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# Energy Efficiency of Repetition Coding and Parallel Coding Relaying Under Partial Secrecy Regime

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**ABSTRACT** This paper evaluates the secure energy efficiency (SEE) of a cooperative network subject to partial secrecy requirements, implemented through a fractional equivocation parameter $\theta \in (0, 1]$ that allows partial secrecy when $\theta < 1$. We assume that only the channel state information (CSI) of the legitimate channel is available, while the CSI with respect to the eavesdropper is unknown. Then, we propose a CSI-aided decode-and-forward (DF) scheme, in which the transmitter uses the available CSI in order to choose between direct and cooperative paths. Moreover, the relay employs either repetition coding (CSI-RC), *i.e.*, source and relay use the same codebook, or parallel coding (CSI-PC), when different codebooks are used. By resorting to the Dinkelbach algorithm, we propose a joint power allocation scheme, which also optimizes $\theta$ to maximize the SEE. Our schemes are compared with the traditional DF, amplify-and-forward, and cooperative jamming (CJ). In most scenarios, CSI-RC performs best in terms of SEE. Nevertheless, we observe that CSI-PC achieves the highest SEE when $\theta \rightarrow 1$ and if the relay is close to either the transmitter or the receiver. Moreover, CJ also stands out to maximize the SEE if the relay is placed closer to the eavesdropper. In addition, the influence of $\theta$ in the system performance is evaluated, showing that a joint $\theta$ and power optimization considerably improves the SEE.

**INDEX TERMS** Cooperative systems, security, energy efficiency.

## I. INTRODUCTION

The growing demand for wireless communications systems makes security an important and difficult design task. With the technological advances, cryptography based in the computational capacity of an eavesdropper may need enhancements or complements, and one alternative is to achieve security at the Physical Layer (PHY) [1], [2]. The PHY security exploits the fluctuations of the wireless channel to allow a secure communication between a pair of legitimate nodes (Alice and Bob) in the presence of an eavesdropper (Eve), model referred to as wiretap channel [3].

The measures associated with PHY security are usually related to the level of channel state information (CSI) available at Alice, in which three different approaches are possible. First, considering global CSI at Alice, it is possible to adapt the rate of the wiretap code, so that the information is never leaked to Eve and perfect secrecy is achieved [4]. The security measure associated with this scenario is the secrecy capacity and is considered, *e.g.*, in [5]–[7]. However, the global CSI assumption is too strong since it may require Eve's collaboration to feedback her CSI to Alice. Thus, a more practical approach considers a probabilistic view in which Alice has only CSI of the legitimate channel and communicates with a fixed secure transmission rate. In this case, security is measured by the secrecy outage probability, *i.e.*, the probability that the fixed secrecy rate is above the secrecy capacity of the channel, which is considered, *e.g.*, in [8]–[10]. Finally, another possible scenario is when Alice has no CSI at all, as in [11]–[13], or in the case of channel estimation errors [14] or outdated CSI [15].

In the cases when Alice has CSI only with respect to the legitimate nodes, secrecy outage probability is usually employed with the constraint that the information leakage to Eve must tend to zero, *i.e.*, the outage probability at Eve tends to one. However, the authors in [16] noted that systems may have different levels of secrecy requirements, and then propose a partial security regime by relaxing the conditions of decoding error probability at Eve. Therefore, a generalized secrecy outage probability is represented by the probability that Eve's equivocation, $\Delta$, is no less than a specified value $\theta \in (0, 1]$, which represents the minimum acceptable equivocation. A perfect security regime would require that $\Delta = 1$, so that $\theta = 1$ encompasses the conventional secrecy outage probability as a special case. Yet, with the generalized approach one can relax security according to the system requirements by the proper tuning of $\theta$, which may improve other metrics of interest.

Moreover, PHY security can be increased by means of cooperation, commonly used to obtain spatial diversity in wireless fading channels [17], in which the communication between Alice and Bob is assisted by a relay node. Initial studies about the secrecy capacity of cooperative schemes can be found in [18] and [19], while more recent studies still show that cooperative techniques are important to improve security [20], [21], specially in scenarios composed of many nodes, such as multicasting or wireless sensor networks. Some traditional cooperative schemes applied to the context of PHY security are the Decode-and-Forward (DF), Amplify-and-Forward (AF) and Cooperative Jamming (CJ) protocols. For instance, [8], [22], [23] compare these protocols for different positions of Eve, with the general conclusion being that AF performs best in most cases, except when Eve is very close to the legitimate nodes, when CJ is more advantageous. Additionally, in [24] the authors exploit the available CSI of the legitimate channel to perform a joint cooperative beamforming and jamming transmission in a scenario with multiple relays to enhance security. Similarly, in [25] the available CSI is used by the relay to choose between cooperation or jamming.

In addition, power allocation between Alice and the relay is often employed to increase the performance of cooperative protocols. For instance, [26] proposes a power allocation scheme for a non-cooperative block fading scenario, in which the transmitter may or may not have CSI feedback. Considering a cooperative scenario, an extension of [8] is given by [22], in which an optimal power allocation scheme is proposed to minimize the secrecy outage probability of DF and CJ schemes. However, the approaches proposed by [22] and [26] are complex, since an exhaustive search is employed in order to obtain the optimal power allocation.

Alternatively, an iterative and distributed manner to allocate power is through the Dinkelbach algorithm [27], which was developed to optimize the ratio between functions of the same variable, coming in handy when energy efficiency is the metric of interest [28]. For example, [29]–[32] employ Dinkelbach-based algorithms to maximize energy efficiency-related metrics. In [29] the authors aim at maximizing the energy efficiency in multiple antenna systems, achieving similar results in comparison with an exhaustive search approach, but with much reduced complexity. Recently, [30] analyzes the secure energy efficiency (SEE) in a multi-relay DF scenario, in which a subset of relays that correctly decoded Alice's message cooperate at the second time slot. Additionally, [31] studies resource allocation to maximize the SEE in a scenario with multiple antennas at the legitimate transmitter with different CSI assumptions. Finnaly, in [32] we maximize the SEE of a CSI-aided scheme, in which Alice exploits the CSI of the legitimate channel to choose the best path to communicate with Bob, *i.e.*, directly or through the relay.

Against this background, the contributions of this paper are listed in what follows:

- We extend [32] by employing the partial secrecy modeling from [16] in order to consider different levels of secrecy requirements through the parameter $\theta$. We show that, if a partial secrecy regime is tolerated, the proper allocation of $\theta$ leads to important energy efficiency gains.
- We also consider that either Repetition Coding (CSI-RC) – when Alice and relay use the same codebook – or Parallel Coding (CSI-PC) – employing different codebooks – can be used [33]. Interestingly, and differently from [33] in which PC outperforms RC in a non-secrecy scenario, in a PHY security context our results show that the CSI-RC outperforms CSI-PC in most situations. Moreover, in order to obtain a closed-form expression for the CSI-PC secrecy outage probability, a useful approximation is developed, which turns out to be very tight.
- Instead of adopting exhaustive search approaches as in [22] and [26], our proposed scheme allocates power through a much less complex Dinkelbach-based algorithm, which is combined with a golden search algorithm to jointly allocate $\theta$. Then, the algorithm proposed here allocates power at Alice and at the relay, as well as chooses the best $\theta$ in order to maximize the SEE.
- Differently from the literature (as in [8], [22], and [23]), our results show that AF is not the best strategy when the CSI of the legitimate channel is available at Alice. Due to the proper exploitation of the available CSI to choose the optimum transmission path, CSI-RC presents the best performance for most situations relative to the position of Eve. However, when Eve is closer to the relay, CJ outperforms other cooperative schemes. By its turn, CSI-PC can bring important performance improvements in terms of SEE when $\theta$ cannot be relaxed (*i.e.*, $\theta \to 1$).

In the sequel, Section II presents the system model and preliminary definitions. Section III formulates the generalized secrecy outage probability for the protocols of interest. Section IV introduces the joint $\theta$ and power allocation algorithm and Section V gives some numerical examples. Finally, Section VI concludes the paper.

## II. PRELIMINARIES

### A. SYSTEM MODEL

We consider two legitimate users, Alice (A) and Bob (B), communicating with the help of a relay (R) node in the

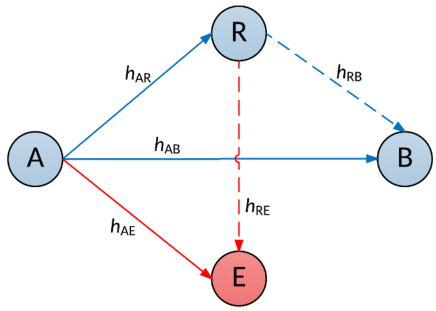


**FIGURE 1. System model: the communication occurs in the presence of an eavesdropper and the legitimate nodes are helped by a relay node.**

presence of an eavesdropper (E), as shown in Figure 1. In particular, we assume a scenario of a wireless sensor network (WSN), which has limited resources in size and energy; therefore, all nodes are considered to be single antenna devices and our goal is to improve the energy efficiency subject to secrecy constraints, denoted as secure energy efficiency (SEE).

The frame received at any node $j \in \{\mathrm{R}, \mathrm{B}, \mathrm{E}\}$ from the transmission of $i \in \{\mathrm{A}, \mathrm{R}\}$, $i \neq j$, is given by

$$\mathbf{y}_{ij} = \sqrt{\kappa_{ij} P_i} h_{ij} \mathbf{x}_i + \mathbf{w}_{ij}, \tag{1}$$

where $P_i$ is the transmit power of the node $i$, $\mathbf{x}_i$ is the unit average energy transmitted symbol vector, $\mathbf{w}_{ij}$ is the zero-mean complex Gaussian noise vector with variance $N_0/2$ per dimension and $h_{ij}$ is the zero-mean and unit-variance quasi-static fading, whose envelop is Rayleigh distributed. Moreover, $\kappa_{ij}$ is the path-loss between $i$ and $j$, given by [34]

$$\kappa_{ij} = \frac{G}{(4\pi f_c/c)^2 d_{ij}^{\upsilon} M_l N_f}, \tag{2}$$

where $G$ is the total antenna gain, $f_c$ is the carrier frequency, $c$ is the speed of light in vacuum, $d_{ij}$ is the distance between $i$ and $j$, $\upsilon$ is the path-loss exponent, $M_l$ is the link margin and $N_f$ is the noise figure at the receiver. In addition, we assume half-duplex communication between the nodes and time orthogonal transmissions.

The instantaneous signal-to-noise ratio (SNR) at any pair of nodes can be written as

$$\gamma_{ij} = |h_{ij}|^2 \bar{\gamma}_{ij}, \tag{3}$$

where $\bar{\gamma}_{ij} = (\kappa_{ij} P_i)/N$ is the average SNR, $N = N_0 B$ is the noise power and $B$ is the system bandwidth.

