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Robust Decentralized Static Output-Feedback Control Design for Large-Scale Nonlinear Systems Using Takagi-Sugeno Fuzzy Models

ZHIXIONG ZHONG[1], YANZHENG ZHU[2], AND TING YANG[3], (Member, IEEE)
[1]Key High-voltage Laboratory of Fujian Province, Xiamen University of Technology, Xiamen 361024, China
[2]College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China
[3]School of Automation, Northwestern Polytechnical University, Xi'an 710072, China

Corresponding author: Y. Zhu (yanzhengzhu@sdust.edu.cn)

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ABSTRACT This paper is concerned with the problem of robust decentralized output-feedback control for a class of continuous-time large-scale nonlinear systems. Each nonlinear subsystem, described by a Takagi–Sugeno model, involves in the interconnections and parametric uncertainties of the large-scale systems. The main focus of this paper is to design a robust decentralized static output-feedback (SOF) fuzzy controller, such that the resulting closed-loop system is asymptotically stable with a prescribed $\mathscr{H}_\infty$ disturbance attenuation level. Based on some matrix inequality linearization techniques and the descriptor system approach, sufficient conditions for the existence of a robust decentralized SOF $\mathscr{H}_\infty$ fuzzy controller are presented in terms of linear matrix inequalities. From different perspectives, the desired controller is designed to analyze the degree of conservatism induced by considering various limitations. The effectiveness and superiority of the proposed method are finally demonstrated by two numerical examples.

INDEX TERMS Decentralized control, static output-feedback (SOF) control, descriptor system, large-scale systems, robust control, Takagi-Sugeno (T-S) model.

I. INTRODUCTION

Many practical systems, for example, communication networks, transportation systems, industrial processes, and power systems, are increasingly large in the dimensionality and strongly interconnected in the structure, and such complex systems can be considered as a class of large-scale systems [1]–[4]. Generally speaking, a large-scale system is comprised of several subsystems with evident interconnections, and two essential difficulties: high dimensionality and strong interconnections. A natural way is to decompose the overall system into several subsystems as well as their interconnections, such that the control of the overall system can be implemented by a cluster of independent controllers instead of a single controller, which referred as decentralized control approach [5]. During the past few decades, as an effective control approach of large-scale systems, the decentralized control has attracted a great deal of attention from control communities, and a large number of results have been reported for large-scale systems. See for instance [6]–[9] and the references therein.

On the other hand, it has been confirmed that Takagi-Sugeno (T-S) model is a powerful tool to approximate any smoothly nonlinear systems with arbitrarily high accuracy [10]–[13]. The main advantage of T-S fuzzy models is that the fuzzy logic theory can be combined with linear system theory as an unified framework. Firstly, the T-S fuzzy model makes use of a family of IF-THEN fuzzy rules to describe the local linear input-output information about a nonlinear system. The global dynamics of the nonlinear system can be represented by these local linear models that are smoothly blended in virtue of fuzzy membership functions. Then, a variety of linear control methods are developed to fulfill the control design of the presented T-S fuzzy model. Up to now, the T-S fuzzy approach has been extensively studied in the literature [14]–[20]. More recently, the T-S fuzzy method has been extended to the area of large-scale nonlinear systems, and some significant results have been reported in the open literature [21]–[27]. To mention a few, the problem of decentralized state-feedback controller design for large-scale T-S fuzzy systems has been investigated in [21]–[24]. The authors in [25] and [26] presented the delay-dependent stability criterion and decentralized $\mathscr{H}_\infty$ filtering design result for large-scale T-S fuzzy systems with time-varying delay, respectively. Moreover, the decentralized $\mathscr{H}_\infty$ filtering design for large-scale T-S fuzzy systems with multiple constant time delays has been conducted in [27].

In addition, it is well-known that usually only the measurement output information, rather than the full state information, is available for feedback control design of dynamic systems [28]. Thus, the static output-feedback (SOF) controller is more realistic and useful than the state-feedback one in practical applications. There have been, however, surprisingly few attempts to address the robust decentralized SOF $\mathscr{H}_\infty$ fuzzy controller design for the large-scale T-S fuzzy systems, which is the first motivation of this study. Moreover, it is also noted that the SOF control design is often formulated as a nonconvex problem representing the form of bilinear matrix inequalities (BMIs) or nonlinear matrix inequalities, which is, in general, difficult to solve using the existing numerical software. Recently, there have been some valuable results on the SOF $\mathscr{H}_\infty$ controller design for T-S fuzzy dynamic systems in the form of linear matrix inequalities (LMIs) [29]–[32]. However, these results given in [29]–[32] are obtained by imposing some constraints on the systematic input or output matrices. For instance, it is assumed that all local linear models share a common output matrix in [29] and [30]. In [31], the strict limitation that the output matrices are common is relaxed, while the output matrices must satisfy some matrix-equality constraints. It seems that the problem of SOF controller design for T-S fuzzy dynamic systems can be solved efficiently via LMIs technique in [32], nevertheless, the proposed approaches are not applicable to the case when uncertainties emerge in the system input and output matrices. Hence, it remains an open issue on how to obtain LMI conditions for robust decentralized SOF fuzzy controller design without making any restrictive assumptions on system matrices, which is the second motivation of the present study.

In this paper, we focus on the problem of robust decentralized SOF $\mathscr{H}_\infty$ control for a class of continuous-time large-scale nonlinear systems. Each nonlinear subsystem is of interconnections and parametric uncertainties in the large-scale systems, and can be represented by a T-S model. First, we will make use of some constraints on the system matrices, and the decentralized SOF $\mathscr{H}_\infty$ fuzzy controller design issue will be reformulated in the form of LMIs using some linearization techniques of matrix inequalities. Then, in order to relax all restrictive assumptions as much as possible, the descriptor system approach is used in this paper such that the closed-loop fuzzy control system could be represented in the form of descriptor systems, the corresponding LMI conditions will be derived to design the robust decentralized SOF $\mathscr{H}_\infty$ fuzzy controller for the large-scale T-S fuzzy system under consideration. Finally, two numerical examples are given to illustrate the effectiveness of the proposed method.

The remainder of this paper is organized as follows. Section II formulates the problem under consideration. The main results for the robust decentralized SOF $\mathscr{H}_\infty$ fuzzy controller design are given in Section III. Two numerical examples are presented in Section IV to demonstrate the effectiveness of the proposed methods, which is followed by some conclusions in Section V.

Notations: The notations used in this paper are standard. $\Re^n$ and $\Re^{n\times m}$ are the n-dimensional Euclidean space and the set of $n \times m$ matrices, respectively. The matrix $P \in \Re^{n\times n}$, $P > 0\,(\geq 0)$ denotes P being positive definite (or positive semidefinite). Sym$\{A\}$ denotes $A + A^T$. $\mathbf{I}_n$ and $0_{m\times n}$ are the $n \times n$ dimensional identity matrix and $m \times n$ dimensional zero matrix, respectively. The subscripts n and $m \times n$ could be omitted when the size can be directly determined in accordance with the context. For a matrix $A \in \Re^{n\times n}$, A^{-1} and A^T denote the inverse and transpose of the matrix A, respectively. diag$\{\cdots\}$ is a block-diagonal matrix. $L_2[0,\infty)$ denotes the space of square integrable vector functions over $[0,\infty)$. The notation $\star$ indicates the terms that can be induced by symmetry.

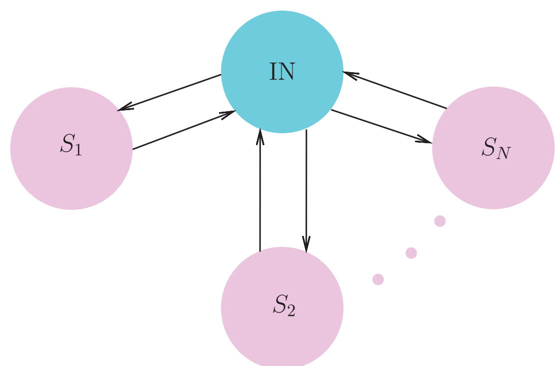

FIGURE 1. Large-scale interconnected systems.

II. MODEL DESCRIPTION AND PROBLEM FORMULATION

In this section, we consider a class of continuous-time large-scale nonlinear systems, which is composed of N nonlinear subsystems with interconnections and parametric uncertainties, as shown in Fig. 1. Here, the i-th nonlinear subsystem could be represented by the following T-S fuzzy model:

Plant Rule $\mathscr{R}_i^l$: **IF** $\zeta_{i1}(t)$ is $\mathscr{F}_{i1}^l$, $\zeta_{i2}(t)$ is $\mathscr{F}_{i2}^l, \cdots$, and $\zeta_{ig}(t)$ is $\mathscr{F}_{ig}^l$, **THEN**

$$\begin{cases} \dot{x}_i(t) = \tilde{A}_{il}x_i(t) + \sum_{k=1,k\neq i}^{N} \bar{A}_{ikl}x_k(t) + \tilde{B}_{il}u_i(t) \\ \qquad + D_{il}w_i(t) \\ y_i(t) = C_{il}x_i(t) + M_{il}w_i(t) \\ z_i(t) = L_{il}x_i(t) + F_{il}u_i(t), \quad l \in \mathscr{L}_i := \{1, 2, \ldots, r_i\} \end{cases} \tag{1}$$

where $\tilde{A}_{il} := A_{il} + \Delta A_{il}$, $\tilde{B}_{il} := B_{il} + \Delta B_{il}$, $i \in \mathcal{N} := \{1, 2, \ldots, N\}$. For the i-th nonlinear subsystem, $\mathscr{R}_i^l$ denotes the l-th fuzzy inference rule; r_i is the number of inference rules; $\mathscr{F}_{i\phi}^l$ $(\phi = 1, 2, \ldots, g)$ are fuzzy sets; $x_i(t) \in \Re^{n_{xi}}$ denotes the system state; $u_i(t) \in \Re^{n_{ui}}$ is the control input; $y_i(t) \in \Re^{n_{yi}}$ is the measurement output; $z_i(t) \in \Re^{n_{zi}}$ is the regulated output; $w_i(t) \in \Re^{n_{wi}}$ is the disturbance input belonging to $L_2[0, \infty)$; $\zeta_i(t) := [\zeta_{i1}(t), \zeta_{i2}(t), \ldots, \zeta_{ig}(t)]$ are some measurable variables; the pair $(A_{il}, B_{il}, D_{il}, C_{il}, M_{il}, L_{il}, F_{il})$ denotes the l-th local model; $\bar{A}_{ikl}$ denotes the interconnection matrix between the i-th and the k-th subsystems; ΔA_{il} and ΔB_{il} denote the uncertainty terms of the l-th local model satisfying

$$\begin{bmatrix} \Delta A_{il} & \Delta B_{il} \end{bmatrix} = H_{1il}\Delta_{il}(t)\begin{bmatrix} H_{2il} & H_{3il} \end{bmatrix}, \quad l \in \mathscr{L}_i \tag{2}$$

where H_{1il}, H_{2il}, and H_{3il} are known real matrices with appropriate dimensions. As studied in [33], $\Delta_{il}(t) \in \Re^{s_1 \times s_2}$ are unknown time-varying matrix functions with Lebesgue measurable elements satisfying

$$\Delta_{il}^T(t)\Delta_{il}(t) \leq \mathbf{I}_{s_2}, \quad l \in \mathscr{L}_i. \tag{3}$$

Remark 1: In this paper, for brevity, we only consider the uncertainty terms ΔA_{il} and $\Delta B_{il}, l \in \mathscr{L}_i$. However, the methods proposed in this paper can be easily extended to the case where the uncertainty terms simultaneously appear in the system matrices $\bar{A}_{ikl}, D_{il}, C_{il}, M_{il}, L_{il}$, and $F_{il}, l \in \mathscr{L}_i$.

