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Adaptive Neural LMI-Based H-Infinity Control for MEMS Gyroscope

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ABSTRACT A novel adaptive radial basis function neural network H-infinity control strategy with robust feedback compensator using linear matrix inequality (LMI) approach is proposed for micro electro mechanical systems vibratory gyroscopes involving parametric uncertainties and external disturbances. The proposed system is comprised of a neural network controller, which is designed to mimic an equivalent control law aimed at relaxing the requirement of exact mathematical model and a robust feedback controller, which is derived to eliminate the effect of modeling error and external disturbances. Based on the Lyapunov stability theorem, it is shown that H-infinity tracking performance of the gyroscope system can be achieved, all variables of the closed-loop system are bounded, and the effect due to external disturbances on the tracking error can be attenuated effectively. Numerical simulations are investigated to demonstrate that the satisfactory tracking performance and strong robustness against external disturbances can be obtained using the proposed adaptive neural H-infinity control strategy with robust feedback compensator by LMI technique.

INDEX TERMS Adaptive control, H-Infinity Control, neural network control.

I. INTRODUCTION

Gyroscopes are popular sensors to measure angular velocity in automobile, navigation etc because of their advantages in size and cost. However, damping effects and cross stiffness, the temperature disturbances and other noise sources such as time varying system parameters, thermal, mechanical noise, and sensing circuitry noise may deteriorate the dynamic performance. During the last few years, a variety of advanced control approaches have been proposed to solve these challenges, enhancing the performance of MEMS gyroscope and robustness in the presence of model uncertainties and external disturbances. Recent years, adaptive controls [1]–[7] methods have achieved great development in the control of MEMS gyroscopes because they can automatically adjust parameters of the controller according to the changing system dynamics and eliminate the effect of model uncertainties and external disturbances on the tracking performance. However, accurate gyroscope model is required since the adaptive controller is not “ robust” in the presence of model uncertainties and external disturbances, making it difficult to design an appropriate adaptive control scheme for time-varying and nonlinear systems. As we know, the neural network has the ability to approximate any continuous smooth function as accurately as possible, making it a useful tool to approximate systems with nonlinearity and variable dynamics and compensate the effect of model uncertainties and external disturbances. Neural network controls have made good progress in uncertain MIMO nonlinear state time-varying delay systems [8], robotic manipulators [9], small rotary-wing unmanned aircraft [10], n-link rigid robot manipulator [11], nonholonomic mobile robot [12], MEMS gyroscope [13].

The strategy of H-infinity robust control has also achieved a great deal of attention in the last decades. An adaptive H-infinity tracking control equipped with a variable structure control (VSC) algorithm is proposed for a class of nonlinear MIMO systems represented by input–output models involving parametric uncertainties, unmodeled perturbations and external disturbances in [14]. In [15], an adaptive H-infinity performance index is defined to describe the disturbance attenuation performance of systems with time-varying parameter estimations. However, the H-infinity robust controller also requires strict constrained conditions and prior knowledge of the entire plant model so that it is difficult to implement the H-infinity robust controller on nonlinear systems. To solve the problem, neural network H-infinity robust controller has been applied to a class of nonlinear systems to approximate system dynamics. Zuo et al. [16] proposed a novel neural-network-based robust H-infinity control strategy for the trajectory following problem of robot manipulators. An adaptive neural robust controller based on H-infinity method was presented for trajectory of uncertain robot manipulators in [17]. Nowadays, Riccati equation and LMI have been two popular tools to ensure the closed-loop systems to be robust regional stable and possess H-infinity performance. In [18], a nonlinear H-infinity control of relative motion in space via the state-dependent Riccati equations was developed. Based on LMI, the operation of the closed-loop supply chain networks with the third party reverse providers were studied in [19].

