Received October 5, 2016, accepted October 19, 2016, date of publication November 1, 2016, date of current version November 18, 2016.

Digital Object Identifier 10.1109/ACCESS.2016.2623772

An Overview of Low-Rank Channel Estimation for Massive MIMO Systems

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This work was supported by the National Natural Science Foundation of China under Grant 61422109, Grant 61531011, and Grant 61625106.

ABSTRACT Massive multiple-input multiple-output is a promising physical layer technology for 5G wireless communications due to its capability of high spectrum and energy efficiency, high spatial resolution, and simple transceiver design. To embrace its potential gains, the acquisition of channel state information is crucial, which unfortunately faces a number of challenges, such as the uplink pilot contamination, the overhead of downlink training and feedback, and the computational complexity. In order to reduce the effective channel dimensions, researchers have been investigating the low-rank (sparse) properties of channel environments from different viewpoints. This paper then provides a general overview of the current low-rank channel estimation approaches, including their basic assumptions, key results, as well as pros and cons on addressing the aforementioned tricky challenges. Comparisons among all these methods are provided for better understanding and some future research prospects for these low-rank approaches are also forecasted.

INDEX TERMS Massive MIMO, channel estimation, low-rank property, channel sparsity, angle reciprocity.

I. INTRODUCTION

Large-scale multiple-input multiple-output (MIMO) or ‘‘massive MIMO’’, a new technique that employs hundreds or even thousands of antennas at base station (BS) to simultaneously serve multiple users, has been widely investigated for its numerous merits, such as high spectrum and energy efficiency, high spatial resolution, and simple transceiver design [1]. Meanwhile, massive MIMO faces many practical challenges, including the sophisticated channel modeling, the high-dimensional channel state information (CSI), the scheduling of numerous accessing users, the limited radio frequency (RF) chains, etc. Among all these challenges, the CSI acquisition is generally recognized as the bottleneck to embrace the potential gains promised by massive MIMO:

A. UPLINK (UL) PILOT CONTAMINATION

From the conventional orthogonal training strategy [2], the required number of orthogonal training sequences as well as the length of the training sequences should be at least the number of transmit antennas. Hence, when the number of users (possibly with more than one antenna) grows dramatically, there may not exist sufficient orthogonal training sequences to separate the UL channel estimation from different users. If the same training sequences are reused or non-orthogonal training sequences are adopted, then the inter-user interference will arise during the channel estimation stage, which is known as pilot contamination [1].[1]

B. OVERHEAD OF DOWNLINK (DL) TRAINING AND FEEDBACK

Similar to UL cases, the required number of training for DL will be comparable to the large number of antennas at BS, and thus BS may not have sufficient number of orthogonal training sequences to separate the DL channels. Even if it does, the conventional DL training strategy could fail due to a shorter channel coherence time. Meanwhile, the amount of CSI feedback from users to BS needs to be scaled with the number of antennas to control the quantization error, which is a much heavy burden for practice concern.

[1]In this paper, we do not discriminate between inter-cell pilot contamination and intra-cell pilot contamination [3], as they have the same rationale and the mathematical formats.

C. HIGH COMPUTATIONAL COMPLEXITY

As the size of channel matrices increases, matrix operations involved in the channel estimation, including multiplication, inversion, eigenvalue decomposition (EVD), or singular value decomposition (SVD), will induce much higher computational complexity in the practical implementation.

D. ACQUISITION OF CHANNEL COVARIANCE MATRICES (CCMs)

CCMs have been widely utilized to improve the CSI accuracy as well as reducing the effective dimensions of MIMO channels. Note that CCM is different from signal covariance where the former can only be obtained from the accumulation of channel estimates. Meanwhile the sample size to construct CCM has to increase linearly with the channel dimensions, making the accuracy or even the acquisition of the estimated CCMs questionable for massive MIMO. Especially, it is much difficult to obtain the downlink CCMs for all users as the cost of training and feedback is hardly affordable.

E. CHANNEL NON-RECIPROCITY

To release the heavy burden of DL training and feedback, many works proposed to utilize the channel reciprocity for time division duplexing (TDD) systems, where the downlink CSI can be immediately obtained from the uplink CSI, provided that the latter can be obtained through certain manner. Unfortunately, channel reciprocity is not applicable for frequency division duplexing (FDD) systems, which is still a dominant transmission mode in most communications systems [1].

