(n) = \sum_{m=-\infty}^{+\infty} x(m) \times k(2n - m) \tag{3}$$

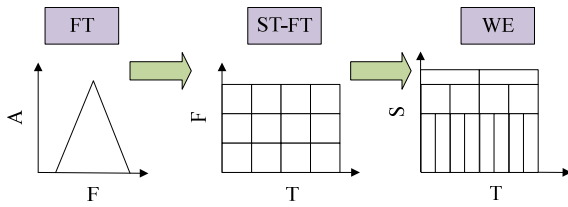


FIGURE 3. Three generations of signal processing.

Here  $m$  is a temporary variable. The coefficients  $C_l$  and  $C_k$  are also known as the approximation coefficient and detail coefficient. One level decomposition is finished till now. For multi-level decomposition, the approximation coefficient is decomposed further into its corresponding approximation and detail coefficient, as shown in Figure 4.

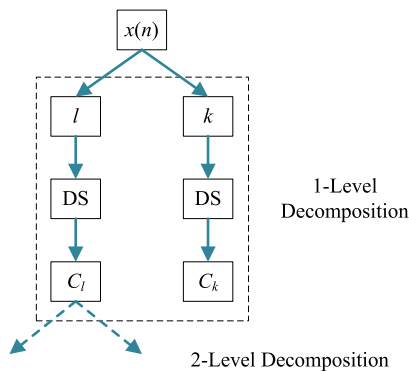


FIGURE 4. Illustration of DWT (DS represents a 2x downsampling operation).

Note that for 2D image processing, we do not use  $C_l$  and  $C_k$  to represent the coefficient subbands of each decomposition. Instead, we have four coefficient subbands at each level, since the low-pass filter and high-pass filter will be used at both horizontal and vertical directions. Following standard convention, we used  $C_a, C_h, C_v,$  and  $C_d$  to denote these four coefficient subbands.

**B. BIORTHOGONAL WAVELET ENTROPY**

Two shortcomings exist for using DWT to extract features from face expression images. First, the traditional wavelet suffers from complicated calculation. Second, the DWT may yield too many features, which will hurdle the following classification.

To solve above two issues, we proposed two improvements. In the first improvement, we revisit the development of wavelet family as shown in Figure 5. We observe that wavelet transform was replaced with orthogonal wavelet transform (abbreviated as OWT), and OWT was recently replaced with biorthogonal wavelet transform (abbreviated as BWT).

Two advantages exist for OWT: (i) its associated wavelet transform is orthogonal [23], i.e., its inverse wavelet transform is the adjoint of the wavelet transform. (ii) OWT can be defined merely on the basis of the scaling filter [24] (i.e., the low-pass filter). BWT inherits the first

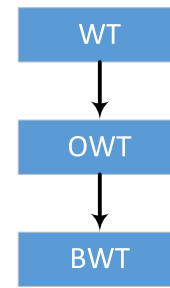


FIGURE 5. Development from WT to OWT to BWT (WT = wavelet transform, OWT = orthogonal wavelet transform; BWT = biorthogonal wavelet transform).

advantage of OWT [25], but its definition is based on both low-pass filter and high-pass filter. It has another advantage that it allows more degrees of freedom than OWT [26].

In the second improvement, we combined the Shannon entropy [27], [28] with BWT, and thus proposed a novel multiscale feature extraction technique named as biorthogonal wavelet entropy (BWE). The Shannon entropy was performed over each coefficient subband, which is regarded as a gray-level image. The Shannon entropy is implemented with following equation:

$$N(Y) = - \sum_{i=1}^L P(y_i) \log P(y_i) \tag{4}$$

Here  $Y$  is a coefficient subband.  $y_i$  is the  $i$ -th gray-level of  $Y$ .  $P$  represent the probability density function.  $L$  is the total number of gray-values.  $N$  is the entropy operation.