### B. GENERALIZED METRIC OF SECRECY OUTAGE PROBABILITY

Considering that systems may have different secrecy requirements, a partial secrecy regime can be used to relax the security constraints in order to maximize the energy efficiency. According to [16], the decoding error probability at Eve can be lower bounded by the fractional equivocation, represented by $\Delta$, which is a random variable (RV) described as [16]

$$\Delta = \begin{cases} 1, & \text{if } C_{\mathrm{E}} \leq C_{\mathrm{B}} - \mathcal{R} \\ \dfrac{(C_{\mathrm{B}} - C_{\mathrm{E}})}{\mathcal{R}}, & \text{if } C_{\mathrm{B}} - \mathcal{R} < C_{\mathrm{E}} < C_{\mathrm{B}} \\ 0, & \text{if } C_{\mathrm{B}} \leq C_{\mathrm{E}}, \end{cases} \tag{4}$$

where $C_{\mathrm{B}}$ and $C_{\mathrm{E}}$ represent the capacities of legitimate and Eve's channels, respectively, and $\mathcal{R}$ is the secrecy rate. Notice that these three conditions for $\Delta$ represent different levels of confusion at Eve. First, when $C_{\mathrm{E}} \leq C_{\mathrm{B}} - \mathcal{R}$, the equivocation $\Delta = 1$ indicates that no information leaks to Eve and she can just randomly guess about the transmitted message. The opposite condition yielding $\Delta = 0$, that is associated with $C_{\mathrm{B}} \leq C_{\mathrm{E}}$, implies that secure communication is not possible. Finally, the intermediate case when $\Delta = \frac{(C_{\mathrm{B}} - C_{\mathrm{E}})}{\mathcal{R}}$ represents a partial secrecy regime, in which only a fraction of the communication is secure.

Therefore, a generalized form to write the secrecy outage probability, representing different levels of security requirements, is [16]

$$p_{\mathrm{gso}}^{(\mathrm{sch})} = \Pr\{\Delta < \theta\}, \tag{5}$$

where the superscript $^{(\mathrm{sch})}$ represents each cooperative scheme studied in this paper, and $\theta \in (0, 1]$ is the minimum acceptable value for the fractional equivocation. Note that the usual formulation for the secrecy outage probability only considers $C_{\mathrm{E}} \leq C_{\mathrm{B}} - \mathcal{R}$, which corresponds to the case when $\theta = 1$.

Then, using (4) we can re-write (5), so that

$$\begin{aligned} p_{\mathrm{gso}}^{(\mathrm{sch})} &= \Pr\left\{0 < \theta \bigcap C_{\mathrm{B}} \leq C_{\mathrm{E}}\right\} \\ &\quad + \Pr\left\{\frac{C_{\mathrm{B}} - C_{\mathrm{E}}}{\mathcal{R}} < \theta \bigcap C_{\mathrm{B}} - \mathcal{R} < C_{\mathrm{E}} < C_{\mathrm{B}}\right\} \\ &\quad + \Pr\left\{1 < \theta \bigcap C_{\mathrm{E}} \leq C_{\mathrm{B}} - \mathcal{R}\right\}. \end{aligned} \tag{6}$$

Nevertheless, since $\theta \in (0, 1]$, one has that $\Pr\{0 < \theta\} = 1$ and $\Pr\{1 < \theta\} = 0$, so that (6) becomes

$$\begin{aligned} p_{\mathrm{gso}}^{(\mathrm{sch})} &= \Pr\{C_{\mathrm{B}} \leq C_{\mathrm{E}}\} \\ &\quad + \Pr\left\{C_{\mathrm{B}} - C_{\mathrm{E}} < \theta\mathcal{R} \bigcap C_{\mathrm{B}} - \mathcal{R} < C_{\mathrm{E}} < C_{\mathrm{B}}\right\}. \end{aligned} \tag{7}$$

### C. SECURE ENERGY EFFICIENCY (SEE)

The SEE is defined as the ratio between the amount of successfully secure transmitted bits, denoted here as secure throughput, and the total power used to perform such transmission. The secure throughput, as in [12] with the addition of the equivocation parameter $\theta$, can be written as

$$\tau_{\mathrm{s}}^{(\mathrm{sch})} = \theta\mathcal{R}\left(1 - p_{\mathrm{gso}}^{(\mathrm{sch})}\right), \tag{8}$$

yielding to the definition of the SEE as

$$\eta_{\mathrm{s}}^{(\mathrm{sch})} = \frac{\tau_{\mathrm{s}}^{(\mathrm{sch})}}{P_{\mathrm{total}}^{(\mathrm{sch})}}, \tag{9}$$

where $P_{\mathrm{total}}^{(\mathrm{sch})}$ is the total power consumed by the scheme sch.

## III. GENERALIZED SECRECY OUTAGE PROBABILITY

In this section we derive closed-form expressions to the generalized secrecy outage probability of CSI-RC and CSI-PC schemes. Moreover, for comparison purposes, we also extend the usual formulation for the DF, AF and CJ schemes. Note that all schemes are subjected to the same condition where Alice has CSI of the legitimate channel only, without any knowledge about the CSI with respect to Eve's channel.

### A. CSI-AIDED DF WITH REPETITION CODING (CSI-RC)

In the scenario where Alice has perfect CSI with respect to the legitimate nodes, we observe that the traditional cooperative schemes, as the Fixed Decode-and-Forward employed in [8], [22], and [23], do not fully exploit such available CSI. Knowing the CSI of the legitimate channels, Alice can select the most secure path towards Bob *a priori*, *i.e.*, transmitting directly or using the relay whenever it is more advantageous. Then, if the cooperative path is chosen by Alice, we assume here that the relay employs a conventional DF relaying using the same codebook of Alice, denoted by [33] as repetition coding. Thus, the capacity of the legitimate channel of CSI-RC, represented by the maximum between direct or cooperative path, is

$$C_{\mathrm{B}}^{(\mathrm{CSI\text{-}RC})} = \frac{1}{2}\max\Big\{\log_2\left(1+\gamma'_{\mathrm{AB}}\right), \min\left\{\log_2\left(1+\gamma_{\mathrm{AR}}\right), \log_2\left(1+\gamma_{\mathrm{B}}\right)\right\}\Big\}, \quad (10)$$

where $\gamma_{\mathrm{B}} = \gamma_{\mathrm{AB}} + \gamma_{\mathrm{RB}}$, and $\gamma'_{\mathrm{AB}} = \gamma_{\mathrm{AB},1} + \gamma_{\mathrm{AB},2}$ represents that Alice transmits the same information in two time slots even when the direct communication is employed, as in [17], in order to make the comparison fair in terms of multiplexing loss to the other cooperative schemes.

Then, the capacity of Eve's channel depends on Alice's choice of the best path, so that

$$C_{\mathrm{E}}^{(\mathrm{CSI\text{-}RC})} = \begin{cases} \frac{1}{2}\log_2(1+\gamma'_{\mathrm{AE}}), & \text{if Alice transmits directly,} \\ \frac{1}{2}\log_2(1+\gamma_{\mathrm{E}}), & \text{otherwise,} \end{cases} \quad (11)$$

where $\gamma'_{\mathrm{AE}} = \gamma_{\mathrm{AE},1} + \gamma_{\mathrm{AE},2}$ and $\gamma_{\mathrm{E}} = \gamma_{\mathrm{AE}} + \gamma_{\mathrm{RE}}$.

However, obtaining a closed-form expression for the generalized secrecy outage probability of CSI-RC is quite difficult, mainly due to the maximum between $\log_2\left(1+\gamma'_{\mathrm{AB}}\right)$ and $\min\left\{\log_2\left(1+\gamma_{\mathrm{AR}}\right), \log_2\left(1+\gamma_{\mathrm{B}}\right)\right\}$ in (10). Then, we resort to an approximation by assuming that the relay is placed at an intermediate position between Alice and Bob, so that we consider that Alice chooses the direct transmission whenever $\gamma_{\mathrm{AB}} \geq \gamma_{\mathrm{AR}}$, while cooperation occurs if $\gamma_{\mathrm{AR}} > \gamma_{\mathrm{AB}}$. Such approximation has been also considered in [32] for the case when $\theta = 1$, in which we show by numerical results that the impact in the overall secrecy outage probability is very small independently of the position of relay between Alice and Bob. Due to space limitation, we omit such comparison here and we refer to [32] for further information.

*Theorem 1: The generalized secrecy outage probability of the CSI-RC scheme can be well approximated by*

$$p_{gso}^{(CSI\text{-}RC)} \approx \frac{4^{2\mathcal{R}\theta} e^{\frac{1-4^{-\mathcal{R}\theta}}{(2\bar{\gamma}_{AE})}} \bar{\gamma}_{AB}\bar{\gamma}_{AE}^2}{(\bar{\gamma}_{AB}+4^{\mathcal{R}\theta}\bar{\gamma}_{AE})(\bar{\gamma}_{AB}\bar{\gamma}_{AR}+4^{\mathcal{R}\theta}\bar{\gamma}_{AE}(\bar{\gamma}_{AB}+\bar{\gamma}_{RE}))} + \frac{4^{\mathcal{R}\theta} e^{-\frac{4^{-\mathcal{R}\theta}(\bar{\gamma}_{RE}+\bar{\gamma}_{AE})}{(\bar{\gamma}_{RE}\bar{\gamma}_{AE})}}}{(\bar{\gamma}_{RE}-\bar{\gamma}_{AE})}\Bigg[\frac{\bar{\gamma}_{AR}}{(\bar{\gamma}_{RB}-\bar{\gamma}_{AB})(\bar{\gamma}_{AB}+\bar{\gamma}_{AR})} \times \left(e^{\frac{1}{\bar{\gamma}_{RE}}+\frac{4^{-\mathcal{R}\theta}}{\bar{\gamma}_{AE}}}\bar{\gamma}_{RE}^2\varsigma(\bar{\gamma}_{RE}) - e^{\frac{1}{\bar{\gamma}_{AE}}+\frac{4^{-\mathcal{R}\theta}}{\bar{\gamma}_{RE}}}\bar{\gamma}_{AE}^2\varsigma(\bar{\gamma}_{AE})\right) + \frac{e^{\frac{1}{\bar{\gamma}_{RE}}+\frac{4^{-\mathcal{R}\theta}}{\bar{\gamma}_{AE}}}\bar{\gamma}_{RE}^2\chi(\bar{\gamma}_{RE}) - e^{\frac{1}{\bar{\gamma}_{AE}}+\frac{4^{-\mathcal{R}\theta}}{\bar{\gamma}_{RE}}}\bar{\gamma}_{AE}^2\chi(\bar{\gamma}_{AE})}{(\bar{\gamma}_{AB}-\bar{\gamma}_{RB})}\Bigg], \quad (12)$$

*where* $\varsigma(x) = \frac{\bar{\gamma}_{RB}(\bar{\gamma}_{AB}+\bar{\gamma}_{AR})}{\bar{\gamma}_{RB}\bar{\gamma}_{AR}+4^{\mathcal{R}\theta}x(\bar{\gamma}_{RB}+\bar{\gamma}_{AR})} + \frac{\bar{\gamma}_{AB}(\bar{\gamma}_{AB}+\bar{\gamma}_{AR})}{\bar{\gamma}_{AB}\bar{\gamma}_{AR}+4^{\mathcal{R}\theta}x(\bar{\gamma}_{AB}+\bar{\gamma}_{AR})} + \frac{\bar{\gamma}_{AB}^2}{\bar{\gamma}_{AB}\bar{\gamma}_{AR}+4^{\mathcal{R}\theta}x(\bar{\gamma}_{AB}+2\bar{\gamma}_{AR})} + \frac{\bar{\gamma}_{RB}\bar{\gamma}_{AB}^2}{\bar{\gamma}_{RB}\bar{\gamma}_{AB}\bar{\gamma}_{AR}+4^{\mathcal{R}\theta}x[\bar{\gamma}_{AB}\bar{\gamma}_{AR}+\bar{\gamma}_{RB}(\bar{\gamma}_{AB}+\bar{\gamma}_{AR})]}$ *and* $\chi(x) = \frac{\bar{\gamma}_{AB}^2}{\bar{\gamma}_{AB}\bar{\gamma}_{AR}+4^{\mathcal{R}\theta}x(\bar{\gamma}_{AB}+2\bar{\gamma}_{AR})} - \frac{\bar{\gamma}_{AB}^2}{\bar{\gamma}_{AB}\bar{\gamma}_{AR}+4^{\mathcal{R}\theta}x(\bar{\gamma}_{AB}+\bar{\gamma}_{AR})} + \frac{\bar{\gamma}_{RB}^2}{\bar{\gamma}_{RB}\bar{\gamma}_{AR}+4^{\mathcal{R}\theta}x(\bar{\gamma}_{RB}+\bar{\gamma}_{AR})} - \frac{\bar{\gamma}_{RB}^2}{\bar{\gamma}_{RB}\bar{\gamma}_{AB}\bar{\gamma}_{AR}+4^{\mathcal{R}\theta}x[\bar{\gamma}_{AB}\bar{\gamma}_{AR}+\bar{\gamma}_{RB}(\bar{\gamma}_{AB}+\bar{\gamma}_{AR})]}$.