A direct way to face the above dimension-originated challenge is to reduce the effective channel dimension. Such a consideration is possible due to two facts: (i) The antenna spacing of massive antenna array is usually as small as half-wave length in order to keep the whole array aperture small [4], [6], [7]; (ii) BS with large-scale antenna array has to be elevated at the top of high buildings such that there are few local scattering [4]; especially, when the carrier frequency goes to millimeter wave band, the severe path loss makes sure that only a few reflecting paths could arrive at BS [8]. In this case, the channel contains strong sparsity and the signals received from massive antennas would have very high correlation. Equivalently, the CCM would be rank deficiency and possess low-rank property. Different channel estimation approaches were then proposed, including CCM based method, compressive sensing (CS) based method, and antenna array theory based method, each with its own unique perspective to exploit the channel sparsity or low-rank property.

The objective of this paper is to provide a summary of these low-rank approaches for massive MIMO channel estimation, focusing on their basic ideas, detailed realization scheme, as well as the merits and drawbacks on addressing the foregoing challenges. Through detailed comparison, we expect to highlight the unique strengths of each kind of methods and thus provide a guideline to select a better channel methods for different scenarios.

The rest of the paper is organized as follows. In Section II, we introduce two universal low-rank approaches that can be applied for any number of receive antennas, i.e., CCM based method and CS based method. In Section III, we present the antenna array theory based low-rank channel method that is specifically suitable for massive number of antennas. Then some enlightening discussions on the massive MIMO systems are provided in Section IV. The comparisons among all these low-rank approaches and some future research prospects are given in Section V, followed by the conclusion in Section VI.

II. CCM AND CS BASED LOW-RANK CHANNEL ESTIMATION

The conventional rank-deficient channel estimation methods exploit either the low-rank properties inside statistical CCM or the sparsity inside the instantaneous channels, which can also be applied to massive MIMO systems.

A. LOW-RANK CCM BASED METHODS

1) CHANNEL MODEL

For MIMO system with M antennas at BS, the channel between a single-antenna user and BS can be represented from the antenna array theory as [4]

$$\mathbf{h}_k = \int_{\theta\in\mathcal{A}_k} \alpha_k(\theta)\mathbf{a}(\theta)d\theta, \tag{1}$$

where θ denotes the direction of arrival (DOA) of each ray insides the incident signal and $\mathcal{A}_k$ is the angular spread (AS) of the incident signal from user-k. Moreover, $\alpha_k(\theta)$ is the complex gain of the incident ray at DOA θ and $\mathbf{a}(\theta)$ is the array manifold vector (AMV), whose expression is dependent on the array structure. When a uniform linear array (ULA) is adopted, there is[2]

$$\mathbf{a}(\theta) = \left[1, e^{j\frac{2\pi d}{\lambda}\sin\theta}, \ldots, e^{j\frac{2\pi d}{\lambda}(M-1)\sin\theta}\right]^T, \tag{2}$$

where d is the antenna spacing and λ is the signal wavelength.

The CCM of user-k, denoted as $\mathbf{R}_k$, can be expressed as [5]

$$\mathbf{R}_k = \mathbb{E}\{\mathbf{h}_k\mathbf{h}_k^H\} = \int_{\theta\in\mathcal{A}_k} \mathbb{E}\{|\alpha_k(\theta)|^2\}\mathbf{a}(\theta)\mathbf{a}(\theta)^H d\theta. \tag{3}$$

2) NARROW ANGULAR SPREAD

Similar to the conventional approaches, if $r \triangleq \text{rank}\{\mathbf{R}_k\} \ll M$, then the channels can be expanded by r dominant eigenvectors that correspond to the r nonzero eigenvalues, which would reduce the channel dimensions from M to r.

To this end, authors of [6] and [7] considered a finite scattering environment for massive MIMO systems and assumed that the AS of each user is restricted within a narrow region, as

[2] This is the typical AMV expression for far-field narrow-band signals, while the discussion for the near-field or wide-band cases could be similar obtained.

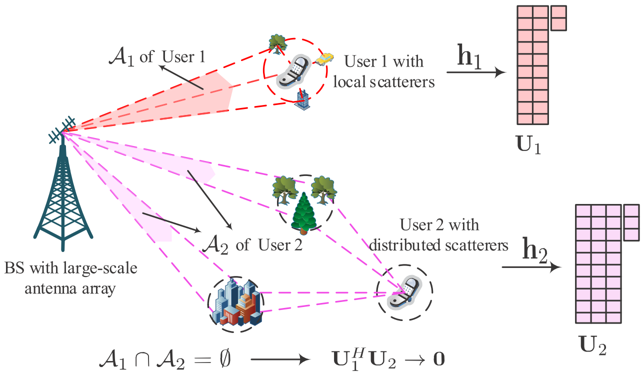


FIGURE 1. Finite scattering model, where user 1 is surrounded by local scatterers ($\mathcal{A}_1$ includes only one continuous AS interval), while user 2 experiences several distributed scatterers ($\mathcal{A}_2$ contains several segmented continuous AS intervals). Both AS intervals of two users are narrow for limited scattering surrounding BS.