Defining the inferred fuzzy set $\mathscr{F}_i^l := \prod_{\phi=1}^{g} \mathscr{F}_{i\phi}^l$ and the normalized membership function $\mu_{il}[\zeta_i(t)]$, it yields that

$$\mu_{il}[\zeta_i(t)] := \frac{\prod_{\phi=1}^{g} \mu_{il\phi}[\zeta_{i\phi}(t)]}{\sum_{\varsigma=1}^{r_i} \prod_{\phi=1}^{g} \mu_{i\varsigma\phi}[\zeta_{i\phi}(t)]} \geq 0, \tag{4}$$

where $\mu_{il\phi}[\zeta_{i\phi}(t)]$ is the grade of membership of $\zeta_{i\phi}(t)$ in $\mathscr{F}_{i\phi}^l$, and $\sum_{l=1}^{r_i} \mu_{il}[\zeta_i(t)] = 1$. In the following we will denote $\mu_{il} := \mu_{il}[\zeta_i(t)]$ for the sake of convenience.

By fuzzy blending, the global T-S fuzzy dynamic model of the i-th subsystem can be obtained as follows:

$$\begin{cases} \dot{x}_i(t) = (A_i(\mu_i) + \Delta A_i(\mu_i))\, x_i(t) + \sum\limits_{k=1,k\neq i}^{N} \bar{A}_{ik}(\mu_i)x_k(t) \\ \qquad + (B_i(\mu_i) + \Delta B_i(\mu_i))\, u_i(t) + D_i(\mu_i)w_i(t) \\ y_i(t) = C_i(\mu_i)x_i(t) + M_i(\mu_i)w_i(t) \\ z_i(t) = L_i(\mu_i)x_i(t) + F_i(\mu_i)u_i(t), \quad i \in \mathcal{N} \end{cases} \tag{5}$$

where

$$\begin{cases} A_i(\mu_i) := \sum\limits_{l=1}^{r_i} \mu_{il}A_{il}, & \Delta A_i(\mu_i) := \sum\limits_{l=1}^{r_i} \mu_{il}\Delta A_{il}, \\ \bar{A}_{ik}(\mu_i) := \sum\limits_{l=1}^{r_i} \mu_{il}\bar{A}_{ikl}, & B_i(\mu_i) := \sum\limits_{l=1}^{r_i} \mu_{il}B_{il}, \\ \Delta B_i(\mu_i) := \sum\limits_{l=1}^{r_i} \mu_{il}\Delta B_{il}, & D_i(\mu_i) := \sum\limits_{l=1}^{r_i} \mu_{il}D_{il}, \\ C_i(\mu_i) := \sum\limits_{l=1}^{r_i} \mu_{il}C_{il}, & M_i(\mu_i) := \sum\limits_{l=1}^{r_i} \mu_{il}M_{il}, \\ L_i(\mu_i) := \sum\limits_{l=1}^{r_i} \mu_{il}L_{il}, & F_i(\mu_i) := \sum\limits_{l=1}^{r_i} \mu_{il}F_{il}. \end{cases} \tag{6}$$

Given the large-scale T-S fuzzy system in (5), a class of decentralized static-output-feedback (SOF) fuzzy controller is considered in the following:

Controller Rule $\mathscr{R}_i^l$: **IF** $\zeta_{i1}(t)$ is $\mathscr{F}_{i1}^l$, $\zeta_{i2}(t)$ is $\mathscr{F}_{i2}^l, \cdots$, $\zeta_{ig}(t)$ is $\mathscr{F}_{ig}^l$, **THEN**

$$u_i(t) = K_{il}y_i(t), \quad l \in \mathscr{L}_i \tag{7}$$

where $K_{il} \in \Re^{n_{ui} \times n_{yi}}, l \in \mathscr{L}_i, i \in \mathcal{N}$ are controller gains to be determined.

Similarly, the overall SOF fuzzy controller of the i-th subsystem can be inferred as follows:

$$u_i(t) = K_i(\mu_i)y_i(t), \quad i \in \mathcal{N} \tag{8}$$

where $K_i(\mu_i) := \sum\limits_{l=1}^{r_i} \mu_{il}K_{il}, i \in \mathcal{N}$.

Let $\tilde{z}(t) = \begin{bmatrix} z_1^T(t) & z_2^T(t) & \cdots & z_N^T(t) \end{bmatrix}^T$, and $\tilde{w}(t) = \begin{bmatrix} w_1^T(t) & w_2^T(t) & \cdots & w_N^T(t) \end{bmatrix}^T$. The robust decentralized SOF $\mathscr{H}_\infty$ fuzzy control problem of the large-scale T-S fuzzy system (5) to be addressed in this paper can be formulated as:

Given the large-scale T-S fuzzy system in (5), and for a prescribed disturbance attenuation level $\gamma > 0$, design a decentralized SOF fuzzy controller in the form of (8) such that the closed-loop fuzzy control system is asymptotically stable, and under zero initial conditions the $\mathscr{H}_\infty$ disturbance attenuation level γ satisfies

$$\int_0^\infty \tilde{z}^T(t)\tilde{z}(t)dt < \gamma^2 \int_0^\infty \tilde{w}^T(t)\tilde{w}(t)dt \tag{9}$$

for any nonzero $\tilde{w} \in L_2[0\ \infty)$.

III. MAIN RESULTS

In this section, a series of results on robust decentralized SOF $\mathscr{H}_\infty$ fuzzy controller design are presented for the continuous-time large-scale T-S fuzzy system in (5) by using different approaches.

In order to facilitate the SOF fuzzy controller design, we first consider a special case of the large-scale T-S fuzzy system (5), in which the assumptions on the measurement output of each subsystem are given as follows:

A1. The measurement outputs $y_i(t)$ are noisy-free, i.e., $M_{il} = 0, l \in \mathscr{L}_i$;

A2. the output matrices $C_{il}, l \in \mathscr{L}_i$ are common, i.e., $C_{il} = C_i$;

A3. the parametric uncertainties do not appear in the output matrices C_i;

A4. the output matrices C_i are of full row rank.

For $i \in \mathcal{N}$, since the output matrices C_i are of full row rank for each subsystem, there exist nonsingular transformation matrices $T_{ci} \in \Re^{n_{xi}\times n_{xi}}$ satisfying [34]

$$C_i T_{ci} = \begin{bmatrix} \mathbf{I}_{n_{yi}} & 0_{n_{yi}\times(n_{xi}-n_{yi})} \end{bmatrix}. \tag{10}$$

Based on the above assumptions, and by combining the large-scale T-S fuzzy system in (5), with the decentralized SOF fuzzy controller in (8), the closed-loop fuzzy control system can be expressed as

$$\begin{cases} \dot{x}_i(t) = \mathscr{A}_i(\mu_i)x_i(t) + \sum_{k=1, k\neq i}^{N} \bar{A}_{ik}(\mu_i)x_k(t) + D_i(\mu_i)w_i(t) \\ z_i(t) = \mathscr{C}_i(\mu_i)x_i(t), \quad i \in \mathcal{N} \end{cases} \tag{11}$$

where

$$\begin{cases} \mathscr{A}_i(\mu_i) = A_i(\mu_i) + \Delta A_i(\mu_i) + (B_i(\mu_i) + \Delta B_i(\mu_i)) \\ \qquad\qquad \times K_i(\mu_i)C_i \\ \mathscr{C}_i(\mu_i) = L_i(\mu_i) + F_i(\mu_i)K_i(\mu_i)C_i. \end{cases} \tag{12}$$

Now, on the basis of the closed-loop fuzzy control system in (11), we will present the following synthesis result.

Theorem 1: Consider the large-scale T-S fuzzy system in (5) with the assumptions A1-A4. A decentralized SOF fuzzy controller in the form (8) exists, and can guarantee the asymptotic stability of the closed-loop fuzzy control system (11) with an $\mathscr{H}_\infty$ *disturbance attenuation level* γ*, if there exist matrices* $0 < X_{i(1)} = X_{i(1)}^T \in \Re^{n_{yi}\times n_{yi}}, 0 < X_{i(2)} = X_{i(2)}^T \in \Re^{(n_{xi}-n_{yi})\times(n_{xi}-n_{yi})}$, $\bar{K}_{il} \in \Re^{n_{ui}\times n_{yi}}$*, and scalars* $0 < \tau_0 \leq \tau_{il}, 0 < \varepsilon_{ilj}$*, such that for all* $i \in \mathcal{N}$ *the following LMIs hold:*

$$\bar{\Phi}_{ill} < 0, \quad 1 \leq l \leq r_i, \tag{13}$$
$$\bar{\Phi}_{ilj} + \bar{\Phi}_{ijl} < 0, \quad 1 \leq l < j \leq r_i \tag{14}$$

where

$$\begin{cases} \bar{\Phi}_{ilj} = \begin{bmatrix} \Phi_{ilj}^{(1)} + \varepsilon_{ilj}\mathbb{H}_{1il}\mathbb{H}_{1il}^T & \mathbb{H}_{2ilj}^T \\ \star & -\varepsilon_{ilj}\mathbf{I} \end{bmatrix}, \\ \Phi_{ilj}^{(1)} = \begin{bmatrix} -\tau_{i0}(N-1)^{-1}\mathscr{E} & 0 & \bar{\mathcal{A}}_{ki}T_{ci}X_iT_{ci}^T \\ \star & -\gamma^2\mathbf{I} & \Phi_{ilj}^{(22)} \\ \star & \star & \Phi_{ilj}^{(33)} \end{bmatrix}, \\ \mathbb{H}_{1il}^T = \begin{bmatrix} 0 & 0 & H_{1il}^T \end{bmatrix}, \Phi_{ilj}^{(22)} = L_{il}T_{ci}X_iT_{ci}^T + F_{il}\mathbb{K}_{ij}T_{ci}^T, \\ \mathbb{H}_{2ilj} = \begin{bmatrix} 0 & 0 & H_{2il}T_{ci}X_iT_{ci}^T + H_{3il}\mathbb{K}_{ij}T_{ci}^T \end{bmatrix}, \\ \Phi_{ilj}^{(33)} = \mathrm{Sym}\left\{A_{il}T_{ci}X_iT_{ci}^T + B_{il}\mathbb{K}_{ij}T_{ci}^T\right\} + \tau_{il}\mathbf{I} + D_{il}D_{il}^T, \\ \mathscr{E} = \mathrm{diag}\underbrace{\left\{\mathbf{I}_{n_{xi}} \cdots \mathbf{I}_{n_{xi}}\right\}}_{N-1}, \mathbb{K}_{ij} = \begin{bmatrix} \bar{K}_{ij} & 0 \end{bmatrix}, \\ X_i = \begin{bmatrix} X_{i(1)} & 0 \\ 0 & X_{i(2)} \end{bmatrix}, \bar{\mathcal{A}}_{ki} = \underbrace{\begin{bmatrix} \bar{A}_{1i}^T \cdots \bar{A}_{ki,k\neq i}^T \cdots \bar{A}_{Ni}^T \end{bmatrix}}_{N-1}^T. \end{cases} \tag{15}$$

Moreover, the corresponding controller gains are given by

$$K_{il} = \bar{K}_{il}X_{i(1)}^{-1}, \quad l \in \mathscr{L}_i, \quad i \in \mathcal{N}. \tag{16}$$

Proof: Consider the following Lyapunov function,

$$V(t) = \sum_{i=1}^{N} V_i(t) = \sum_{i=1}^{N} x_i^T(t)P_ix_i(t) \tag{17}$$

where $0 < P_i = P_i^T \in \Re^{n_{xi}\times n_{xi}}, i \in \mathcal{N}$.

