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Energy Efficiency Maximization for Device-to-Device Communication Underlaying Cellular Networks on Multiple Bands

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ABSTRACT As green communication becomes an inevitable trend for future 5G wireless networks, how to maximize the energy efficiency (EE) of device-to-device (D2D) communication has drawn extensive attention recently. However, most of existing works only optimize the EE in the single-cell scenario, while little attention is paid to maximizing the EE of the whole cellular network underlaid with D2D communication with randomly distributed users on multiple bands. In this paper, we first consider the whole cellular network underlaid with D2D communication on multiple bands and derive the exact expressions of the successful transmission probabilities, the average sum rate and the EE based on stochastic geometry theory. Then, we formulate the optimization problem of maximizing the EE subject to four constraints regarding to transmission power and outage probabilities, and the non-convexity of this problem is also verified. After that, by exploiting the objective function property of being the sum of several functions, we propose a derivative-based algorithm to solve this non-convex optimization problem. Our theoretical analysis shows that the computational complexity of the proposed algorithm is significantly lower than that of the conventional branch and bound algorithm. Finally, simulation results demonstrate that the proposed algorithm can achieve the near-optimal EE with much better performance than the conventional algorithm.

INDEX TERMS 5G, D2D communication, energy efficiency, stochastic geometry.

I. INTRODUCTION

Device-to-device (D2D) communication is widely recognized as one of the key enablers for 5G wireless networks, in which many future concepts like internet of things and smart cities will come into reality [1]. In D2D communication, the communication between spatially closely located devices can be established directly [2], which can enhance the network throughput, reduce the transmission latency, improve the spectrum efficiency (SE) and the energy efficiency (EE) [3]. However, as D2D communication reuses the frequency resources of existing cellular networks, extra interference will be introduced to the network and impair the communication quality. As a result, a certain part of the total power should be used to mitigate the interference, leading to a reduction of the power used for transmission. Hence, it is crucial to allocate the power appropriately to strike a balance between the interference coordination and the transmission efficiency [4].

A widely used performance indicator in the literature to evaluate the power allocation schemes is the EE [5]–[8]. As more and more attention is paid to green communication [9], the EE maximization of D2D communication has attracted extensive interests recently [10]–[12]. Specifically, the authors in [10] proposed an iterative algorithm to maximize the EE of D2D communication in the single-cell scenario, where multiple cellular users and D2D users are considered. Besides, a distributed resource allocation algorithm was proposed in [11] to make a tradeoff between EE and SE of D2D communication in the uplink single-cell scenario on multiple bands. Furthermore, the authors in [12] considered the D2D communication underlaying cellular networks on multiple bands in a single-cell system, and adopted the branch and bound (BB) algorithm to maximize the EE. However, most of existing works only consider the EE in the single-cell scenario on multiple bands, while little attention is paid to the EE optimization of the whole cellular network underlaid with D2D communication with randomly distributed users on multiple bands. In the whole cellular network underlaid with D2D communication, apart from the interference inside each cell, we also need to coordinate the mutual interference of different cells, which is more practical in future 5G wireless networks, yet more difficult to investigate. In addition, the channel fading coefficients may vary on different bands, therefore, each band will have a different effect on the network performance. Hence, it is indispensable to design an effective solution to optimally allocate the power on different bands so as to maximize the EE of the whole cellular network underlaid with D2D communication on multiple bands.

In this paper, we formulate the EE optimization problem of the whole cellular network underlaid with D2D communication on multiple bands based on stochastic geometry theory, and propose a derivative-based algorithm to maximize the EE with the computational complexity significantly lower than that of the conventional BB algorithm.[1] Specifically, the spatial random distribution of users in the network is modeled as a homogeneous Poisson point process (PPP), from which the successful transmission probabilities, the average sum rate (ASR), and the EE of D2D communication on multiple bands are derived. Then, the optimization problem of maximizing the EE subject to four constraints regarding to transmission power and outage probabilities is formulated, which is proved to be a non-convex problem. To solve this challenging problem, we propose a derivative-based algorithm by exploiting the objective function property of being the sum of several functions. Our theoretical analysis shows that the computational complexity of the derivative-based algorithm is substantially lower than that of the conventional BB algorithm. Simulation results demonstrate that the proposed derivative-based algorithm can achieve the near-optimal EE with remarkably better performance than the conventional BB algorithm.

The rest of this paper is organized as follows. The system model is briefly introduced in Section II. Then in Section III, we derive the exact expressions of the successful transmission probabilities, the ASR, and the EE of D2D communication on multiple bands, based on which the EE optimization problem is also formulated. Section IV presents the proposed derivative-based algorithm to solve the optimization problem in details, together with the computational complexity comparison with the conventional BB algorithm. Simulation results and the corresponding analysis are provided in Section V, followed by the final conclusions in Section VI.

Notation: $\Pr(\cdot)$ denotes the probability; $\Gamma(\cdot)$ stands for the gamma function, i.e., $\Gamma(z) = \int_0^{+\infty} t^{z-1}e^{-t}dt$; $\mathcal{L}_{f(x)}(s)$ represents the Laplace transformation (LT) of $f(x)$, where s is the independent variable of the function we obtain after transformation; Finally, $E(x)$ denotes the expectation of a random variable x.

[1] Simulation codes are provided to reproduce the results presented in this paper: http://oa.ee.tsinghua.edu.cn/dailinglong/.

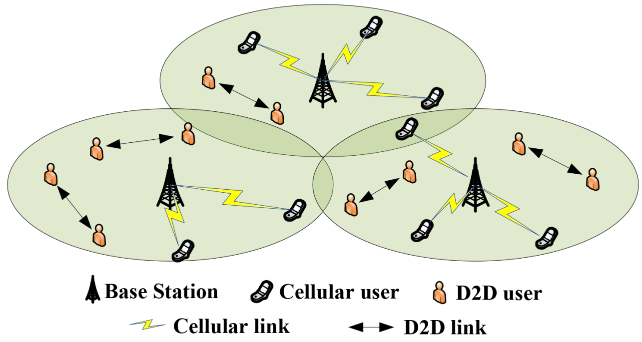

FIGURE 1. System model of the whole cellular network underlaid with D2D communication.

II. SYSTEM MODEL

As illustrated in Fig. 1, in this paper we consider the general scenario that the whole cellular network underlaid with D2D communication, where D2D communication shares the uplink frequency resources of the existing cellular networks. The base station (BS) is in charge of the resource allocation of the whole cellular network underlaid with D2D communication.

