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Impact of Factor Graph on Average Sum Rate for Uplink Sparse Code Multiple Access Systems

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ABSTRACT In this paper, we first study the average sum rate of sparse code multiple access (SCMA) systems, where a general scenario is considered under the assumption that the distances between the mobile users and the base station are not necessarily identical. Closed-form analytical results are derived for the average sum rate based on which an optimal factor graph matrix is designed for maximizing the capacity of the SCMA systems. Moreover, we propose a low-complexity iterative algorithm to facilitate the design of the optimal graph matrix. Finally, Monte Carlo simulations are provided to corroborate the accuracy of the theoretical results and the efficiency of the proposed iterative algorithm.

INDEX TERMS Multiple access, average sum rate, optimal factor graph.

I. INTRODUCTION

Recently, sparse code multiple access (SCMA) has been proposed as a promising non-orthogonal multiple access technology for fifth generation (5G) mobile networks, since it has ability to support high throughput, low latency, and massive connectivity [1]. The key idea of SCMA is that the coded bits are directly mapped to multi-dimensional sparse codewords and codewords from different users are overlaid non-orthogonally using sparse spreading. The receiver performs a message passing algorithm for data recovering [2].

Recently, there are a few studies devoted to the analysis and the design of SCMA systems. Specifically, an energy efficient uplink approach was proposed for SCMA systems in [3]. A heuristic algorithm for subcarrier and power allocation was devised in [4] by maximizing the weighted sum rate of the uplink SCMA. In [5], the single-input multiple-output antenna technique was applied to a downlink multi-user SCMA system while a coordinated multipoint joint transmission scheme was studied in [6]. To facilitate analysis, all existing works on SCMA considered a simple scenario assuming the distances between base station (BS) and mobile users are all equal.

In this letter, we consider a more general and practical scenario for an uplink SCMA system, where the user-BS distances are not necessarily identical. The goal of this letter is to design an optimal factor graph matrix to maximize the average sum rate for the SCMA systems. Firstly, an exact cumulative distribution function (CDF) of a sum of Rayleigh random variables with distinct expectations [7] is applied to derive a closed-form expression for the average sum rate of SCMA systems. By elaborating the developed analytical results, it is revealed that the maximal average sum rate can be achieved when orthogonal resources are allocated for adjacent users. We then propose an iterative algorithm with lower implementation complexity to design an optimal factor graph matrix for SCMA systems. Finally, numerical and simulation results are presented to illustrate the accuracy of the analysis and the validity of the proposed algorithm.

II. SYSTEM MODEL

Consider an uplink SCMA communication scenario with M users or codebooks, where the length of the codeword is set K, and the number of the non-zeros elements in each codeword is N. Denote by d_i the distance between BS and user U_i, and assume that all users know the distances d_i, $i = 1, 2, \cdots, M$. Without loss of generality, we assume that $d_1 \leq d_2 \leq \cdots \leq d_M$. Because M users are multiplexed over K subcarriers, the received signal over all subcarriers

$y = [y_1\, y_2 \cdots y_K]^T$ at BS can be represented as

$$y = \sum_{m=1}^{M} \sqrt{P_m/N}\mathrm{diag}(f_m)\mathrm{diag}(h_m)x_m + w, \quad (1)$$

where $x_m = [x_{m1}\, x_{m2} \cdots x_{mK}]^T$ is the transmit symbols or codeword of user U_m and P_m is the transmission power of user U_m. Let $h_m = [h_{m1}\, h_{m2} \cdots h_{mK}]^T$ be the channel coefficient vector for user U_m, with $h_{mk} = g_{mk}/d_m^{\frac{\alpha}{2}}$, where $g_{mk} \sim \mathcal{CN}(0, 1)$ is the Rayleigh fading channel gain between the m-th user on the k-th subcarrier, and α is the path loss exponent. $f_m = [f_{m1}\, f_{m2} \cdots f_{mK}]^T$ denotes the binary indicator vector of the m-th user, where f_{mk} is the subcarrier index with $f_{mk} = 1$, if $x_{mk} \neq 0$, and $f_{mk} = 0$, if $x_{mk} = 0$. $w \sim \mathcal{CN}(0, I)$ is the additive white Gaussian noise (AWGN).