In this study, an adaptive neural network H-infinity control strategy with robust feedback compensator using LMI is firstly proposed for the control of MEMS gyroscopes to relax the requirement of accurate dynamic model associated with conventional control and to achieve desired H-infinity tracking performance. The RBF neural network control and the LMI-based H-infinity robust control are successfully integrated with MEMS gyroscopes so that satisfactory dynamic behavior and strong robustness in the presence of parameter uncertainties and inevitable disturbances can be achieved, convergence of tracking error to zero can be guaranteed and chattering can be eliminated effectively. The advantages of the proposed control strategy are summarized as follows compared to existing ones:

(1) The powerful approximation property of RBF NN is used to mimic the ideal control law so that accurate gyroscope model is not required and effect of modeling error and nonlinearity on tracking performance can be eliminated effectively. A robust feedback compensator is incorporated into the neural network H-infinity control to improve the tracking performance and enhance the robustness against model uncertainties and external disturbances..

(2) Adaptive control, neural network control, LMI-based H-infinity robust control are combined in the MEMS gyroscopes which is the innovative development of robust feedback compensator incorporated into the conventional neural network control for MEMS gyroscopes, so that satisfactory H-infinity tracking performance and robustness in the presence of model uncertainties and inevitable disturbances can be obtained. Moreover, using the proposed adaptive neural H-infinity control, chattering can be eliminated effectively, avoiding deteriorating the robustness property and precision

II. DYNAMICS OF MEMS GYROSCOPE

The structure of $z-$ axis MEMS gyroscope is shown in Fig. 1. Referring to [20], by some assumptions, the governing equation is:

$$\begin{aligned} m\ddot{x} + d_{xx}\dot{x} + d_{xy}\dot{y} + k_{xx}x + k_{xy}y &= u_x + 2m\Omega_z\dot{y} \\ m\ddot{y} + d_{xy}\dot{x} + d_{yy}\dot{y} + k_{xy}x + k_{yy}y &= u_y - 2m\Omega_z\dot{x} \end{aligned} \quad (1)$$

where d_{xx} and d_{yy} are damping; k_{xx} and k_{yy} are spring coefficients; d_{xy} and k_{xy} called quadrature errors are coupled damping and spring terms.

Dividing both sides of (1) by a reference mass m, a reference length q_0, ω_0^2, and considering lumped external

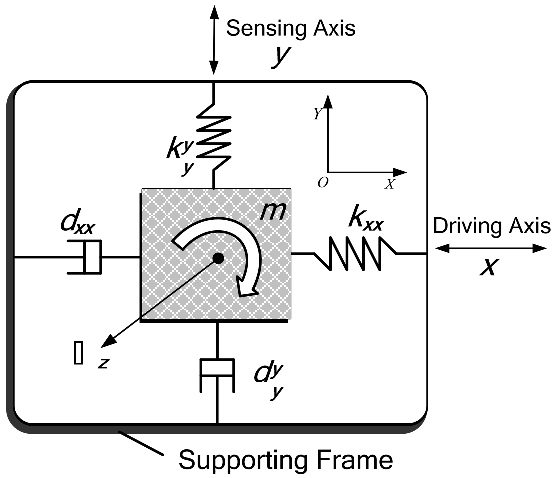

FIGURE 1. Structure of $z-$axis MEMS gyroscope.

disturbances d_1, d_2, we get the non-dimensional form as

$$\begin{aligned} \ddot{x} + d_{xx}\dot{x} + d_{xy}\dot{y} + \omega_x^2 x + \omega_{xy}y &= u_x + 2\Omega_z\dot{y} + d_1 \\ \ddot{y} + d_{xy}\dot{x} + d_{yy}\dot{y} + \omega_{xy}x + \omega_y^2 y &= u_y - 2\Omega_z\dot{x} + d_2 \end{aligned} \quad (2)$$

where $\frac{d_{xx}}{m\omega_0} \to d_{xx}$, $\frac{d_{xy}}{m\omega_0} \to d_{xy}$, $\frac{d_{yy}}{m\omega_0} \to d_{yy}$,

$$\frac{\Omega_z}{\omega_0} \to \Omega_z, \sqrt{\frac{k_{xx}}{m\omega_0^2}} \to \omega_x, \sqrt{\frac{k_{yy}}{m\omega_0^2}} \to \omega_y,$$

$$\frac{k_{xy}}{m\omega_0^2} \to \omega_{xy}, \frac{u_x}{m\omega_0} \to u_x \frac{u_y}{m\omega_0} \to u_y.$$