Unlike our previous work which only investigated the power allocation problem on a single band [13], here we consider the power allocation on multiple bands. The spectrum of the whole cellular network is divided into K bands, and the bandwidth of the ith band is W_i. In what follows, the subscript i in the variables denotes the ith band and $i = 1, 2, \cdots, K$. Based on stochastic geometry theory, the spatial random distribution of cellular users in the ith band can be modeled as a homogeneous PPP $\Phi_{c,i}$ with density $\lambda_{c,i}$ on the two-dimensional plane $\Re$ [14]. The transmission power of cellular users in the ith band is $P_{c,i}$, and the total transmission power of cellular users is P_c. Hence, we have

$$\sum_{i=1}^{K} P_{c,i} = P_c. \tag{1}$$

Similarly, the spatial random distribution of D2D users in the ith band can also be modeled as a homogeneous PPP $\Phi_{d,i}$ with density $\lambda_{d,i}$ on $\Re$. The transmission power of D2D users in the ith band is $P_{d,i}$, and the total transmission power of D2D users is P_d, then we have

$$\sum_{i=1}^{K} P_{d,i} = P_d. \tag{2}$$

According to Palm theory [15], the typical receiver at the origin does not influence the statistics of the PPP. To analyze the performance of the whole cellular network underlaid with D2D communication, without loss of generality, we can focus on a typical receiver located at the origin of $\Re$, namely a typical BS for cellular uplink transmission or a typical D2D receiver for D2D communication.

By considering both the large-scale path loss and the small-scale Rayleigh fading, the received power P_r for cellular users or D2D users can be expressed as

$$P_r = P_t \delta R^{-\alpha}, \tag{3}$$

where P_t is the transmission power, δ represents the Rayleigh fading coefficient that follows an independent exponential distribution with unit mean for every communication link in the network, R stands for the distance between the transmitter and the receiver, and α denotes the path loss exponent.

III. EE OPTIMIZATION PROBLEM FORMULATION

In this section, we first derive the successful transmission probability (STP) of typical receivers. Then, the exact expressions of the ASR and the EE of D2D communication on multiple bands are also obtained, followed by the formulation of the EE optimization problem.

A. SUCCESSFUL TRANSMISSION PROBABILITY

The typical receiver suffers from the interference introduced by both cellular transmission and D2D communication. Thus, the signal-to-interference plus noise-ratio (SINR) of the typical BS in the ith band is

$$\mathrm{SINR}_{c,i} = \frac{P_{c,i}\delta_{c,00}R_{c,00,i}^{-\alpha}}{\sum_{j\in\Phi_{c,i}} P_{c,i}\delta_{c,j0}R_{c,j0,i}^{-\alpha} + \sum_{l\in\Phi_{d,i}} P_{d,i}\delta_{d,l0}R_{d,l0,i}^{-\alpha} + N_0}, \tag{4}$$

where $\delta_{c,00}$ and $R_{c,00,i}$ denote the Rayleigh fading coefficient and the distance between the typical BS and the corresponding cellular user in the ith band, respectively. Similarly, $\delta_{c,j0}$ and $R_{c,j0,i}$ stand for the Rayleigh fading coefficient and the distance between the jth cellular user and the typical BS in the ith band. $\delta_{d,l0}$ and $R_{d,l0,i}$ are the counterparts for D2D communication. Finally, N_0 is the thermal noise.

Since the interference caused by spectrum sharing is usually much larger than the thermal noise, the SINR in (4) becomes the signal-to-interference-ratio (SIR) as

$$\mathrm{SIR}_{c,i} = \frac{\delta_{c,00}R_{c,00,i}^{-\alpha}}{I_{c,c0,i} + I_{c,d0,i}}, \tag{5}$$

where $I_{c,c0,i} = \sum_{j\in\Phi_{c,i}} \delta_{c,j0}R_{c,j0,i}^{-\alpha}$, $I_{c,d0,i} = \sum_{l\in\Phi_{d,i}} \frac{P_{d,i}}{P_{c,i}}\delta_{d,l0}R_{d,l0,i}^{-\alpha}$. Then, the STP of the typical BS is derived in the following **Lemma 1**.

Lemma 1: The successful transmission probability of the typical BS in the ith band satisfies:

$$\Pr\left(SIR_{c,i} \geq T_{c,i}\right) = \exp\left\{-\varsigma_{c,i}\left[\lambda_{c,i} + \lambda_{d,i}\left(\frac{P_{d,i}}{P_{c,i}}\right)^{\frac{2}{\alpha}}\right]\right\} \tag{6}$$

where $T_{c,i}$ represents the SIR threshold of cellular uplink transmission and $\varsigma_{c,i} = \pi T_{c,i}^{\frac{2}{\alpha}} R_{c,00,i}^2 \Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)$.

Proof: Since $\delta_{c,00}$ follows an independent exponential distribution with unit mean as mentioned in Section II, by considering (5), the STP of the typical BS can be expressed as

$$\begin{aligned}
&\Pr\left(\mathrm{SIR}_{c,i} \geq T_{c,i}\right) \\
&= \Pr\left(\frac{\delta_{c,00}R_{c,00,i}^{-\alpha}}{I_{c,c0,i}+I_{c,d0,i}} \geq T_{c,i}\right) \\
&= \Pr\left[\delta_{c,00} \geq T_{c,i}R_{c,00,i}^{\alpha}\left(I_{c,c0,i}+I_{c,d0,i}\right)\right] \\
&= E\left(\prod_{j\in\Phi_{c,i}} \exp\left(-T_{c,i}R_{c,00,i}^{\alpha}\delta_{c,j0}R_{c,j0,i}^{-\alpha}\right)\right) \\
&\quad \times E\left(\prod_{l\in\Phi_{d,i}} \exp\left(-T_{c,i}R_{c,00,i}^{\alpha}\frac{P_{d,i}}{P_{c,i}}\delta_{d,l0}R_{d,l0,i}^{-\alpha}\right)\right) \\
&= \mathcal{L}_{I_{c,c0,i}(\delta_{c,j0})}\left(T_{c,i}R_{c,00,i}^{\alpha}\right)\mathcal{L}_{I_{c,d0,i}(\delta_{d,l0})}\left(T_{c,i}R_{c,00,i}^{\alpha}\right).
\end{aligned} \tag{7}$$

According to the definition of LT and stochastic geometry theory [14], we have

$$\begin{aligned}
&\mathcal{L}_{I_{c,c0,i}(\delta_{c,j0})}\left(T_{c,i}R_{c,00,i}^{\alpha}\right) \\
&= \exp\left[-\lambda_{c,i}\int_0^{+\infty} E\left(\delta_{c,j0}\right)\left(1-e^{-T_{c,i}R_{c,00,i}^{\alpha}r^{-\alpha}}\right)dr\right] \\
&= \exp\left[-\lambda_{c,i}\pi T_{c,i}^{\frac{2}{\alpha}}R_{c,00,i}^2\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)\right],
\end{aligned} \tag{8}$$

$$\begin{aligned}
&\mathcal{L}_{I_{c,d0,i}(\delta_{d,l0})}\left(T_{c,i}R_{c,00,i}^{\alpha}\right) \\
&= \exp\left[-\lambda_{d,i}\left(\frac{P_{d,i}}{P_{c,i}}\right)^{\frac{2}{\alpha}}\pi T_{c,i}^{\frac{2}{\alpha}}R_{c,00,i}^2\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)\right].
\end{aligned} \tag{9}$$

Let $\varsigma_{c,i} = \pi T_{c,i}^{\frac{2}{\alpha}}R_{c,00,i}^2\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)$ and then substitute (8) and (9) into (7), we can have (6). ■

Lemma 1 reveals how the key network parameters impact the STP of the typical BS. Specifically, if the threshold $T_{c,i}$ increases, the STP decreases because the inequality $\mathrm{SIR}_{c,i} \geq T_{c,i}$ is more difficult to satisfy. In addition, the growth in $R_{c,00,i}$ will result in a reduction in the STP. The reason is that, the channel fading becomes more serious when the distance increases. Besides, the STP increases as the densities of cellular users $\lambda_{c,i}$ or D2D users $\lambda_{d,i}$ become sparser, which can be attributed to the mitigation of interference caused by different users. Furthermore, if we increase $P_{d,i}$, the STP will decrease since the transmission power of D2D communication will introduce interference to cellular transmission. Finally, $\mathrm{SIR}_{c,i}$ will increase if more power is used for cellular transmission, which means the increase in $P_{c,i}$ will lead to a higher STP.