Based on (1), the average sum rate of the uplink SCMA systems can be then given by [4]

$$\begin{aligned}\overline{R} &= \sum_{k=1}^{K} \mathbf{E}\Big(\log_2 \Big(1 + \frac{1}{N}\sum_{m=1}^{M} f_{mk}P_m|g_{mk}|^2 d_m^{-\alpha}\Big)\Big) \\ &= \sum_{k=1}^{K} \overline{R}_k. \end{aligned} \quad (2)$$

By using (2), we can evaluate the average sum rate of uplink SCMA systems, which will be studied in the next Section.

III. AVERAGE SUM RATE OF UPLINK SCMA SYSTEMS

In this Section, we first derive the average sum rate of uplink SCMA systems, then we obtain the optimal factor graph matrix by elaborating the results.

Theorem *1:* The average sum rate of SCMA systems with different distances between users and BS is given as

$$\begin{aligned}\overline{R} = \frac{1}{\ln 2}\sum_{k=1}^{K}\Big(\prod_{j=1}^{b}\beta_{jk}^{r_{jk}}\Big)\sum_{i=1}^{b}\sum_{l=1}^{r_{ik}}\frac{\Psi_{il}(-\beta_{ik})e^{\beta_{ik}}}{(r_{ik}-l)!(l-1)!} \\ \times \sum_{n=0}^{r_{ik}-l}\binom{r_{ik}-l}{n}(-1)^{r_{ik}-l-n}\frac{1}{\beta_{ik}^{n}}\Gamma\big(n, \beta_{ik}\big),\end{aligned} \quad (3)$$

where $\Gamma(s, x)$ is upper incomplete Gamma function [8, eq. (8.350)], and $\Psi_{il}(t) = -\frac{d^{l-1}}{dt^{l-1}}\prod_{j=0,j\neq i}^{b}(\beta_{jk} + t)^{-r_{jk}}$, where b is the number of distinct β_{ik} for a given subcarrier k, and $\beta_{ik} = Nd_{ik}^{\alpha}/P_i \triangleq Nd_i^{\alpha}/P_i$, i.e., the user U_i is allowed to access to subcarrier k. Note that β_{ik} is the inverse of link signal-to-noise ratio (SNR) for U_i at subcarrier k.

Proof: See Appendix. ■

Note that the result in (3) is mainly depended on the factor graph matrix of the SCMA systems, which indicates that the subcarrier allocation pattern will significantly affect the average sum rate of the system. To illustrate how to obtain an optimal factor graph matrix maximizing the average sum rate of SCMA systems in (3), we focus on the typical parameters $M = 6, K = 4, N = 2$ with three users multiplexing on each subcarrier [2]. Without loss of generality, we take the first subcarrier as an example, i.e., $k = 1$ in (3), and analyse how the link SNR of users affects its average rate. Furthermore, assume that the distances between the three users and the BS are d_i, d_j, and d_l, with the transmission power P_i, P_j, and P_l, respectively. To reveal some important insights, the following four cases are considered:

- There are two users with the same link SNR, which are smaller than the other one. Therefore, we can assume that $Nd_i^{\alpha}/P_i < Nd_j^{\alpha}/P_j = Nd_l^{\alpha}/P_l$, i.e., $\beta_{11} \triangleq Nd_i^{\alpha}/P_i$, $\beta_{21} \triangleq Nd_j^{\alpha}/P_j$.

In this case, $\overline{R_1}$ in (3) can be rewritten as

$$\begin{aligned}\overline{R_1} =& \frac{1}{\ln 2}\frac{\beta_{21}^{2\alpha}e^{\beta_{11}^{\alpha}}}{(\beta_{21}^{\alpha}-\beta_{11}^{\alpha})^2}\mathrm{E}_1\big(\beta_{11}^{\alpha}\big) \\ &+ \frac{e^{\beta_{21}^{\alpha}}}{\ln 2}\frac{\beta_{11}^{\alpha}\beta_{21}^{\alpha}}{(\beta_{11}^{\alpha}-\beta_{21}^{\alpha})}\Big(-\mathrm{E}_1\big(\beta_{21}^{\alpha}\big) + \frac{1}{\beta_{21}^{\alpha}}e^{-\beta_{11}^{\alpha}}\Big) \\ &+ \frac{1}{\ln 2}\Big(\frac{\beta_{11}^{\alpha}}{(\beta_{11}^{\alpha}-\beta_{21}^{\alpha})} - \frac{\beta_{11}^{\alpha}\beta_{21}^{\alpha}}{(\beta_{11}^{\alpha}-\beta_{21}^{\alpha})^2}\Big)e^{\beta_{21}^{\alpha}}\mathrm{E}_1\big(\beta_{21}^{\alpha}\big),\end{aligned} \quad (4)$$