The vector form of MEMS gyroscope dynamics can be written as

$$\ddot{q} = -(D + 2\Omega)\dot{q} - K_b q + u + d \quad (3)$$

where

$$q = \begin{bmatrix} x \\ y \end{bmatrix}, u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \Omega = \begin{bmatrix} 0 & -\Omega_z \\ \Omega_z & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} d_{xx} & d_{xy} \\ d_{xy} & d_{yy} \end{bmatrix}, K_b = \begin{bmatrix} \omega_x^2 & \omega_{xy} \\ \omega_{xy} & \omega_y^2 \end{bmatrix}, d = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

represents the matched lumped modeling errors and external disturbance.

The desired reference model for system (3) is given by

$$\ddot{q}_m + k_m q_m = 0 \quad (4)$$

where $q_m = \begin{bmatrix} x_m \\ y_m \end{bmatrix}$, $k_m = \begin{bmatrix} w_1^2 & 0 \\ 0 & w_2^2 \end{bmatrix}$. Our control target for MEMS gyroscope is to find a control law so that the proof mass position $q(t)$ can track the desired trajectory $q_m(t)$.

Suppose that the dynamic models of MEMS gyroscopes are known , and model uncertainties and external disturbances which may degrade dynamic performance of the gyroscopes are excluded, i.e. $d = 0$. At this time, the ideal control law can be designed as

$$u^* = (D + 2\Omega)\dot{q} + K_b q + \ddot{q}_m + k_p e + k_v \dot{e} \quad (5)$$

where k_p and k_v are positive definite matrices, and tracking error is $e = q_m - q$.

Substituting (5) into (3) yields

$$\ddot{e} + k_v\dot{e} + k_p e = 0 \tag{6}$$

It can be obviously found that the tracking errors will tend to zero sympathetically and the stability of the control system can be ensured if favorable proportional gain k_p and derivative gain k_v are chosen. Here we choose k_v, k_p as $k_p = \begin{bmatrix} \alpha^2 & 0 \\ 0 & \alpha^2 \end{bmatrix}$, $k_v = \begin{bmatrix} 2\alpha & 0 \\ 0 & 2\alpha \end{bmatrix}$ to obtain satisfactory dynamic characteristic and robustness ,where α is a positive contstant.

However, the control law(5) is difficult to implement without exact known knowledge of model dynamics. Then an adaptive neural network controller is needed to mimic the ideal controller (5) by using the powerful approximation property of RBF neural networks.

III. ADAPTIVE BACKSTEPPING LMI-BASED DYNAMIC SLIDING MODE CONTROLLER

To relax the requirement of strict constrained conditions and prior system knowledge, the adaptive neural network H-infinity control strategy with robust feedback compensator using LMI, which integrates the advantages of robustness of robust control and powerful approximation property of RBF neural network control, is proposed to apply to a MEMS gyroscope to solve the optimal problem of convergence time and get satisfactory tracking performance and robustness due to model uncertainties and external disturbances. The detailed block diagram of the proposed approach is showed in Fig.2.

The following RBF NN scheme with architecture as Fig. 3 is used to approximate the ideal controller in (5):

$$u_c(X|\theta) = \hat{\theta}^{\mathrm{T}}\phi(X) \tag{7}$$

where, $u_c \in \Re^2$ is the output of the RBF NN, $\theta \in \Re^{m\times 2}$ is the weight from the hidden layer to the output layer, in which $m$is the number of hidden nodes $\theta^{\mathrm{T}} = [\omega_{ij}]$. The vector $\phi(X) \in R^m$č $\phi(X) = [\phi_i(X)]$ is the radical basis