shown in Fig. 1. With this assumption, one could mathematically demonstrate the low-rank property of CCM; namely, the CCM $\mathbf{R}_k$ can be expressed as

$$\mathbf{R}_k = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}_k^H, \tag{4}$$

where $\mathbf{U}_k$ is the signal subspace eigen-matrix of size $M \times r$ and $\mathbf{\Lambda}_k$ is the nonzero eigenvalue matrix of size $r \times r$.

A key inspection from [6] and [7] is that the CCMs of any two users with non-overlapped AS are asymptotically orthogonal to each other, say,

$$\mathbf{U}_k^H \mathbf{U}_l \rightarrow \mathbf{0}, \quad \text{for } \mathcal{A}_k \cap \mathcal{A}_l = \emptyset, \text{ as } M \rightarrow \infty, \tag{5}$$

and hence the pilot contamination could be removed for these users with non-overlapped AS even if they employ the same training sequence. Then [6] proposed a DL joint spatial division multiplexing (JSDM) scheme, where a classical multiuser precoder was adopted to restrict each user's beamforming vectors within the orthogonal complement of the channel subspaces of the others. Meanwhile, [7] directly applied UL channel training via the minimum mean square error (MMSE) estimator and proved that channels with non-overlapped AS can be estimated free of interference.

3) MERITS AND DRAWBACKS

By leveraging the low-rank property of CCMs and reducing the effective dimensions of channels, UL pilot contamination as well as DL training and feedback overhead can be significantly reduced. Meanwhile, with the real eigen-direction of the channel statistics, the subsequent channel estimation would possess very high accuracy. However, the acquisition of CCM is a difficult task for multi-user massive MIMO systems, especially for FDD mode, where the channel non-reciprocity says that the uplink CCMs cannot be directly used as downlink CCM. Thus each user's high-dimensional downlink CCM at BS has to be separately estimated and fed back. Furthermore, the accompanied computational complexity involved in the SVD of high-dimensional CCMs for multiple users is hardly affordable.

B. CS BASED METHODS

1) NUCLEAR NORM REGULARIZATION VIA SEMIDEFINITE PROGRAMMING (SDP)

Adopting the same finite scattering models as [6] and [7], authors of [9] investigated the channel estimation for a TDD multiuser massive MIMO system. It was shown that the degree of freedom (DoF) of the multiuser channel matrix was absolutely small due to the limited number of multi-path propagation components. Utilizing this sparsity, a CS-based approximation technique was proposed for multiuser channel estimation, which aimed at solving the relaxation version of rank minimization problem, i.e., the minimization of the nuclear norm of channel matrix (see [9, eq. (7)]). After several mathematical operations, this CSI acquisition problem was further translated to a quadratic SDP problem and was solved efficiently with readily available polynomial SDP approach.

This rank minimization method directly exploits the low-rank properties of channel matrices with the aid of CS theory, without the need of any additional knowledge about the statistical distribution or physical parameters of the propagation channels.

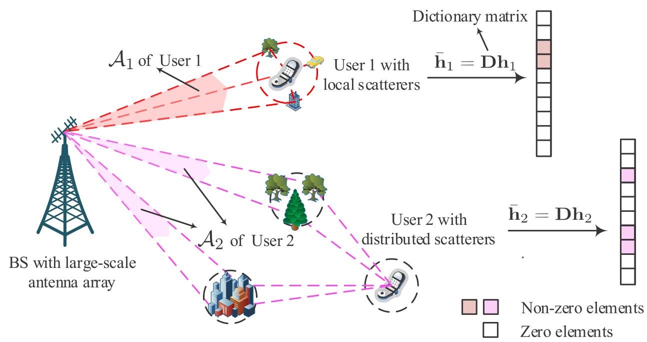


FIGURE 2. Channels exhibit sparsity under certain dictionary matrices.

2) CHANNEL SPARSITY BASED JOINT CSI RECOVERY

Employing a massive ULA at BS, channels can be represented by a virtual channel representation with a given unitary dictionary matrix [10]. It is claimed that transformed channels possess sparsity due to the limited local scattering at BS (see Fig. 2), and that different user channels have a partially common sparsity support due to the shared scatterers in the propagation environments. A joint CSI estimation approach was then proposed for both CSI training and feedback reduction, where the CS technology with a nonlinear recursive optimization is adopted to recover the sparsity support identification, followed by the oracle least square (LS) channel estimation.