where $\mathrm{E}_1(z)$ is exponential integral [8].

When $\beta_{11} \ll \beta_{21}$,

$$\overline{R_1} \approx \frac{1}{\ln 2}e^{\beta_{11}^{\alpha}}\mathrm{E}_1\big(\beta_{11}^{\alpha}\big), \quad (5)$$

which means $\overline{R_1}$ mainly depends on β_{11}, i.e., better link SNR. Furthermore, the derivation of $\overline{R_1}$ in (5) with respect to β_{11} is given by

$$\frac{\partial \overline{R_1}}{\partial \beta_{11}} = \frac{\alpha}{\beta_{11}}\big(\beta_{11}^{\alpha}e^{\beta_{11}^{\alpha}}\mathrm{E}_1(\beta_{11}^{\alpha}) - 1\big) < 0, \quad (6)$$

where the inequality is always true due to the fact that $xe^x\mathrm{E}_1(x) < x\ln(1+\frac{1}{x}) < 1$ [8]. Therefore, $\overline{R_1}$ is a decreasing function of β_{11}.

- There are two users with the same link SNR, which is larger than the other one. Therefore, we can assume that $Nd_i^{\alpha}/P_i = Nd_j^{\alpha}/P_j < Nd_l^{\alpha}/P_l$, i.e., $\beta_{11} \triangleq Nd_i^{\alpha}/P_i$, $\beta_{21}{=}Nd_l^{\alpha}/P_l$.

In this case, $\overline{R_1}$ in (3) can be rewritten as

$$\begin{aligned}\overline{R_1} =& \frac{1}{\ln 2}\frac{\beta_{11}^{\alpha}\beta_{21}^{\alpha}e^{\beta_{11}^{\alpha}}}{(\beta_{21}^{\alpha}-\beta_{11}^{\alpha})}\Big(-\mathrm{E}_1\big(\beta_{11}^{\alpha}\big) + \frac{e^{-\beta_{11}^{\alpha}}}{\beta_{11}^{\alpha}}\Big) \\ &+ \frac{1}{\ln 2}\Big(\frac{\beta_{21}^{\alpha}}{(\beta_{21}^{\alpha}-\beta_{11}^{\alpha})} - \frac{\beta_{11}^{\alpha}\beta_{21}^{\alpha}}{(\beta_{21}^{\alpha}-\beta_{11}^{\alpha})^2}\Big)e^{\beta_{11}^{\alpha}}\mathrm{E}_1\big(\beta_{11}^{\alpha}\big) \\ &+ \frac{1}{\ln 2}\frac{\beta_{11}^{2\alpha}}{(\beta_{11}^{\alpha}-\beta_{21}^{\alpha})^2}e^{\beta_{21}^{\alpha}}\mathrm{E}_1\big(\beta_{21}^{\alpha}\big).\end{aligned} \quad (7)$$

When $\beta_{11} \ll \beta_{21}$,

$$\overline{R_1} \approx \frac{1}{\ln 2} + \frac{1}{\ln 2}e^{\beta_{11}^{\alpha}}\mathrm{E}_1\big(\beta_{11}^{\alpha}\big). \quad (8)$$

Note that $\overline{R_1}$ also relies on the user with better link SNR, and $\overline{R_1}$ is a decreasing function of β_{11}.

- The three users have different link SNR. Therefore, we can assume that $Nd_i^{\alpha}/P_i < Nd_j^{\alpha}/P_j < Nd_l^{\alpha}/P_l$, i.e., $\beta_{11} \triangleq Nd_i^{\alpha}/P_i$, $\beta_{21} \triangleq Nd_j^{\alpha}/P_j$, $\beta_{31} \triangleq Nd_l^{\alpha}/P_l$.