Following the same way of obtaining the SIR of the typical BS in the ith band, namely $\mathrm{SIR}_{c,i}$ in (5), the SIR of the typical D2D receiver in the ith band can be written as

$$\mathrm{SIR}_{d,i} = \frac{\delta_{d,00}R_{d,00,i}^{-\alpha}}{I_{d,c0,i} + I_{d,d0,i}}, \tag{10}$$

where $\delta_{d,00}$ and $R_{d,00,i}$ denote the Rayleigh fading coefficient and the distance between the typical D2D receiver and the corresponding D2D transmitter in the ith band, respectively, $I_{d,c0,i} = \sum_{j\in\Phi_{c,i}} \frac{P_{c,i}}{P_{d,i}}\delta_{c,j0}R_{c,j0,i}^{-\alpha}$ and $I_{d,d0,i} = \sum_{l\in\Phi_{d,i}} \delta_{d,l0}R_{d,l0,i}^{-\alpha}$. Then, we present the STP of the typical D2D receiver in the following **Lemma 2**.

Lemma 2: The successful transmission probability of the typical D2D receiver in the ith band satisfies:

$$\Pr\left(SIR_{d,i} \geq T_{d,i}\right) = \exp\left\{-\varsigma_{d,i}\left[\lambda_{c,i}\left(\frac{P_{c,i}}{P_{d,i}}\right)^{\frac{2}{\alpha}} + \lambda_{d,i}\right]\right\}, \tag{11}$$

where $T_{d,i}$ represents the SIR threshold of D2D communication, and $\varsigma_{d,i} = \pi T_{d,i}^{\frac{2}{\alpha}} R_{d,00,i}^2 \Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)$.

Proof: The proof is similar to the proof of **Lemma 1**. ■

According to **Lemma 2**, the STP of the typical D2D receiver is influenced by the key network parameters. Particularly, if we decrease $P_{d,i}$ and increase $T_{d,i}$, $R_{d,00,i}$, $\lambda_{c,i}$, $\lambda_{d,i}$, and $P_{c,i}$, then the STP will decrease, which can be explained by similar reasons mentioned before.

Up to now, we have completed the derivation of the STP of typical receivers, which is essential to the formulation of the optimization problem. In the next subsection, we discuss another two important network performance indicators, namely the ASR and the EE.

B. AVERAGE SUM RATE AND ENERGY EFFICIENCY OF D2D COMMUNICATION

Let $R_{d,i}$ denote the average rate of D2D communication in the ith band, if we obtain the value of the SIR threshold $T_{d,i}$, then we have [16]

$$R_{d,i} = W_i \log_2\left(1+T_{d,i}\right)\Pr\left(\text{SIR}_{d,i} \geq T_{d,i}\right). \tag{12}$$

According to **Lemma 2**, (12) can be written as

$$R_{d,i} = W_i \log_2(1+T_{d,i})\exp\left\{-\varsigma_{d,i}\left[\lambda_{c,i}\left(\frac{P_{c,i}}{P_{d,i}}\right)^{\frac{2}{\alpha}} + \lambda_{d,i}\right]\right\}. \tag{13}$$

Thus, the ASR of D2D communication in the ith band is

$$\begin{aligned}\text{ASR}_{d,i} &= \lambda_{d,i}R_{d,i} \\ &= \lambda_{d,i}W_i\log_2(1+T_{d,i}) \\ &\quad\times\exp\left\{-\varsigma_{d,i}\left[\lambda_{c,i}\left(\frac{P_{c,i}}{P_{d,i}}\right)^{\frac{2}{\alpha}} + \lambda_{d,i}\right]\right\}.\end{aligned} \tag{14}$$

The EE is defined as the ASR divided by the total power consumption [17]. Here we consider the power consumed per unit area for D2D communication in the ith band, which can be expressed as $\lambda_{d,i}P_{d,i}$ [18]. Accordingly, the EE of D2D communication in the ith band can be defined as

$$\begin{aligned}\text{EE}_{d,i} &= \frac{\text{ASR}_{d,i}}{\lambda_{d,i}P_{d,i}} \\ &= \frac{W_i}{P_{d,i}}\log_2\left(1+T_{d,i}\right) \\ &\quad\times\exp\left\{-\varsigma_{d,i}\left[\lambda_{c,i}\left(\frac{P_{c,i}}{P_{d,i}}\right)^{\frac{2}{\alpha}} + \lambda_{d,i}\right]\right\},\end{aligned} \tag{15}$$

and the total EE of D2D communication is

$$\text{EE}_d = \sum_{i=1}^{K}\text{EE}_{d,i}. \tag{16}$$

The total EE of D2D communication is the objective function of our optimization problem, which is formulated in the next subsection.

C. EE OPTIMIZATION PROBLEM

In this subsection, we discuss the constraints to formulate the EE optimization problem, and proved that the EE optimization problem is non-convex.

To ensure the high quality of communication, the outage probabilities of cellular transmission and D2D communication should be less than certain thresholds, i.e.,

$$1-\Pr\left(\text{SIR}_{c,i} \geq T_{c,i}\right) \leq \theta_{c,i}, \tag{17}$$

$$1-\Pr\left(\text{SIR}_{d,i} \geq T_{d,i}\right) \leq \theta_{d,i}, \tag{18}$$

where $\theta_{c,i}$ and $\theta_{d,i}$ represent the outage thresholds of cellular transmission and D2D communication in the ith band, respectively. It should be noted that, if (17) or (18) cannot hold for any of the $P_{d,i}$ in the domain determined by other constraints, then this means that the interference in the network is too severe to be coordinated. Under such circumstance, the BS will reduce the number of D2D users that are permitted to access the network until the outage probabilities are sufficiently small.

For D2D communication, the sum of the power of all bands should equal the total transmission power, which is determined by D2D terminals. Thus, (2) should be satisfied.