Instead of assuming the common sparsity supports between separate users' channels, the authors of [11] look into the common sparsity support shared by subchannels of different subcarriers in the orthogonal frequency-division multiplexing (OFDM) systems. Similar to the training procedure of [10], the compressive training signals are transmitted over multiple pilot tones and the compressive measurements are fed back. Then BS recovers all the subchannel vectors for each user with CS technology. Moreover, the authors also devised a close-loop channel tracking scheme to simplify the subsequent CSI acquisition by using the preceding sparsity supports under the premise that the sparsity supports keep unchanged during the following transmission.

3) MERITS AND DRAWBACKS

Channel sparsity together with various CS algorithms could reduce the training overhead for massive MIMO systems and meanwhile eliminate the requirement of CCMs. Nevertheless, the hypothetically exact sparse channel models may be too ideal, considering both the possible power leakage [14] during the transformation procedure and the low signal-to-noise ratio (SNR) conditions. Meanwhile, the underlying requirement of perfect measurement feedback for users to BS is also a matter of concern. Furthermore, the design of a dictionary matrix that could provide a better sparsity and meanwhile satisfy the restricted isometry property (RIP) seems to be difficult, since such a matrix is both related with the array structure as well as the instant incoming DOAs of the signal.

III. ANTENNA ARRAY THEORY BASED LOW-RANK CHANNEL ESTIMATION

Now that the angular information is crucial for the low-rank channel estimation, a natural question arise: why don't we directly achieve such angular information via certain canonical means, say, array signal processing? However, there are three main reasons that the conventional MUSIC [12] and ESPRIT [13] are not applicable here: (i) They may suffer from very high computational complexity due to their SVD operation with massive antennas; (ii) They are designed for the scenario when the incoming signals do not have AS and would suffer from performance degradation with surrounding scattering; (ii) They are blind approaches originally designed for Radar application but do not utilize the training sequences embedded in communications systems. Hence, a new array signal processing approach that could efficiently obtain the angular information specifically for massive MIMO system was proposed in [14].

A. SPATIAL BASIS EXPANSION MODEL (SBEM)

1) INITIAL AS ESTIMATION VIA DFT

The large antenna number at BS, namely large spatial sampling points, will greatly enhance the resolution of discrete Fourier transform (DFT) and thus render the possibility to immediately achieve AS of the incoming signals via spatial DFT operation at BS [14]. Interestingly, for ULA with $M \gg 1$ antennas, the DFT of the UL channel yields an equivalent evenly spaced subchannels in the beamspace. The nonzero points of DFT reflect the beamspace subchannels that concentrate around the central DOA of the incident signals, while the width of these nonzero points corresponds to AS of the incident signals. With narrow AS condition [6], [7], the number of beamspace subchannels is limited and thus the original channel would exhibit the sparsity in beamspace for massive MIMO system.[3] Such a sparse representation is equivalent to spanning the original channel in the beamspace with limited basis vectors (subchannels) and is thus named as SBEM.

Remark 1: The initial AS estimation with DFT could be efficiently implemented via the fast Fourier transform (FFT), making it much appealing for massive MIMO scenarios.

2) ENHANCING CHANNEL SPARSITY VIA SPATIAL ROTATION

In practical application, the number of antennas at BS cannot be ideally infinite, and hence the DFT resolution is still limited. In this case, the power of the beamspace subchannels would leak to their neighbors outside the direction of the incident signals. Consequently, there would be much more non-zero beamspace subchannels beyond those truly representing the AS of the incident signals, which would deteriorate the sparsity of the beamspace representation of the channel and would increase the burden of the subsequent training. The authors of [14] then proposed to use *spatial rotation* to mitigate this power leakage by rotating the channel by an appropriate spatial phase such that the beamspace subchannels would point towards the incident signal in a better format. With spatial rotation, the channel sparsity after DFT operation can be dramatically enhanced, and one example is given in Fig. 3.

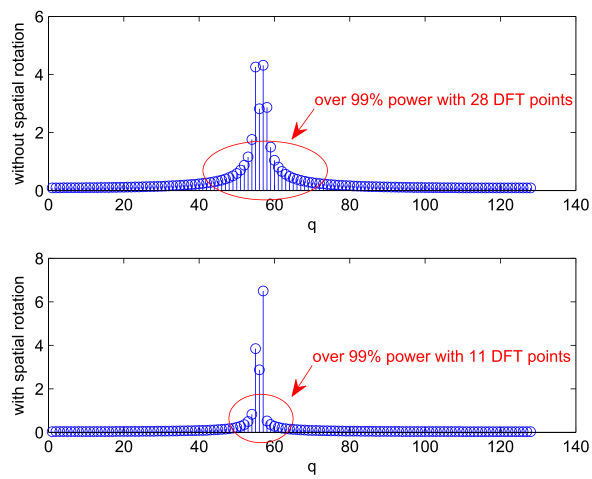


FIGURE 3. Illustration of spatial rotation with AS = 4° and M = 128. After spatial rotation, the number of DFT points gathering over 99% channel power is significantly reduced from 28 to 11.