In this case, $\overline{R_1}$ in (3) can be rewritten as

$$\overline{R_1} = \frac{1}{\ln 2}\frac{\beta_{21}\beta_{31}}{(\beta_{21}-\beta_{11})(\beta_{31}-\beta_{11})}e^{\beta_{11}}\mathrm{E}_1(\beta_{11}) + \frac{1}{\ln 2}\frac{\beta_{11}\beta_{31}}{(\beta_{11}-\beta_{21})(\beta_{31}-\beta_{21})}e^{\beta_{21}}\mathrm{E}_1(\beta_{21}) + \frac{1}{\ln 2}\frac{\beta_{11}\beta_{21}}{(\beta_{11}-\beta_{31})(\beta_{21}-\beta_{31})}e^{\beta_{31}}\mathrm{E}_1(\beta_{31}). \quad (9)$$

When $\beta_{11} < \beta_{21} \ll \beta_{31}$,

$$\overline{R_1} \approx \frac{1}{\ln 2}\frac{\beta_{21}e^{\beta_{11}}\mathrm{E}_1(\beta_{11})}{(\beta_{21}-\beta_{11})} + \frac{1}{\ln 2}\frac{\beta_{11}e^{\beta_{21}}\mathrm{E}_1(\beta_{21})}{(\beta_{11}-\beta_{21})}. \quad (10)$$

Furthermore, if $\beta_{11} \ll \beta_{21}$,

$$\overline{R_1} \approx \frac{1}{\ln 2}e^{\beta_{11}}\mathrm{E}_1(\beta_{11}). \quad (11)$$

From (10) and (11), we can also see that the average rate is relevant to the user with the strong channel condition, and $\overline{R_1}$ is a decreasing function of β_{11}.

The fourth case is the three users have the same link SNR. Since the link SNR of all users are the same, the subcarrier allocation has no impact on the average sum rate for regular SCMA systems.

The details to design an optimal factor graph matrix to maximize the average sum rate for the SCMA systems is discussed as follows. Assume that the highest link SNR in each subcarrier is $\rho^i_{\max}$ $i = 1, 2, 3, 4$, and $\rho^1_{\max} \geq \rho^2_{\max} \geq \rho^3_{\max} \geq \rho^4_{\max}$. For all $K = 4$ subcarriers, based on (5), (8) and (11), the average sum rate of uplink SCMA systems is approximated as follows:

$$\overline{R} \approx \sum_{k=1}^{4}\overline{R_k}(\{\rho^i_{\max}\}_{i=1}^4). \quad (12)$$

Note that $\overline{R}$ is an increasing function of $\{\rho^i_{\max}, i = 1, \cdots, 4\}$, since each $\overline{R_k}$ is a decreasing function of $1/\rho^k_{\max}$. Therefore, when $\rho^1_{\max} = \rho^2_{\max} = \rho^3_{\max} = \rho^4_{\max}$, we can obtain the maximal of $\overline{R}$.

Furthermore, the optimal average sum rate is attained when the top four link SNRs $\rho_1, \rho_2, \rho_3, \rho_4$ should be separately corresponded to different subcarriers. This results can be proved by contradiction. Assume that the optimal subcarrier allocation satisfies the condition that ρ_3 and ρ_4 are in the same subcarrier 3, and ρ_1 and ρ_2 occupy the subcarrier 1 and 2, respectively. In this case, omitting the worse users' rate, the average sum rate $\overline{R}_0$ satisfies the following condition:

$$\overline{R}_0 \approx \frac{1}{\ln 2}\left(e^{\rho_1}\mathrm{E}_1(\rho_1) + e^{\rho_2}\mathrm{E}_1(\rho_2) + e^{\rho_3}\mathrm{E}_1(\rho_3)\right) < \frac{1}{\ln 2}\left(e^{\rho_1}\mathrm{E}_1(\rho_1) + e^{\rho_2}\mathrm{E}_1(\rho_2) + e^{\rho_3}\mathrm{E}_1(\rho_3) + e^{\rho_4}\mathrm{E}_1(\rho_4)\right). \quad (13)$$

Obviously, $\overline{R}_0$ is not optimal, which contradicts to the assumption. Similarly, we can verify the remaining cases, i.e., at least one scbcarrier is shared by two users with better link SNRs, are also not optimal.