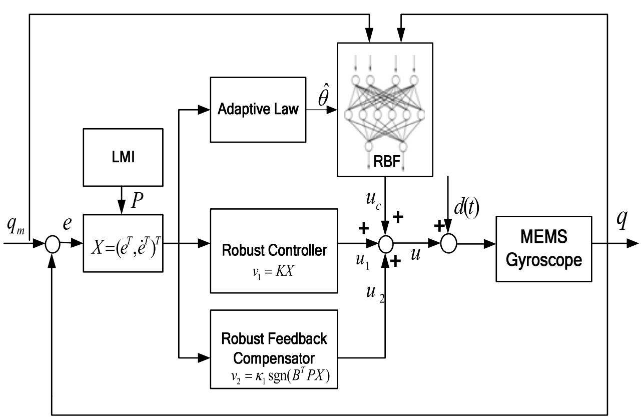

FIGURE 2. **Structure of adaptive NN LMI-based H-infinity control.**

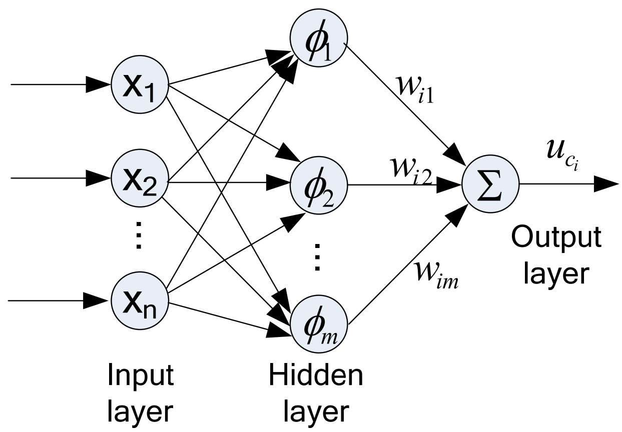

FIGURE 3. **Architecture of RBF network**

function defined as

$$\phi_i(x) = \exp\left(\frac{-\|x - c_i\|^2}{b_i^2}\right),\ \ i = 1, 2, \ldots, m \tag{8}$$

where $c_i \in \Re^m$ is the jth center, with m denoting the number of dimensions of the input vector. b_i is the width of the jth hidden nodes.

Our control target is to seek an appropriate control law based on the neural networks such that joint motions of the MEMS gyroscope system (3) can follow the desired trajectory. Overall control law becomes

$$u = u_c(X|\hat{\theta}) + u_r \tag{9}$$

where u_c is the RBF network controller and u_r is the robust controller..

The RBF network controller is designed to real-timely approximate the ideal control law (5) to relax the requirement of strict mathematical model. The robust controller is developed to achieve a specified H-infinity robust tracking performance.

Substituting Eq.(9) into Eq.(3) yields

$$\ddot{q} = -(D + 2\Omega)\dot{q} - K_b q + u_c(X|\hat{\theta}) + u_r + d \tag{10}$$

Rewriting Eq.(5) can be written as:

$$\ddot{q}_m = u^* - (D + 2\Omega)\dot{q} - K_b q - k_p e - k_v\dot{e} \tag{11}$$

Using (10) and (11), one can get the closed loop system as

$$\ddot{e} = u^* - k_p e - k_v\dot{e} - u_c(X|\hat{\theta}) - u_r - d \tag{12}$$

Suppose that state vector is designed as $X = (e^T, \dot{e}^T)^T$,

the state space-equation has the form as

$$\dot{X} = AX + B[u^* - u_c(X|\hat{\theta}) - u_r - d] \tag{13}$$

where $A = \begin{bmatrix} 0 & I \\ -k_v & -k_p \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$.

We make the following assumptions on the basis of the universal approximation properties of RBF network [21].

Assumption 1: Assume there exists a parameter value θ^*, known as the optimal approximation parameter, such that $u_c(X|\theta^*)$ can approximate $u^*(e,\dot{e})$ as close as possible, that is, given an arbitrary small positive constant w, there exists an optimal weight θ^* so that the approximation error satisfies

$$\|w(e,\dot{e})\| = \left\|u^*(e,\dot{e}) - u_c(X|\theta^*)\right\| < w_1 \tag{14}$$

Where $\|w(e,\dot{e})\| = \|u^*(e,\dot{e}) - u_c(X|\theta^*)\| < w_1$ is a small constant.