Figure 6 shows the block diagram of calculating a 2-level BWE. In the first step, a facial expression image was imported. Next, we performed 1-level BWT and obtained four coefficient subbands, i.e.,  $C_a(1), C_h(1), C_v(1),$  and  $C_d(1)$ . Here ( $j$ ) represents the  $j$ -level decomposition. In the third step, we performed a 2-level BWT, that means, we decomposed the  $C_a(1)$  into four higher-level subbands of  $C_a(2), C_h(2), C_v(2)$  and  $C_d(2)$ . Finally, the Shannon entropy in equation (4) was implemented over these seven coefficient subbands.

**IV. CLASSIFICATION**

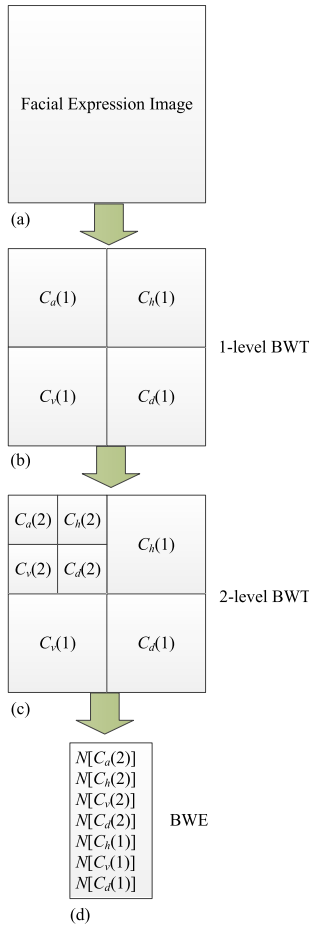
**A. SUPPORT VECTOR MACHINE**

In the field of supervised learning, Support vector machine (SVM) is one of the most influential approaches. It has been successfully applied into identify brains [29], spatio-temporal activity [30], Alzheimer’s disease [31], online review [32], multiple sclerosis [33], etc. Suppose we have two classes of +1 and -1, and the dataset is

$$\{(p_n, t_n) | p_n \in \mathbb{R}^d\}, \quad n = 1, \dots, N \tag{5}$$

where  $N$  represents the number of samples,  $d$  is the dimension of input features,  $p_n$  is the feature vector of  $n$ -th sample data, and  $t_n$  is its corresponding target label.

$$t_n = \begin{cases} +1 & \text{class} = +1 \\ -1 & \text{class} = -1 \end{cases} \tag{6}$$



**FIGURE 6. Block Diagram of a 2-level BWE: (a) A facial expression image; (b) 1-level BWT; (c) 2-level BWT; (d) Shannon entropy calculation. In this figure, (j) represents the j-level decomposition.**

The aim of SVM is to create a hyperplane with dimension of  $(d-1)$ , which can separate the two classes. The model of SVM is similar to logistic regression, since both are driven by a linear function

$$\mathbf{w}^T p - b = 0 \tag{7}$$

which is also the form of the hyperplane to be created.  $\mathbf{w}$  and  $b$  have the similar physical meanings as in logistic function.  $\mathbf{w}$  means the weights and  $b$  means the bias. The mathematical solution of obtaining the  $\mathbf{w}$  and  $b$  are as follows:

$$\begin{aligned} \min_{\mathbf{w}, b} & \frac{1}{2} \|\mathbf{w}\|^2 \\ \text{s.t. } & t_n (\mathbf{w}^T p_n - b) \geq 1, \quad n = 1, \dots, N \end{aligned} \tag{8}$$

If the input features contain noises, we can use the “soft margin” technique. After adding the positive slack vector  $\rho = (\rho_1, \dots, \rho_n, \dots, \rho_N)$ , equation (8) is changed to:

$$\begin{aligned} \min_{\mathbf{w}, \rho, b} & \frac{1}{2} \|\mathbf{w}\|^2 + Le^T \rho \\ \text{s.t. } & \begin{cases} t_n (\mathbf{w}^T p_n - b) \geq 1 - \rho_n \\ \rho_n \geq 0 \end{cases}, \quad n = 1, \dots, N \end{aligned} \tag{9}$$

where  $e$  is a vector of ones of  $N$ -dimension, and  $L$  is the error penalty.