*Proof:* Please refer to Appendix A. ■

### B. CSI-AIDED DF WITH PARALLEL CODING (CSI-PC)

Similarly to the CSI-RC scheme, Alice selects the best path towards Bob in order to increase the SEE using the available CSI. However, with the CSI-PC scheme we assume that the transmission occurs employing parallel coding [33]. Therefore, the information sent during the second time slot is encoded using a different codebook, independent of the codebook employed by Alice at first time slot, so that the capacity of the legitimate channel, following [33], becomes

$$\begin{aligned} C_{\mathrm{B}}^{(\mathrm{CSI\text{-}PC})} &= \frac{1}{2}\max\Big\{\sum_{i=1}^{2}\log_2\left(1+\gamma_{\mathrm{AB},i}\right), \min\left\{\log_2\left(1+\gamma_{\mathrm{AR}}\right), \log_2\left(1+\gamma_{\mathrm{AB}}\right)+\log_2\left(1+\gamma_{\mathrm{RB}}\right)\right\}\Big\} \\ &= \frac{1}{2}\max\Big\{\sum_{i=1}^{2}\log_2\left(1+\gamma_{\mathrm{AB},i}\right), \min\left\{\log_2\left(1+\gamma_{\mathrm{AR}}\right), \log_2\left(1+\Phi_{\mathrm{B}}\right)\right\}\Big\}, \end{aligned} \quad (13)$$

where $\Phi_{\mathrm{B}} = \gamma_{\mathrm{AB}} + \gamma_{\mathrm{RB}} + \gamma_{\mathrm{AB}}\gamma_{\mathrm{RB}}$.

The capacity of Eve's channel, similarly to CSI-RC scheme, depends of the choice between direct or cooperative transmission and is given by

$$C_{\mathrm{E}}^{(\mathrm{CSI\text{-}PC})} = \begin{cases} \frac{1}{2}\sum_{i=1}^{2}\log_2(1+\gamma_{\mathrm{AE},i}), & \text{w/ direct transm.,} \\ \frac{1}{2}\log_2(1+\Phi_{\mathrm{E}}), & \text{otherwise,} \end{cases} \quad (14)$$

where $\Phi_{\mathrm{E}} = \gamma_{\mathrm{AE}} + \gamma_{\mathrm{RE}} + \gamma_{\mathrm{AE}}\gamma_{\mathrm{RE}}$.

Then in order to obtain the secrecy outage probability of the CSI-PC scheme, we need first to obtain the pdf of $\Phi_{\mathrm{B}}$ and $\Phi_{\mathrm{E}}$, which is quite difficult to obtain in an exact form due to the multiplication and summation of the two involved RVs. Thus, in the following Lemma we propose a useful approximation.

*Lemma 1: The RV $\Phi_j = \gamma_{Aj}+\gamma_{Rj}+\gamma_{Aj}\gamma_{Rj}$, with $j \in \{B, E\}$, can be well approximated by a single Gamma distributed RV, whose pdf is given by*

$$f_{\Phi_j} \approx \frac{\Phi_j^{m_j-1} e^{-\frac{\Phi_j m_j}{\Omega_j}}}{\Gamma(m_j)\left(\frac{\Omega_j}{m_j}\right)^{m_j}}, \tag{15}$$

*where* $m_j = \frac{\Omega_j{}^2}{\bar{\gamma}_{Rj}^2+\bar{\gamma}_{Aj}^2+\left(\sqrt{2}\bar{\gamma}_{Aj}\bar{\gamma}_{Rj}\right)^2}$ *and* $\Omega_j = \bar{\gamma}_{Rj} + \bar{\gamma}_{Aj} + \left(\sqrt{2}\bar{\gamma}_{Aj}\bar{\gamma}_{Rj}\right)$.

*Proof:* Please refer to Appendix B. ■

Building upon the results of Lemma 1, we propose the following approximation to the generalized secrecy outage probability of the CSI-PC scheme.

*Theorem 2: The generalized secrecy outage probability of the CSI-PC scheme can be well approximated by*

$$\begin{aligned} p_{gso}^{(CSI\text{-}PC)} &\approx \frac{2^{2\mathcal{R}\theta}\bar{\gamma}_{AB}\bar{\gamma}_{AE}^2 e^{-\frac{2^{-\mathcal{R}\theta}-1}{\bar{\gamma}_{AE}}}\left(\bar{\gamma}_{AB}+2^{\mathcal{R}\theta}\bar{\gamma}_{AE}\right)^{-1}}{\left(\bar{\gamma}_{AB}\bar{\gamma}_{AR}+2^{\mathcal{R}\theta}\bar{\gamma}_{AE}\left(\bar{\gamma}_{AB}+\bar{\gamma}_{AR}\right)\right)} \\ &\quad - \frac{\bar{\gamma}_{AR}^2}{(\bar{\gamma}_{AB}+\bar{\gamma}_{AR})^2}\left[\frac{\nu(\bar{\gamma}_{AB})(\bar{\gamma}_{AB}+\bar{\gamma}_{AR})+\nu(\bar{\gamma}_{AB}+\bar{\gamma}_{AR})\bar{\gamma}_{AB}}{\bar{\gamma}_{AR}}\right. \\ &\quad - 2^{2\mathcal{R}\theta m_B}\left(\frac{m_B}{m_E}\right)^{m_B}\left(\frac{\Omega_E}{\Omega_B}\right)^{m_B}\frac{\Gamma(m_B+m_E)}{\Gamma(m_E)} \\ &\quad \left.\times {}_2F_1\left(m_E, m_B+m_E; 1+m_B; -\frac{2^{2\mathcal{R}\theta}m_B\Omega_E}{m_E\Omega_B}\right)-1\right], \end{aligned} \tag{16}$$

*where* $\nu(x) = \left(1+\frac{2^{2\mathcal{R}\theta}\Omega_E x}{\gamma_{AR} m_E}\right)^{-m_E}$ *and* ${}_2F_1(\alpha, \beta; \gamma; z)$ *is the Gauss hypergeometric function [35, eq. (9.111)].*

*Proof:* Please refer to Appendix C. ■

### C. DECODE-AND-FORWARD (DF)

For comparison purposes, let us now consider the classical DF as usually found in the literature (*e.g.*, as in [8], [19], [22], and [23]). Alice broadcasts in the first time-slot, while the relay decodes the message received from Alice before forwarding it to Bob. Considering MRC at Bob we have [19]

$$C_{\mathrm{B}}^{(\mathrm{DF})} = \frac{1}{2}\min\left\{\log_2\left(1+\gamma_{\mathrm{AR}}\right), \log_2\left(1+\gamma_{\mathrm{B}}\right)\right\} \tag{17}$$

and

$$C_{\mathrm{E}}^{(\mathrm{DF})} = \frac{1}{2}\log_2\left(1+\gamma_{\mathrm{E}}\right). \tag{18}$$

*Proposition 1: The generalized secrecy outage probability of DF is given by*

$$p_{gso}^{(DF)} \leq \frac{4^{\mathcal{R}\theta}\left[\mathcal{K}(\bar{\gamma}_{RE})-\mathcal{K}(\bar{\gamma}_{AE})\right]}{\bar{\gamma}_{RE}-\bar{\gamma}_{AE}}, \tag{19}$$

*where* $\mathcal{K}(x) = \frac{e^{\frac{(1-4^{\mathcal{R}\theta})}{x}}x^2\left[\bar{\gamma}_{RB}\bar{\gamma}_{AB}\bar{\gamma}_{AR}+4^{\mathcal{R}\theta}x(\bar{\gamma}_{RB}+\bar{\gamma}_{AR})(\bar{\gamma}_{AB}+\bar{\gamma}_{AR})\right]}{\left[\bar{\gamma}_{RB}\bar{\gamma}_{AR}+4^{\mathcal{R}\theta}x(\bar{\gamma}_{RB}+\bar{\gamma}_{AR})\right]\left[\bar{\gamma}_{AB}\bar{\gamma}_{AR}+4^{\mathcal{R}\theta}x(\bar{\gamma}_{AB}+\bar{\gamma}_{AR})\right]}$.

*Proof:* Please refer to Appendix D. ■

Let us remark that (19) simplifies to the expression obtained in [22] when $\theta = 1$.

### D. AMPLIFY-AND-FORWARD (AF)

In AF, Alice broadcasts in the first time slot, while in the second time slot the relay only applies a power gain upon the signal received from Alice and forwards it to Bob, which, in turn, employs MRC on both received frames. The capacities of the legitimate and Eve's channels are given by [19]

$$C_{\mathrm{B}}^{(\mathrm{AF})} = \frac{1}{2}\log_2\left(1+\gamma_{\mathrm{AB}}+\frac{\gamma_{\mathrm{AR}}\gamma_{\mathrm{RB}}}{1+\gamma_{\mathrm{AR}}+\gamma_{\mathrm{RB}}}\right) \tag{20}$$

and

$$C_{\mathrm{E}}^{(\mathrm{AF})} = \frac{1}{2}\log_2\left(1+\gamma_{\mathrm{AE}}+\frac{\gamma_{\mathrm{AR}}\gamma_{\mathrm{RE}}}{1+\gamma_{\mathrm{AR}}+\gamma_{\mathrm{RE}}}\right). \tag{21}$$

*Proposition 2: The generalized secrecy outage probability of AF can be approximated by*

$$\begin{aligned} p_{gso}^{(AF)} &\approx \frac{\bar{\gamma}_B''\left[\mathcal{B}(\bar{\gamma}_B'', \bar{\gamma}_E'')-\mathcal{B}(\bar{\gamma}_B'', \bar{\gamma}_{AE})\right]}{(\bar{\gamma}_E''-\bar{\gamma}_{AE})(\bar{\gamma}_B''-\bar{\gamma}_{AB})} \\ &\quad - \frac{\bar{\gamma}_{AB}\left[\mathcal{B}(\bar{\gamma}_{AB}, \bar{\gamma}_E'')-\mathcal{B}(\bar{\gamma}_{AB}, \bar{\gamma}_{AE})\right]}{(\bar{\gamma}_E''-\bar{\gamma}_{AE})(\bar{\gamma}_B''-\bar{\gamma}_{AB})}, \end{aligned} \tag{22}$$

*where* $\mathcal{B}(x, y) = \frac{y^2}{x2^{-2\theta\mathcal{R}}+y}e^{-\frac{(2^{-2\theta\mathcal{R}}-1)}{y}}$ *and* $\bar{\gamma}_j'' = \frac{\bar{\gamma}_{AR}\bar{\gamma}_{Rj}}{\bar{\gamma}_{AR}+\bar{\gamma}_{Rj}}$ *with* $j \in \{B, E\}$.