Taking the time derivative of $V_i(t)$ along the trajectories of the closed-loop fuzzy control system in (11), one has

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{N} 2\left\{\left[\mathscr{A}_i(\mu_i)x_i(t) + \sum_{k=1, k\neq i}^{N} \bar{A}_{ik}(\mu_i)x_k(t) + D_i(\mu_i)w_i(t)\right]^T P_ix_i(t)\right\} \\ &= \sum_{i=1}^{N} \left\{2\left[\mathscr{A}_i(\mu_i)x_i(t) + D_i(\mu_i)w_i(t)\right]^T P_ix_i(t)\right\} \\ &\quad + \sum_{i=1}^{N} \left\{2\left[\sum_{k=1, k\neq i}^{N} \bar{A}_{ik}(\mu_i)x_k(t)\right]^T P_ix_i(t)\right\}. \end{aligned} \tag{18}$$

Note that

$$2\bar{x}^T\bar{y} \leq \kappa^{-1}\bar{x}^T\bar{x} + \kappa\bar{y}^T\bar{y} \tag{19}$$

where $\bar{x}, \bar{y} \in \Re^n$ and the scalar $\kappa > 0$.

In addition, define $\bar{A}_{ik} \geq \left\|\bar{A}_{ik}(\mu_i)\right\|$. Then, by using Lemma 1 as given in the Appendix, we also have

$$\begin{aligned} &\sum_{i=1}^{N} \left\{\left[\sum_{k=1, k\neq i}^{N} \bar{A}_{ik}(\mu_i)x_k(t)\right]^T \left[\sum_{k=1, k\neq i}^{N} \bar{A}_{ik}(\mu_i)x_k(t)\right]\right\} \\ &\leq \sum_{i=1}^{N} \left\{\left[\sum_{k=1, k\neq i}^{N} \bar{A}_{ik}x_k(t)\right]^T \left[\sum_{k=1, k\neq i}^{N} \bar{A}_{ik}x_k(t)\right]\right\} \\ &= \sum_{i=1}^{N} \left\{\left[\sum_{k=1, k\neq i}^{N} \bar{A}_{ki}x_i(t)\right]^T \left[\sum_{k=1, k\neq i}^{N} \bar{A}_{ki}x_i(t)\right]\right\} \\ &\leq \sum_{i=1}^{N} \left\{(N-1)\sum_{k=1, k\neq i}^{N} x_i^T(t)\bar{A}_{ki}^T\bar{A}_{ki}x_i(t)\right\}. \end{aligned} \tag{20}$$

Based on the inequalities (19) and (20), and by introducing scalar parameters $0 < \tau_0 \leq \tau_i(\mu_i)$, $\tau_i(\mu_i) := \sum_{l=1}^{r_i} \mu_{il}\tau_{il}$,

$i \in \mathcal{N}$, it yields

$$\begin{aligned}\dot{V}(t) \leq & \sum_{i=1}^{N}\left\{2\left[\mathscr{A}_i(\mu_i)x_i(t) + D_i(\mu_i)w_i(t)\right]^T P_i x_i(t)\right\} \\ & + \sum_{i=1}^{N}\left\{\tau_i^{-1}(\mu_i)\left[\sum_{\substack{k=1\\k\neq i}}^{N}\bar{A}_{ik}(\mu_i)x_k(t)\right]^T \times\left[\sum_{\substack{k=1\\k\neq i}}^{N}\bar{A}_{ik}(\mu_i)x_k(t)\right]\right\} \\ & + \sum_{i=1}^{N}\left\{\tau_i(\mu_i)x_i^T(t)P_iP_ix_i(t)\right\} \\ \leq & \sum_{i=1}^{N}\left\{2\left[\mathscr{A}_i(\mu_i)x_i(t) + D_i(\mu_i)w_i(t)\right]^T P_i x_i(t)\right\} \\ & + \sum_{i=1}^{N}\left\{\tau_0^{-1}(N-1)\sum_{\substack{k=1\\k\neq i}}^{N}x_i^T(t)\bar{A}_{ki}^T\bar{A}_{ki}x_i(t)\right\} \\ & + \sum_{i=1}^{N}\left\{\tau_i(\mu_i)x_i^T(t)P_iP_ix_i(t)\right\}. \end{aligned} \tag{21}$$

Consider the following performance index

$$\begin{aligned} J(t) &= \sum_{i=1}^{N} J_i(t) \\ &= \sum_{i=1}^{N}\int_0^{\infty}\left\{\gamma^{-2}z_i^T(t)z_i(t) - w_i^T(t)w_i(t)\right\}dt. \end{aligned} \tag{22}$$

Under zero initial conditions, we have $V_i(0) = 0$ and $V_i(\infty) \geq 0$. Then, it follows from (21) and (22) that

$$\begin{aligned} J(t) &\leq \sum_{i=1}^{N} J_i(t) + V_i(\infty) - V_i(0) \\ &= \sum_{i=1}^{N}\int_0^{\infty}\left\{\dot{V}_i(t) + \gamma^{-2}z_i^T(t)z_i(t) - w_i^T(t)w_i(t)\right\}dt \\ &\leq \sum_{i=1}^{N}\int_0^{\infty}\tilde{x}_i^T(t)\begin{bmatrix}\Theta_i(\mu_i) & P_iD_i(\mu_i)\\ \star & -\mathbf{I}\end{bmatrix}\tilde{x}_i(t)\,dt, \end{aligned} \tag{23}$$

where $\tilde{x}_i(t) = \left[x_i^T(t)\ \ w_i^T(t)\right]^T$, and

$$\begin{aligned}\Theta_i(\mu_i) = & \operatorname{Sym}\{P_i\mathscr{A}_i(\mu_i)\} + \tau_0^{-1}(N-1)\sum_{\substack{k=1\\k\neq i}}^{N}\bar{A}_{ki}^T\bar{A}_{ki} \\ & + \tau_i(\mu_i)P_iP_i + \gamma^{-2}\mathscr{C}_i^T(\mu_i)\mathscr{C}_i(\mu_i). \end{aligned} \tag{24}$$

It is easy to see from (23) that the resulting closed-loop system in (11) is asymptotically stable with an $\mathscr{H}_\infty$ disturbance attenuation level γ under zero initial conditions for any nonzero $\tilde{w} \in L_2[0\ \infty)$, if the following inequalities hold:

$$\begin{bmatrix}\Theta_i(\mu_i) & P_iD_i(\mu_i)\\ \star & -\mathbf{I}\end{bmatrix} < 0, \quad i \in \mathcal{N}. \tag{25}$$

By applying Schur complement to (25), we have

$$\begin{bmatrix}-\tau_0(N-1)^{-1}\mathscr{E} & 0 & \bar{\mathcal{A}}_{ki}\\ \star & -\gamma^2\mathbf{I} & \mathscr{C}_i(\mu_i)\\ \star & \star & \Theta(3,3)\end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned}\Theta(3,3) &= \operatorname{Sym}\{P_i\mathscr{A}_i(\mu_i)\} + \tau_i(\mu_i)P_iP_i + P_iD_i(\mu_i) \times D_i^T(\mu_i)P_i, \\ \bar{\mathcal{A}}_{ki} &= \big[\underbrace{\bar{A}_{1i}^T\cdots\bar{A}_{ki,k\neq i}^T\cdots\bar{A}_{Ni}^T}_{N-1}\big]^T, \\ \mathscr{E} &= \operatorname{diag}\{\underbrace{\mathbf{I}_{n_{xi}}\cdots\mathbf{I}_{n_{xi}}}_{N-1}\}. \end{aligned} \tag{27}$$

Then, by defining $\Gamma := \operatorname{diag}\left\{\mathscr{E}\ \mathbf{I}\ P_i^{-1}\right\}$ and performing a congruence transformation to (26) by Γ, it yields

$$\begin{bmatrix}-\tau_0(N-1)^{-1}\mathscr{E} & 0 & \bar{\mathcal{A}}_{ki}P_i^{-1}\\ \star & -\gamma^2\mathbf{I} & \mathscr{C}_i(\mu_i)P_i^{-1}\\ \star & \star & \Upsilon(3,3)\end{bmatrix} < 0. \tag{28}$$

where $\Upsilon(3,3) \triangleq \operatorname{Sym}\left\{\mathscr{A}_i(\mu_i)P_i^{-1}\right\} + \tau_i(\mu_i)\mathbf{I} + D_i(\mu_i)D_i^T(\mu_i)$.

For $\forall i \in \mathcal{N}$, to carry out the SOF fuzzy controller design, we specify P_i^{-1} as

$$P_i^{-1} = T_{ci}X_iT_{ci}^T \tag{29}$$

with

$$X_i = \begin{bmatrix}X_{i(1)} & 0\\ 0 & X_{i(2)}\end{bmatrix}, \tag{30}$$

where $0 < X_{i(1)} = X_{i(1)}^T \in \Re^{n_{yi}\times n_{yi}}$, $0 < X_{i(2)} = X_{i(2)}^T \in \Re^{(n_{xi}-n_{yi})\times(n_{xi}-n_{yi})}$, and T_{ci} are defined in (10).

Now, by substituting P_i^{-1} given by (29) into (28) and extracting the fuzzy basis functions, the inequality (28) can be rewritten as

$$\sum_{l=1}^{r_i}\mu_{il}^2\Phi_{ill} + \sum_{l=1}^{r_i-1}\sum_{j=l+1}^{r_i}\mu_{il}\mu_{ij}\left[\Phi_{ilj} + \Phi_{ijl}\right] < 0, \tag{31}$$

where

$$\begin{cases}\Phi_{ilj} = \begin{bmatrix}-\tau_0(N-1)^{-1}\mathscr{E} & 0 & \bar{\mathcal{A}}_{ki}T_{ci}X_iT_{ci}^T\\ \star & -\gamma^2\mathbf{I} & \mathscr{C}_{ilj}T_{ci}X_iT_{ci}^T\\ \star & \star & \Phi_{ilj}(3,3)\end{bmatrix}, \\ \mathscr{A}_{ilj} = A_{il} + \Delta A_{il} + (B_{il} + \Delta B_{il})K_{ij}C_i, \\ \Phi_{ilj}(3,3) = \operatorname{Sym}\left\{\mathscr{A}_{ilj}T_{ci}X_iT_{ci}^T\right\} + \tau_{il}\mathbf{I} + D_{il}D_{il}^T, \\ \mathscr{C}_{ilj} = L_{il} + F_{il}K_{ij}C_i. \end{cases} \tag{32}$$

It follows from (10), (30), and (32) that

$$K_{ij}C_iT_{ci}X_i = K_{ij}\left[\mathbf{I}\ 0\right]\begin{bmatrix}X_{i(1)} & 0\\ 0 & X_{i(2)}\end{bmatrix}$$

$$
\begin{aligned}
&= \left[\, K_{ij} X_{i(1)} \; 0 \,\right] \\
&= \left[\, \bar{K}_{ij} \; 0 \,\right] \\
&= \mathbb{K}_{ij}, j \in \mathscr{L}_i, i \in \mathcal{N}. \qquad (33)
\end{aligned}
$$