From the above discussions, the binary indicator vector of the first two best link SNRs should be orthogonal to obtain the maximum average sum rate. Furthermore, it can be observed from (10) that the average sum rate depends on the former two better link SNRs, which means the best link SNR should be grouped with third or fourth link SNR, while the second link SNR should be grouped with third or fourth link SNR. In other words, third link SNR and fourth link SNR should be orthogonal. The remaining two link SNRs are also orthogonal. Therefore, we can obtain two representative factor graph matrices as follows:

$$\mathbf{F_1} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix},$$

and

$$\mathbf{F_2} = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

Note that by changing any two orthogonal columns in $\mathbf{F_1}$ or $\mathbf{F_2}$, together with the row transformation, we can obtain $\mathbf{F_2}$ or $\mathbf{F_1}$. Furthermore, according to combinatorial mathematics, there are totally 720 different factor graph matrices, out of which 48 matrices can have the same performance as $\mathbf{F_1}$ or $\mathbf{F_2}$. Therefore, we only need to pay attention to $\mathbf{F_1}$ and $\mathbf{F_2}$.

Based on the above discussions, we can conclude that we should assign orthogonal resources to the adjacent link SNRs in order to optimize the average sum rate of SCMA systems. With more flexible system parameters, we should assign the adjacent link SNRs to the orthogonal binary indicator vectors as possible. In the next Section, we propose an iterative algorithm to achieve this.

IV. OPTIMIZATION PROBLEM

As described in Section III, finding a good factor graph matrix is essential to improve the average sum rate in SCMA systems, especially when the link SNRs among users are significantly distinct. In the following, we discuss how to design an optimal factor graph matrix in a general SCMA system with only the knowledge of the average CSI. Different from the algorithm in [4] which studied the design of an irregular factor graph matrix in low density signature-orthogonal frequency division multiplexing (LDS-OFDM) systems, this paper focuses on designing a regular factor graph matrix for SCMA systems. The optimization problem can be formulated as

$$\max_{\{f_{ik}, \forall k, \forall i\}} \ \overline{R} \quad (14a)$$

$$\text{s.t.} \quad \sum_{k=1}^{K} f_{mk} = N, \quad (14b)$$

$$f_{mk} \in \mathbf{F} = \{0, 1\}^K, \quad (14c)$$

$$\mathbf{f}_m \neq \mathbf{f}_{m'}, \quad (14d)$$

$$\forall m, m' \in \mathcal{M}, \ m' \neq m, \ \forall k \in \mathcal{K}, \quad (14e)$$

where $\mathbf{f}_m = [f_{m1}, \cdots, f_{mK}]^T$, $\mathcal{M}$ and $\mathcal{K}$ denote the set of users and subcarrier respectively.

The above optimization problem is intractable as the objective function and the constraints are combinational with factor graph matrix $\mathbf{F}$, which proves to be NP-hard [9]. In addition, it can be observed that the expression in (3) is very complex and hard to transform. However, by applying Jensen's inequality, the expectation in (2) can be lower bounded by

$$\overline{R} = \sum_{k=1}^{K} \mathbf{E}\left(\log_2 \left(1 + \frac{1}{N}\sum_{m=1}^{M} f_{mk} P_m d_m^{-\alpha} e^{\ln(|g_{mk}|^2)}\right)\right) \quad (15a)$$

$$\geq \sum_{k=1}^{K} \log_2 \left(1 + \frac{1}{N}\sum_{m=1}^{M} f_{mk} P_m d_m^{-\alpha} e^{\mathbf{E}[\ln(|g_{mk}|^2)]}\right) \quad (15b)$$

$$= \sum_{k=1}^{K} \log_2 \left(1 + \sum_{m=1}^{M} e^{-\gamma} f_{mk} P_m d_m^{-\alpha}/N\right), \quad (15c)$$

where $\mathbf{E}[\ln(|g_{mk}|^2)] = \int_0^\infty e^{-x}\ln(x)\mathrm{d}x = -\gamma$ with $\gamma = 0.577$ being the Euler's constant. Regarding (15) as the relaxation of the average sum rate in (2), we propose a heuristic algorithm as follows.