Besides, the power in the ith band should not be less than zero or exceed the upper bound of the power of that band, which is denoted as $P_{d,i,\text{up}}$. Hence, we have

$$0 \leq P_{d,i} \leq P_{d,i,\text{up}}. \tag{19}$$

The ultimate goal of the optimal resource allocation scheme is to maximize EE_d in (16) with respect to $P_{d,i}$ subject to constraints (2), (17), (18), (19), which can be formulated as the following optimization problem

$$\begin{aligned}&\max_{P_{d,i}}\ \text{EE}_d = \sum_{i=1}^{K}\text{EE}_{d,i} \\ &\text{s.t. } (2), (17), (18), (19).\end{aligned} \tag{20}$$

However, the constraints in (20) are complicated, rendering the optimization problem intractable. In what follows, we will transform the inequality constraints in (20) into the feasible regions of $P_{d,i}$ to simplify the optimization problem.

Based on (6), (11), (17) and (18), we have

$$P_{d,i} \geq P_{c,i}\left(\frac{-\ln\left(1-\theta_{d,i}\right)}{\lambda_{c,i}\varsigma_{d,i}} - \frac{\lambda_{d,i}}{\lambda_{c,i}}\right)^{-\frac{\alpha}{2}}, \tag{21}$$

$$P_{d,i} \leq P_{c,i}\left(\frac{-\ln\left(1-\theta_{c,i}\right)}{\lambda_{d,i}\varsigma_{c,i}} - \frac{\lambda_{c,i}}{\lambda_{d,i}}\right)^{\frac{\alpha}{2}}. \tag{22}$$

Let $P_{d,i,\text{low}} = P_{c,i}\left(\frac{-\ln(1-\theta_{d,i})}{\lambda_{c,i}\varsigma_{d,i}} - \frac{\lambda_{d,i}}{\lambda_{c,i}}\right)^{-\frac{\alpha}{2}}$ and $P_{d,i,\text{high}} = P_{c,i}\left(\frac{-\ln(1-\theta_{c,i})}{\lambda_{d,i}\varsigma_{c,i}} - \frac{\lambda_{c,i}}{\lambda_{d,i}}\right)^{\frac{\alpha}{2}}$, then (19), (21) and (22) actually determine the feasible region of $P_{d,i}$. If the lower bound and upper bound of the feasible region of $P_{d,i}$ are denoted as $P_{d,i,\inf}$ and $P_{d,i,\sup}$, respectively, we have $P_{d,i,\inf} = \max\left\{0, P_{d,i,\text{low}}\right\}$ and $P_{d,i,\sup} = \min\left\{P_{d,i,\text{up}}, P_{d,i,\text{high}}\right\}$. Thus, (20) can be transformed into the following form

$$\max_{P_{d,i}} \ \text{EE}_d = \sum_{i=1}^{K} \text{EE}_{d,i}$$
$$\text{s.t.} \begin{cases} P_{d,i,\inf} \leq P_{d,i} \leq P_{d,i,\sup}, \\ \sum\limits_{i=1}^{K} P_{d,i} = P_d. \end{cases} \tag{23}$$

This completes the optimization problem formulation of maximizing the EE of the whole cellular network underlaid with D2D communication, which differs significantly from the previous works that consider the EE in the single-cell scenario [10]–[12].

In our previous work [13], where only a single band is considered in the optimization problem, the objective function is convex, therefore, the problem can be solved by convex optimization theory. However, the objective function EE_d in (23) is non-convex, which is verified below.

Let $A_i = W_i\log_2\left(1+T_{d,i}\right)\exp\left(-\varsigma_{d,i}\lambda_{d,i}\right)$ and $B_i = \varsigma_{d,i}\lambda_{c,i}\left(P_{c,i}\right)^{\frac{2}{\alpha}}$, then we can rewrite $\text{EE}_{d,i}$ in (15) as

$$\text{EE}_{d,i} \triangleq f_i\left(P_{d,i}\right) = \frac{A_i}{P_{d,i}}\exp\left(-B_i\left(\frac{1}{P_{d,i}}\right)^{\frac{2}{\alpha}}\right). \tag{24}$$

Then, we present the intervals on which $f_i\left(P_{d,i}\right)$ is convex or concave in the following **Lemma 3**.

Lemma 3: $f_i\left(P_{d,i}\right)$ is convex on the interval $\left(0, t_{1,i}^{\frac{\alpha}{2}}\right) \cup \left(t_{2,i}^{\frac{\alpha}{2}}, +\infty\right)$ but concave on the interval $\left(t_{1,i}^{\frac{\alpha}{2}}, t_{2,i}^{\frac{\alpha}{2}}\right)$, where

$$t_{1,i} = \frac{B_i}{2\alpha^2}\left(2+3\alpha-\sqrt{\left(\alpha^2+12\alpha+4\right)}\right),$$
$$t_{2,i} = \frac{B_i}{2\alpha^2}\left(2+3\alpha+\sqrt{\left(\alpha^2+12\alpha+4\right)}\right).$$

Proof: The proof is given in Appendix A. ■

It should be noted that the standard form of convex optimization problems is minimizing a convex function, which is equivalent to maximizing a concave function. To optimize the problem based on convex optimization theory, the objective function EE_d in (23) needs to be concave on the feasible region, i.e., according to **Lemma 3**, for $i = 1, 2, \cdots, K$, $\left(P_{d,i,\inf}, P_{d,i,\sup}\right)$ should be the subinterval of $\left(t_{1,i}^{\frac{\alpha}{2}}, t_{2,i}^{\frac{\alpha}{2}}\right)$, which is not true in general. In fact, if we set the aforementioned network parameters as the typical values in practical wireless networks (see Table 2 in Section V), $t_{2,i}^{\frac{\alpha}{2}}$ is just slightly greater than zero, which means the length of the interval on which $f_i\left(P_{d,i}\right)$ is concave is negligible. Hence, unlike the optimization problem in our previous work where only a single band is considered [13], the new optimization problem (23) considering multiple bands is non-convex, which cannot be solved by convex optimization theory. In view of this, we propose a derivative-based algorithm to solve (23) in the next section.

IV. PROPOSED DERIVATIVE-BASED ALGORITHM

In this section, we describe and compare two algorithms that can be used to solve non-convex optimization problems. Specifically, we first briefly describe the implementation process of the conventional BB algorithm. Then, the derivative-based algorithm is proposed to solve the non-convex EE optimization problem (23) in Section III. Finally, we analyze and compare the computational complexity of these two algorithms.

A. CONVENTIONAL BRANCH AND BOUND ALGORITHM

A conventional algorithm commonly used to solve non-convex optimization problems is the BB algorithm [19], which has also been widely adopted to solve some challenging optimization problems in wireless communication networks [12], [20]. The BB algorithm can be essentially perceived to be an improved version of the exhaustive enumeration method, where the candidate solutions to the problem are enumerated systematically in order to find the optimal solution that maximize the objective function. Specifically, we can interpret the set of candidate solutions as a rooted tree, where the root and the branches represent the full set and subsets of the solution set, respectively. All branches of the tree are explored and before enumerating the candidate solutions of a branch, we estimate the upper bound of the objective function in this branch. If the upper bound is not greater than the best function value found so far, then this branch is discarded, namely pruned from the search space. After all branches are explored, the solution that yields the maximum value is regarded as the final optimal solution.