*Proof:* Please refer to Appendix E. ■

Note that (22) is very similar to the expression given in [36], with the difference that we consider $\theta\mathcal{R}$ instead of $\mathcal{R}$.

### E. COOPERATIVE JAMMING (CJ)

In the CJ scheme, the relay does not help the communication between the legitimate nodes. Instead, it injects Gaussian noise with the intention of confusing Eve.[1] Thus, Alice and the relay transmit at the same time, so that the capacities of legitimate and Eve's channels are [9]

$$C_{\mathrm{B}}^{(\mathrm{CJ})} = \log_2\left(1+\frac{\gamma_{\mathrm{AB}}}{1+\gamma_{\mathrm{RB}}}\right) \tag{23}$$

and

$$C_{\mathrm{E}}^{(\mathrm{CJ})} = \log_2\left(1+\frac{\gamma_{\mathrm{AE}}}{1+\gamma_{\mathrm{RE}}}\right). \tag{24}$$

Notice that the noise injected by the relay also affects Bob, once it appears as interference in (23).

[1] Let us remark that the performance of CJ could be improved with the help of multiple antennas or multiple jammers [37], [38]. However, we consider a scenario of a WSN with size and cost constraints [8], [22], and we leave scenarios with multiple antennas or multiple relays for future investigation.

*Proposition 3: The generalized secrecy outage probability of the CJ scheme is*

$$p_{gso}^{(CJ)} = 1 + \frac{e^{-b}}{\bar{\gamma}_{RE}\bar{\gamma}_{RB}\varepsilon\alpha}\left[\left(1-\frac{1}{\alpha l\varepsilon}\right)\mathcal{F}(\varepsilon+\varepsilon\alpha) + \left(\frac{1}{\alpha l\varepsilon}+\frac{1}{\alpha}\right)\mathcal{F}\left(\frac{1+\alpha}{\alpha\bar{\gamma}_{RE}}\right) - \bar{\gamma}_{RE}\right], \quad (25)$$

*where* $b = \frac{2^{\theta\mathcal{R}}-1}{\bar{\gamma}_{AB}}$, $\varepsilon = \frac{1+\bar{\gamma}_{RB}b}{\bar{\gamma}_{RB}}$, $\alpha = \frac{\bar{\gamma}_{AB}}{\bar{\gamma}_{AE}(1+\bar{\gamma}_{AB}b)}$, $l = 1-\frac{1}{\bar{\gamma}_{RE}\varepsilon\alpha}$, $\mathcal{F}(x) = e^{x}E_1(x)$, *and* $E_1(x) = \int_x^{\infty}\frac{e^{-t}}{t}dt$ *is the exponential integral.*

*Proof:* Please refer to Appendix F. ■

Note that (25) is very similar to the expression obtained in [22]. However, the term b here depends on $\theta\mathcal{R}$, instead of only $\mathcal{R}$ as in [22].

## IV. SEE MAXIMIZATION ALGORITHM

In this work, we are interested in maximizing the SEE of the network. To that end, here we propose an algorithm to jointly allocate power at Alice and at the relay, as well as to find the best $\theta$ in order to save energy.[2] The problem can be stated as follows:

$$\begin{aligned} \max_{(P_A,P_R,\theta)} \ & \eta^{(sch)} = \frac{\tau^{(sch)}}{P_{total}^{(sch)}} \\ \text{s.t. } & 0 < P_i \le P_{max}, \quad \text{with } i \in \{A, R\}, \\ & 0 < \theta \le 1, \end{aligned} \quad (26)$$

where $P_{max}$ represents the maximum transmit power (assumed to be the same for both A and R).

### A. TOTAL POWER CONSUMPTION

The total power consumed by each cooperative protocol follows the same model adopted in [39], which accounts for the power used by both Alice and the relay, $P_A$ and $P_R$, besides the power consumed by transmission and reception circuitry, $P_{TX}$ and $P_{RX}$. Additionally, we also consider the energy spent at the power amplifier, represented by $\delta$.

Then, the total power consumption of the CSI-RC and CSI-PC schemes is given by

$$\begin{aligned} P_{total}^{(sch)} &= 2\left[(1+\delta)P_A + P_{TX} + P_{RX}\right] \times \Pr\{\gamma_{AB} \ge \gamma_{AR}\} \\ &+ ((1+\delta)(P_A+P_R) + 2P_{TX} + 3P_{RX}) \times \\ &\times \Pr\{\gamma_{AR} > \gamma_{AB}\}, \end{aligned} \quad (27)$$

where sch $\in$ {CSI-RC, CSI-PC} and $\Pr\{\gamma_{AR} > \gamma_{AB}\} = \frac{\bar{\gamma}_{AR}}{\bar{\gamma}_{AR}+\bar{\gamma}_{AB}}$. Note that (27) depends on Alice's choice between direct and cooperative transmission. Whenever $\gamma_{AB} \ge \gamma_{AR}$ the transmission is direct, so that the power consumption is given by the power used by Alice in both time slots, as well as the power associated with the transmission circuitry at Alice and the reception circuitry at Bob. On the other hand, when $\gamma_{AR} > \gamma_{AB}$ cooperation occurs, so that the total power consumption is given by the power employed by Alice and by the relay, as well as the associated transmit circuitry consumption, and the receive circuitry consumption of the relay in the first time slot and of Bob in both time slots.

Next, for the AF and DF cooperative schemes we have

$$P_{total}^{(AF)} = P_{total}^{(DF)} = (1+\delta)(P_A+P_R) + 2P_{TX} + 3P_{RX}, \quad (28)$$

which is very similar to (27) when the relay cooperates. However, let us remark that although we have the same expression for AF and DF and for CSI-Aided schemes, the power allocated to each transmitter can be very different for each scheme.

Finally, the total consumption of the CJ scheme is

$$P_{total}^{(CJ)} = (1+\delta)(P_A+P_R) + 2P_{TX} + P_{RX}, \quad (29)$$

which accounts for the power used by Alice to transmit information and by the relay to inject Gaussian noise. Besides that, $P_{total}^{(CJ)}$ includes the transmit circuitry, at Alice and at the relay, and the receive circuitry, at Bob.

### B. PROPOSED ALGORITHM

The proposed scheme to maximize the SEE optimizes power and $\theta$ through a Dinkelbach-based approach combined with golden search. A theoretical foundation of Dinkelbach algorithm and golden section search with parabolic interpolation are given on IV-B1 and IV-B2, respectively.

In order to optimize each parameter, we resort to an algorithm comprised of one outer-loop and three inner-loops. The outer-loop is related to the stop criterion of the algorithm and is iterated until the increase in SEE, due to the allocation of $P_A$, $P_R$ and $\theta$, is smaller than a predefined threshold $\epsilon$. Then, the inner-loops are associated with the allocation of the powers and $\theta$. Therefore, two inner-loops use a Dinkelbach-based approach to allocate power at Alice and at the relay while a third inner-loop finds the best $\theta$, for the given values of $P_A$ and $P_R$, using a golden section search algorithm with parabolic interpolation. The first two inner-loops stop when a tolerance $\epsilon_P$ is achieved, while $\epsilon_\theta$ defines the stop criterion for the third inner-loop. The proposed algorithm to solve (26) is presented in Algorithm 1 and is discussed in detail next.

#### 1) OPTIMIZING THE POWER ALLOCATION

Due to the complexity of the $p_{gso}^{(sch)}$ expressions, the optimization problem in (26) is very hard to be solved in closed form. As an iterative and distributed alternative to optimize the ratio between functions of the same variable (fractional programming), that contrasts to the time-consuming exhaustive searching, is the Dinkelbach algorithm [27], [28]. A fractional program is defined in a general form as

$$\max_{\mathbf{x}\in\mathbf{S}} q(x) = \frac{f_1(x)}{f_2(x)}, \quad (30)$$

where $\mathbf{S} \subseteq \mathbb{R}^n$ is a convex set, $f_1, f_2 : \mathbf{S} \to \mathbb{R}$, being $f_1(x)$ concave and $f_2(x) > 0$ convex. Moreover, following [28], even if the function (30) is pseudo-concave, meaning that although not strictly concave there exists an inflection point $m$. So that,

[2] Let us remark that the range in which $\theta$ can be optimized depends on the particular system security constraints. Nevertheless, as Section V shows, even a small relaxation of $\theta$ may allow important SEE savings.

**Algorithm 1** Proposed Allocation Algorithm

**Input:** $\eta_{\mathrm{s}}^{(\mathrm{sch})}$, and tolerances $\epsilon$, $\epsilon_{\mathrm{P}}$, $\epsilon_{\theta}$
**Initialize:** $m=1$, $\eta_{\mathrm{s},0}^{(\mathrm{sch})}=0$ and $\eta_{\mathrm{s},1}^{(\mathrm{sch})}=\eta_{\mathrm{s}}^{(\mathrm{sch})}$
**while** $\eta_{\mathrm{s},m}^{(\mathrm{sch})}-\eta_{\mathrm{s},m-1}^{(\mathrm{sch})}\geq\epsilon$ **do**
  ; Power Allocation of $P_{\mathrm{A}}$;
  **Initialize:** $\lambda_0=0$, $n=0$
  **while** $|F(\lambda_n)|\geq\epsilon_{\mathrm{P}}$ **do**
    Use $\lambda=\lambda_n$ in (34) to obtain $P_{\mathrm{A}_n}$;
    $\lambda_{n+1}=\frac{f_1(P_{\mathrm{A}_n})}{f_2(P_{\mathrm{A}_n})}$;
    $n++$;
  **end**
  $P^{\star}_{\mathrm{A}_m}=P_{\mathrm{A}_n}$;
  ; Power Allocation of $P_{\mathrm{R}}$;
  **Initialize:** $n=0$
  **while** $|F(\lambda_n)|\geq\epsilon_{\mathrm{P}}$ **do**
    Use $\lambda=\lambda_n$ in (34) to obtain $P_{\mathrm{R}_n}$;
    $\lambda_{n+1}=\frac{f_1(P_{\mathrm{R}_n})}{f_2(P_{\mathrm{R}_n})}$;
    $n++$;
  **end**
  $P^{\star}_{\mathrm{R}_m}=P_{\mathrm{R}_n}$;
  ; Allocation of $\theta$;
  **Initialize:** $\vartheta_0=(\theta_1,\theta_3,\theta_2)$, $n=1$
  **while** $|\theta_1-\theta_2|\geq\epsilon_{\theta}$ **do**
    Find $\theta_4$ by parabolic interpolation;
    **if** $\eta_{\mathrm{s}}^{(\mathrm{sch})}(\theta_4)>\eta_{\mathrm{s}}^{(\mathrm{sch})}(\theta_3)$: $\theta_1=\theta_3$ and $\theta_3=\theta_4$;
    **else**: $\theta_2=\theta_4$;
    Compute the triplet $\vartheta_n$ using the new $(\theta_1,\theta_3,\theta_2)$;
    $n++$
  **end**
  $\theta^{\star}_m=\theta_4$;
  $m++$;
  Compute $\eta_{\mathrm{s},m}^{(\mathrm{sch})}$ using $P^{\star}_{\mathrm{A}_{m-1}}$, $P^{\star}_{\mathrm{R}_{m-1}}$ and $\theta^{\star}_{m-1}$;
**end**

for any other point $t\leq m$ the function is non-decreasing while for $t\geq m$ the function is non-increasing, it is still possible to solve it by rewriting (30) as [27], [28]

$$\max_{\mathbf{x}\in\mathbf{S},\lambda\in\mathbb{R}}\ \lambda \quad \text{with } f_1(x)-\lambda f_2(x)\geq 0. \tag{31}$$

Furthermore, we can still modify (31) to write it as [28]

$$F(\lambda)=\max_{\mathbf{x}\in\mathbf{S}} f_1(x)-\lambda f_2(x), \tag{32}$$

wherein $f_1(x)$ is maximized while $f_2(x)$ is minimized, with the parameter $\lambda$ determining the weight associated with the denominator.