Let $R_{sum}[m]$ denotes the sum rate including the rates of the first m users. $\triangle[m] = R_{sum}[m] - R_{sum}[m-1]$ is the increment of the system sum rate when the m-th user get accessed, which denotes the achieved rate of the m-th user. For the m-th user, the subcarriers are allocated satisfying $(f_{m1}^*, \cdots, f_{mN}^*) = \arg\max \triangle[m]$. Denote by $\mathbf{f}_m^* = (f_{m1}^*, \cdots, f_{mN}^*)^T$ for convenience. Two cases are discussed as follows:

1) When $Nm \leq K$, $\mathbf{f}_m^*$ can be obtained by allocating orthogonal column of $\mathbf{S}$.

2) When $Nm > K$, $\triangle[m]$ can be given by

$$\triangle[m] = \sum_{k=1}^{K} \log\left(1 + \sum_{i=1}^{m} e^{-\gamma} P_m d_m^{-\alpha} f_{ik}/N\right) - \sum_{k=1}^{K} \log\left(1 + \sum_{i=1}^{m-1} e^{-\gamma} P_m d_m^{-\alpha} f_{ik}/N\right). \quad (16)$$

For the m-th user, the optimization problem can be transformed as

$$\max_{\{f_{mk}, \forall k\}} \quad \triangle[m]$$
$$\text{s.t. } (14b) \;\&\; (14c) \;\&\; (14d)$$
$$k \in \mathcal{K} \;\&\; m' = 1, \cdots, m-1. \quad (17)$$

Inspired by the above discussions, an iterative algorithm is proposed, where subcarriers for each user can be allocated successively, which is summarized in Algorithm 1.

The complexity of Algorithm 1 is dependent on the relationship between MN and K. Specifically, if $MN \leq K$, the optimal solution can be found with M iterations. Otherwise, the optimal solution can be found after performing $(M - \lfloor \frac{K}{N} \rfloor)!$ iterations. For example, when $M = 6$, $K = 4$, $N = 2$, the complexity can be decreased 30 times compared with the exhaustive search.

Algorithm 1 Subcarrier Allocation Algorithm

Initialization: $m = 0$, $\mathbf{F}^* = \emptyset$. Generate a factor matrix $\mathbf{S}$ randomly.
while $m \leq M$ **do**
  Update $m = m + 1$.
  if $mN \leq K$ **then**
    Select $\mathbf{f}_m^*$ with $\{\mathbf{f}_m^*\} \cap \mathbf{F}^* = \emptyset$, $\{\mathbf{f}_m^*\} \in \mathbf{S}$.
    $\mathbf{F}^* = \mathbf{F}^* \cup \{\mathbf{f}_m^*\}$, $\mathbf{S} = \mathbf{S} \setminus \{\mathbf{f}_m^*\}$.
  end if
  Solve (17) with domain $\mathbf{S}$, store one of the optimal solution $\mathbf{f}_m^*$.
  $\mathbf{F}^* = \mathbf{F}^* \cup \{\mathbf{f}_m^*\}$, $\mathbf{S} = \mathbf{S} \setminus \{\mathbf{f}_m^*\}$.
end while
Output the optimal subcarrier allocation: $\mathbf{F}^*$.

TABLE 1. Simulation results for outage probability of SCMA based on $\mathbf{F_1}$ and $\mathbf{F_2}$, respectively, $\alpha = 2$. Case 1 denotes that the distances between users and base station $\mathbf{D_1}$ = [1 2 3 4 5 6], while Case 2 corresponds to $\mathbf{D_2}$ = [2 4 6 8 10 12].