Obviously, whether a branch will be pruned or not is not predictable. For instance, if the optimal solution is acquired in the first branch, then all the unexplored branches will be pruned, thus reducing computational complexity significantly. However, most of the branches will not be pruned if the optimal solution is in the last branch, which means the computational complexity may approach that of the exhaustive enumeration method in this case. Hence, the computational complexity of the BB algorithm is not fixed, and even worse, the BB algorithm may degenerate into the exhaustive enumeration method. In view of the limited performance of the conventional BB algorithm, we propose another algorithm, namely the derivative-based algorithm in the next subsection.

B. PROPOSED DERIVATIVE-BASED ALGORITHM

To solve (23), we propose a derivative-based algorithm by exploiting the objective function property of being the sum of several functions.

Let $P_{d,i,\max}$ denote the global maximum point of $f_i\left(P_{d,i}\right)$ on the interval $\left[P_{d,i,\inf}, P_{d,i,\sup}\right]$, i.e., $f_i\left(P_{d,i}\right)$ achieves the maximum value when $P_{d,i} = P_{d,i,\max}$. From (16), we know that EE_d is the sum of K functions. If the equality constraint in (23), i.e., $\sum_{i=1}^{K} P_{d,i} = P_d$ is removed, then every $P_{d,i}$ is mutually independent, therefore, the maximum value of EE_d can be obtained when every $\mathrm{EE}_{d,i}$ achieves its maximum value on the feasible region. Apparently, maximizing every $\mathrm{EE}_{d,i}$ individually is much more tractable than solving (23). Thus, we can calculate $P_{d,i,\max}$ for $i = 1, 2, \cdots, K$ and let $P_{d,i} = P_{d,i,\max}$ in the first place, and then adjust the value of $P_{d,i}$ so as to meet the equality constraint $\sum_{i=1}^{K} P_{d,i} = P_d$ in a way that causes the least reduction in EE_d. This is the core idea of our proposed algorithm, and the implementation details are stated below.

Firstly, the calculation of $P_{d,i,\max}$ is given by the following **Theorem 1**.

Theorem 1: The global maximum point of $f_i\left(P_{d,i}\right)$ *on the feasible region* $\left[P_{d,i,\inf}, P_{d,i,\sup}\right]$ *is*

$$P_{d,i,\max} = \begin{cases} P_{d,i,\sup}, & P_{d,i,\sup} \le \left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}, \\ \left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}, & P_{d,i,\inf} < \left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}} < P_{d,i,\sup}, \\ P_{d,i,\inf}, & P_{d,i,\inf} \ge \left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}. \end{cases} \quad (25)$$

Proof: The proof is given in Appendix B. ■

From **Theorem 1**, we know that $P_{d,i,\max}$ is determined by the relationship between $\left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}$ and the feasible region $\left[P_{d,i,\inf}, P_{d,i,\sup}\right]$ of $f_i\left(P_{d,i}\right)$.

In what follows, the current value of $P_{d,i}$ is denoted by $P_{d,i,\mathrm{cur}}$. Consider the method we adopt to adjust the value of $P_{d,i}$ after assigning $P_{d,i,\max}$ to $P_{d,i}$. Let $d = P_d - \sum_{i=1}^{K} P_{d,i,\max}$, $\Delta = \frac{d}{n}$, where Δ is the adjustment step of $P_{d,i}$, and n is the parameter that controls Δ. Since the adjustment commences at the global maximum point of $f_i\left(P_{d,i}\right)$ on the feasible region, the adjustment process will certainly make $P_{d,i}$ deviate from $P_{d,i,\max}$, i.e., we have

$$f_i\left(P_{d,i,\mathrm{cur}}\right) > f_i\left(P_{d,i,\mathrm{cur}} + \Delta\right). \quad (26)$$

To meet the equality constraint while keeping the reduction in EE_d as little as possible, we need to adjust the $P_{d,i}$ whose function value decreases the least after adjustment. According to Taylor's theorem [21], the approximation of $f_i\left(P_{d,i}\right)$ at $P_{d,i} = P_{d,i,\mathrm{cur}}$ by the first order Taylor polynomial is

$$f_i\left(P_{d,i,\mathrm{cur}} + \Delta\right) \approx f_i\left(P_{d,i,\mathrm{cur}}\right) + f_i'\left(P_{d,i,\mathrm{cur}}\right)\Delta. \quad (27)$$

If we adjust the value of $P_{d,i}$ from $P_{d,i,\mathrm{cur}}$ to $P_{d,i,\mathrm{cur}} + \Delta$, EE_d will decrease by $\left|f_i\left(P_{d,i,\mathrm{cur}}\right) - f_i\left(P_{d,i,\mathrm{cur}} + \Delta\right)\right|$, which is mainly determined by $\left|f_i'\left(P_{d,i,\mathrm{cur}}\right)\right|$ according to (27). Hence, we can calculate $\left|f_i'\left(P_{d,i,\mathrm{cur}}\right)\right|$ for $i = 1, 2, \cdots, K$ and adjust the value of $P_{d,j}$ from $P_{d,j,\mathrm{cur}}$ to $P_{d,j,\mathrm{cur}} + \Delta$, where j satisfies

$$\left|f_j'\left(P_{d,j,\mathrm{cur}}\right)\right| \le \left|f_i'\left(P_{d,i,\mathrm{cur}}\right)\right|, \quad \forall i = 1, 2, \cdots, K. \quad (28)$$

Repeat such process for at least n times, then the equality constraint $\sum_{i=1}^{K} P_{d,i} = P_d$ can be satisfied, and the near-optimal solution to (23) is obtained.

Based on the aforementioned analysis, the proposed derivative-based algorithm is shown in **Algorithm 1**, and we explain several key steps as follows.

Algorithm 1 The Proposed Derivative-Based Algorithm

Input: K; W_i; α; $\theta_{c,i}$; $\theta_{d,i}$; $T_{c,i}$; $T_{d,i}$; $R_{c,00,i}$; $R_{d,00,i}$; $\lambda_{c,i}$; $\lambda_{d,i}$; $P_{c,i}$; $P_{d,i,\mathrm{up}}$; P_d.

Output: EE_d.

1: Initialize the tolerance ε that controls the loop;
2: Calculate $P_{d,i,\max}$ based on **Theorem 1**;
3: $P_{d,i} = P_{d,i,\max}$;
4: $d = P_d - \sum_{i=1}^{K} P_{d,i}$;
5: $\Delta = \frac{d}{n}$;
6: $der_i = \left|f_i'\left(P_{d,i}\right)\right|$;
7: **while** $\left|P_d - \sum_{i=1}^{K} P_{d,i}\right| \ge \varepsilon$ **do**
8:     $j = \arg\min_i \{der_i\}$;
9:     **if** $P_{d,j} + \Delta > P_{d,j,\sup}$ **or** $P_{d,j} + \Delta < P_{d,j,\inf}$ **then**
10:         $der_j = +\infty$;
11:     **else**
12:         $P_{d,j} = P_{d,j} + \Delta$;
13:         $der_j = \left|f_j'\left(P_{d,j}\right)\right|$;
14:     **end if**
15: **end while**
16: **return** $\mathrm{EE}_d = \sum_{i=1}^{K} f_i\left(P_{d,i}\right)$.