Substituting (33) into Φ_{ilj} given by (32), together with (2) and (3), it gets

$$
\Phi_{ilj} = \Phi_{ilj}^{(1)} + \mathrm{Sym}\left\{\mathbb{H}_{1il}\Delta_{il}(t)\mathbb{H}_{2ilj}\right\}, \qquad (34)
$$

where

$$
\begin{cases}
\Phi_{ilj}^{(1)} = \begin{bmatrix} \Phi_{ilj}^{(1)}(1,1) & 0 & \bar{A}_{ki}T_{ci}X_iT_{ci}^T \\ \star & -\gamma^2\mathbf{I} & \Phi_{ilj}^{(1)}(2,3) \\ \star & \star & \Phi_{ilj}^{(1)}(3,3) \end{bmatrix}, \\
\Phi_{ilj}^{(1)}(3,3) = \mathrm{Sym}\left\{A_{il}T_{ci}X_iT_{ci}^T + B_{il}\mathbb{K}_{ij}T_{ci}^T\right\} + \tau_{il}\mathbf{I} \\
\quad + D_{il}D_{il}^T, \; \Phi_{ilj}^{(1)}(2,3) = L_{il}T_{ci}X_iT_{ci}^T + F_{il}\mathbb{K}_{ij}T_{ci}^T, \\
\mathbb{H}_{1il}^T = \left[\, 0 \; 0 \; H_{1il}^T \,\right], \mathbb{K}_{ij} = \left[\, \bar{K}_{ij} \; 0 \,\right], \\
\Phi_{ilj}^{(1)}(1,1) = -\tau_0 (N-1)^{-1}\mathscr{E}, \\
\mathbb{H}_{2ilj} = \left[\, 0 \; 0 \; H_{2il}T_{ci}X_iT_{ci}^T + H_{3il}\mathbb{K}_{ij}T_{ci}^T \,\right], \\
\mathscr{E} = \mathrm{diag}\underbrace{\left\{\mathbf{I}_{n_{xi}} \cdots \mathbf{I}_{n_{xi}}\right\}}_{N-1}, \; \bar{A}_{ki} = \underbrace{\left[\bar{A}_{1i}^T \cdots \bar{A}_{ki,k\neq i}^T \cdots \bar{A}_{Ni}^T\right]}_{N-1}^T.
\end{cases}
\qquad (35)
$$

In addition, by introducing scalar parameters $\varepsilon_{ilj} > 0$, $(l,j) \in \mathscr{L}_i$, $i \in \mathcal{N}$ and using Schur complement lemma and Lemma 2 given in the Appendix, the inequalities (13) and (14) can be obtained, thus completing this proof. ■

It is noted that the LMI conditions given by Theorem 1 are based on some assumptions on the measurement output of the large-scale T-S fuzzy system (5). We can also make the following assumptions on the input of each subsystem to derive the LMI conditions similar to Theorem 1.

A5. The regulated outputs $z_i(t)$ do not contain the input signal information, that is $F_{il} = 0, l \in \mathscr{L}_i$;

A6. the input matrices $B_{il}, l \in \mathscr{L}_i$ are common, i.e., $B_{il} = B_i$;

A7. the parametric uncertainties do not appear in the input matrices B_i ;

A8. the input matrices B_i are of full column rank.

Due to the assumption that the input matrices $B_i, i \in \mathcal{N}$ are of full column rank, there exist nonsingular transformation matrices $T_{bi} \in \Re^{n_{xi}\times n_{xi}}, i \in \mathcal{N}$ satisfying [32]

$$
T_{bi}B_i = \begin{bmatrix} \mathbf{I}_{n_{ui}} \\ 0_{(n_{xi}-n_{ui})\times n_{ui}} \end{bmatrix}, i \in \mathcal{N}. \qquad (36)
$$

It follows from the assumptions in A5-A8 that the closed-loop fuzzy control system consisting (5) and (8) can be expressed as,

$$
\begin{cases}
\dot{x}_i(t) = \hat{\mathscr{A}}_i(\mu_i)x_i(t) + \sum_{k=1,k\neq i}^N \bar{A}_{ik}x_k(t) + \hat{\mathscr{B}}_i(\mu_i)w_i(t) \\
z_i(t) = L_i(\mu_i)x_i(t), i \in \mathcal{N}
\end{cases}
\qquad (37)
$$

where

$$
\begin{cases}
\hat{\mathscr{A}}_i(\mu_i) = A_i(\mu_i) + \Delta A_i(\mu_i) + B_iK_i(\mu_i)C_i(\mu_i) \\
\hat{\mathscr{B}}_i(\mu_i) = B_iK_i(\mu_i)M_i(\mu_i) + D_i(\mu_i).
\end{cases}
\qquad (38)
$$

Based on the closed-loop control system in (37), it is easy to derive the LMI-based results similar to Theorem 1. Due to the page length consideration, the corresponding results are omitted here.

Remark 2: It is noted that the assumptions A1-A8 on the system matrices are very restrictive in practical applications, such as a well-known fuzzy model, i.e., inverted pendulum on a cart, do not share a common input matrix. Thus, apart from assumptions A1-A8, the method considering these system matrix constraints is inapplicable to the decentralized SOF fuzzy controller design.

In order to relax all restrictive assumptions on the system matrices, a descriptor system approach [35], [36] is developed to derive the LMI-based results for the robust decentralized SOF $\mathscr{H}_\infty$ fuzzy controller design.

By introducing virtual dynamics in the measurement output, and using the descriptor system approach, the closed-loop system consisting of (5) and (8) can be rewritten as

$$
\begin{cases}
\dot{x}_i(t) = \left(A_i(\mu_i) + \Delta A_i(\mu_i)\right)x_i(t) + \sum\limits_{k=1,k\neq i}^{N} \bar{A}_{ik}(\mu_i)x_k(t) \\
\qquad + \left(B_i(\mu_i) + \Delta B_i(\mu_i)\right)K_i(\mu_i)y_i(t) + D_i(\mu_i)w_i(t) \\
0\cdot \dot{y}_i(t) = C_i(\mu_i)x_i(t) - y_i(t) + M_i(\mu_i)w_i(t) \\
z_i(t) = L_i(\mu_i)x_i(t) + F_i(\mu_i)K_i(\mu_i)y_i(t), \quad i \in \mathcal{N}.
\end{cases}
\qquad (39)
$$

Define $\bar{x}_i(t) = \left[\, x_i^T(t) \; y_i^T(t) \,\right]^T$, the closed-loop dynamics in (39) can be expressed as the following descriptor system:

$$
\begin{cases}
E_1\dot{\bar{x}}_i(t) = \bar{\mathscr{A}}_i(\mu_i)\bar{x}_i(t) + R_1\sum_{k=1,k\neq i}^N \bar{A}_{ik}(\mu_i)x_k(t) \\
\qquad + \bar{\mathscr{B}}_i(\mu_i)w_i(t) \\
z_i(t) = \bar{\mathscr{C}}_i(\mu_i)\bar{x}_i(t), i \in \mathcal{N}
\end{cases}
\qquad (40)
$$

where

$$
\begin{cases}
\bar{\mathscr{A}}_i(\mu_i) = \begin{bmatrix} A_i(\mu_i) + \Delta A_i(\mu_i) & \bar{\mathscr{A}}_i^{(1,2)}(\mu_i) \\ C_i(\mu_i) & -\mathbf{I} \end{bmatrix}, \\
E_1 = \begin{bmatrix} \mathbf{I} & 0 \\ 0 & 0 \end{bmatrix}, \quad R_1 = \begin{bmatrix} \mathbf{I} \\ 0 \end{bmatrix}, \quad \bar{\mathscr{B}}_i(\mu_i) = \begin{bmatrix} D_i(\mu_i) \\ M_i(\mu_i) \end{bmatrix}, \\
\bar{\mathscr{C}}_i(\mu_i) = \left[\, L_i(\mu_i) \; F_i(\mu_i)K_i(\mu_i) \,\right], \\
\bar{\mathscr{A}}_i^{(1,2)}(\mu_i) = \left(B_i(\mu_i) + \Delta B_i(\mu_i)\right)K_i(\mu_i).
\end{cases}
\qquad (41)
$$

In accordance with the descriptor system in (40), we have the following synthesis result.

Theorem 2: Consider the large-scale T-S fuzzy system in (5). Then, given matrices $J_i \in \Re^{n_{yi}\times n_{xi}}, i \in \mathcal{N}$, a decentralized SOF fuzzy controller in the form (8) exists, and can guarantee the asymptotic stability of the closed-loop fuzzy control system with an $\mathscr{H}_\infty$ disturbance attenuation level γ, if there exist matrices $0 < \bar{X}_{i(1)} = \bar{X}_{i(1)}^T \in \Re^{n_{xi}\times n_{xi}}, \bar{X}_{i(2)} \in$

$\Re^{n_{yi}\times n_{yi}}$, $\bar{K}_{il}\in\Re^{n_{ui}\times n_{yi}}$, *and scalars* $0<\eta_0\leq\eta_{il}$, $0<\epsilon_{ilj}$, *such that for all* $i\in\mathcal{N}$ *the following LMIs hold:*

$$\bar{\Psi}_{ill}<0,\quad 1\leq l\leq r_i \tag{42}$$

$$\bar{\Psi}_{ilj}+\bar{\Psi}_{ijl}<0,\quad 1\leq l<j\leq r_i \tag{43}$$

where

$$\begin{cases}\bar{\Psi}_{ilj}=\begin{bmatrix}\Psi_{ilj}^{(1)}+\epsilon_{ilj}\bar{\mathbb{H}}_{1il}\bar{\mathbb{H}}_{1il}^T & \bar{\mathbb{H}}_{2ilj}^T\\ \star & -\epsilon_{ilj}\mathbf{I}\end{bmatrix},\\ \Psi_{ilj}^{(1)}=\begin{bmatrix}-\eta_0(N-1)^{-1}\mathscr{E} & 0 & \bar{\mathcal{A}}_{ki}\begin{bmatrix}\bar{X}_{i(1)} & 0\end{bmatrix}\\ \star & -\gamma^2\mathbf{I} & \Psi_{ilj}^{(11)}\\ \star & \star & \Psi_{ilj}^{(12)}\end{bmatrix},\\ \Psi_{ilj}^{(11)}=\begin{bmatrix}L_{il}\bar{X}_{i(1)}+F_{il}\bar{K}_{ij}J_i & F_{il}\bar{K}_{ij}\end{bmatrix},\\ \Psi_{ilj}^{(12)}=\begin{bmatrix}D_{il}D_{il}^T+\eta_{il}\mathbf{I} & D_{il}M_{il}^T\\ \star & M_{il}M_{il}^T+\eta_{il}\mathbf{I}\end{bmatrix}\\ \qquad+\mathrm{Sym}\left\{\begin{bmatrix}A_{il}\bar{X}_{i(1)}+B_{il}\bar{K}_{ij}J_i & B_{il}\bar{K}_{ij}\\ C_{il}\bar{X}_{i(1)}-\bar{X}_{i(2)}J_i & -\bar{X}_{i(2)}\end{bmatrix}\right\},\\ \bar{\mathbb{H}}_{1il}^T=\begin{bmatrix}0 & 0 & \bar{H}_{1il}^T\end{bmatrix},\quad \bar{H}_{1il}^T=\begin{bmatrix}H_{1il}^T & 0\end{bmatrix},\end{cases}$$

$$\bar{\mathbb{H}}_{2ilj}=\begin{bmatrix}0 & 0 & \bar{H}_{2ilj}\end{bmatrix},\quad \mathscr{E}=\mathrm{diag}\underbrace{\{\mathbf{I}_{n_{xi}}\cdots\mathbf{I}_{n_{xi}}\}}_{N-1},$$

$$\bar{H}_{2ilj}=\begin{bmatrix}H_{2il}X_{i(1)}+H_{3il}\bar{K}_{ij}J_i & H_{3il}\bar{K}_{ij}\end{bmatrix},$$

$$\bar{\mathcal{A}}_{ki}=\underbrace{\begin{bmatrix}\bar{A}_{1i}^T\cdots\bar{A}_{ki,k\neq i}^T\cdots\bar{A}_{Ni}^T\end{bmatrix}}_{N-1}^T. \tag{44}$$

Moreover, the corresponding controller gains are given by

$$K_{il}=\bar{K}_{il}\bar{X}_{i(2)}^{-1},\quad l\in\mathscr{L}_i,\quad i\in\mathcal{N}. \tag{45}$$

Proof: Consider the following Lyapunov function,

$$V(t)=\sum_{i=1}^N V_i(t)=\sum_{i=1}^N\bar{x}_i^T(t)E_1^T\bar{P}_i\bar{x}_i(t) \tag{46}$$

with

$$\bar{P}_i=\begin{bmatrix}\bar{P}_{i(1)} & 0\\ \bar{P}_{i(2)} & \bar{P}_{i(3)}\end{bmatrix},\quad i\in\mathcal{N} \tag{47}$$

where $0<\bar{P}_{i(1)}=\bar{P}_{i(1)}^T\in\Re^{n_{xi}\times n_{xi}}$, $\bar{P}_{i(2)}\in\Re^{n_{yi}\times n_{xi}}$, $\bar{P}_{i(3)}\in\Re^{n_{yi}\times n_{yi}}$, $i\in\mathcal{N}$.