Assumption 2: Let $w(e,\dot{e}) = u^*(e,\dot{e}) - u_c(X|\theta^*)$. Without a loss of generality, assume there exists a constant $\kappa > 0$ such that $\|(w(e,\dot{e}))_i\| \leq \kappa$ for all $1 \leq i \leq n$.

Substituting Eq.(14) and Eq.(7) into Eq.(13) generates

$$\dot{X} = AX - B\tilde{\theta}^T\phi(X) + Bw(e,\dot{e}) - Bu_r - Bd \tag{15}$$

where $\tilde{\theta} = \hat{\theta} - \theta^*$ is the estimated error of θ.

Theorem 1: Consider the nonlinear gyroscope system defined in (3). The control law is designed as follows:

$$u = u_c(X|\theta) + u_r \tag{16}$$

where the NN controller is given as $u_c(X|\theta) = \hat{\theta}^T\phi(X)$, and the robust controller is defined as

$$u_r = v_1 + v_2 \tag{17}$$

with

$$\dot{\hat{\theta}} = \dot{\tilde{\theta}} = \eta\xi X^T PB \tag{18}$$

$$v_1 = KX \tag{19}$$

$$v_2 = \kappa_1 sgn(B^T PX) \tag{20}$$

where κ_1 is positive constants, the matrix K and the symmetric positive definite matrix $P = P^T$ satisfy the following LMI condition:

$$A^TP + PA - PBK - K^TB^TP + \frac{1}{\rho^2}PBB^TP + 2Q < 0 \tag{21}$$

Using the proposed control law, all the variables of the closed-loop system (3) are bounded, the global stability and the robustness of the closed-loop gyroscope system can be obtained and the following H-infinity tracking performance can be achieved:

$$\int_0^T X^TQX \leq X^T(0)PX(0) + \frac{1}{\eta}\tilde{\theta}^T(0)\tilde{\theta}(0) + \rho^2\int_0^T \|d\|t \tag{22}$$

where $X(0)$ and $\tilde{\theta}(0)$ is the initial value of X and $\tilde{\theta}$ respectively.

Proof: Choose a Lyapunov function candidate as follows:

$$V = \frac{1}{2}X^TPX + \frac{1}{2\eta}tr(\tilde{\theta}^T\tilde{\theta}) \tag{23}$$

where $tr(\bullet)$ is the trace operator, and η is the positive constant.

The time derivative of the Lyapunov function along the state trajectory (15) can be rewritten as:

$$\begin{aligned}\dot{V} &= \frac{1}{2}\dot{X}^TPX + \frac{1}{2}X^TP\dot{X} + \frac{1}{\eta}tr(\tilde{\theta}^T\dot{\tilde{\theta}}) \\ &= \frac{1}{2}[X^TA^T - \phi^T\tilde{\theta}B^T + w^T(e,\dot{e})B^T - u_r^TB^T - d^TB^T]PX \\ &\quad + \frac{1}{2}X^TP\left[AX - B\tilde{\theta}^T\phi + Bw(e,\dot{e}) - Bu_r - Bd\right] \\ &\quad + \frac{1}{\eta}tr(\tilde{\theta}^T\dot{\tilde{\theta}})\end{aligned} \tag{24}$$

Substituting (17), (19) and (20) into Eq.(24) yields

$$\begin{aligned}\dot{V} &= \frac{1}{2}X^T(A^TP + PA - K^TB^TP - PBK)X + X^TPBw \\ &\quad - X^TPBd - \kappa_1 X^TPBsgn(B^TPX) \\ &\quad + tr\left[\tilde{\theta}^T(\frac{1}{\eta}\dot{\tilde{\theta}} - \phi X^TPB)\right]\end{aligned} \tag{25}$$

Then, substituting Eq.(18) into Eq.(25), one can get

$$\begin{aligned}\dot{V} &= \frac{1}{2}X^T(A^TP + PA - K^TB^TP - PBK)X \\ &\quad + X^TPBw - X^TPBd - \kappa_1 X^TPBsgn(B^TPX)\end{aligned} \tag{26}$$