In step 5, n is a parameter that controls the balance between computational complexity and the performance, to which we can assign a suitable value in accordance with the practical requirement.

In step 6, we set a variable der_i to save the value of $\left|f_i'\left(P_{d,i}\right)\right|$. This variable is used for selecting the appropriate $P_{d,j}$ in step 8, and may be updated in step 10 or step 13 in every iteration.

In step 7, if the $P_{d,j}$ selected in the current iteration will exceed the feasible region after adjustment, then we need to choose another $P_{d,i}$ to adjust after this iteration. Since whether this situation will happen or not is unpredictable, the number of iterations is not determined either. Thus, we set a tolerance threshold ε instead of a counter to decide when to exit the loop.

In steps 9 and 10, if $P_{d,j}$ will overstep the feasible region after adjustment, then we set der_j to infinite so that j will not be chosen in step 8 again. By doing so, we can preclude the occurrence of an endless loop.

In step 13, we update der_j instead of every der_i because for $i = 1, 2, \cdots, K$ and $i \neq j$, der_i remains unchanged after adjusting $P_{d,j}$.

C. COMPUTATIONAL COMPLEXITY ANALYSIS

In this subsection, we compare the computational complexity of the conventional BB algorithm and the proposed derivative-based algorithm.

As mentioned before, the BB algorithm can be regarded as an improved version of the exhaustive enumeration method, therefore, we will investigate the computational complexity of the exhaustive enumeration method in the first place. Let Δ_{BB} denote the step length of each loop. Then in the ith band whose bandwidth is W_i, we need to enumerate $\frac{W_i}{\Delta_{\mathrm{BB}}}$ candidate solutions. Hence, the total computational complexity of the exhaustive enumeration method is the product of the computational complexity of every band, namely $\mathcal{O}\left(\prod_{i=1}^{K-1} W_i\left(\frac{1}{\Delta_{\mathrm{BB}}}\right)^{K-1}\right)$. It should be noted that the exponent on $\frac{1}{\Delta_{\mathrm{BB}}}$ is $K-1$ instead of K because according to the equality constraint in (23), only $K-1$ variables in (23) are mutually independent. For the BB algorithm, some of the branches are pruned from the search space based on the estimation of the upper bound. However, the exact number of the discarded branches is not fixed, so the computational complexity of the BB algorithm cannot be determined accurately, which can be expressed approximately as $\mathcal{O}\left(\beta\prod_{i=1}^{K-1} W_i\left(\frac{1}{\Delta_{\mathrm{BB}}}\right)^{K-1}\right)$, where factor β is a positive number. In the worst case, the BB algorithm enumerates candidate solutions in all branches and degenerates into the exhaustive enumeration method, whose computational complexity is unbearably high.

Next, we analyze the computational complexity of the proposed derivative-based algorithm. From steps 9 and 10 in **Algorithm 1**, we know that if the $P_{d,j}$ selected in the current iteration will exceed the feasible region after adjustment, then der_j is set to infinite, which means that this $P_{d,j}$ will not be chosen again, since we always select the j with the minimum der_j in step 8. As a result, for a particular j, the situation that the currently selected $P_{d,j}$ cannot be adjusted because of the restriction of the feasible region, will happen at most once. Hence, in the best case, i.e., all the $P_{d,j}$ selected in the iterations can be adjusted, after adjusting $P_{d,i}$ for n times, the equality constraint in (23) will be satisfied, so the computational complexity is $\mathcal{O}(n)$. In the worst case, all of the selected $P_{d,j}$ reach the boundaries of the feasible regions, therefore, apart from the necessary n iterations for adjustment, there are K iterations in which we do nothing except set the corresponding der_j to infinite. Thus, the computational complexity is $\mathcal{O}(n+K)$. It is worth pointing out that K is negligible compared with n in general, i.e., $n+K \approx n$, so the computational complexity of the proposed derivative-based algorithm is $\mathcal{O}(n)$ in the general case.

For fair comparison of the computational complexity, we set identical adjustment step for these two algorithms, i.e., $\Delta_{\mathrm{BB}} = \Delta = \frac{d}{n}$. Consequently, the computational complexity of the BB algorithm becomes $\mathcal{O}\left(\mu n^{K-1}\right)$, where $\mu = \beta\prod_{i=1}^{K-1} W_i\left(\frac{1}{d}\right)^{K-1}$.

Table 1 compares the computational complexity of the conventional BB algorithm and the proposed derivative-based algorithm, where we set $K = 5$ and $\mu = 5 \times 10^{-4}$ as a typical example. We can see that the proposed derivative-based algorithm has significantly lower computational complexity than the conventional BB algorithm. Besides, it is also noticeable that as n increases, the computational complexity of the conventional BB algorithm increases exponentially, while that of the proposed derivative-based algorithm only increases linearly.

TABLE 1. The comparison of computational complexity.

Iterative number n	Conventional BB algorithm $\mathcal{O}\left(\mu n^{K-1}\right)$	Proposed derivative-based algorithm $\mathcal{O}(n)$
100	5×10^4	100
500	3.125×10^7	500
1000	5×10^8	1000
5000	3.125×10^{11}	5000
10000	5×10^{12}	10000

TABLE 2. Simulation parameters.

Parameter	Value
K	5
W_i	20MHz
α	4
$\theta_{c,i}$	0.1
$\theta_{d,i}$	0.1
$T_{c,i}$	0dB
$T_{d,i}$	0dB
$[R_{c,00,1}, R_{c,00,2}, \cdots, R_{c,00,5}]$	$[50, 60, 70, 80, 90]$ m
$[R_{d,00,1}, R_{d,00,2}, \cdots, R_{d,00,5}]$	$[10, 20, 30, 20, 10]$ m
$[\lambda_{c,1}, \lambda_{c,2}, \cdots, \lambda_{c,5}]$	$[10, 1, 10, 10, 10] \times 10^{-5}$user/m^2
$[\lambda_{d,1}, \lambda_{d,2}, \cdots, \lambda_{d,5}]$	$[10, 1, 10, 10, 10] \times 10^{-4}$user/m^2
$P_{c,i}$	100mW
P_d	60mW
$P_{d,i,\mathrm{up}}$	20mW
ε	1×10^{-3}

V. SIMULATION RESULTS

In this section, we investigate the EE performance of the derivative-based algorithm as well as the BB algorithm. The EE of D2D communication under different network parameters is also obtained and analyzed. The main simulation parameters, including the bandwidth of the ith band W_i, the total transmission power of D2D users P_d, are given in Table 2 [10].