3) SBEM BASED CHANNEL ESTIMATION

With the DFT operation and the spatial rotation, the channel can be represented as [14]

$$\mathbf{h}_k = \sum_{q=1}^{\tau} \beta_{k,q} \mathbf{\Phi}(\phi_k) \mathbf{a}(\theta_{k,q}) \tag{6}$$

[3] This property is the direct reason that the CS method could utilize DFT matrix as a good dictionary matrix for sparsity but is presented from antenna theory viewpoint in [14] for a vivid understanding.

where $\mathbf{\Phi}(\phi_k) = \operatorname{diag}\left\{[1, e^{j\phi_k}, \cdots, e^{j(M-1)\phi_k}]\right\}$ is the spatial rotation matrix and $\phi_k \in [-\frac{\pi}{M}, \frac{\pi}{M}]$ is a spatial rotation parameter; $\mathbf{a}(\theta_{k,q})$'s represent τ subchannels in the beamspace and are orthogonal to each other. If ULA is adopted, $\mathbf{a}(\theta_{k,q})$ would be one column of the DFT matrix. Moreover, $\beta_{k,q}$ is the complex gain of the corresponding beamspace subchannel.

Obviously, the channel estimation in massive MIMO can be decomposed into two parts: AS estimation (for the $\theta_{k,q}$ in (6)) and gain estimation (for the $\beta_{k,q}$ in (6)). The former can be immediately achieved via the previously mentioned DFT approach, and the remaining limited number of channel gains can be simply obtained via linear estimation method, e.g., LS, while the amount of training is equivalent to the number of non-zero beamspace subchannels in (6). Furthermore, the angular information of each user would vary much slower than the channel itself since the physical position of any terminal changes negligibly in one channel coherence time as compared to its distance from BS. Hence, one would safely treat the AS information as unchanged within a much longer period while only update the channel gains for every channel coherence time. Meanwhile, it was proved in [7] that users with non-overlapped AS would have orthogonal channels, which can also be observed from SBEM (6), where non-overlapped AS results in orthogonal beamspace subchannels.[4] One could then assign users with non-overlapped AS into one group and let them reuse the same training sequence for UL channel estimation without causing pilot contamination.

4) ANGLE RECIPROCITY FOR DOWNLINK CHANNEL ESTIMATION IN FDD SYSTEMS

The channel non-reciprocity isolates UL and DL training, resulting in heavy resource overhead and computational complexity for DL training in FDD systems. The antenna array theory based approach [14] proposed to utilize the *angle reciprocity* [15] to help the DL channel estimation for FDD systems. Angle reciprocity says that only those signal waves that physically reverse the UL paths could arrive at users during DL transmission, and this reciprocity between uplink and downlink AS holds true even for FDD system as long as the UL carrier frequency is not far from the DL carrier frequency, say less than several Giga Hz [16]. Hence, the UL angular information can be directly viewed as its DL counterpart and thus only the remaining DL channel gains need to be re-estimated through linear method. Such an approach significantly reduces the cost of DL training and feedback for massive MIMO systems. Moreover, since that angle reciprocity holds for both TDD and FDD system, it helps to establish a unified UL/DL channel estimation protocol for both TDD and FDD massive MIMO systems.

[4] Though different users may have different phase rotation and would suffer from some inter-subchannel interference, such kind of interference would be negligible with massive number of antennas and with certain guard interval among users [14].

5) DATA-AIDED BLIND AS TRACKING

The AS information of all users could be estimated with a relatively long preamble at the start of the transmission and should be re-estimated once there appear significant changes of users' positions. Nevertheless, by grouping the users with non-overlapped AS together, each user's AS information could be blindly tracked through DFT-aided approach even during data transmission period. This helps to further reduce the training demands for AS estimation.