The constraint optimization problem in equation (9) is solved by the “Lagrange multiplier” technique as:

$$\begin{aligned} \min_{\mathbf{w}, \rho, b} \max_{\varphi, \gamma} & \left\{ \frac{1}{2} \|\mathbf{w}\|^2 + Le^T \rho \sum_{n=1}^N \varphi_n \left[ t_n (\mathbf{w}^T p_n - b) + \rho_n - 1 \right] \right. \\ & \left. - \sum_{n=1}^N \rho_n \gamma_n \right\} \end{aligned} \tag{10}$$

The *dual form* technique is used to solve equation (10), since the min-max problem is difficult to solve. In this way, we have:

$$\begin{aligned} \max_{\varphi} & \sum_{n=1}^N \varphi_n - \frac{1}{2} \sum_{n=1}^N \sum_{m=1}^N \varphi_m \varphi_n t_m t_n p_m^T p_n \\ \text{s.t. } & \begin{cases} L \geq \varphi_n \geq 0 \\ \sum_{n=1}^N t_n \varphi_n = 0 \end{cases}, \quad n = 1, \dots, N \end{aligned} \tag{11}$$

**B. MULTICLASS SVM**

Traditionally, SVMs were developed for two-class problem [34]. In this study, we need to handle a seven-class classification problem: happy, sadness, surprise, anger, disgust, fear, and neutral. Hence, we used the winner-takes-all (WTA) technique [35] to break the 7-class task into multiple 2-class tasks [36].

That means, we need to create 7 individual SVMs, each one was trained to detect one class from the remaining classes. For example, the first SVM will judge whether a new sample is Class 1 or not Class 1, the second SVM judges a new sample is Class 2 or not Class 2, etc. We label the seven individual SVMs as “1v1”, “2v2”, “3v3”, “4v4”, “5v5”, “6v6”, and “7v7”, respectively. Here “*ivi*” represents the corresponding individual SVM detect Class  $i$  from other classes (not Class  $i$ ).

A score  $S_k$  is associated with the output of  $k$ -th individual SVM “ $k$ v ~~$k$~~ ”. The class label set  $\mathcal{K}$  is defined as

$$\mathcal{K} = [1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7] \tag{12}$$

Finally, the final output of the combined classifier is

$$O(p) = \arg \max_{k \in \mathcal{K}} S_k(p) \tag{13}$$

Figure 7 shows the diagram of 7-class SVM using WTA. Here assume the score of each individual classifier is  $-0.2, 0.6, 0.5, -0.3, -0.8, 0.9$ , and  $-0.7$ , respectively. The argmax function outputs 6, i.e., the new facial expression image is identified as Class 6.

**C. FUZZY MEMBERSHIP FUNCTION**

The fuzzy logic [37] is introduced here, namely, we applies the fuzzy membership function (FMF) [38] to each training data. The original training data contains the input and target, and the fuzzy training data contains the input, the target,

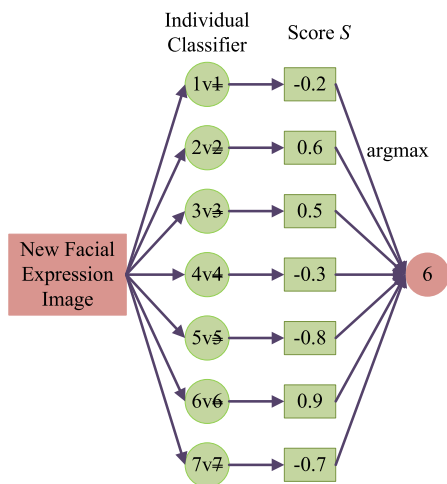


FIGURE 7. Diagram of a 7-class SVM using WTA technique.

and the fuzzy altitude. Mathematically, equation (5) is transformed to

$$\begin{cases} (p_n, t_n, a_{n,k}) | p_n \in \mathbb{R}^d \\ 0 < a_{n,k} \leq 1 \end{cases} \quad k = 1, 2, \dots, K, \quad n = 1, \dots, N \quad (14)$$

where  $a_{n,k}$  represents the fuzzy altitude [39] of sample  $p_n$  towards class  $t_n$  when training  $k$ -th individual classifier. Remember the two classes of  $k$ -th individual are defined as either  $+k$  or  $-k$ , viz., we have seven different fuzzy altitudes for each training data.