Then, by taking the time derivative of $V_i(t)$ along the trajectory of the descriptor system in (40), one has

$$\begin{aligned}\dot{V}(t)&=\sum_{i=1}^N\left\{\dot{\bar{x}}_i^T(t)E_1^T\bar{P}_i\bar{x}_i(t)+\bar{x}_i^T(t)E_1^T\bar{P}_i\dot{\bar{x}}_i(t)\right\}\\&=\sum_{i=1}^N 2\left\{\left[\bar{\mathscr{A}}_i(\mu_i)\bar{x}_i(t)+R_1\sum_{\substack{k=1\\k\neq i}}^N\bar{A}_{ik}(\mu_i)x_k(t)+\bar{\mathscr{B}}_i(\mu_i)w_i(t)^T\bar{P}_i\bar{x}_i(t)\right]\right\}\\&=\sum_{i=1}^N\left\{2\left[\bar{\mathscr{A}}_i(\mu_i)\bar{x}_i(t)+\bar{\mathscr{B}}_i(\mu_i)w_i(t)\right]^T\bar{P}_i\bar{x}_i(t)\right\}\\&\quad+\sum_{i=1}^N\left\{2\left[R_1\sum_{\substack{k=1\\k\neq i}}^N\bar{A}_{ik}(\mu_i)x_k(t)\right]^T\bar{P}_i\bar{x}_i(t)\right\}.\end{aligned} \tag{48}$$

Introducing scalar parameters $0<\eta_{i0}\leq\eta_i(\mu_i)$, $\eta_i(\mu_i):=\sum_{l=1}^{r_i}\mu_{il}\eta_{il}$, $i\in\mathcal{N}$, and follows (19) and (20) that (48) can be rewritten as

$$\begin{aligned}\dot{V}(t)&\leq\sum_{i=1}^N\left\{2\left[\bar{\mathscr{A}}_i(\mu_i)\bar{x}_i(t)+\bar{\mathscr{B}}_i(\mu_i)w_i(t)\right]^T\bar{P}_i\bar{x}_i(t)\right\}\\&\quad+\sum_{i=1}^N\left\{\eta_i^{-1}(\mu_i)\left[R_1\sum_{\substack{k=1\\k\neq i}}^N\bar{A}_{ik}(\mu_i)x_k(t)\right]^T\left[R_1\times\sum_{\substack{k=1\\k\neq i}}^N\bar{A}_{ik}(\mu_i)x_k(t)\right]\right\}\\&\quad+\sum_{i=1}^N\left\{\eta_i(\mu_i)\bar{x}_i^T(t)\bar{P}_i^T\bar{P}_i\bar{x}_i(t)\right\}\\&\leq\sum_{i=1}^N\left\{2\left[\bar{\mathscr{A}}_i(\mu_i)\bar{x}_i(t)+\bar{\mathscr{B}}_i(\mu_i)w_i(t)\right]^T\bar{P}_i\bar{x}_i(t)\right\}\\&\quad+\sum_{i=1}^N\left\{\eta_0^{-1}(N-1)\sum_{\substack{k=1\\k\neq i}}^N x_i^T(t)\bar{A}_{ki}^T\bar{A}_{ki}x_i(t)\right\}\\&\quad+\sum_{i=1}^N\left\{\eta_i(\mu_i)\bar{x}_i^T(t)\bar{P}_i^T\bar{P}_i\bar{x}_i(t)\right\}.\end{aligned} \tag{49}$$

Furthermore, it follows a similar line as the proof process in Theorem 1, we have

$$\begin{aligned}J(t)&\leq\sum_{i=1}^N J_i(t)+V_i(\infty)-V_i(0)\\&=\sum_{i=1}^N\int_0^\infty\left\{\dot{V}_i(t)+\gamma^{-2}z_i^T(t)z_i(t)-w_i^T(t)w_i(t)\right\}dt\\&\leq\sum_{i=1}^N\int_0^\infty\tilde{x}_i^T(t)\left\{\begin{bmatrix}\bar{\Theta}_i(\mu_i) & \bar{P}_i^T\bar{\mathscr{B}}_i(\mu_i)\\ \star & -\mathbf{I}\end{bmatrix}\right\}\tilde{x}_i(t)\,dt,\end{aligned} \tag{50}$$

where $\tilde{x}_i(t)=\begin{bmatrix}x_i^T(t) & w_i^T(t)\end{bmatrix}^T$, and

$$\begin{aligned}\bar{\Theta}_i(\mu_i)&=\mathrm{Sym}\left\{\bar{\mathscr{A}}_i^T(\mu_i)\bar{P}_i\right\}+\eta_0^{-1}(N-1)\sum_{\substack{k=1\\k\neq i}}^N R_1\bar{A}_{ki}^T\\&\quad\times\bar{A}_{ki}R_1^T+\gamma^{-2}\bar{\mathscr{C}}_i^T(\mu_i)\bar{\mathscr{C}}_i(\mu_i)+\eta_i(\mu_i)\bar{P}_i^T\bar{P}_i.\end{aligned} \tag{51}$$

It is clear to see from (50) that the closed-loop fuzzy control system consisting of (5) and (8) is asymptotically stable with a prescribed $\mathscr{H}_\infty$ disturbance attenuation level γ under zero initial conditions for any nonzero $\tilde{w} \in L_2[0\ \infty)$, if the following inequalities hold:

$$\begin{bmatrix} \bar{\Theta}_i(\mu_i) & \bar{P}_i^T\bar{\mathscr{B}}_i(\mu_i) \\ \star & -\mathbf{I} \end{bmatrix} < 0, i \in \mathcal{N}. \tag{52}$$

By applying Schur complement, the inequality in (52) can be rewritten as

$$\begin{bmatrix} -\eta_0 (N-1)^{-1}\mathscr{E} & 0 & \bar{A}_{ki}R_1^T \\ \star & -\gamma^2\mathbf{I} & \bar{\mathscr{C}}_i(\mu_i) \\ \star & \star & \Gamma(3,3) \end{bmatrix} < 0, \tag{53}$$

where

$$\begin{aligned} \Gamma(3,3) &= Sym\left\{\bar{\mathscr{A}}_i^T(\mu_i)\bar{P}_i\right\} + \bar{P}_i^T\bar{\mathscr{B}}_i(\mu_i)\bar{\mathscr{B}}_i^T(\mu_i)\bar{P}_i \\ &\quad + \eta_i(\mu_i)\bar{P}_i^T\bar{P}_i, \\ \bar{A}_{ki} &= \underbrace{\left[\bar{A}_{1i}^T \cdots \bar{A}_{ki,k\neq i}^T \cdots \bar{A}_{Ni}^T\right]}_{N-1}^T, \\ \mathscr{E} &= \text{diag}\underbrace{\left\{\mathbf{I}_{n_{xi}} \cdots \mathbf{I}_{n_{xi}}\right\}}_{N-1}. \end{aligned} \tag{54}$$

In addition, it follows from (41) and (47) that

$$\bar{\mathscr{A}}_i^T(\mu_i)\bar{P}_i = \begin{bmatrix} A_i(\mu_i) + \Delta A_i(\mu_i) & \tilde{B}_i(\mu_i) \\ C_i(\mu_i) & -\mathbf{I} \end{bmatrix}^T \tag{55}$$

$$\times \begin{bmatrix} \bar{P}_{i(1)} & 0 \\ \bar{P}_{i(2)} & \bar{P}_{i(3)} \end{bmatrix} = \begin{bmatrix} \tilde{A}_i^T(\mu_i) & C_i^T(\mu_i)\bar{P}_{i(3)} \\ \tilde{K}_i^T(\mu_i) & -\bar{P}_{i(3)} \end{bmatrix}. \tag{56}$$

where $\tilde{B}_i(\mu_i) = (B_i(\mu_i) + \Delta B_i(\mu_i))K_i(\mu_i)$, $\tilde{A}_i^T(\mu_i) = \left(A_i^T(\mu_i) + \Delta A_i^T(\mu_i)\right)\bar{P}_{i(1)} + C_i^T(\mu_i)\bar{P}_{i(2)}$, $\tilde{K}_i^T(\mu_i) = K_i^T(\mu_i)\left(B_i^T(\mu_i) + \Delta B_i^T(\mu_i)\right)\bar{P}_{i(1)} - \bar{P}_{i(2)}$.

Then, it is not hard to get from (55) that the inequality (53) implies $-\bar{P}_{i(3)} - \bar{P}_{i(3)}^T < 0$, thus the Lyapunov matrices $\bar{P}_i$, $i \in \mathcal{N}$ are nonsingular. Now, defining $\bar{\Gamma} := \text{diag}\left\{\mathscr{E}\ \mathbf{I}\ \bar{P}_i^{-T}\right\}$, and performing the congruence transformation to (53) by $\bar{\Gamma}$ and $\bar{\Gamma}^T$, respectively, it yields

$$\begin{bmatrix} -\eta_0 (N-1)^{-1}\mathscr{E} & 0 & \bar{A}_{ki}R_1^T\bar{P}_i^{-1} \\ \star & -\gamma^2\mathbf{I} & \bar{\mathscr{C}}_i(\mu_i)\bar{P}_i^{-1} \\ \star & \star & \Xi(3,3) \end{bmatrix} < 0. \tag{57}$$

where $\Xi(3,3) = \text{Sym}\left\{\bar{P}_i^{-T}\bar{\mathscr{A}}_i^T(\mu_i)\right\} + \bar{\mathscr{B}}_i(\mu_i)\bar{\mathscr{B}}_i^T(\mu_i) + \eta_i(\mu_i)\mathbf{I}$.

In order to cast the inequality (56) into LMI conditions, we define $\bar{X}_i = \bar{P}_i^{-1}$, and it follows from (47) that $\bar{X}_i$ is specified as

$$\bar{X}_i = \begin{bmatrix} \bar{X}_{i(1)} & 0 \\ \bar{X}_{i(2)}J_i & \bar{X}_{i(2)} \end{bmatrix}, i \in \mathcal{N} \tag{58}$$

where $0 < \bar{X}_{i(1)} = \bar{X}_{i(1)}^T \in \Re^{n_{xi}\times n_{xi}}$, $J_i \in \Re^{n_{yi}\times n_{xi}}$, $\bar{X}_{i(2)} \in \Re^{n_{yi}\times n_{yi}}$, $i \in \mathcal{N}$.