Then, the optimum value of this function is found when

$$F(\lambda)=0 \iff \lambda=q^{\star}, \tag{33}$$

where $q^{\star}$ is the optimum value of (30). Therefore, solving (30) is equivalent to finding the root of

$$F(\lambda^{\star})=\max_{\mathbf{x}\in\mathbf{S}} f_1(x)-\lambda f_2(x)=0. \tag{34}$$

Thereby, the Dinkelbach algorithm [27], [28] is an efficient form to find $F(\lambda)=0$. This approach is based on Newton's method to calculate $\lambda$ for each $(n+1)$-th iteration as

$$\lambda_{n+1}=\lambda_n-\frac{F(\lambda_n)}{F'(\lambda_n)}=\frac{f_1(x_n^{\star})}{f_2(x_n^{\star})}. \tag{35}$$

#### 2) OPTIMIZING THE FRACTIONAL EQUIVOCATION PARAMETER $\theta$

In order to find the fractional equivocation parameter $\theta$ that optimizes $\eta_{\mathrm{s}}^{(\mathrm{sch})}$, we resort to a golden section search algorithm with parabolic interpolation, which finds the maximum of an unimodal function by narrowing the range of values inside an interval [40]. Considering the initial range for $\theta\in(0,1]$, we first choose an initial triplet $\vartheta_0=(\theta_1,\theta_3,\theta_2)$, where $\theta_1<\theta_3<\theta_2$, and we interpolate $\vartheta_0$ by a parabola, whose maximum is given by $\theta_4$. Then, we compute the energy efficiency using $\theta_4$, and if $\eta_{\mathrm{s}}^{(\mathrm{sch})}(\theta_4)>\eta_{\mathrm{s}}^{(\mathrm{sch})}(\theta_3)$ the new triplet is defined by $\vartheta_1=(\theta_3,\theta_4,\theta_2)$, otherwise, $\vartheta_1=(\theta_1,\theta_3,\theta_4)$. Fig. 2 illustrates the idea, where the SEE function is represented in solid blue, while the parabolic interpolation of $\vartheta_0=(\theta_1,\theta_3,\theta_2)$ is depicted in dashed red. Since the SEE when using $\theta_4$ is higher than that when adopting $\theta_3$ in this example, the algorithm narrows the interval by choosing $\vartheta_1=(\theta_3,\theta_4,\theta_2)$ as a new triplet. Then, at each iteration the interval becomes smaller, until the algorithm stops when a predefined tolerance for the size of the interval is achieved.

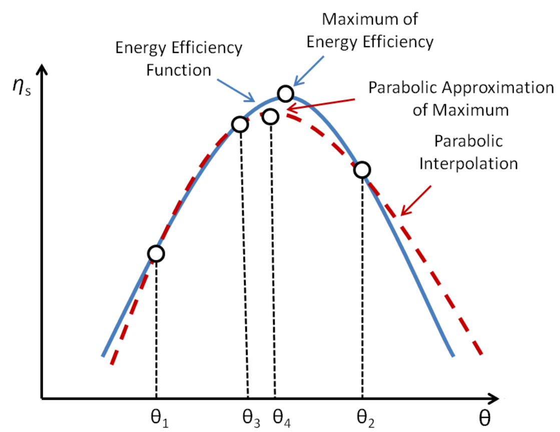


**FIGURE 2.** Allocation of $\theta$ using the golden section search algorithm with parabolic interpolation.

## V. NUMERICAL RESULTS

In this section we provide some numerical examples aiming at evaluating the SEE of the aforementioned cooperative schemes. We consider $\mathcal{R}=3$ bps/Hz, $d_{\mathrm{AB}}=100$ m and $\upsilon=3$. Moreover, as in [39], we employ the following parameters representative of a WSN, with $P_{\mathrm{TX}}=112.2$ mW, $P_{\mathrm{RX}}=97.9$ mW, $\delta=1.86$, $B=10$ kHz and $N_0=-174$ dBm/Hz. Additionally, we consider a link margin of $M_l=20$ dB, total antenna gain $G=5$ dBi, noise figure $N_f=10$ dB and carrier frequency $f_c=2.5$ GHz.

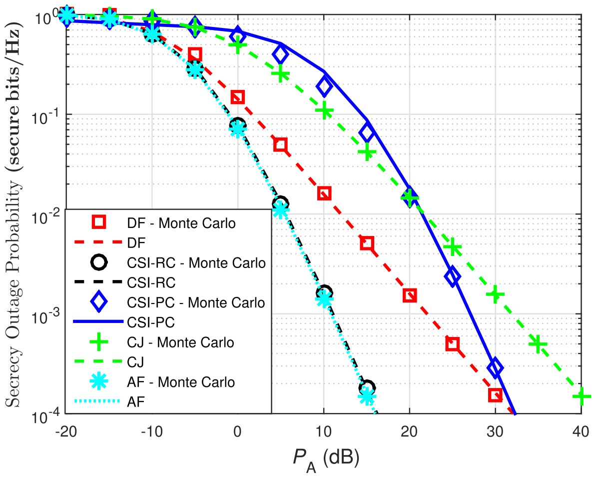


**FIGURE 3.** Closed-form secrecy outage probability expressions compared with Monte Carlo simulation results considering $d_{AR} = 0.5\, d_{AB}$.

### A. NUMERICAL VALIDATION

First, Fig. 3 compares the derived secrecy outage probability expressions with Monte Carlo simulations, where we observe very good agreement between each pair of curves. In particular, Fig. 3 shows that the approximation provided by Lemma 1, for the CSI-PC scheme, is very tight. In addition, although the figure only considers the case when the relay is placed an a intermediate position between Alice and Bob ($d_{AR} = 0.5\, d_{AB}$), the same agreement between theoretic and simulation results is observed for different positions of the relay.

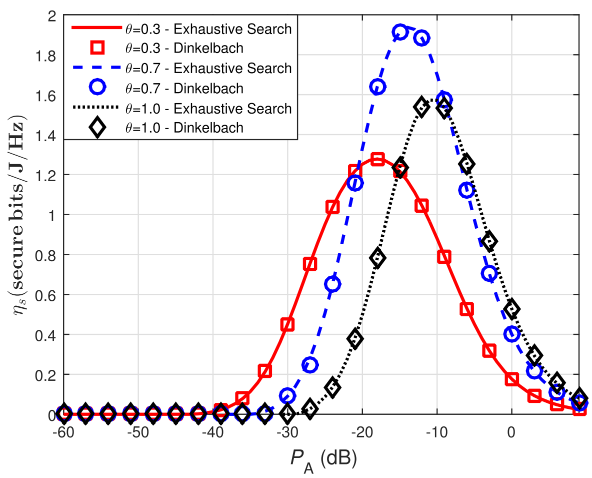


**FIGURE 4.** Secure energy efficiency of CSI-RC for fixed $\theta$, and power allocation using the Dinkelbach algorithm and an exhaustive search-based approach.

Next, in Fig. 4 we compare the SEE of the proposed CSI-RC scheme for fixed values of $\theta \in \{0.3, 0.7, 1.0\}$. We consider the relay at the midpoint between Alice and Bob, while we vary the SNR of the Alice-Bob channel ($\bar{\gamma}_{AB}$) for a fixed SNR at Eve, with $\bar{\gamma}_{AE} = 9$ dB and $\bar{\gamma}_{RE} = 13$ dB. Then, for each $\bar{\gamma}_{AB}$ we allocate the power at the relay using the Dinkelbach algorithm and with an exhaustive search-based approach. As we can observe, the Dinkelbach algorithm agrees very well with the exhaustive search solution, with the advantage of being implemented with low complexity and converging with super-linear rate, as shown in [28]. Moreover, it is interesting to observe that $\eta_s$ varies considerably with $\theta$. Furthermore, different values for $\theta$ imply maximum points at different $\bar{\gamma}_{AB}$, indicating that a joint optimization of $\theta$ and power allocation may considerably improve the SEE of the schemes.

### B. POWER OPTIMIZATION

Extending the analysis for different cooperative schemes, considering power optimization at Alice and at the relay, in Fig. 5 we compare the SEE of CSI-RC, CSI-PC, DF, AF and CJ as a function of $\theta$. Note that AF and DF are always outperformed by CSI-RC and CSI-PC, which is a consequence of fully exploiting the available CSI to choose between transmitting directly or through the relay.[3] In addition, Fig. 5 also shows that CSI-PC is not always the best strategy. Differently from [33], in which parallel coding outperforms repetition coding when secrecy is not considered, Fig. 5 demonstrates that CSI-RC and CSI-PC outperform each other for different ranges of $\theta$. This is due to the fact that the capacities with respect to Bob and with respect to Eve improve in different proportions. Therefore, when a security metric that depends on $\theta$ is considered, CSI-PC can be more beneficial to Eve than to Bob in some situations, making CSI-RC better in these cases. Moreover, since CJ performs best in some situations, we further analyze the performance of CSI-RC, CSI-PC and CJ for different scenarios in the sequel.

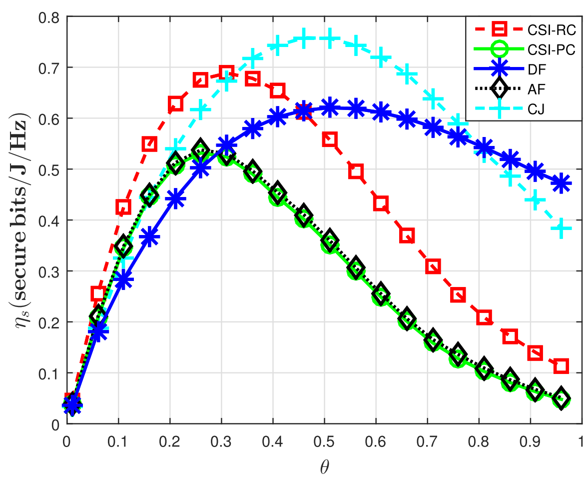


**FIGURE 5.** Secure energy efficiency of CJ, AF, DF, CSI-RC and CSI-PC as a function of $\theta$ for $d_{RE} = 1.3\, d_{AB}$ and $d_{AR} = 0.2\, d_{AB}$.