Suppose the mean of class  $+k$  is  $m_{+k}$ , and the mean of class  $-k$  is  $m_{-k}$ . Mathematically,

$$m_{+k} = \text{mean}_n (p_n | t_n = +k) \quad (16)$$

$$m_{-k} = \text{mean}_n (p_n | t_n = -k) \quad (17)$$

We can deduce the radius of Class  $+k$  and Class  $-k$  as:

$$r_{+k} = \max_{\{p_n: t_n = +k\}} |m_{+k} - p_n| \quad (18)$$

$$r_{-k} = \max_{\{p_n: t_n = -k\}} |m_{-k} - p_n| \quad (19)$$

The FMF is defined based on the means and radii of both classes

$$a_{n,k} = \begin{cases} 1 - |m_{+k} - p_n| / (r_{+k} + \nu) & t_n = +k \\ 1 - |m_{-k} - p_n| / (r_{-k} + \nu) & t_n = -k \end{cases} \quad (20)$$

Here  $\nu$  is a positive parameter, so as to guarantee the fuzzy altitude is greater than zero.

#### D. FUZZY MULTICLASS SVM

After definition of FMF in Section IV-C, we can arrive at the mathematical form of the individual FSVM. When  $k$ -th individual classifier is an FSVM, the two classes of processed dataset are defined as either  $+k$  or  $-k$ , assume vector  $\mathbf{a}$  is the membership vector of all data

$$a_k = (a_{1,k}, a_{2,k}, \dots, a_{n,k}, \dots, a_{N,k}) \quad (21)$$

We can draw the hyperplane of individual FSVM [40] as

$$\begin{aligned} \min_{\mathbf{w}, \rho, b} & \frac{1}{2} \|\mathbf{w}\|^2 + La_k^T \rho \\ \text{s.t.} & \begin{cases} t_n (\mathbf{w}^T p_n - b) \geq 1 - \rho_n \\ \rho_n \geq 0 \end{cases}, \quad n = 1, \dots, N \end{aligned} \quad (22)$$

A smaller  $a_{n,k}$  decreases the influence of the slack vector  $\rho_n$ , such that the corresponding sample  $p_n$  is regarded less substantial [41]. The Lagrangian [42] is obtained in a similar way as

$$\begin{aligned} \min_{\mathbf{w}, \rho, b} \max_{\varphi, \gamma} & \left\{ \frac{1}{2} \|\mathbf{w}\|^2 + La_k^T \rho - \sum_{n=1}^N \varphi_n [t_n (\mathbf{w}^T p_n - b) + \rho_n - 1] \right. \\ & \left. - \sum_{n=1}^N \rho_n \gamma_n \right\} \end{aligned} \quad (23)$$

The dual form is obtained as

$$\begin{aligned} \max_{\varphi} & \sum_{n=1}^N \varphi_n - \frac{1}{2} \sum_{n=1}^N \sum_{m=1}^N \varphi_m \varphi_n t_m t_n p_m^T p_n \\ \text{s.t.} & \begin{cases} 0 \leq \varphi_n \leq a_{n,k} L \\ \sum_{n=1}^N t_n \varphi_n = 0 \end{cases}, \quad n = 1, \dots, N \end{aligned} \quad (24)$$

#### E. MODEL VALIDATION

A 10-fold stratified cross validation was used to validate our model. Since the 700-image model contains 100 image for each emotion, the stratification splits the folds in the way that each fold contains the same distribution of emotion images as shown in Table 1.

TABLE 1. Stratification in cross validation.