Now, by substituting $\bar{X}_i$ given by (57) into (56) and extracting the fuzzy basis functions, the inequality (56) can be rewritten as

$$\sum_{l=1}^{r_i}\mu_{il}^2\Psi_{ill} + \sum_{l=1}^{r_i-1}\sum_{j=l+1}^{r_i}\mu_{il}\mu_{ij}\left[\Psi_{ilj} + \Psi_{ijl}\right] < 0, \tag{59}$$

where

$$\begin{cases} \Psi_{ilj} = \begin{bmatrix} -\eta_0 (N-1)^{-1}\mathscr{E} & 0 & \bar{A}_{ki}R_1^T\bar{X}_i \\ \star & -\gamma^2\mathbf{I} & \bar{\mathscr{C}}_{ilj}\bar{X}_i \\ \star & \star & \Psi_{ilj}(3,3) \end{bmatrix}, \\ \bar{\mathscr{A}}_{ilj} = \begin{bmatrix} A_{il} + \Delta A_{il} & (B_{il} + \Delta B_{il})K_{ij} \\ C_{il} & -\mathbf{I} \end{bmatrix}, \\ \bar{\mathscr{B}}_{il} = \begin{bmatrix} D_{il} \\ M_{il} \end{bmatrix}, \quad \bar{\mathscr{C}}_{ilj} = \begin{bmatrix} L_{il} & F_{il}K_{ij} \end{bmatrix}. \end{cases} \tag{60}$$

where $\Psi_{ilj}(3,3) = \text{Sym}\left\{\bar{X}_i^T\bar{\mathscr{A}}_{ilj}^T\right\} + \bar{\mathscr{B}}_{il}\bar{\mathscr{B}}^T{}_{il} + \eta_{il}\mathbf{I}$.

It can be seen from (57) and (59) that the matrix $\bar{X}_{i(2)}$ can be absorbed by the controller gain variable K_{ij} by introducing

$$\bar{K}_{ij} = K_{ij}\bar{X}_{i(2)}, j \in \mathscr{L}_i, i \in \mathcal{N}. \tag{61}$$

Then, by introducing scalar parameters $\epsilon_{ilj} > 0$, $(l,j) \in \mathscr{L}_i$, $i \in \mathcal{N}$, and it follows a similar line as in the proof of Theorem 1 that the inequalities (42) and (43) can be obtained, thus completing this proof. ■

In the followings, motivated by [37], we will introduce an alternative descriptor representation to the robust decentralized SOF $\mathscr{H}_\infty$ controller design. An auxiliary system is firstly given by

$$0 \cdot \dot{x}_i(t) = x_i(t) - x_i(t). \tag{62}$$

Combining with (39) and (61), we can also obtain the following descriptor representation,

$$\begin{cases} \dot{x}_i(t) = (A_i(\mu_i) + \Delta A_i(\mu_i))x_i(t) + \sum\limits_{k=1,k\neq i}^{N}\bar{A}_{ik}(\mu_i)x_k(t) \\ \qquad + (B_i(\mu_i) + \Delta B_i(\mu_i))K_i(\mu_i)y_i(t) + D_i(\mu_i)w_i(t) \\ 0 \cdot \dot{x}_i(t) = x_i(t) - x_i(t) \\ 0 \cdot \dot{y}_i(t) = C_i(\mu_i)x_i(t) - y_i(t) + M_i(\mu_i)w_i(t) \\ z_i(t) = L_i(\mu_i)x_i(t) + F_i(\mu_i)K_i(\mu_i)y_i(t), \quad i \in \mathcal{N}. \end{cases} \tag{63}$$

Define $\tilde{x}_i(t) = \left[x_i^T(t)\ x_i^T(t)\ y_i^T(t)\right]^T$, the closed-loop dynamics in (62) is rewritten as

$$\begin{cases} E_2\dot{\tilde{x}}_i(t) = \tilde{\mathscr{A}}_i(\mu_i)\tilde{x}_i(t) + R_2\sum_{k=1,k\neq i}^{N}\bar{A}_{ik}(\mu_i)x_k(t) \\ \qquad + \tilde{\mathscr{B}}_i(\mu_i)w_i(t) \\ z_i(t) = \tilde{\mathscr{C}}_i(\mu_i)\tilde{x}_i(t), \quad i \in \mathcal{N} \end{cases} \tag{64}$$

where

$$\begin{cases} \tilde{\mathscr{A}}_i(\mu_i) = \begin{bmatrix} 0 & \tilde{\mathscr{A}}_i(\mu_i)(1,2) & \tilde{\mathscr{A}}_i(\mu_i)(1,3) \\ \mathbf{I} & -\mathbf{I} & 0 \\ C_i(\mu_i) & 0 & -\mathbf{I} \end{bmatrix}, \\ \tilde{\mathscr{A}}_i(\mu_i)(1,2) = A_i(\mu_i) + \Delta A_i(\mu_i), \\ R_2 = \begin{bmatrix} \mathbf{I} & 0 & 0 \end{bmatrix}^T, \\ \tilde{\mathscr{A}}_i(\mu_i)(1,3) = (B_i(\mu_i) + \Delta B_i(\mu_i))\, K_i(\mu_i), \\ E_2 = \begin{bmatrix} \mathbf{I} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \tilde{\mathscr{B}}_i(\mu_i) = \begin{bmatrix} D_i(\mu_i) \\ 0 \\ M_i(\mu_i) \end{bmatrix}, \\ \tilde{\mathscr{C}}_i(\mu_i) = \begin{bmatrix} 0 & L_i(\mu_i) & F_i(\mu_i)K_i(\mu_i) \end{bmatrix}. \end{cases} \tag{65}$$

In terms of the descriptor system in (63), we consider the following Lyapunov function:

$$V(t) = \sum_{i=1}^{N} V_i(t) = \sum_{i=1}^{N} \tilde{x}_i^T(t) E_2^T \tilde{P}_i \tilde{x}_i(t) \tag{66}$$

with

$$\tilde{P}_i = \begin{bmatrix} \tilde{P}_{i(1)} & 0 & 0 \\ \tilde{P}_{i(2)} & \tilde{P}_{i(3)} & \tilde{P}_{i(4)} \\ \tilde{P}_{i(5)} & \tilde{P}_{i(6)} & \tilde{P}_{i(7)} \end{bmatrix}, \quad i \in \mathcal{N} \tag{67}$$

where $0 < \tilde{P}_{i(1)} = \tilde{P}_{i(1)}^T$, $\left\{\tilde{P}_{i(1)}, \tilde{P}_{i(2)}, \tilde{P}_{i(3)}\right\} \in \Re^{n_{xi} \times n_{xi}}$, $\tilde{P}_{i(4)} \in \Re^{n_{xi} \times n_{yi}}$, $\left\{\tilde{P}_{i(5)}, \tilde{P}_{i(6)}\right\} \in \Re^{n_{yi} \times n_{xi}}$, $\tilde{P}_{i(7)} \in \Re^{n_{yi} \times n_{yi}}$, $i \in \mathcal{N}$.

Similarly, it can be known that the Lyapunov matrices $\tilde{P}_i, i \in \mathcal{N}$ are nonsingular. Now, in order to derive an LMI-based result, we define $\tilde{X}_i = \tilde{P}_i^{-1}$, and specify $\tilde{X}_i$ as

$$\tilde{X}_i = \begin{bmatrix} \tilde{X}_{i(1)} & 0 & 0 \\ \tilde{X}_{i(2)} & \tilde{X}_{i(3)} & \tilde{X}_{i(4)} \\ \tilde{X}_{i(5)}\bar{J}_i & \tilde{X}_{i(5)}\bar{J}_i & \tilde{X}_{i(5)} \end{bmatrix}, \quad i \in \mathcal{N} \tag{68}$$

where $0 < \tilde{X}_{i(1)} = \tilde{X}_{i(1)}^T$, $\left\{\tilde{X}_{i(1)}, \tilde{X}_{i(2)}, \tilde{X}_{i(3)}\right\} \in \Re^{n_{xi} \times n_{xi}}$, $\tilde{X}_{i(4)} \in \Re^{n_{xi} \times n_{yi}}$, $\bar{J}_i \in \Re^{n_{yi} \times n_{xi}}$, $\tilde{X}_{i(5)} \in \Re^{n_{yi} \times n_{yi}}$, $i \in \mathcal{N}$.

Based on (63)-(67), and following a similar line as the derivations in Theorem 2, we can readily obtain the succeeding results on the decentralized robust SOF $\mathscr{H}_\infty$ fuzzy controller design for the large-scale T-S fuzzy system (5).

Theorem 3: Consider the large-scale T-S fuzzy system in (5). Then, given matrices $\bar{J}_i \in \Re^{n_{yi} \times n_{xi}}, i \in \mathcal{N}$, a decentralized SOF fuzzy controller in the form (8) exists, and can guarantee the asymptotic stability of the closed-loop fuzzy control system with an $\mathscr{H}_\infty$ disturbance attenuation level γ, if there exist matrices $0 < \tilde{X}_{i(1)} = \tilde{X}_{i(1)}^T \in \Re^{n_{xi} \times n_{xi}}$, $\left\{\tilde{X}_{i(2)}, \tilde{X}_{i(3)}\right\} \in \Re^{n_{xi} \times n_{xi}}$, $\tilde{X}_{i(4)} \in \Re^{n_{xi} \times n_{yi}}$, $\tilde{X}_{i(5)} \in \Re^{n_{yi} \times n_{yi}}$, $\bar{K}_{il} \in \Re^{n_{ui} \times n_{yi}}$, and scalars $0 < \rho_0 \leq \rho_{il}$, $0 < \nu_{ilj}$, such that for all $i \in \mathcal{N}$ the following LMIs hold:

$$\bar{\Omega}_{ill} < 0, \quad 1 \leq l \leq r_i \tag{69}$$

$$\bar{\Omega}_{ilj} + \bar{\Omega}_{ijl} < 0, \quad 1 \leq l < j \leq r_i \tag{70}$$

where

$$\begin{cases} \bar{\Omega}_{ilj} = \begin{bmatrix} \Omega_{ilj}^{(1)} + \nu_{ilj}\bar{\mathbb{H}}_{1il}\bar{\mathbb{H}}_{1il}^T & \bar{\mathbb{H}}_{2ilj}^T \\ \star & -\nu_{ilj}\mathbf{I} \end{bmatrix}, \\ \Omega_{ilj}^{(1)} = \begin{bmatrix} \Omega_{ilj}^{(1)}(1,1) & 0 & \bar{\mathcal{A}}_{ki}\begin{bmatrix} \tilde{X}_{i(1)} & 0 & 0 \end{bmatrix} \\ \star & -\gamma^2\mathbf{I} & \Omega_{ilj}^{(11)} \\ \star & \star & \Omega_{ilj}^{(12)} \end{bmatrix}, \\ \Omega_{ilj}^{(1)}(1,1) = -\rho_0 (N-1)^{-1} \mathscr{E}, \\ \Omega_{ilj}^{(11)} = \begin{bmatrix} \Omega_{ilj}^{(11)}(1,1) & \Omega_{ilj}^{(11)}(1,2) & L_{il}\tilde{X}_{i(4)} + F_{il}\bar{K}_{ij} \end{bmatrix}, \\ \Omega_{ilj}^{(11)}(1,1) = L_{il}\tilde{X}_{i(2)} + F_{il}\bar{K}_{ij}\bar{J}_i, \\ \Omega_{ilj}^{(11)}(1,2) = L_{il}\tilde{X}_{i(3)} + F_{il}\bar{K}_{ij}\bar{J}_i, \\ \Omega_{ilj}^{(12)} = \begin{bmatrix} D_{il}D_{il}^T + \rho_{il}\mathbf{I} & 0 & D_{il}M_{il}^T \\ \star & \rho_{il}\mathbf{I} & 0 \\ \star & \star & M_{il}M_{il}^T + \rho_{il}\mathbf{I} \end{bmatrix} \\ \quad + \mathrm{Sym}\left\{\begin{bmatrix} \Omega_{ilj}^{(12)}(1,1) & \Omega_{ilj}^{(12)}(1,2) & \Omega_{ilj}^{(12)}(1,3) \\ \tilde{X}_{i(1)} - \tilde{X}_{i(2)} & -\tilde{X}_{i(3)} & -\tilde{X}_{i(4)} \\ \Omega_{ilj}^{(12)}(3,1) & -\tilde{X}_{i(5)}\bar{J}_i & -\tilde{X}_{i(5)} \end{bmatrix}\right\}, \\ \Omega_{ilj}^{(12)}(1,1) = A_{il}\tilde{X}_{i(2)} + B_{il}\bar{K}_{ij}\bar{J}_i, \\ \Omega_{ilj}^{(12)}(1,2) = A_{il}\tilde{X}_{i(3)} + B_{il}\bar{K}_{ij}\bar{J}_i, \\ \Omega_{ilj}^{(12)}(1,3) = A_{il}\tilde{X}_{i(4)} + B_{il}\bar{K}_{ij}, \\ \Omega_{ilj}^{(12)}(3,1) = C_{il}\tilde{X}_{i(1)} - \tilde{X}_{i(5)}\bar{J}_i, \\ \bar{\mathbb{H}}_{1il}^T = \begin{bmatrix} 0 & 0 & \tilde{H}_{1il}^T \end{bmatrix}, \quad \tilde{H}_{1il}^T = \begin{bmatrix} H_{1il}^T & 0 & 0 \end{bmatrix}, \\ \bar{\mathbb{H}}_{2ilj} = \begin{bmatrix} 0 & 0 & \tilde{H}_{2ilj} \end{bmatrix}, \\ \tilde{H}_{2ilj}(1,1) = H_{2il}\tilde{X}_{i(2)} + H_{3il}\bar{K}_{ij}\bar{J}_i, \\ \tilde{H}_{2ilj}(1,2) = H_{2il}\tilde{X}_{i(3)} + H_{3il}\bar{K}_{ij}\bar{J}_i, \\ \tilde{H}_{2ilj}(1,3) = H_{2il}\tilde{X}_{i(4)} + H_{3il}\bar{K}_{ij}, \\ \tilde{H}_{2ilj} = \begin{bmatrix} \tilde{H}_{2ilj}(1,1) & \tilde{H}_{2ilj}(1,2) & \tilde{H}_{2ilj}(1,3) \end{bmatrix}, \\ \mathscr{E} = \mathrm{diag}\underbrace{\left\{\mathbf{I}_{n_{xi}} \ldots \mathbf{I}_{n_{xi}}\right\}}_{N-1}, \quad \bar{\mathcal{A}}_{ki} = \underbrace{\begin{bmatrix} \bar{A}_{1i}^T \ldots \bar{A}_{ki,k\neq i}^T \ldots \bar{A}_{Ni}^T \end{bmatrix}^T}_{N-1}. \end{cases} \tag{71}$$