Figs. 6 and 7 plot $\eta_s$ for these three schemes as a function of $d_{RE}$ and $\theta$ when $d_{AR} = 0.2 d_{AB}$ (Fig. 6) and $d_{AR} = 0.8 d_{AB}$ (Fig. 7). As we can observe, the performance of each

[3] Let us remark that we do not consider a non-cooperative (direct) transmission scheme in our comparison, since CSI-RC and CSI-PC already accounts for the maximum between direct and cooperative transmission.

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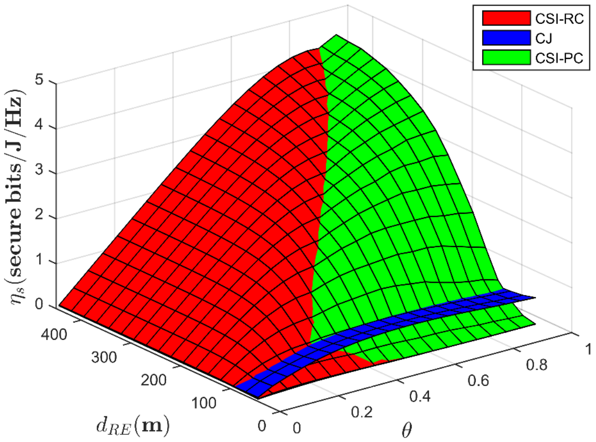


**FIGURE 6.** $\eta_s^{(\text{CSI-RC})}$, $\eta_s^{(\text{CSI-PC})}$ and $\eta_s^{(\text{CJ})}$ as a function of $d_{\text{RE}}$ and $\theta$ for $d_{\text{AR}} = 0.2\, d_{\text{AB}}$.

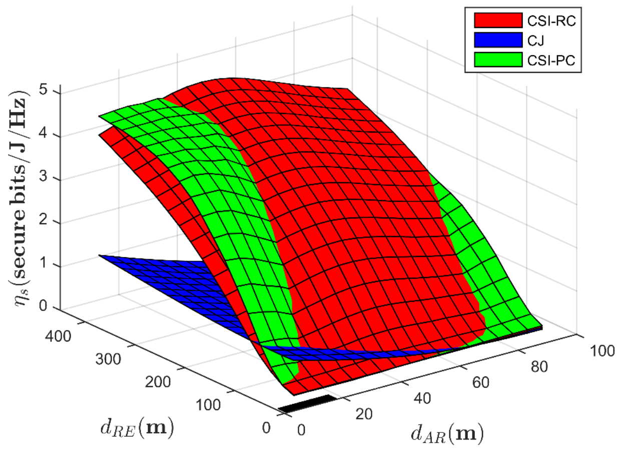


**FIGURE 8.** Secure energy efficiency as a function of $d_{\text{RE}}$ and $d_{\text{AR}}$ by jointly optimizing $\theta$ and power through the proposed algorithm.

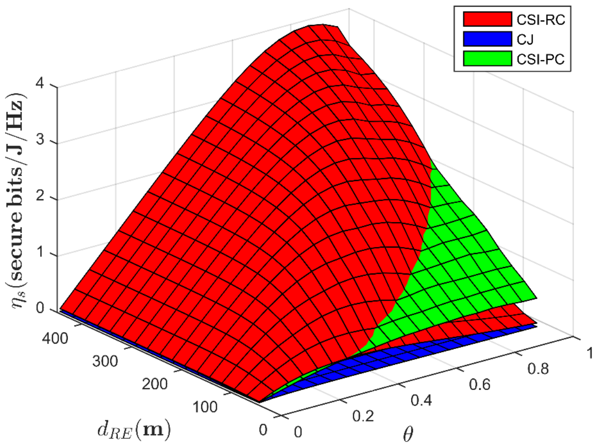


**FIGURE 7.** $\eta_s^{(\text{CSI-RC})}$, $\eta_s^{(\text{CSI-PC})}$ and $\eta_s^{(\text{CJ})}$ as a function of $d_{\text{RE}}$ and $\theta$ for $d_{\text{AR}} = 0.8\, d_{\text{AB}}$.

cooperative scheme depends considerably on both relay and Eve positioning. When the relay is closer to Alice, CSI-RC, CSI-PC and CJ have different regions for which each scheme performs better. For instance, when Eve is closer to the legitimate nodes, it is better for the relay to attack Eve though jamming than to help Alice by cooperating. Therefore, CJ has increased performance, as Fig. 6 shows in the low $d_{\text{RE}}$ region. However, if the relay is closer to Bob, the Gaussian noise injected by the CJ scheme also affect Bob's performance, considerably decreasing the SEE of CJ in Fig. 7. The CSI-PC scheme, by its turn, allows increasing the secrecy capacity of the system when $\theta \to 1$ and the relay: *i)* is closer to Alice (Fig. 6); *ii)* is closer to Bob, and Eve is closer to relay (Fig. 7); however, when $\theta$ can be relaxed the CSI-RC has important performance improvements in comparison with CSI-PC.

### C. JOINT OPTIMIZATION OF POWER AND $\theta$

In order to better visualize the intersections between CSI-RC, CSI-PC and CJ, Fig. 8 plots $\eta_s^{(\text{CSI-RC})}$, $\eta_s^{(\text{CSI-PC})}$ and $\eta_s^{(\text{CJ})}$ by jointly optimizing $\theta$ and power, employing Algorithm 1 proposed in Section IV, as a function of $d_{\text{RE}}$ and $d_{\text{AR}}$. As we can observe, CSI-PC outperforms CSI-RC only when the relay is at one of the extremes, very close to Alice or to Bob. Nevertheless, a significant performance improvement is obtained in this region. Additionally, CJ outperforms the other schemes when Eve is closer to the legitimate nodes.

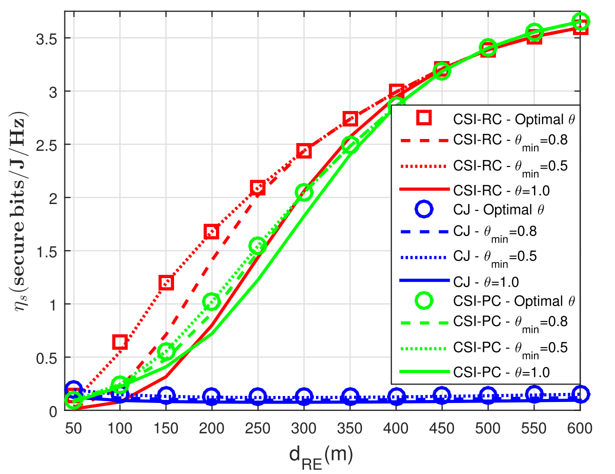


**FIGURE 9.** SEE as a function of $d_{\text{RE}}$ by jointly optimizing $\theta$ and power, with different minimum requirements of secrecy at the system. The relay is at $d_{\text{AR}} = 0.5 d_{\text{AB}}$.

Regarding Fig. 8, we also observe that a joint allocation of $\theta$ and power allows to increase the SEE of the system. However, depending on the particular application, an optimal $\theta$ cannot be applied, due to a minimum level of secrecy requirement that has to be met. Therefore, in Fig. 9 we compare $\eta_s^{(\text{CSI-RC})}$, $\eta_s^{(\text{CSI-PC})}$ and $\eta_s^{(\text{CJ})}$ by jointly optimizing $\theta$ and power, considering different minimum values of secrecy requirements. We consider the case of power optimization with fixed $\theta = 1$, representing a scenario where the security constraints cannot be relaxed, two scenarios when the minimum requirement is $\theta \in \{0.5, 0.8\}$, and the case with unrestricted $\theta$. As we can notice, the general conclusions in terms of SEE are maintained. However, it is interesting to note that, for CSI-RC and CSI-PC schemes, an acceptable $\theta = 0.8$ allows to obtain values of SEE considerably larger than the case with $\theta = 1$. Additionally, a minimum acceptable value of $\theta = 0.5$ ensures SEE close to the unrestricted case. Note that, by its turn, CJ has very small performance changes in these different scenarios, obtaining a better performance only when the relay is closest to Alice.

Finally, the behavior of $\theta^\star$, the $\theta$ that maximizes the SEE, is shown in Fig. 10 as a function of $d_{\mathrm{RE}}$. As we can observe, $\theta^\star$ for CSI-RC and CSI-PC increases when Eve is farther away from the legitimate nodes, once $p_{\mathrm{gso}}^{(\mathrm{CSI\text{-}RC})}$ and $p_{\mathrm{gso}}^{(\mathrm{CSI\text{-}PC})}$ decrease when $d_{\mathrm{RE}}$ increases, so that the SEE increases with $\theta$. On the other hand, $\theta^\star$ is rather constant for the CJ scheme, regardless of the position of Eve. This is due to the fact that the outage performance of CJ is limited by the interference caused by the relay at Bob, and thus the distance with respect to Eve has smaller impact in the overall SEE.

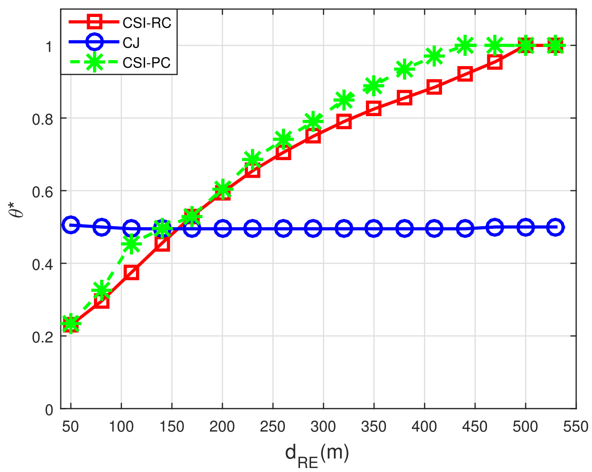


**FIGURE 10.** Optimal $\theta^\star$ that maximizes the secure energy efficiency of CSI-RC, CSI-PC and CJ as a function of $d_{\mathrm{RE}}$. The relay is at $d_{\mathrm{AR}} = 0.2d_{\mathrm{AB}}$.

## VI. CONCLUSIONS

In this paper we investigated the secure energy efficiency of some cooperative protocols considering a generalized metric for the secrecy outage probability, which allows different levels of secrecy requirements, defined by a fractional equivocation parameter $\theta \in (0, 1]$. Considering cooperative schemes that fully exploit the CSI available at the transmitter side, we proposed two methods based on DF relaying, the CSI-Aided DF with Repetition Coding (CSI-RC) and the CSI-Aided DF with Parallel Coding (CSI-PC), assuming that only the CSI with respect to the legitimate nodes is known. Moreover, we also proposed an energy efficient algorithm to jointly allocate power at Alice and at the relay, based on the Dinkelbach algorithm, as well as to find the best $\theta$ that maximizes the energy efficiency by employing a golden section search with parabolic interpolation. Our results show that CSI-RC outperforms the other cooperative schemes for most scenarios, except if the relay is positioned very close to either Alice or Bob, when CSI-PC becomes more advantageous if $\theta \to 1$. Additionally, CJ performs better if Eve is close to the relay and the relay is close to Alice, independently of $\theta$.