SNR (dB)	0	10	20	30	40
Case 1-$\mathbf{F_1}$	1.6368	7.5053	18.5585	31.5127	44.7671
Case 1-$\mathbf{F_2}$	1.6378	7.5021	18.5516	31.5114	44.7604
Case 2-$\mathbf{F_1}$	0.4919	3.2521	11.4573	23.6342	36.7817
Case 2-$\mathbf{F_2}$	0.4919	3.2516	11.4539	23.6213	36.7745

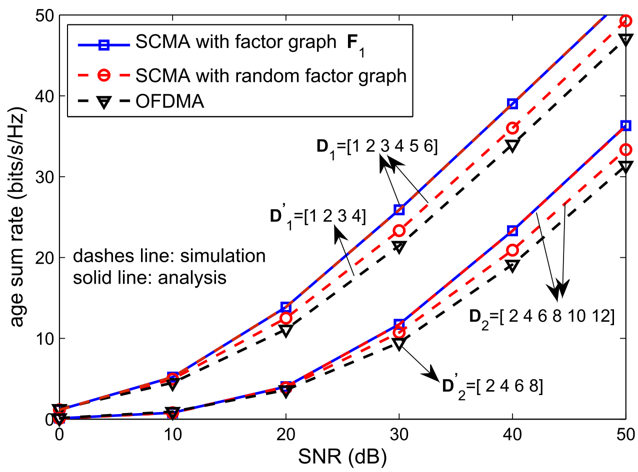

FIGURE 1. Sum rate comparison of SCMA and OFDMA systems with $\alpha = 4$ and total power is 6ρ, where $P_1 = \rho$, $P_2 = 1.1\rho$, $P_3 = 0.9\rho$, $P_4 = 0.7\rho$, $P_5 = 0.8\rho$, $P_6 = 1.5\rho$, ρ is the transmission SNR.

V. NUMERICAL AND SIMULATION RESULTS

In this Section, computer simulation results are provided to verify the accuracy of analytical results and the efficiency of the proposed optimization algorithm. In the simulation, the number of the total subcarriers, the effective subcarriers and the users are denoted as (K, N, M), for convenience. In Table I and Fig. 1, we focus on the parameters (4, 2, 6). Table I shows the average sum rate of SCMA with the factor graph matrices $\mathbf{F_1}$ and $\mathbf{F_2}$. It can be observed that the average sum rate with $\mathbf{F_1}$ is negligibly greater than that with $\mathbf{F_2}$.

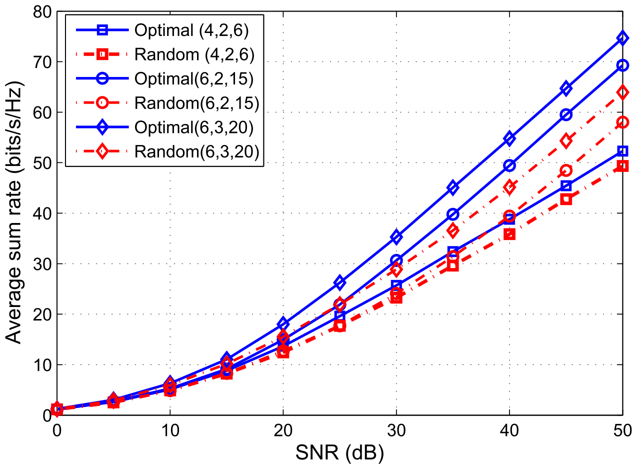

FIGURE 2. Average sum rate of SCMA systems with different factor matrices with $\alpha = 4$.

Fig. 1 illustrates the performance comparison of SCMA and OFDMA as a function of SNR. It can be observed that the average sum rate of SCMA with factor graph matrix F_1 is greater than that with random factor graph. Moreover, as expected, SCMA can achieve superior average sum rate to OFDMA for all presented cases. This comes from the fact that the frequency resource is more efficiently used in SCMA than in OFDMA systems. In addition, it is worth noting that the analytical and simulated results are closely matched for different parameters.

The impact of the factor graph matrix is plotted in Fig. 2 based on the distance information of the users is selected from a simple arithmetical sequence with $\mathcal{D} = \{d_m | d_m \in \{1, 2, \cdots, M\}\}$. It can be obviously seen from the figure that the optimized factor graph matrix can significantly enhance the SCMA system performance especially at high SNR region. In addition, it shows that the performance gap between the optimal factor matrix and the random factor matrix gets larger as the overloading factor defined as $\lambda = \frac{M}{K}$ increases. This implies that the subcarrier allocation becomes more important when SCMA system accommodates more users.