Fold	Anger	Disgust	Fear	Happy	Neutral	Sadness	Surprise
I	10	10	10	10	10	10	10
II	10	10	10	10	10	10	10
III	10	10	10	10	10	10	10
IV	10	10	10	10	10	10	10
V	10	10	10	10	10	10	10
VI	10	10	10	10	10	10	10
VII	10	10	10	10	10	10	10
VIII	10	10	10	10	10	10	10
IX	10	10	10	10	10	10	10
X	10	10	10	10	10	10	10

Although SVM has closed-form solution, we used iterative gradient-based learning to train its weights and biases. This can bring us an advantage, that the training shall be stopped if the error on validation set increases. Figure 8 shows the setting of 10-fold cross validation, where eight folds are employed for training, one for validation, and the rest for test. The confusion matrix of 10 trials were summed together, and then we reported the performance of current run. An ideal



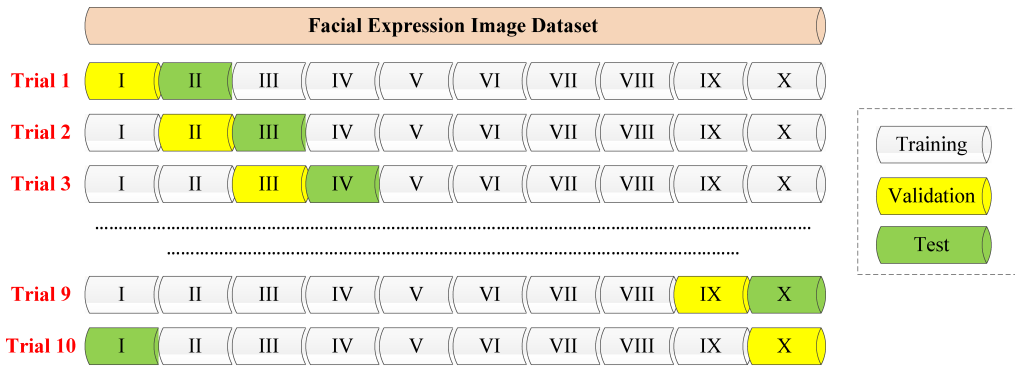


FIGURE 8. A run of 10-fold cross validation.

cost matrix  $\mathbb{C}$  is listed below:

$$\mathbb{C} = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix} \quad (25)$$

which means all classes are identified correctly for the seven classes if there is a perfect classifier. In real condition, the classifier shall make mistakes, hence, we measure the classifier by the sensitivity (SEN) of each class. Suppose  $\mathbb{C} = [C_{ij}]$ ,  $i \in [1, 2, \dots, 7], j \in [1, 2, \dots, 7]$ , we defined the sensitivity of Class  $k$  as

$$SEN(k) = \frac{C_{kk}}{\sum_j C_{kj}} \quad (26)$$

Another important measure is the overall accuracy (OA) which is defined as

$$OA = \frac{\sum_i C_{ii}}{\sum_j \sum_i C_{ij}} \quad (27)$$

To further remove the randomness, we carried out the 10-fold stratified cross validation 10 runs. Each run, the fold segmentation was performed independently. In this way, we can report the mean and standard deviation of  $SEN(k)$  and OA.

## V. RESULTS AND DISCUSSIONS

### A. DWT VERSUS BWT

We used the center image in the first row in Figure 2 as an illustration. Figure 9(a-b) presents its 1-level, and 2-level DWT decomposition, respectively. For fair comparison, we used BWT decomposition over the same image, and the corresponding results are shown in Figure 10.

As we can observe from Figure 9 and Figure 10, both DWT and BWT can analyze original facial expression image in multi-scales, as the decomposition level increases. The higher the decomposition level is, the more detailed information