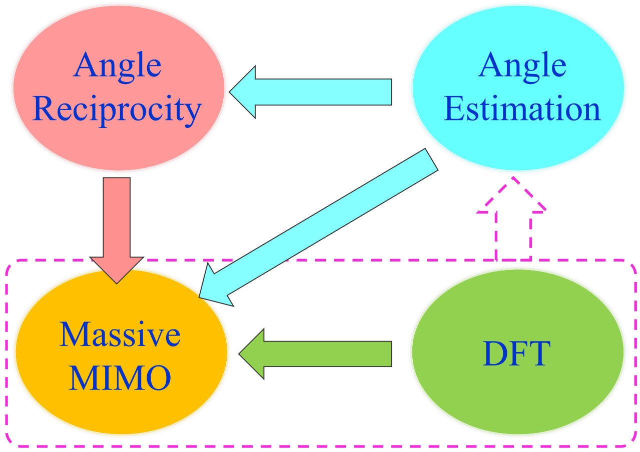


FIGURE 4. **Rationale of Array Signal Processing aided Channel Estimation.**

B. RATIONALE OF SBEM WITH MASSIVE MIMO

In fact, SBEM based channel estimation is closely related with antenna theory and with the massive MIMO system, whose rationale is shown in Fig. 4. More precisely:

- With massive MIMO, channel estimation could be decomposed into angle estimation and gain estimation, whereas the conventional MIMO system would have low resolution and cannot separate angular information from the channel.
- With massive MIMO, angle estimation can be effectively achieved by DFT approach due to the improved spatial resolution.
- With the estimated angular information, one can utilize the angle reciprocity to simplify the DL channel estimation, especially for FDD system.

Remark 2: It can be easily realized that all the above angle related benefits do not exist for conventional small-scale antennas system because it cannot formulate the narrow beams to utilize such angular information. For example, pointing a wide beam towards the user direction would suffer from severe power leakage and this why the beamforming in conventional MIMO system should stay in eigen-space but not the beamspace. In other words, SBEM could be treated as a specifically designed scheme for massive MIMO system. In contrast, CCM or CS based approaches are universal for any MIMO systems as long as sparsity or the low rank property holds. Hence, one would expect a better performance from SBEM in terms of accuracy or complexity than CCM and CS based methods under massive MIMO systems.

IV. DISCUSSIONS

A. WHERE DOES LOW-RANK PROPERTY HOLD TRUE?

Obviously, the above works all rely on the assumption of finite scattering environments, i.e., the narrow AS for low-rank property of CCMs or the sparsity of channels. Generally, two scenarios have been accepted to support this key assumption:

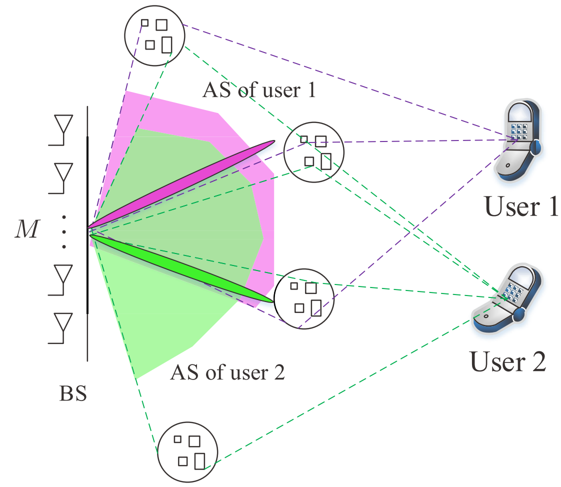
- BS equipped with a large number of antennas is always elevated at a very high altitude, say on the top of a high building, a dedicated tower, or an unmanned aerial vehicle (UAV) platform, such that there are few surrounding scatterers [4]. Meanwhile, the array is normally formulated by half-wavelength spaced antennas and this smaller aperture further reduce the possibility to see many surrounding scatterers. Hence, the AS seen by BS is quite small and the number of incoming signal path is limited.
- When massive MIMO system is employed at the millimeter-wave band, e.g., 60GHz, the high path loss will lead the primary propagation to be only the line-of-sight (LOS) or the first order reflection, such that the number of incoming signal path is limited [8].

Remark 3: Obviously, the sparsity in spatial domain, either the low-rank properties of CCMs or sparsity of channels is the only way to reduce the channel spatial dimensionality for massive MIMO systems.