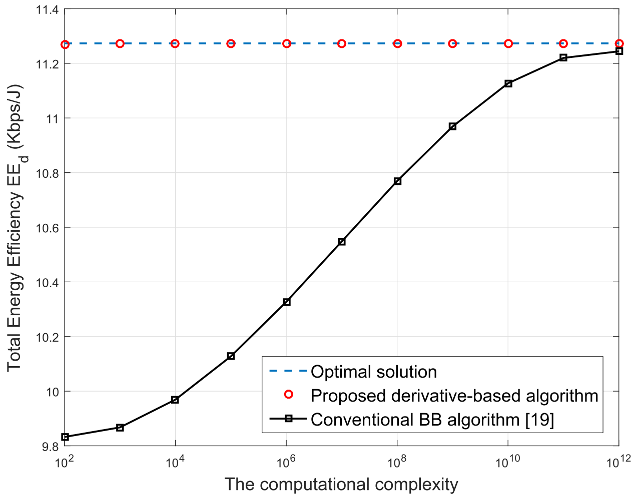

FIGURE 2. The EE of D2D communication achieved by different algorithms.

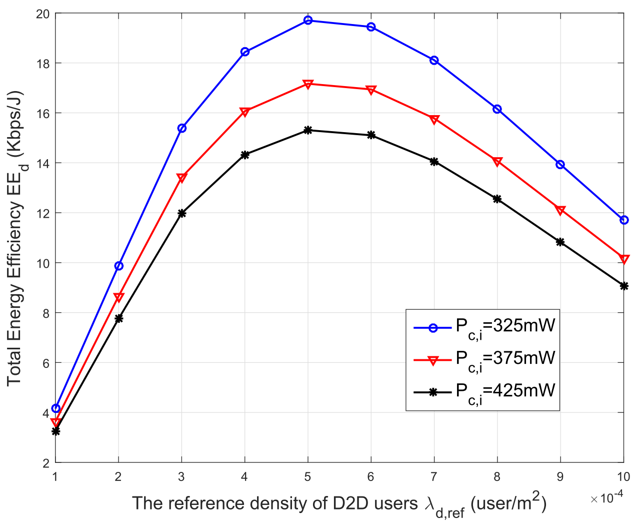

FIGURE 3. The EE of D2D communication against the reference density of D2D users $\lambda_{d,\mathrm{ref}}$.

Fig. 2 shows the EE performance comparison of different algorithms. The dash line denotes the EE performance obtained by the optimal solution (the exhaustive enumeration method), which serves as a benchmark for comparison. We can see that the EE achieved by the proposed derivative-based algorithm is almost identical with the optimal solution, while a significant performance gap exists between the conventional BB algorithm and the optimal solution. Thus, we can conclude that the proposed derivative-based algorithm is near-optimal and remarkably outperforms the conventional BB algorithm.

Fig. 3 demonstrates the EE under different densities of D2D users and transmission power of cellular users achieved by the proposed derivative-based algorithm, where we set $\left[\lambda_{d,1}, \lambda_{d,2}, \cdots, \lambda_{d,5}\right]$=$\lambda_{d,\mathrm{ref}} \times$ [10,1,10,10,10]. We can find that the EE rises at first and then declines as $\lambda_{d,\mathrm{ref}}$ increases. The reason for this phenomenon is that when $\lambda_{d,\mathrm{ref}}$ is relatively small, the interference caused by spectrum sharing is

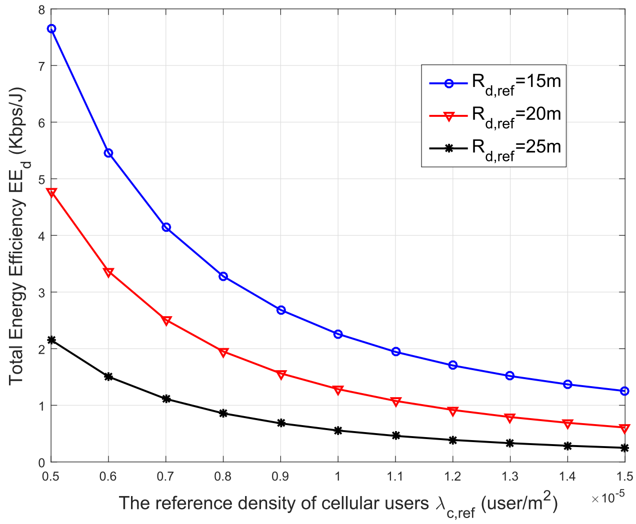

FIGURE 4. The EE of D2D communication against the reference density of cellular users $\lambda_{c,\mathrm{ref}}$.

slight. Thus, if $\lambda_{d,\mathrm{ref}}$ increases, compared with the increase in the ASR, the growth of interference is insignificant, which results in a higher EE. However, the interference becomes more and more serious as $\lambda_{d,\mathrm{ref}}$ continues increasing, which means more energy will be consumed to coordinate the interference, leading to a decrease in the EE. In addition, the tendency that the EE declines as the transmission power of cellular users $P_{c,i}$ increases is also revealed in Fig. 3, which can be attributed to the growing interference caused by cellular transmission. The increase in $P_{c,i}$ leads to more serious interference to D2D communication. Consequently, the EE decreases because more power is used to coordinate the interference.

Fig. 4 illustrates the EE under different densities of cellular users and distances of D2D users obtained by the proposed derivative-based algorithm. The simulation parameters are set as: $\left[R_{d,00,1}, R_{d,00,2}, \cdots, R_{d,00,5}\right] = R_{d,\mathrm{ref}} \times [1,2,3,2,1]$, $\left[\lambda_{c,1}, \lambda_{c,2}, \cdots, \lambda_{c,5}\right] = \lambda_{c,\mathrm{ref}} \times [10, 1, 10, 10, 10]$, $P_{c,i} =$ 300mW, $P_d =$ 80mW. It can be seen that the EE decreases as $\lambda_{c,\mathrm{ref}}$ increases, which is a consequence of the exacerbation of interference introduced by cellular transmission. More power consumed for D2D communication will be used to coordinate the interference caused by the growing numbers of cellular users, which results in a reduction in the EE. Another noticeable trend in Fig. 4 is that as the reference distance of D2D users $R_{d,\mathrm{ref}}$ increases, a decrease in the EE can be observed. According to (3), we know that the channel fading becomes more serious as the distance increases, leading to a decrease in $\mathrm{SIR}_{d,i}$. Thus, the ASR decreases, which results in the decrease in the EE.

VI. CONCLUSIONS

In this paper, we have proposed a derivative-based algorithm to maximize the EE of the whole cellular network underlaid with D2D communication on multiple bands. Particularly, the performance of the whole cellular network underlaid with

D2D communication has been analyzed at first based on stochastic geometry theory, where the exact expressions of the successful transmission probabilities, the ASR, and the EE of D2D communication on multiple bands have been derived. Then, we have formulated the optimization problem of maximizing the EE and proved that the corresponding objective function is non-convex. After that, by utilizing the objective function property of being the sum of several functions, we have proposed a derivative-based algorithm to iteratively achieve the near-optimal solution to this non-convex EE optimization problem. We have shown that the computational complexity of the proposed derivative-based algorithm is significantly lower than that of the conventional BB algorithm. Simulation results have verified the near-optimal performance of the proposed algorithm, which is conducive to the realization of energy-efficient D2D communication in future 5G wireless networks. Particularly, we will investigate the optimization of both the SE and the EE of the whole cellular network underlaid with D2D communication on multiple bands in our future works.