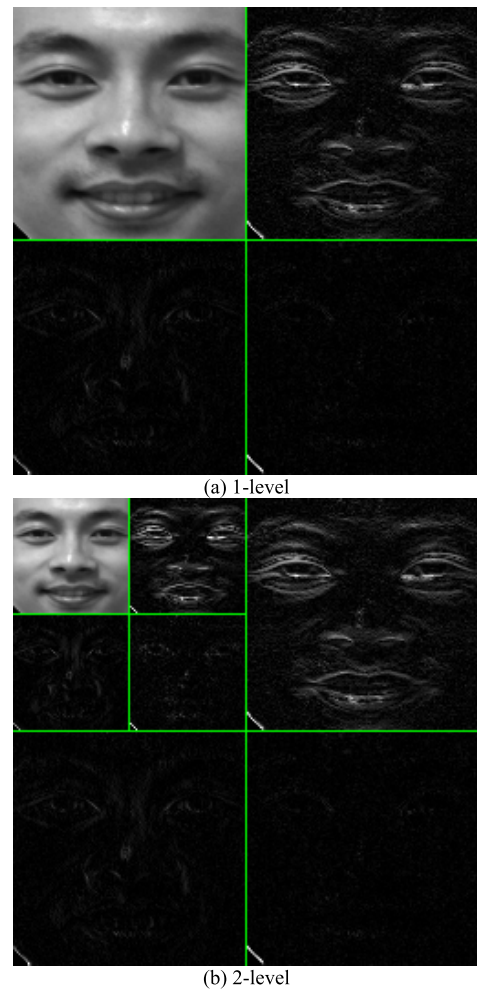


FIGURE 9. DWT result. (a) 1-level. (b) 2-level.

can be obtained. Nevertheless, the BWT subbands are more sparse than DWT, as shown in the  $C_h(1)$  and  $C_h(2)$  subbands, i.e., the 1-level and 2-level horizontal decomposition subbands. Other famous feature extraction techniques, such as Curve-like structure [43], [44], image moment [45] will be used in the future.

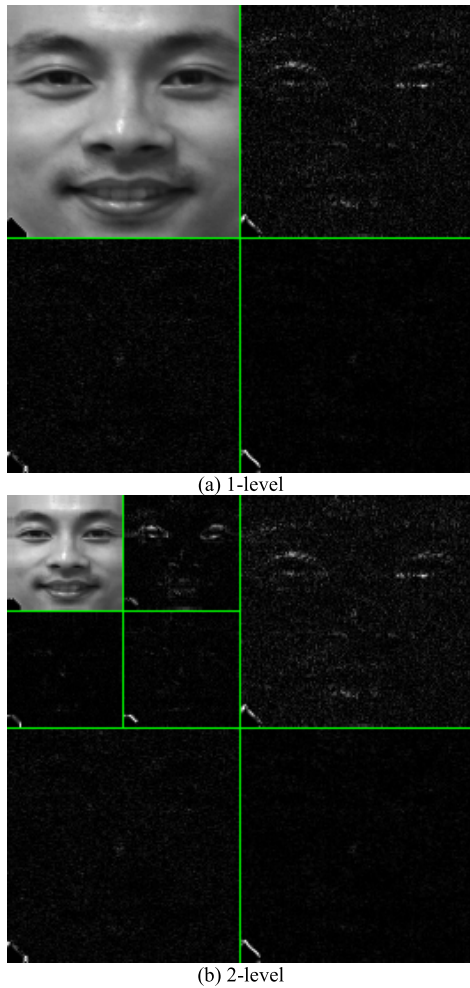


FIGURE 10. Biorthogonal wavelet transform result of the same image in Figure 9. (a) 1-level. (b) 2-level.

**B. STATISTICAL ANALYSIS**

The 10-fold stratified cross validation results are offered in Table 2. Here the sensitivities over anger, disgust, fear, happy, neutral, sadness, and surprise are  $98.00 \pm 1.08\%$ ,  $95.60 \pm 0.97\%$ ,  $96.10 \pm 0.99\%$ ,  $98.50 \pm 0.85\%$ ,  $97.30 \pm 0.70\%$ ,  $96.60 \pm 1.33\%$ , and  $95.30 \pm 1.64$ , respectively.

TABLE 2. Statistical analysis on the sensitivities of each class.