## APPENDIX A
## PROOF OF THEOREM 1

To compute the generalized secrecy outage probability of CSI-RC, one must consider the maximum between $\log_2\left(1+\gamma'_{\mathrm{AB}}\right)$ and $\min\left\{\log_2\left(1+\gamma_{\mathrm{AR}}\right), \log_2\left(1+\gamma_{\mathrm{B}}\right)\right\}$, *i.e.*, the direct and the relayed paths, respectively. However, such analytic solution is cumbersome, so that we resort to an approximation by assuming that the relay is placed at an intermediate position between Alice and Bob. Then, we consider that Alice chooses the direct transmission whenever $\gamma_{\mathrm{AB}} \geq \gamma_{\mathrm{AR}}$. Conversely, cooperation occurs if $\gamma_{\mathrm{AR}} > \gamma_{\mathrm{AB}}$. Therefore, three sub-cases must be considered: *i.* $\mathcal{E}_1 = \{\gamma_{\mathrm{AB}} \geq \gamma_{\mathrm{AR}}\}$, indicating the Alice's choice for the direct transmission; *ii.* $\mathcal{E}_2 = \{\gamma_{\mathrm{AB}} < \gamma_{\mathrm{AR}} \bigcap \gamma_{\mathrm{AR}} < \gamma_{\mathrm{B}}\}$, indicating the choice for the cooperative transmission with the capacity limited by the A-R link; *iii.* $\mathcal{E}_3 = \{\gamma_{\mathrm{AB}} < \gamma_{\mathrm{AR}} \bigcap \gamma_{\mathrm{AR}} \geq \gamma_{\mathrm{B}}\}$, indicating cooperation with the capacity limited by the MRC at Bob.

Following (7), $p_{\mathrm{gso}}^{(\mathrm{CSI\text{-}RC})}$ can be written as a sum of two terms, $A_1$ and $A_2$, where $A_1 = \Pr\{C_{\mathrm{B}} \leq C_{\mathrm{E}}\}$ and $A_2 = \Pr\left\{C_{\mathrm{B}} - C_{\mathrm{E}} < 2\theta\mathcal{R} \bigcap C_{\mathrm{B}} - 2\mathcal{R} < C_{\mathrm{E}} < C_{\mathrm{B}}\right\}$, noting that $2\mathcal{R}$ is considered due to the multiplexing loss of the cooperative transmission. Note also that we drop the superscript (CSI-RC) only to simplify the notation. Then, the solution of $A_1$ yields

$$\begin{aligned} A_1 &= \Pr\left\{C_{\mathrm{B}} \leq C_{\mathrm{E}} \bigcap \mathcal{E}_1\right\} + \Pr\left\{C_{\mathrm{B}} \leq C_{\mathrm{E}} \bigcap \mathcal{E}_2\right\} \\ &\quad + \Pr\left\{C_{\mathrm{B}} \leq C_{\mathrm{E}} \bigcap \mathcal{E}_3\right\} \\ &= \int_0^\infty \int_0^{\gamma_{\mathrm{AB}}} \int_{\gamma_{\mathrm{AB}}}^\infty f_{\gamma_{\mathrm{AB}}} f_{\gamma_{\mathrm{AR}}} f_{\gamma_{\mathrm{AE}}} \mathrm{d}\gamma_{\mathrm{AE}} \mathrm{d}\gamma_{\mathrm{AR}} \mathrm{d}\gamma_{\mathrm{AB}} \\ &\quad + \int_0^\infty \int_0^{\gamma_{\mathrm{AR}}} \int_{\gamma_{\mathrm{AR}}}^\infty \int_{\gamma_{\mathrm{AR}}}^\infty f_{\gamma_{\mathrm{E}}} f_{\gamma_{\mathrm{B}}} f_{\gamma_{\mathrm{AR}}} f_{\gamma_{\mathrm{AB}}} \mathrm{d}\gamma_{\mathrm{E}} \mathrm{d}\gamma_{\mathrm{B}} \mathrm{d}\gamma_{\mathrm{AB}} \mathrm{d}\gamma_{\mathrm{AR}} \\ &\quad + \int_0^\infty \int_{\gamma_{\mathrm{AB}}}^\infty \int_0^{\gamma_{\mathrm{AR}}} \int_{\gamma_{\mathrm{B}}}^\infty f_{\gamma_{\mathrm{E}}} f_{\gamma_{\mathrm{B}}} f_{\gamma_{\mathrm{AR}}} f_{\gamma_{\mathrm{AB}}} \mathrm{d}\gamma_{\mathrm{E}} \mathrm{d}\gamma_{\mathrm{B}} \mathrm{d}\gamma_{\mathrm{AR}} \mathrm{d}\gamma_{\mathrm{AB}}, \end{aligned} \tag{36}$$

where, since we assume Rayleigh fading channels, $\gamma_{ij}$'s are exponentially distributed random variables, whose probability density function (pdf) is [34]

$$f_{\gamma_{ij}} = \frac{1}{\bar{\gamma}_{ij}} e^{-\gamma_{ij}/\bar{\gamma}_{ij}}, \tag{37}$$

and with the pdf of the equivalent SNRs at Bob and Eve ($\gamma_j = \gamma_{\mathrm{A}j} + \gamma_{\mathrm{R}j}$, with $j \in \{\mathrm{B}, \mathrm{E}\}$) given by [41]

$$f_{\gamma_j} = \frac{1}{\bar{\gamma}_{\mathrm{R}j} - \bar{\gamma}_{\mathrm{A}j}} \left(e^{-\gamma_{\mathrm{R}j}/\bar{\gamma}_{\mathrm{R}j}} - e^{-\gamma_{\mathrm{A}j}/\bar{\gamma}_{\mathrm{A}j}}\right). \tag{38}$$

Moreover, note that in the case of $\gamma'_{AB}$ and $\gamma'_{AE}$, since the same channel statistics are assumed for both transmission from Alice, we can define $\bar{\gamma}'_{AB} = 2\bar{\gamma}_{AB}$ and $\bar{\gamma}'_{AE} = 2\bar{\gamma}_{AE}$.

Next, $A_2$ can be written as $A_{2a} - A_{2b}$, with $A_{2a} = \Pr\left\{C_B - C_E < 2\theta\mathcal{R} \bigcap C_B - 2\mathcal{R} < C_E\right\}$ and $A_{2b} = \Pr\left\{C_B - C_E < 2\theta\mathcal{R} \bigcap C_E \geq C_B\right\}$, so that

$$\begin{aligned} A_{2a} &= \Pr\left\{C_B - C_E < 2\theta\mathcal{R} \bigcap C_B - 2\mathcal{R} < C_E \bigcap \mathcal{E}_1\right\} \\ &\quad + \Pr\left\{C_B - C_E < 2\theta\mathcal{R} \bigcap C_B - 2\mathcal{R} < C_E \bigcap \mathcal{E}_2\right\} \\ &\quad + \Pr\left\{C_B - C_E < 2\theta\mathcal{R} \bigcap C_B - 2\mathcal{R} < C_E \bigcap \mathcal{E}_3\right\} \\ &= \int_0^\infty \int_{\gamma_{AR}}^\infty \int_\rho^\infty f_{\gamma_{AE}} f_{\gamma_{AB}} f_{\gamma_{AR}} \mathrm{d}\gamma_{AE} \mathrm{d}\gamma_{AB} \mathrm{d}\gamma_{AR} \\ &\quad + \int_0^\infty \int_{\gamma_{AB}}^\infty \int_{\gamma_{AR}}^\infty \int_{\psi(\gamma_{AR})}^\infty f_{\gamma_E} f_{\gamma_B} f_{\gamma_{AB}} f_{\gamma_{AR}} \mathrm{d}\gamma_E \mathrm{d}\gamma_B \mathrm{d}\gamma_{AR} \mathrm{d}\gamma_{AB} \\ &\quad + \int_0^\infty \int_{\gamma_{AB}}^\infty \int_0^{\gamma_{AR}} \int_{\psi(\gamma_B)}^\infty f_{\gamma_E} f_{\gamma_B} f_{\gamma_{AB}} f_{\gamma_{AR}} \mathrm{d}\gamma_E \mathrm{d}\gamma_B \mathrm{d}\gamma_{AR} \mathrm{d}\gamma_{AB}, \end{aligned} \tag{39}$$

where $\rho = \frac{2^{-2\theta\mathcal{R}}(1+\gamma'_{AB})-1}{2}$, $\psi(x) = 2^{-2\theta\mathcal{R}}(1+x) - 1$ and

$$\begin{aligned} A_{2b} &= \Pr\left\{C_B - C_E < 2\theta\mathcal{R} \bigcap C_E \geq C_B \bigcap \mathcal{E}_1\right\} \\ &\quad + \Pr\left\{C_B - C_E < 2\theta\mathcal{R} \bigcap C_E \geq C_B \bigcap \mathcal{E}_2\right\} \\ &\quad + \Pr\left\{C_B - C_E < 2\theta\mathcal{R} \bigcap C_E \geq C_B \bigcap \mathcal{E}_3\right\} \\ &= \int_0^\infty \int_{\gamma_{AR}}^\infty \int_{\gamma_{AB}}^\infty f_{\gamma_{AE}} f_{\gamma_{AB}} f_{\gamma_{AR}} \mathrm{d}\gamma_{AE} \mathrm{d}\gamma_{AB} \mathrm{d}\gamma_{AR} \\ &\quad + \int_0^\infty \int_{\gamma_{AB}}^\infty \int_{\gamma_{AR}}^\infty \int_{\gamma_{AR}}^\infty f_{\gamma_E} f_{\gamma_B} f_{\gamma_{AB}} f_{\gamma_{AR}} \mathrm{d}\gamma_E \mathrm{d}\gamma_B \mathrm{d}\gamma_{AR} \mathrm{d}\gamma_{AB} \\ &\quad + \int_0^\infty \int_{\gamma_{AB}}^\infty \int_0^{\gamma_{AR}} \int_{\gamma_B}^\infty f_{\gamma_E} f_{\gamma_B} f_{\gamma_{AB}} f_{\gamma_{AR}} \mathrm{d}\gamma_E \mathrm{d}\gamma_B \mathrm{d}\gamma_{AR} \mathrm{d}\gamma_{AB}. \end{aligned} \tag{40}$$

Finally, after a few algebraic manipulations with $A_1 + A_{2a} - A_{2b}$ we have that

$$\begin{aligned} p_{gso}^{(\text{CSI-RC})} &\approx \int_0^\infty \int_{\gamma_{AR}}^\infty \int_\rho^\infty f_{\gamma_{AE}} f_{\gamma_{AB}} f_{\gamma_{AR}} \mathrm{d}\gamma_{AE} \mathrm{d}\gamma_{AB} \mathrm{d}\gamma_{AR} \\ &\quad + \int_0^\infty \int_{\gamma_{AB}}^\infty \int_{\gamma_{AR}}^\infty \int_{\psi(\gamma_{AR})}^\infty f_{\gamma_E} f_{\gamma_B} f_{\gamma_{AB}} f_{\gamma_{AR}} \mathrm{d}\gamma_E \mathrm{d}\gamma_B \mathrm{d}\gamma_{AR} \mathrm{d}\gamma_{AB} \\ &\quad + \int_0^\infty \int_{\gamma_{AB}}^\infty \int_0^{\gamma_{AR}} \int_{\psi(\gamma_B)}^\infty f_{\gamma_E} f_{\gamma_B} f_{\gamma_{AB}} f_{\gamma_{AR}} \mathrm{d}\gamma_E \mathrm{d}\gamma_B \mathrm{d}\gamma_{AR} \mathrm{d}\gamma_{AB} \end{aligned} \tag{41}$$

whose solution yields (12). Moreover, let us remark that although the solution resorts to an approximation by assuming that the relay is placed at an intermediate position between Alice and Bob, the impact in the overall secrecy outage probability, for different positions of the relay, is small, as shown in [32] for the case when $\theta = 1$.