In summary, by using the LMI in (21) and assumption 2, the derivative of V can be bounded as

$$\begin{aligned}\dot{V} &= \frac{1}{2}X^T(A^TP + PA - K^TB^TP - PBK \\ &\quad + \frac{1}{\rho^2}PBB^TP + 2Q)X \\ &\quad - X^TQX + \frac{1}{2}\rho^2 d^Td - \frac{1}{2}(\frac{1}{\rho}B^TPX + \rho d)^T \\ &\quad \times (\frac{1}{\rho}B^TPX + \rho d) \\ &\quad + X^TPBw - \kappa_1 X^TPBsgn(B^TPX) \\ &< -X^TQX + \frac{1}{2}\rho^2 d^Td - \kappa_1\sum_{i=1}^{2}\left|(B^TPX)_i\right| \\ &\quad + \sum_{i=1}^{2}|w_i|\cdot\left|(B^TPX)_i\right| \\ &\leq -\frac{1}{2}X^TQX + \frac{1}{2}\rho^2\|d\|^2\end{aligned} \tag{27}$$

Integrating the inequality (27) from $t = 0$ to $t = T$ generates

$$V(T) - V(0) \leq -\frac{1}{2}\int_0^T X^TQXdt + \frac{1}{2}\rho^2\int_0^T \|d\|^2dt \tag{28}$$

Since $V(T) \geq 0$, we can obtain the H-infinity tracking performance as in Eq.(22)

$$\begin{aligned}\frac{1}{2}\int_0^T X^TQXdt &\leq V(0) + \frac{1}{2}\rho^2\int_0^T \|d\|^2t \\ &\leq \frac{1}{2}X^T(0)PX(0) + \frac{1}{2\eta}\tilde{\theta}^T(0)\tilde{\theta}(0) \\ &\quad + \frac{1}{2}\rho^2\int_0^T \|d\|^2dt\end{aligned} \tag{29}$$

Let $d \in L_\infty[0, \infty)$, i.e., $\|d\| \le \varepsilon_d$ for some ε_d. From the inequality (27), we can get $\dot{V} \le -(1/2)\lambda_q\|X\|^2 +)(1/2)\rho^2\varepsilon_d^2$ where λ_q denotes the minimum eigenvalue of Q. For any small $\delta > 0$, if we choose , $\lambda_q > (\rho^2\varepsilon_d^2/\delta^2)$ then there is a $\varsigma > 0$ such that $\dot{V} \le -\varsigma\|e\|^2 < 0\ \forall\|e\| > \delta$. This implies the tracking error is uniformly ultimate bounded and all the variables are bounded. Then, using the Barbalat's lemma, one can get the tracking error converges to zero asymptotically .

IV. SIMULATION STUDY

A numerical simulation of a MEMS gyroscope is implemented to verify the effectiveness of the proposed control scheme. Parameters of the adopted MEMS gyroscope are shown as follows [22]:

$$m = 1.8 \times 10^{-7}\text{kg}, k_{xx} = 63.955\text{N}/\text{m}, k_{yy} = 95.92\text{N}/\text{m},$$
$$k_{xy} = 12.779\text{N}/\text{m}, d_{xx} = 1.8 \times 10^{-6}\text{N}\cdot\text{s}/\text{m},$$
$$d_{yy} = 1.8 \times 10^{-6}\text{N}\cdot\text{s}/\text{m}, d_{xy} = 3.6 \times 10^{-7}\text{N}\cdot\text{s}/\text{m}$$

Taking into account that the general displacement range of the MEMS gyroscope in each axis is sub-micrometer level and the usual natural frequency of each axis of a MEMS gyroscope is in the KHz range, it is necessary for us to choose $1\mu m$ as the reference length q_0 and 1KHz as the resonant frequency ω_0. The unknown angular velocity is assumed as $\Omega = 100 rad/s$. Therefore, the non-dimensional values of the MEMS gyroscope parameters are listed as follows:

$$w_x^2 = 355.3, w_y^2 = 532.9, w_{xy} = 70.99,$$
$$d_{xx} = 0.01, d_{yy} = 0.01, d_{xy} = 0.002, \Omega_z = 0.1$$