	Anger	Disgust	Fear	Happy	Neutral	Sadness	Surprise
Run 1	100.00	96.00	97.00	99.00	97.00	98.00	94.00
Run 2	99.00	95.00	95.00	99.00	97.00	97.00	97.00
Run 3	99.00	98.00	97.00	99.00	98.00	98.00	94.00
Run 4	98.00	96.00	96.00	97.00	97.00	99.00	97.00
Run 5	98.00	96.00	95.00	99.00	98.00	97.00	97.00
Run 6	97.00	98.00	97.00	98.00	97.00	97.00	96.00
Run 7	99.00	97.00	95.00	99.00	97.00	96.00	94.00
Run 8	100.00	96.00	97.00	99.00	99.00	94.00	93.00
Run 9	98.00	96.00	97.00	99.00	98.00	97.00	94.00
Run 10	97.00	97.00	95.00	97.00	98.00	97.00	97.00
Average	$98.00 \pm 1.08$	$95.60 \pm 0.97$	$96.10 \pm 0.99$	$98.50 \pm 0.85$	$97.30 \pm 0.70$	$96.60 \pm 1.33$	$95.30 \pm 1.64$

The results in Table 2 showed the sensitivities of Anger and Happy reach as high as  $98.00 \pm 1.08\%$  and  $98.50 \pm 0.85\%$ .

The facial emotion with third largest sensitivity is Neutral with average value of  $97.30 \pm 0.70$ . This result is in line with our initial guess.

First, the Anger facial expression is an intense emotional response when the person thought his/her personal boundaries are violated. Those people usually take following gestures: intense stare with eyes wide open, make sounds, bare the teeth, attempt to physically seem larger, etc. The staring with eyes wide open is a good indicator for both human and computers to identify Anger with other facial emotions. Indeed, there are other facial factors, such as V-shape eyebrows, wrinkled nose, narrowed eyes, forwarded jaws. All those predominant factors help to identify Anger emotion.

Second, the happiness represents an emotional state of well-being, based on emotions from contentment to delirious joy. In a happiness facial expression image, the reader can observe the forehead muscle relaxes and the eyebrows are pulled up slightly. Besides, both the wrinkled outer corners of eyes and pulled up lip corners are distinctive features. Calvo and Beltran [2] once used event-related potential (ERP) to demonstrate why Anger and Happiness are easily detected. Our results are coherent with their findings.

Third, the Neutral facial expression relaxes the facial muscles while other facial expression all need to use a massive of facial muscles. Ali et al. [13] got a perfect Neutral identification in their study. The reason is their dataset (Japanese Female Facial Expression) used professional models, who extensively and routinely used facial muscles in the daily life than common persons used in this study. Therefore, the other six facial expressions in their datasets seem more extreme than the expressions of our enrolled subjects.

TABLE 3. Statistical analysis on the overall accuracies.

Run	OA
1	96.71
2	96.71
3	96.86
4	96.71
5	96.86
6	96.86
7	96.57
8	96.86
9	96.71
10	96.86
Average	$96.77 \pm 0.10$

The overall accuracies of all runs are listed in Table 3. The final overall accuracy is  $96.77 \pm 0.10\%$ .

**C. CONFUSION MATRIX**

The confusion matrix of 10 runs are listed below in Table 4. Here the sum of each row equals to 1000, which represents the number of samples of each class (100) multiplied with the number of runs (10). Take the first row as an example, the data shows 980 Anger samples were correctly identified, and 10 Anger samples were misclassified as Disgust, 3 Anger samples were misclassified as Fear, 4 Anger samples were

misclassified as Happy, 2 Anger samples were misclassified as Sadness, and the left Anger sample was misclassified as Surprise.

TABLE 4. Confusion matrix over 10 runs.

	Anger	Disgust	Fear	Happy	Neutral	Sadness	Surprise
Anger	980	10	3	4	0	2	1
Disgust	28	956	6	1	2	5	2
Fear	5	4	961	0	7	4	19
Happy	1	1	4	985	3	2	4
Neutral	3	2	7	5	973	4	6
Sadness	10	8	6	2	0	966	8
Surprise	2	6	26	3	3	7	953

From Table 4, we can observe two important findings. One is Anger and Disgust are confused easily. The other is Fear and Surprise are confused easily. The reasons may result from the early phase of dynamic facial expressions between Anger and Disgust are similar due to the common transmission of nose wrinkle and lip funneler [46]. For Fear and Surprise, the upper lid raiser and jaw drop may contribute to the confusion [46]. Those two findings support the conclusion in [46].