Moreover, the corresponding controller gains are given by

$$K_{il} = \bar{K}_{il}\tilde{X}_{i(5)}^{-1}, \quad l \in \mathscr{L}_i, \quad i \in \mathcal{N}. \tag{72}$$

Remark 3: Compared with the conditions obtained in Theorem 2, more variables are considered in Theorem 3 due to the introduction of (67), which implies that the results obtained via solving (68)-(71) offer more freedom and will be accordingly less conservative in achieving a better $\mathscr{H}_\infty$ disturbance attenuation performance. The explicit verification on it will be achieved in the next section.

IV. NUMERICAL EXAMPLES

In this section, two illustrative numerical examples are utilized to verify the effectiveness of the developed robust decentralized SOF $\mathscr{H}_\infty$ fuzzy controller design methods in this paper.

Example 1: Consider a double-inverted pendulums system connected by a spring, the modified equations of the motion for the interconnected pendulum are given by [26]:

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = -\dfrac{kr^2}{4J_i}x_{i1} + \dfrac{kr^2}{4J_i}\sin(x_{i1})x_{i2} + \dfrac{2}{J_i}x_{i2} + \dfrac{1}{J_i}u_i \\ \qquad + \sum\limits_{j=1, j\neq i}^{2} \dfrac{kr^2}{8J_i}x_{j1}, i = \{1, 2\} \end{cases}$$

where x_{i1} denotes the angle of the i-th pendulum from the vertical; x_{i2} is the angular velocity of the i-th pendulum.

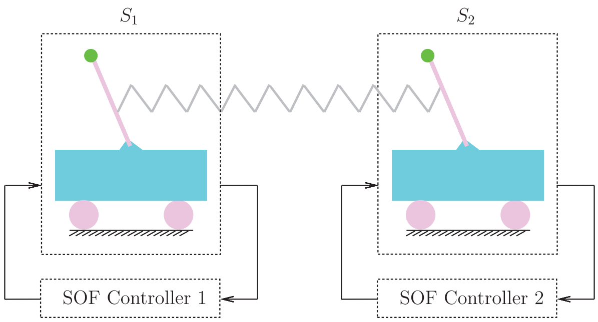

FIGURE 2. **A concise framework on the decentralized SOF control.**

The objective here is to design a robust decentralized SOF fuzzy controller in the form of (8) such that the resulting closed-loop system is asymptotically stable with an $\mathscr{H}_\infty$ disturbance attenuation level γ. A concise framework on the decentralized SOF control is shown in Fig. 2. In this simulation, the masses of two pendulums are chosen as $m_1 = 2$ kg and $m_2 = 2.5$ kg; the moments of inertia are $J_1 = 2$ kg $\cdot$ m^2 and $J_2 = 2.5$ kg$\cdot$m^2; the constant of the connecting torsional spring is $k = 8$ N/m; the length of the pendulum is $r = 1$ m; the gravity constant is $g = 9.8$ m/s^2. We choose two local models, i.e., by linearizing the interconnected pendulum around the origin and $x_{i1} = (\pm 88°, 0)$, respectively, each pendulum can be represented by the following T-S fuzzy model with two fuzzy rules.

Plant Rule $\mathscr{R}_i^l$: IF $x_{i1}(t)$ is $\mathscr{F}_{i1}^l$, THEN

$$\begin{cases} \dot{x}_i(t) = A_{il}x_i(t) + \sum\limits_{\substack{k=1\\k\neq i}}^{2} \bar{A}_{ik}x_k(t) + B_{il}u_i(t) + D_{il}w_i(t) \\ y_i(t) = C_{il}x_i(t) \\ z_i(t) = L_{il}x_i(t) + F_{il}u_i(t), \quad l = \{1, 2\}, \quad i = \{1, 2\} \end{cases}$$

where

$$\begin{bmatrix} A_{11} & A_{12} & \bar{A}_{12} & B_{1l} \end{bmatrix} = \left[\begin{array}{cc|cc|cc|c} 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 8.81 & 0 & 5.38 & 0 & 0.25 & 0 & 0.5 \end{array}\right]$$

$$D_{1l} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad C_{1l} = \begin{bmatrix} 1 & 1 \end{bmatrix},$$

$$L_{1l} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad F_{1l} = 1$$

for the first subsystem, and

$$\begin{bmatrix} A_{21} & A_{22} & \bar{A}_{21} & B_{2l} \end{bmatrix} = \left[\begin{array}{cc|cc|cc|c} 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 9.01 & 0 & 5.58 & 0 & 0.20 & 0 & 0.5 \end{array}\right]$$

$$D_{2l} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad C_{2l} = \begin{bmatrix} 1 & 1 \end{bmatrix},$$

$$L_{2l} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad F_{2l} = 1$$

for the second subsystem.

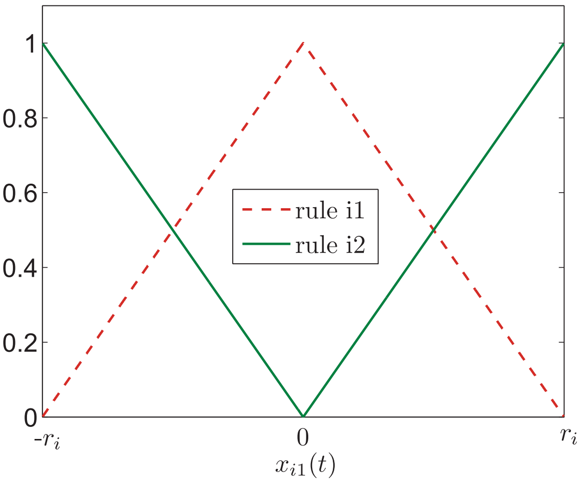

FIGURE 3. **Membership functions in Example 1.**

The normalized membership functions are shown in Fig. 3, where $r_i = 88°$. The parameter uncertainties are assumed to be the form of (2) as follows:

$$\Delta A_{il} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} \sin(t) \begin{bmatrix} 0.2 & 0 \end{bmatrix}, \quad l = \{1, 2\}, \quad i = \{1, 2\}.$$

For the control design result given in Theorems 1-3, designating the transformation matrices $T_{c1} = T_{c2} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$, $J_1 = J_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $\bar{J}_1 = \bar{J}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, and the measurement noises $M_{il} = 0.1$, $(i, l) = \{1, 2\}$, considering the measurement noises or not, the minimum $\mathscr{H}_\infty$ performance $\gamma_{\min}$ and the corresponding controller gains are calculated respectively as listed in Table 1.

Given the initial conditions $x_1(0) = [1.2, 0]^T$, $x_2(0) = [0.8, 0]^T$, Fig. 4 shows that the double-inverted pendulums system is not stable in the open-loop case. Taken the controller gains resolved by Theorem 3, Fig. 5 shows the state responses for the closed-loop large-scale system. Then, considering the external disturbances $w_1(t) = 0.8e^{-0.2t}\sin(0.2t)$ and $w_2(t) = 0.6e^{-0.2t}\sin(0.2t)$, it can be observed from Fig. 6 that the desired $\mathscr{H}_\infty$ performance is satisfactory under zero initial conditions, which fully demonstrates the effectiveness of the developed decentralized SOF $\mathscr{H}_\infty$ fuzzy controller design method.

Example 2: Consider a continuous-time large-scale T-S fuzzy system in the form of (1) with two interconnected subsystems as follows:

TABLE 1. Values of minimum $\mathscr{H}_\infty$ performance indexes and controller gains for different cases in Example 1.

Cases without measurement noises	$[\; K_{11} \quad K_{12} \quad K_{21} \quad K_{22} \;]$	$\gamma_{\min}$
Theorem 1	$[\; -134470 \quad -96951 \quad -5962.4 \quad -4375 \;]$	2.2086
Theorem 2	$[\; -57.9661 \quad -57.4094 \quad -50.0527 \quad -49.4973 \;]$	3.8435
Theorem 3	$[\; -92.7592 \quad -92.7251 \quad -84.4707 \quad -82.6375 \;]$	1.5924
Cases with measurement noises	$[\; K_{11} \quad K_{12} \quad K_{21} \quad K_{22} \;]$	$\gamma_{\min}$
Theorem 1	infeasible	—
Theorem 2	$[\; -48.7841 \quad -46.9999 \quad -39.0832 \quad -37.7832 \;]$	7.3177
Theorem 3	$[\; -51.2052 \quad -45.0926 \quad -53.2875 \quad -47.8572 \;]$	6.9660

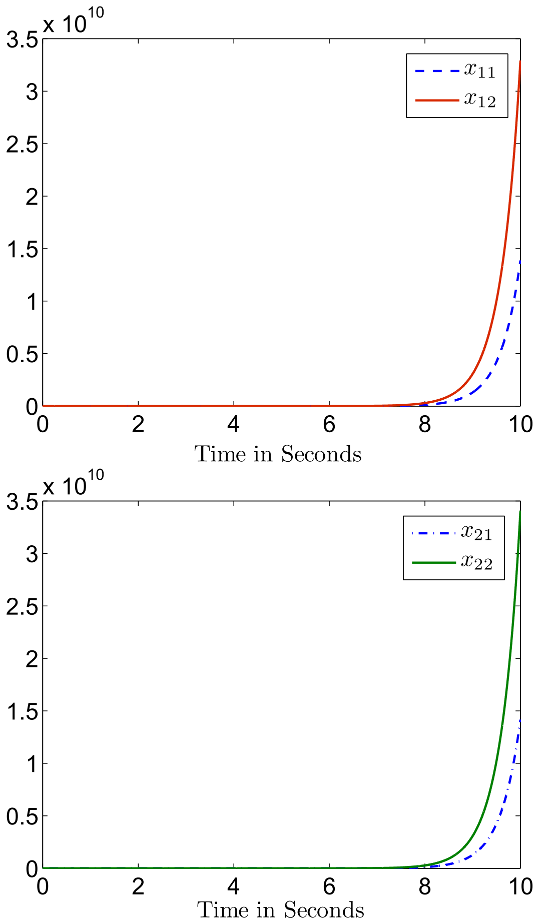

FIGURE 4. State responses for open-loop double-inverted pendulums system.