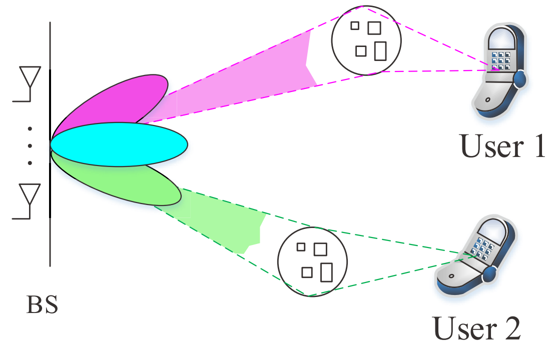
B. THOUGHTS ON "HIGH SPATIAL RESOLUTION" OF MASSIVE MIMO

It has been widely recognized that massive MIMO enjoys the advantage of high spatial resolution. Nevertheless, it must be pointed out that the combination of massive antenna array and the narrow AS condition (or the spatial sparsity) is a must to realize such a high spatial resolution. For example, Fig. 5(a) illustrates the case where BS has massive number of antennas while AS of users are too broad and overlap with each other. In this case, even if the massive antenna array is able to formulate narrow DL beams, it still cannot distinguish the two users totally in beamspace; namely, DL signals sent by two separate narrow beams still cause interferences between users. By contrast, Fig. 5(b) shows the case where BS has a few antennas while the AS of users are narrow and do not overlap with each other. In this case, the DL beams would be too wide to separate different users; namely, the power leakage to the sidelobe would cause interferences between users.

Hence, in order to enjoy the high resolution claimed by massive MIMO, there is a pre-requisite that the incoming channels should be sparse, which makes massive MIMO more suitable for mmWave applications.



(a)



(b)

FIGURE 5. Massive MIMO with wide incoming AS and conventional MIMO with narrow incoming AS. (a) Massive MIMO + broad AS. (b) MIMO + narrow AS.

V. COMPARISONS AND FUTURE RESEARCH

A. COMPARISONS AMONG LOW-RANK APPROACHES

1) SIMILARITIES

- All these methods require narrow AS assumption or the spatial sparsity assumption.
- All these methods are able to mitigate the UL pilot contamination as well as the heavy burden of DL training and feedback.

2) DIFFERENCES

a: CCM-BASED METHOD

- The low-rank CCM approach in [6] and [7] is more accurate than the others since the exact channel statistics are exploited and the accurate channel eigen-spaces are extracted. On the other hand, SBEM method limits its channel in beamspace and would face more number of unknowns. Similarly, CS method would have difficulty to choose a better dictionary matrix and may suffer from higher sparsity level. Nevertheless, achieving the covariance matrices for all users would be a costly task especially for moving users whose AS would change from time to time. Moreover, obtaining the downlink covariance matrices may also suffer from heavy feedback cost.

b: CS-BASED METHOD

- The CS-based method normally started with DL channel estimations and could obtain the channel estimates of

all users simultaneously without user grouping. Nevertheless, such an approach will suffer from performance degradation in order to cover the whole spatial domain with less number of training, and the corresponding performance loss is randomly determined by the users' position. In UL case, however, compressive sensing normally require some prior knowledge, for example whether users have similar AS, such that it can perform user grouping before the channel estimation. Otherwise, the users' training signals will superimpose on top of each other and the sparsity of each channels cannot be observed from the received signal.
- CS-based method does not require the knowledge of CCMs, and thus avoid the accompanied SVD operation.

c: SBEM-BASED METHOD

- SBEM does not require the knowledge of CCMs, and could be efficiently implemented with FFT. By contrast, CCM-based method suffers from high-dimensional SVD and CS-based method requires non-linear optimization. Therefore, SBEM is a more friendly approach for hardware implementation.
- SBEM could strengthen the sparsity of channels via phase rotation, while CS-based method generally assumes perfect sparsity and ignore the power leakage effect.
- With the estimated AS information, SBEM could unify the UL/DL transmission for both TDD and FDD systems by utilizing the angle reciprocity. However, the UL/DL CCMs [6], [7] and sparsity supports [10], [11] are not necessarily reciprocal in FDD systems.
- SBEM could track the angular information of different users during the data transmission via the simple FFT operation.

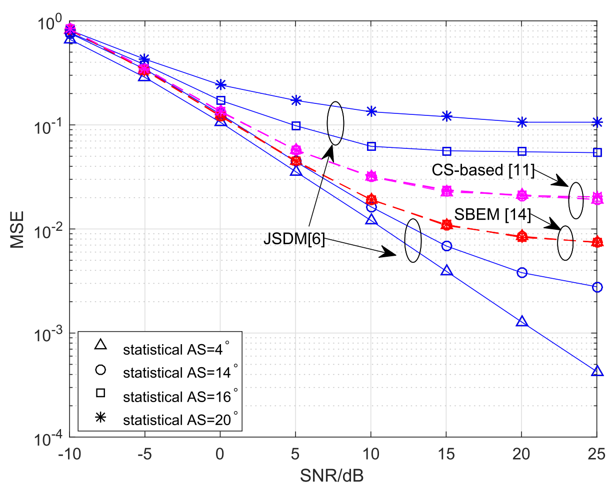


FIGURE 6. The DL MSE performance comparison of these low-rank approaches under the case of user mobility.