TABLE 5. The effect of fuzzy logic in SVM.

	MSVM	FMSVM(Proposed)
Anger	95.10±1.32	98.00±1.08
Disgust	92.10±1.52	95.60±0.97
Fear	94.50±0.97	96.10±0.99
Happy	95.70±1.52	98.50±0.85
Neutral	94.90±1.10	97.30±0.70
Sadness	94.60±1.35	96.60±1.33
Surprise	94.50±0.97	95.30±1.64
OA	94.49±0.14	96.77±0.10

D. FMSVM VERSUS MSVM

In the fourth experiment, we shall test the effectiveness of fuzzy logic. We designed this experiment to compare our fuzzy multiclass SVM (FMSVM) classifier with multiclass SVM (MSVM). All the settings are the same as in Section V-B. The results are offered in Table 5.

We can observe from the data listed in Table 5, that the MSVM without fuzzy logic obtains lower sensitivities in all facial emotion classes and lower overall accuracy than the FMSVM with fuzzy logic. Therefore, this proves the effectiveness of using fuzzy logic.

E. COMPARISON TO STATE-OF-THE-ART APPROACHES

In the fifth experiment, we compared our proposed “BWE + FMSVM” method with three state-of-the-art methods over our dataset. The comparison basis methods include PCA + SVM [11], CSO-GA-PSO + SVM [12], and RT + HOS + SVM [13]. All the settings are the same as in Section V.B. The comparison results are listed in Table 6.

Table 6 shows the PCA + SVM [11] yields an overall accuracy of 89.14±2.91%, the CSO-GA-PSO + SVM [12] yields an overall accuracy of 93.86±0.39%, RT + HOS + SVM [13]

TABLE 6. Comparison to state-of-the-art methods.

Approach	OA
PCA + SVM [11]	89.14±2.91
CSO-GA-PSO + SVM [12]	93.86±0.39
RT + HOS + SVM [13]	83.43±2.15
BWE + FMSVM (Proposed)	96.77±0.10

yields an overall accuracy of 83.43±2.15%, and our method yields the largest overall accuracy of 96.77±0.10%.

Although our develop FER system achieved promising results and performed better than three state-of-the-art approaches, it yet had several shortcomings: (i) we did not consider the geometric transform of faces, such as yaw angles [47]. (ii) We used still image, not video images [48].

In the future, we may use apply our system to help treat and diagnosis patients with emotion problems: Luzzi et al. [49] once requested AD patients to identify happy/sad to measure their disease. Chen et al. [50] also used facial expression to improve perceptions and judgments of autism children. We believed our system can help quantitatively measure the emotional identification ability of AD [51]–[55] and autism [56] patients. Besides, more advanced image processing [57] techniques, artificial intelligence [58], [59] approaches, and swarm intelligence [60], [61] methods shall be tested. Our method can apply to not only facial expression images, but also MR images [62], [63], CT images, remote-sensing images [64], [65], etc.

VI. CONCLUSION

In this study, we proposed a new facial emotion recognition system based on facial expression image. Our methodology chose the biorthogonal wavelet entropy and fuzzy multiclass support vector machine. The strict statistical analysis showed the superiority of our method to other three state-of-the-art methods. In the future, we shall apply our method in recommender system [66], [67], cloud computing [68], big data [69]. Sparse representation [70] is another potential research direction.

VII. CONFLICT OF INTEREST

We have no conflict of interest to disclose, with regard to the subject matter of this paper.

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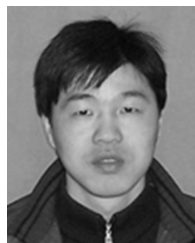


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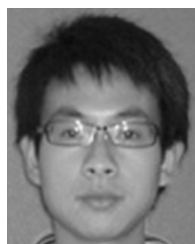
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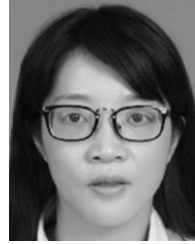


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