VI. CONCLUSIONS

In this letter, we have analysed the average sum rate of the uplink SCMA under the scenario that the users' distances to the BS are not necessarily identical. We have derived a closed-form expression for the average sum rate. Moreover, we have proposed a low complexity iterative algorithm to design the optimal graph matrix which maximizes the average sum rate of the SCMA systems for the general parameters. It has been revealed that the SCMA system can significantly outperform the orthogonal OFDMA technique which is a dominant multiple access scheme in current cellular systems.

APPENDIX

Let $Y_{mk} = P_m |g_{mk}|^2 d_m^{-\alpha}/N$, then the PDF of Y_{mk} is given as $f_{Y_{mk}}(y) = \lambda_{mk} e^{-\lambda_{mk} y},\ y > 0$, where $\lambda_{mk} = N d_m^{\alpha}/P_m$.

Let $Z_k = \sum_{m=1}^{M} Y_{mk}$, assume the b different λ_m are rearranged as $\lambda_{1k} = \cdots = \lambda_{r_{1k}} \triangleq \beta_{1k}$, $\lambda_{r_{1k}+1} = \cdots = \lambda_{r_{1k}+r_{2k}} \triangleq \beta_{2k}$, $\lambda_{r_{1k}+r_{2k}+\cdots+r_{(b-1)k}+1} = \cdots = \lambda_{r_{1k}+r_{2k}+\cdots+r_{bk}} \triangleq \beta_{bk}$, where $r_{mk} \geq 1$ and is an integral, the sum of all r_{mk} is M. Then the CDF of Z_k is given by [7]

$$F_{Z_k}(z) = 1 - \Big(\prod_{j=1}^{2} \beta_{jk}^{r_{jk}}\Big) \sum_{i=1}^{2} \sum_{l=1}^{r_{ik}} \frac{\Psi_{il}(-\beta_{ik}) z^{r_{ik}-l} \exp(-\beta_{ik} z)}{(r_{ik}-l)!(l-1)!}. \tag{18}$$

By using (18), the average sum rate of the k-th subcarrier can be evaluated as

$$\begin{aligned} \overline{R_k} &= \int_0^{\infty} \log_2(1+z) f_{Z_k}(z) dz = \frac{1}{\ln 2} \int_0^{\infty} \frac{1 - F_{Z_k}(z)}{1+z} dz \\ &= \frac{1}{\ln 2} \Big(\prod_{j=1}^{b} \beta_{jk}^{r_{jk}}\Big) \sum_{i=1}^{b} \sum_{l=1}^{r_{ik}} \frac{\Psi_{il}(-\beta_{ik})}{(r_{ik}-l)!(l-1)!} \\ &\quad \times \int_0^{\infty} \frac{z^{r_{ik}-l} \exp(-\beta_{ik} z)}{1+z} dz. \end{aligned} \tag{19}$$

Let $t = 1 + z$, denote the integral in (19) as Q, can be evaluated as

$$Q = e^{\beta_{ik}} \int_1^{\infty} (t-1)^{r_{ik}-l} e^{-\beta_{ik} t} / t dt. \tag{20}$$

Denote the integral in (20) as Q_1, and using binomial Theorem, which can be further evaluated as

$$\begin{aligned} Q_1 &= \sum_{n=0}^{r_{ik}-l} C_{r_{ik}-l}^{n} (-1)^{r_{ik}-l-n} \int_1^{\infty} t^{n-1} e^{-\beta_{ik} t} dt \\ &= \sum_{n=0}^{r_{ik}-l} C_{r_{ik}-l}^{n} (-1)^{r_{ik}-l-n} \Gamma\big(n, \beta_{ik}\big) / \beta_{ik}^{n}. \end{aligned} \tag{21}$$

Substituting (19), (20) and (21) into (2), the $\overline{R}$ can be obtained. The proof is completed.

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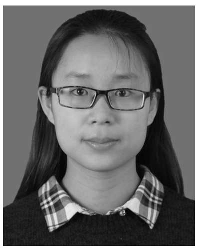

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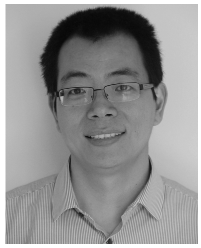

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