The initial state condition is selected as [0 0 0 0]. The reference inputs are given by $x_m = \sin(6.17t)$ and $y_m = \sin(5.11t)$. Random signal is considered as disturbances for $d = 1 \times [\text{randn}(1,1),\text{randn}(1,1)]^T \mu N$. The fixed gain of the robust feedback compensator is set as$\kappa_1 = 500$, and the adaptive gain in Eq.(18) is chosen as $\eta = 500$. The parameters of the LMI in Eq.(21) are set to be $Q = [10\,2; 2\,10], \rho = 8$. Assuming the RBF network has 11 nodes, we took width $b = 1$. The simulation results are shown in Figs. 4∼6.

Fig. 4 and Fig. 5 depict the MEMS gyroscope tracking trajectories and tracking errors along x-axis and y-axis using adaptive neural network H-infinity controller with robust feedback compensator via LMI respectively. It is observed that the actual motion trajectory is consistent with the desired reference trajectory in a short time, and all the tracking errors can converge to zero asymptotically, showing that satisfactory tracking performance can be achieved as expected. Fig.6 depicts the control forces of the MEMS gyroscope along x-axis and y-axis using adaptive neural network H-infinity controller with robust feedback compensator via LMI.

Subsequently, in order to illustrate the effectiveness of the proposed control method, a comparable investigation is implemented between the adaptive neural network H-infinity control using LMI and conventional neural network H-infinity control using Riccati equation.

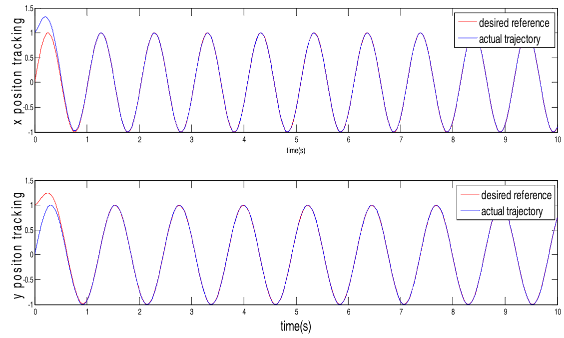

FIGURE 4. Position tracking utilizing the adaptive neural network H-infinity control strategy using LMI

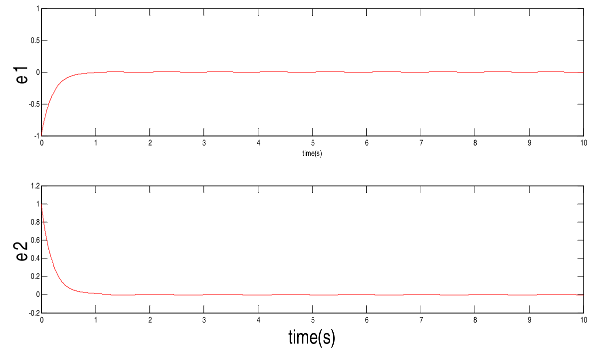

FIGURE 5. Position tracking errors utilizing the adaptive neural network H-infinity control strategy using LMI

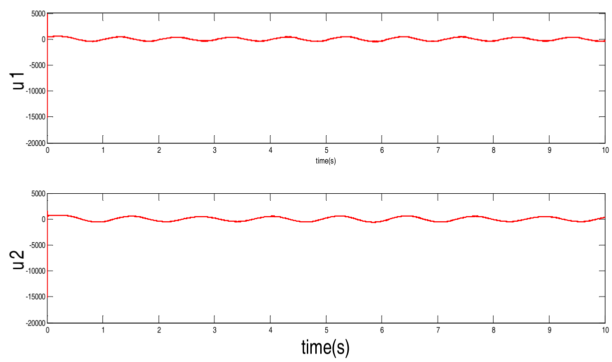

FIGURE 6. Control input signals utilizing the adaptive neural network H-infinity control strategy using LMI

Fig.7 and Fig.8 depict the MEMS gyroscope tracking trajectories and tracking errors along x-axis and y-axis under the conventional neural network H-infinity controller using Riccati equation. Compared with Fig.4 and Fig.5, showing that the positions of x and y axis under the proposed controller using LMI follow the desired reference model much faster than that under the conventional neural H-infinity controller using Riccati equation. The proposed control scheme using LMI has better transient performance.