A numerical example on the mean square errors (MSE) of DL channel estimation from different low-rank approaches versus SNR are shown in Fig. 6, where the channels are formulated by equations (1) and (2) with $M = 128$ and $d = \lambda/2$, and the maximum rank or sparsity level considered for all methods is set as 16. To keep a fair comparison, the total training power for all methods is the same for any given SNR. We consider the situation of user mobility. Specifically, as users move, the instantaneous AS is fixed as 4° at each time moment while the statistical AS, defined as the total range of AS that users cover within a relatively longer period, is set as 4°, 14°, 16°, 20°, respectively. The CCM will be obtained based on statistical AS while the channels in CS and SBEM is obtained based on the instantaneous AS.

As expected, JSDM method with perfect CCM performs the best among all different methods when statistical AS is 4°. Meanwhile, both SBEM and CS methods approach certain error floors due to the power leakage in the spatial domain. As the statistical AS increases, the MSE performances of JSDM deteriorate obviously whereas the MSE curves from CS method and SBEM are not affected so much. The reason lies in that CCM will cover too broad AS and thus are not accurate for the instantaneous channel estimation. Instead, SBEM and CS method are only related with sparsity of instantaneous channels and thus are more suitable for the mobile communications. Meanwhile, SBEM performs better than CS method and has a smaller error floor for the reason that SBEM could strengthen the sparsity representation via spatial rotation, whereas CS method does not harness the antenna array theory to enhance the channel sparsity and thus have a relatively larger sparsity level, resulting in a lower channel estimation accuracy.

TABLE 1. Comparison between existing low-rank approaches.

Challenges \ Approaches	CCM [6][7]	CS [10][11]	SBEM [14]
UL Pilot Decontamination	✓	✓	✓
Reduced DL Training Overhead	✓	✓	✓
No Need for CCMs	×	×	✓
Low Computational Complexity	×	×	✓
Easy Hardware Implementation	×	×	✓
Reciprocity for FDD Systems	×	×	✓
Blind Angle (partial channel) Tracking	×	×	✓

A summarized comparison between different low-rank channel estimation approaches for massive MIMO is shown in Table 1.

B. FUTURE RESEARCH PROSPECTS

Many mature techniques in antenna array theory could be borrowed into massive MIMO system and formulate prospective research directions in the future. Some immediate examples are given here:

1) ARRAY CONFIGURATION DESIGN

Due to space limitation, the large number of antennas may be arranged into various array topologies, like uniform circular arrays (UCA), cylinder arrays, lens arrays or even irregular shape. Extending above low-rank approaches to these new array configurations needs careful investigations. For example, CS-based method should exploit new dictionary matrices for sparse channel representation, while SBEM should obtain efficient algorithms for angle acquisition. On the other side, designing the array shape that could achieve the best angle estimation to help the next channel estimation would also be interesting.

2) ANGLE-BASED BEAMFORMING

Channel matrices based beamforming such as MMSE and zero-forcing could be re-explained by the beamspace transmission in massive MIMO scenario, where the latter physically points to the incident directions of different users. Hence, many conventional angle-based beamforming techniques [17] would certainly be promoted to wireless communications system, such as beam-shape formulation, sidelobe control, optimal beamforming design, etc.

3) ANGLE TRACKING BASED CHANNEL TRACKING

Conventional angle-based target tracking algorithm could be applied in massive MIMO system to help the channel tracking of the mobile users. Nevertheless, the antenna array theory based tracking algorithm is mainly designed for blind scenario where the targets do not cooperate with the BS. One should certainly re-design a more efficient tracking strategy based on the pilot embedded in the massive MIMO systems.

VI. CONCLUSION

In this paper, we have investigated the main challenges faced by high-dimensional CSI acquisition in massive MIMO systems and provided a detailed review on different low-rank channel estimation methods. It was first shown that the narrow angular spread or the spatial sparsity is crucial for all low-rank approaches. We then demonstrated that the CCM method and the CS method are universal ideas while the SBEM approach is specifically designed for massive MIMO systems from the viewpoint of antenna array theory. Detailed comparisons and discussions on all these low-rank methods have been provided, which demonstrated that SBEM method could handle most challenges faced by massive MIMO channel estimation while the other two still have certain limitations. Moreover, SBEM links the transmission of massive MIMO with angle estimation such that many mature techniques in antenna array theory could be borrowed here and formulate prospective research directions in the future.

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