## APPENDIX B
## PROOF OF LEMMA 1

Let us re-write $\Phi_j = \gamma_{Aj} + \gamma_{Rj} + \gamma_{Aj}\gamma_{Rj}$ as

$$\Phi_j = X + Y + \underbrace{XY}_{Z}, \tag{42}$$

where we notice that obtaining the exact expression for the pdf of $\Phi_j$ is a complex task due to the sum and the product of the two independent and non-identically distributed (i.n.i.d) RVs $X$ and $Y$, which follow exponential distributions as $X \sim \text{Exp}\left(\frac{1}{\bar{\gamma}_{Aj}}\right)$ and $Y \sim \text{Exp}\left(\frac{1}{\bar{\gamma}_{Rj}}\right)$. Alternatively, we resort to a two-step approximation in which we first approximate the product $Z = XY$ by a single Gamma distributed RV, then we approximate the sum $X + Y + Z$ again by another Gamma distributed RV.

Starting with the product $Z = XY$, we use the fact that an exponential RV with parameter $\beta$ can be represented by a Gamma RV with parameters $k = 1$ and $\Theta = \frac{1}{\beta}$. Therefore, $X \sim \text{Gamma}\left(k = 1, \bar{\gamma}_{Aj}\right)$ and $Y \sim \text{Gamma}\left(k = 1, \bar{\gamma}_{Rj}\right)$.

Then, we follow a procedure similar to [42] and [43], in which the authors approximate the product of two Nakagami-$m$ distributed RV's into a single Nakagami-$m$ distributed RV using the method of moments. In our case, the goal is approximate $Z$ using a Gamma distribution, so that $Z \sim \text{Gamma}(\xi, \varrho)$. However, we noticed that the direct application of the method of moments as in [43] yields an approximation with a different diversity in the case of the Gamma distribution. Therefore, since the shape parameter $\xi$ is related to the slope of the curve, and since both $X$ and $Y$ have the shape parameter equal to one, we observe that a more precise approximation can be obtained by considering $\xi = 1$ and calculating $\varrho$ according to the second moment of the Gamma distribution.

The $n$-th moment of $Z$ is given by

$$\mathbb{E}\left[Z^n\right] = \frac{\Gamma[\xi + n]\varrho^n}{\Gamma[\xi]}, \tag{43}$$

while the $n$-th joint moment of $XY$ is [44]

$$\mathbb{E}\left[(XY)^n\right] = \frac{\Gamma[k+n](\bar{\gamma}_{Aj})^n}{\Gamma[k]} \times \frac{\Gamma[k+n](\bar{\gamma}_{Rj})^n}{\Gamma[k]}. \tag{44}$$

Then, matching the second moments of (43) with (44) we obtain $\varrho = \sqrt{2}\bar{\gamma}_{Aj}\bar{\gamma}_{Rj}$, so that $Z \sim \text{Gamma}\left(1, \sqrt{2}\bar{\gamma}_{Aj}\bar{\gamma}_{Rj}\right)$.

As a second step, we now approximate the sum $X + Y + Z$ by a single Gamma RV $\Phi_j$. Based on [45], we can make $\Phi_j \sim \text{Gamma}(m_j, \frac{\Omega_j}{m})$, with $\Omega_j = \mathbb{E}[\Phi_j]$ and $m_j = \frac{\Omega_j^2}{\mathrm{E}[\Phi_j^2]-\Omega_j^2}$, where the $n$-th moment of $\Phi_j$ can be obtained using a multinomial expansion represented by [44]

$$\mathbb{E}\left[\Phi_j^n\right] = \sum_{n_1=0}^{n}\sum_{n_2=0}^{n_1}\binom{n}{n_1}\binom{n_1}{n_2} \times \mathbb{E}\left[X^{(n-n_1)}\right]\mathbb{E}\left[Y^{(n_1-n_2)}\right]\mathbb{E}\left[Z^{n_2}\right], \quad (45)$$

yielding

$$m_j = \frac{\Omega_j{}^2}{\bar{\gamma}_{Rj}^2 + \bar{\gamma}_{Aj}^2 + \left(\sqrt{2}\bar{\gamma}_{Aj}\bar{\gamma}_{Rj}\right)^2}, \quad (46)$$

$$\Omega_j = \bar{\gamma}_{Rj} + \bar{\gamma}_{Aj} + \left(\sqrt{2}\bar{\gamma}_{Aj}\bar{\gamma}_{Rj}\right). \quad (47)$$

Finally, the pdf of $\Phi_j$ leads to (15), concluding the proof.

## APPENDIX C
## PROOF OF THEOREM 2

As in the case of CSI-RC, the same approximation is considered to the choice of Alice between direct or cooperative transmission, in which direct transmission occurs whenever $\gamma_{AB} \geq \gamma_{AR}$ and cooperative transmission occurs when $\gamma_{AR} > \gamma_{AB}$. Redefining the three possible sub-cases we have: *i.* $\mathcal{E}_1 = \{\gamma_{AB} \geq \gamma_{AR}\}$ (direct transmission – the same as CSI-RC); *ii.* $\mathcal{E}_2' = \{\gamma_{AB} < \gamma_{AR} \bigcap \gamma_{AR} < \Phi_B\}$ (cooperation with the capacity limited by the A-R link); *iii.* $\mathcal{E}_3' = \{\gamma_{AB} < \gamma_{AR} \bigcap \gamma_{AR} \geq \Phi_B\}$ (cooperation with the capacity limited at Bob), and following the same procedure as in Appendix A, we can easily arrive at

$$\begin{aligned} p_{gso}^{(CSI\text{-}PC)} \approx & \int_0^\infty \int_{\gamma_{AR}}^\infty \int_{\rho'}^\infty f_{\gamma_{AE}} f_{\gamma_{AB}} f_{\gamma_{AR}} d\gamma_{AE} d\gamma_{AB} d\gamma_{AR} \\ & + \int_0^\infty \int_{\gamma_{AB}}^\infty \int_{\gamma_{AR}}^\infty \int_{\psi'(\gamma_{AR})}^\infty f_{\Phi_E} f_{\Phi_B} f_{\gamma_{AB}} f_{\gamma_{AR}} \\ & \times d\Phi_E d\Phi_B d\gamma_{AR} d\gamma_{AB} \\ & + \int_0^\infty \int_{\gamma_{AB}}^\infty \int_0^{\gamma_{AR}} \int_{\psi'(\Phi_B)}^\infty f_{\Phi_E} f_{\Phi_B} f_{\gamma_{AB}} f_{\gamma_{AR}} \\ & \times d\Phi_E d\Phi_B d\gamma_{AR} d\gamma_{AB}, \end{aligned} \quad (48)$$

where $\rho' = 2^{-\theta\mathcal{R}}(1+\gamma_{AB}) - 1$ and let us remark that we also assume that $\log_2(1+x) \approx \log_2(x)$ to simplify the analysis, so that $\psi'(x) = 2^{-2\theta\mathcal{R}}x$ is employed in (48). Such approximation has been validated by extensive simulation showing to be tight in the whole SNR range considered in the numerical results.

Finally, to solve (48) we need to recur to [35, eq. (3.351.1)] and [35, eq. (6.455.2)], and after some algebraic manipulations the solution yields (16).

## APPENDIX D
## PROOF OF PROPOSITION 1

Similarly to Appendix A, we can write $p_{gso}^{(DF)}$ as a sum of two terms and treat each intersection individually. Moreover, note that the solution for DF appears as a sub-case of the solution for CSI-RC, since CSI-RC involves the choice for traditional DF and the direct transmission. Therefore, it is not difficult to show that $p_{gso}^{(DF)}$ reduces to

$$\begin{aligned} p_{gso}^{(DF)} = & \int_0^\infty \int_{\gamma_{AR}}^\infty \int_{\psi(\gamma_{AR})}^\infty f_{\gamma_B} f_{\gamma_E} f_{\gamma_{AR}} d\gamma_E d\gamma_B d\gamma_{AR} \\ & + \int_0^\infty \int_0^{\gamma_{AR}} \int_{\psi(\gamma_B)}^\infty f_{\gamma_B} f_{\gamma_E} f_{\gamma_{AR}} d\gamma_E d\gamma_B d\gamma_{AR}, \end{aligned} \quad (49)$$

whose solution with standard calculus yields (19).

## APPENDIX E
## PROOF OF PROPOSITION 2

First, we re-write $p_{gso}^{(AF)}$ as a sum of $B_1$ and $B_2$. Moreover, as in [36], we must also consider a high SNR assumption to define $\gamma_{B'} = \gamma_{AB} + \frac{\gamma_{AR}\gamma_{RB}}{\gamma_{AR}+\gamma_{RB}}$ and $\gamma_{E'} = \gamma_{AE} + \frac{\gamma_{AR}\gamma_{RE}}{\gamma_{AR}+\gamma_{RE}}$, so that $B_1$ can be approximated by $B_1 \approx \Pr\left\{\gamma_{B'} < \gamma_{E'}\right\}$, while $B_2 = \Pr\left\{C_B - C_E < 2\theta\mathcal{R} \bigcap C_B - 2\mathcal{R} < C_E\right\} - \Pr\left\{C_B - C_E < 2\theta\mathcal{R} \bigcap C_E \geq C_B\right\}$. Then, by distributing the intersections we have

$$p_{gso}^{(AF)} \approx \int_0^\infty \int_0^{2^{2\theta\mathcal{R}}(1+\gamma_{E'})-1} f_{\gamma_{B'}} f_{\gamma_{E'}} d\gamma_{B'} d\gamma_{E'}, \quad (50)$$

whose solution is given by (22).

## APPENDIX F
## PROOF OF PROPOSITION 3

Following the same approach as in Appendix E, diving the solution into sub-cases, it is straightforward to show that

$$p_{gso}^{(CJ)} = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\sigma f_{\gamma_{AB}} f_{\gamma_{RB}} f_{\gamma_{AE}} f_{\gamma_{RE}} d\gamma_{AB} d\gamma_{RB} d\gamma_{AE} d\gamma_{RE}, \quad (51)$$

where $\sigma = \left[\left(1 + \frac{\gamma_{AE}}{1+\gamma_{RE}}\right)2^{\theta\mathcal{R}} - 1\right](1+\gamma_{RB})$, and whose solution is given by (25), concluding the proof.

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