Fig.9 depicts the control forces of the MEMS gyroscope along x-axis and y-axis under the conventional neural H-infinity controller using Riccati equation. Obviously the

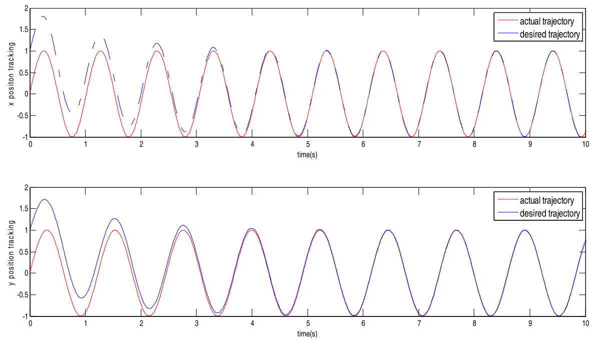

FIGURE 7. Position tracking utilizing the conventional neural network H-infinity controller using Riccati equation

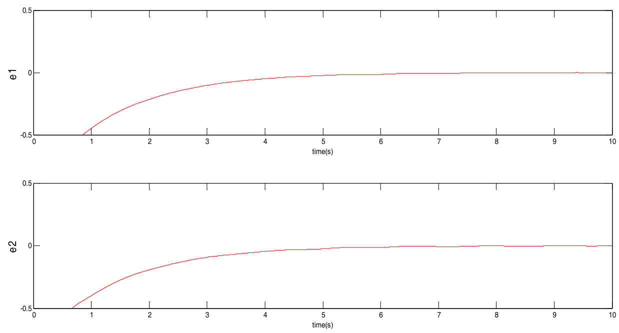

FIGURE 8. Position tracking errors utilizing the conventional neural network H-infinity controller using Riccati equation

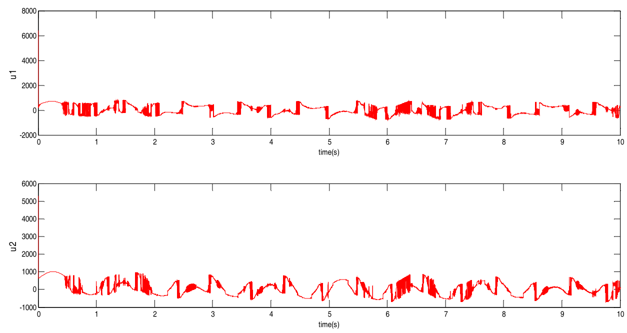

FIGURE 9. Control input signals utilizing the conventional neural network H-infinity controller using Riccati equation

undesirable chattering phenomena are serious due to the model uncertainties and external disturbances. Compared with Fig.6, it is obvious that the chattering phenomenon can be restrained effectively using the adaptive neural H-infinity control strategy with robust feedback compensator via LMI. Therefore, the proposed control scheme using LMI can adapt to the changes of external environment and model parameters and eliminate the chattering effectively.

V. CONCLUSION

In this paper, a novel adaptive neural H-infinity controller with robust feedback compensator using LMI is proposed for the tracking control of MEMS gyroscopes to handle unknown model uncertainties and external disturbances. A RBF network controller is designed to approximate ideal control law, overcoming some shortcomings associated with conventional control strategy; A robust H-infinity feedback compensator using LMI is also built to achieve desired H-infinity tracking performance and improve disturbance rejecting ability. Consequently, compared with the conventional neural network H-infinity control scheme using Riccati equation, the proposed controller can reduce chattering effectively and has better dynamic performance and stronger robustness against modeling error and inevitable disturbances.

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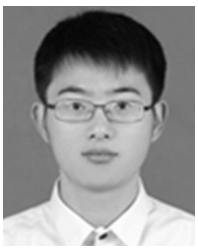

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