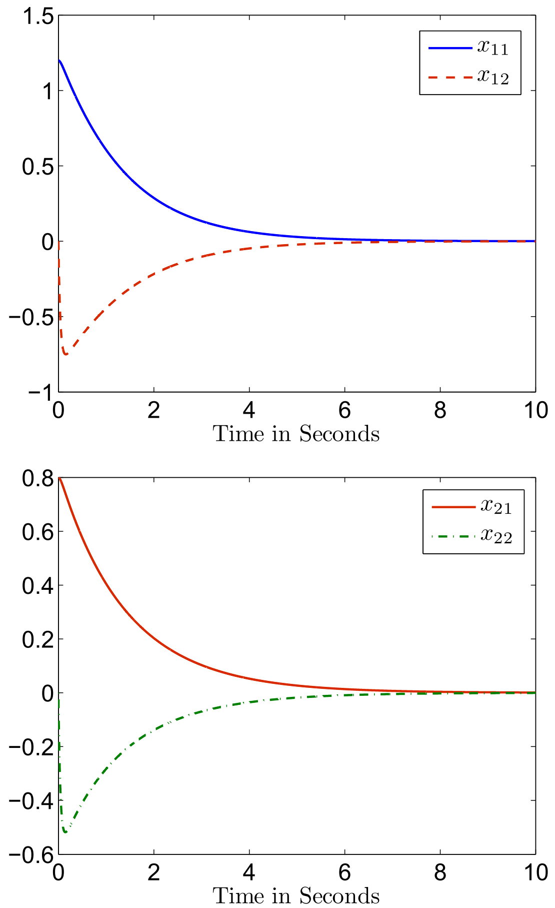

FIGURE 5. State responses for closed-loop double-inverted pendulums system based on Theorem 3.

Plant Rule $\mathscr{R}_i^l$: IF $x_{i1}(t)$ is $\mathscr{F}_{i1}^l$, THEN

$$\begin{cases} \dot{x}_i(t) = (A_{il} + \Delta A_{il})\, x_i(t) + \sum_{k=1,k\neq i}^{2} \bar{A}_{ikl} x_k(t) \\ \qquad\quad + B_{il} u_i(t) + D_{il} w_i(t) \\ y_i(t) = C_{il} x_i(t) \\ z_i(t) = L_{il} x_i(t) + F_{il} u_i(t), \quad l = \{1, 2\}, \quad i = \{1, 2\} \end{cases}$$

where

$$\begin{bmatrix} A_{11} & A_{12} & B_{11} & B_{12} & H_{11l} \\ \bar{A}_{121} & \bar{A}_{122} & D_{11} & D_{12} & \end{bmatrix} = \left[\begin{array}{cc|cc|c|c|c} a & 1.8 & a & 1.6 & 0.4 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.4 & 0.1 \\ \hline 0.15 & 0 & 0.14 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0.5 & 0.4 & \end{array}\right]$$

$$\begin{bmatrix} C_{1l} & L_{1l} & F_{1l} & H_{21l} \end{bmatrix} = \begin{bmatrix} 1 & 0 | & 0.8 & 0 | & 1 | & 0.2 & 0 \end{bmatrix}$$

for the first subsystem, and

$$\begin{bmatrix} A_{21} & A_{22} & B_{21} & B_{22} & H_{12l} \\ \bar{A}_{211} & \bar{A}_{212} & D_{21} & D_{22} & \end{bmatrix} = \left[\begin{array}{cc|cc|c|c|c} a & 1.6 & a & 1.4 & 0.3 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.6 & 0.2 \\ \hline 0.14 & 0 & 0.13 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0.5 & 0.4 & \end{array}\right]$$

$$\begin{bmatrix} C_{2l} & L_{2l} & F_{2l} & H_{22l} \end{bmatrix} = \begin{bmatrix} 1 & 0 | & 0.8 & 0 | & 1 | & 0.3 & 0 \end{bmatrix}$$

for the second subsystem.

TABLE 2. Values of minimum $\mathscr{H}_\infty$ performance indexes and controller gains for different cases in Example 2.

Cases without measurement noises	$[\ K_{11}\quad K_{12}\quad K_{21}\quad K_{22}\ ]$	$\gamma_{\min}$
Theorem 1	infeasible	–
Theorem 2	$[\ -2.3393\quad -2.6441\quad -2.3338\quad -1.6800\ ]$	4.2157
Theorem 3	$[\ -4.7888\quad -5.5571\quad -5.1245\quad -3.9754\ ]$	1.5924
Cases with measurement noises	$[\ K_{11}\quad K_{12}\quad K_{21}\quad K_{22}\ ]$	$\gamma_{\min}$
Theorem 1	infeasible	–
Theorem 2	$[\ -2.3796\quad -2.6901\quad -2.3833\quad -1.7163\ ]$	3.2739
Theorem 3	$[\ -6.2099\quad -7.2946\quad -6.8982\quad -5.4341\ ]$	2.2530

TABLE 3. Comparison of minimum $\mathscr{H}_\infty$ performance indexes for different cases of Example 2.

Methods	Cases	$a=0.1$	$a=0.2$	$a=0.3$	$a=0.4$	$a=0.5$	$a=0.6$	$a=0.7$
Theorem 2	$\gamma_{\min}$	3.2739	6.5389	296.7036	∞	∞	∞	∞
Theorem 3	$\gamma_{\min}$	2.2530	2.4718	2.7150	2.9949	3.3296	3.7448	4.2797

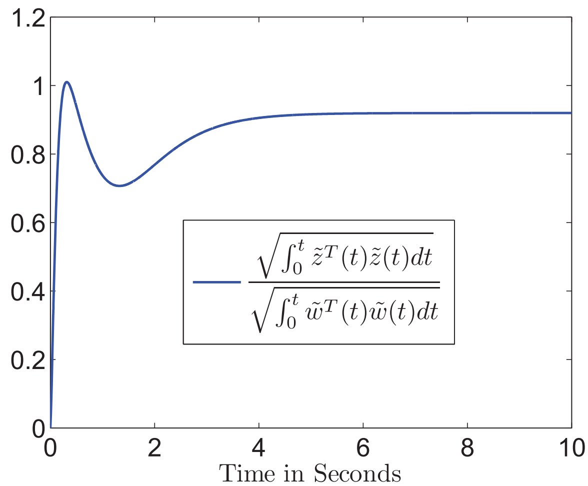

FIGURE 6. Response of $\mathscr{H}_\infty$ performance based on Theorem 3.

It is straightforward to see that these two open-loop subsystems are not asymptotically stable. The objective here is to design a decentralized robust SOF fuzzy controller in the form of (8) such that the closed-loop fuzzy control system is asymptotically stable with an $\mathscr{H}_\infty$ disturbance attenuation level γ. Given $a = 0.1$, $J_1 = J_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $\bar{J}_1 = \bar{J}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, and the measurement noises $M_{il} = 0.1$, $(i, l) = \{1, 2\}$, the values of optimal $\mathscr{H}_\infty$ disturbance attenuation level $\gamma_{\min}$ and the desired controller gains are obtained in Table 2 considering the cases with measurement noises or not, respectively.

For the system with $M_{il} = 0.1$, designating $J_1 = J_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$ and $\bar{J}_1 = \bar{J}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, a more detailed comparison between the obtained minimum $\mathscr{H}_\infty$ performance indexes, which based on different methods, is summarized in Table 3 for different parameters a. It can be seen from Table 3 that Theorem 3 gives much better results than the ones calculated via Theorem 2, which clearly verifies the conjecture given in Remark 3.

V. CONCLUSIONS

In this paper, the robust decentralized SOF $\mathscr{H}_\infty$ fuzzy controller design has been investigated for a class of continuous-time large-scale T-S fuzzy systems. A descriptor system approach by considering different virtual dynamics is adopted to derive the results with less conservatism. Through some matrix inequality linearization techniques, it has been shown that the SOF fuzzy controller gains can be calculated by solving a set of LMIs, and the resulting closed-loop fuzzy control system is asymptotically stable under a prescribed $\mathscr{H}_\infty$ disturbance attenuation level. Two illustrative examples have been provided to verify the effectiveness of the developed methods. An interesting problem for future research is to deal with the robust decentralized SOF control design for large-scale systems with the aid of non-deterministic switched system approach including dwell time switching [38], average dwell time switching [39] and persistent dwell time switching [40].

APPENDIX

Lemma 1 (Jensen's Inequality [41]): For any constant positive semidefinite symmetric matrix $W \in \Re^{n\times n}$, $W^T = W \geq 0$, two positive integers d_2 and d_1 satisfy $d_2 \geq d_1 \geq 1$, the following inequality holds

$$\left(\sum_{k=d_1}^{d_2} x(k)\right)^T W \left(\sum_{k=d_1}^{d_2} x(k)\right) \leq \bar{d} \sum_{k=d_1}^{d_2} x^T(k) Wx(k),$$

where $\bar{d} = d_2 - d_1 + 1$.

Lemma 2 [42]: Let matrices $M = M^T$, S, N, and $\Delta(t)$ be real matrices of appropriate dimensions, the inequality

$$M + \mathrm{Sym}\{S\Delta(t)N\} < 0$$

holds for all $\Delta^T(t)\Delta(t) \leq \mathbf{I}$ if and only if for some positive scalar $\varepsilon > 0$ such that

$$M + \varepsilon SS^T + \varepsilon^{-1}N^T N < 0.$$

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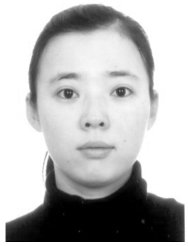
ZHIXIONG ZHONG received the B.Eng. degree in mechatronics and the M.Eng. degree in control theory and control engineering from Fuzhou University, Fuzhou, China, in 2008 and 2012, respectively, the Ph.D. degree in control science and engineering from the Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin, China, in 2015. He joined the School of Electrical Engineering and Automation, Xiamen University of Technology, Xiamen, China. His research interests include fuzzy control and robust filtering and control.

YANZHENG ZHU received the B.S. degree in network engineering from the Shandong University of Science and Technology, Tai'an, China, in 2009, the M.S. degree in pattern recognition and intelligent system from Fuzhou University, Fuzhou, China, in 2012, and the Ph.D. degree in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2016. From 2013 to 2015, he was a Visiting Scholar with the Department of Electrical and Computer Engineering, Ohio State University, Columbus, OH, USA. Since 2016, he has been a Post-Doctoral Researcher with the College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao, China. His current research interests include control and filtering of complex switched systems, networked control systems, and its applications.



TING YANG (M'16) received the B.S. degree in automation from the Tianjin University of Science and Technology, Tianjin, China, in 2009, and the M.S. and Ph.D. degrees in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2011 and 2016, respectively. From 2013 to 2015, she was a Visiting Scholar with the Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, NC, USA. She is currently an Assistant Professor with the School of Automation, Northwest Polytechnical University, Xi'an, China. Her research interests include nondeterministic and stochastic switching systems, fuzzy control systems, model predictive control, and their applications.