APPENDIX A
PROOF OF LEMMA 3

To find the intervals on which $f_i\left(P_{d,i}\right)$ is convex or concave, we need to find the interval on which $\frac{d^2 f_i(P_{d,i})}{dP_{d,i}^2}$ is non-negative or non-positive [22]. Take the second derivative of $f_i\left(P_{d,i}\right)$ as shown in (24), we have

$$\frac{d^2 f_i\left(P_{d,i}\right)}{dP_{d,i}^2} = 2A_i \exp\left(-B_i\left(\frac{1}{P_{d,i}}\right)^{\frac{2}{\alpha}}\right)\left(\frac{1}{P_{d,i}}\right)^{3+\frac{4}{\alpha}} \times\left(P_{d,i}^{\frac{4}{\alpha}} - \frac{B_i}{\alpha}\left(\frac{2}{\alpha}+3\right)P_{d,i}^{\frac{2}{\alpha}} + \frac{2B_i^2}{\alpha^2}\right). \quad (29)$$

In (29), all terms except for the last one are greater than zero, which means that we only need to consider the last term. Let $t_i = P_{d,i}^{\frac{2}{\alpha}}$, then the last term in (29) can be expressed as

$$g_i\left(t_i\right) = t_i^2 - \frac{B_i}{\alpha}\left(\frac{2}{\alpha}+3\right)t_i + \frac{2B_i^2}{\alpha^2}. \quad (30)$$

Let $t_{1,i}$ and $t_{2,i}$ denote the solutions to $g_i\left(t_i\right)=0$, where $t_{1,i} < t_{2,i}$, we have

$$t_{1,2,i} = \frac{B_i}{2\alpha^2}\left(2+3\alpha \pm \sqrt{\left(\alpha^2+12\alpha+4\right)}\right). \quad (31)$$

Thus, $g_i\left(t_i\right)$ is positive on the interval $\left(0, t_{1,i}\right)\cup\left(t_{2,i}, +\infty\right)$ and negative on the interval $\left(t_{1,i}, t_{2,i}\right)$. Then, $\frac{d^2 f_i(P_{d,i})}{dP_{d,i}^2}$ is positive on the interval $\left(0, t_{1,i}^{\frac{\alpha}{2}}\right) \cup \left(t_{2,i}^{\frac{\alpha}{2}}, +\infty\right)$ and negative on the interval $\left(t_{1,i}^{\frac{\alpha}{2}}, t_{2,i}^{\frac{\alpha}{2}}\right)$. Hence, $f_i\left(P_{d,i}\right)$ is convex or concave on the corresponding intervals, and this completes the proof of **Lemma 3**.

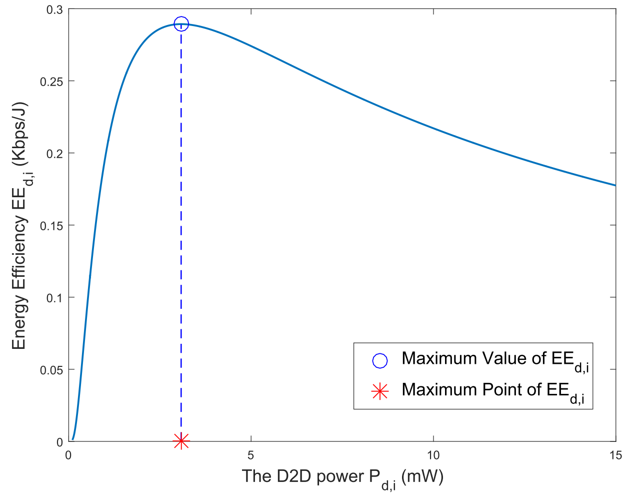

FIGURE 5. $\mathrm{EE}_{d,i}$ **against** $P_{d,i}$.

APPENDIX B
PROOF OF THEOREM 1

Consider the monotone intervals of $f_i\left(P_{d,i}\right)$. Take the derivative of $f_i\left(P_{d,i}\right)$, we have

$$\frac{df_i\left(P_{d,i}\right)}{dP_{d,i}} = \frac{A_i}{P_{d,i}^2}\exp\left(-B_i\left(\frac{1}{P_{d,i}}\right)^{\frac{2}{\alpha}}\right) \times\left(\frac{2B_i}{\alpha}\left(\frac{1}{P_{d,i}}\right)^{\frac{2}{\alpha}} - 1\right). \quad (32)$$

Apparently, the first and second term of (32) are greater than zero, therefore, we only need to consider the last term, from which we know that $\frac{df_i(P_{d,i})}{dP_{d,i}}$ is positive on the interval $\left(0, \left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}\right)$ and negative on the interval $\left(\left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}, +\infty\right)$. Hence, $f_i\left(P_{d,i}\right)$ increases monotonically on the interval $\left(0, \left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}\right)$, decreases monotonically on the interval $\left(\left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}, +\infty\right)$, and reaches the global maximum value at $P_{d,i} = \left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}$. To verify the aforementioned properties of $f_i\left(P_{d,i}\right)$, we plot the curve of $f_i\left(P_{d,i}\right)$ in Fig. 5.

As shown in Fig. 5, we can see that $f_i\left(P_{d,i}\right)$ rises at first and then declines as $P_{d,i}$ increases, and the demarcation point is the maximum point of $f_i\left(P_{d,i}\right)$, namely $\left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}$, which is denoted by the asterisk. Since the monotone intervals obtained by calculating derivatives agree with the curve of $f_i\left(P_{d,i}\right)$, the correctness of our analysis above is proved. Then, the situation can be divided into the following 3 cases according to the relationship between $\left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}$ and the feasible region of $f_i\left(P_{d,i}\right)$.

Firstly, if $P_{d,i,\sup} \leq \left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}$, which means the feasible region is on the left of the dash line in Fig. 5, then $f_i\left(P_{d,i}\right)$

increases monotonically on the feasible region, therefore, $f_i\left(P_{d,i}\right)$ reaches the maximum value at $P_{d,i} = P_{d,i,\sup}$, which makes $P_{d,i,\max} = P_{d,i,\sup}$. Secondly, if $P_{d,i,\inf} < \left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}} < P_{d,i,\sup}$, then the global maximum point of $f_i\left(P_{d,i}\right)$ is within the feasible region, so $P_{d,i,\max} = \left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}$. Thirdly, if $P_{d,i,\inf} \geq \left(\frac{2B_i}{\alpha}\right)^{\frac{\alpha}{2}}$, i.e., the feasible region is on the right of the dash line in Fig. 5, then $f_i\left(P_{d,i}\right)$ decreases monotonically on the feasible region. Hence, the maximum value of $f_i\left(P_{d,i}\right)$ is achieved at $P_{d,i}=P_{d,i,\inf}$, i.e., $P_{d,i,\max} = P_{d,i,\inf}$. Summarize the aforementioned cases in (25), then **Theorem 1** holds.

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