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A Cluster-Based Energy-Efficient Resource Management Scheme for Ultra-Dense Networks

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ABSTRACT Ultra-dense networks (UDNs), which can provide extremely high throughput and data rates, have been considered as one of the key techniques for the fifth generation mobile networks. However, it may cause severe inter-cell interference and significant energy consumption due to numerous base stations (BSs) being randomly deployed. To mitigate the interference and boost energy efficiency (EE) of the UDN effectively, we propose a cluster-based energy-efficient resource allocation scheme in this paper. The proposed scheme has two stages: clustering stage and resource allocation stage. In clustering stage, we use a modified K-means algorithm in BS-clustering process to dynamically adjust the number of BS-clusters based on the density of BSs. Then, in each BS cluster, we divide user equipments (UEs) into multiple UE-groups with minimum intra-cluster interference. In this way, the complexity of resource allocation can be greatly reduced. While in resource allocation stage, we design a two-step resource blocks assignment algorithm and an iterative energy efficient power allocation algorithm based on a non-cooperative game. Furthermore, we implement simulations under the realistic broadband channel propagation conditions and the simulation results show that the proposed approach can effectively mitigate the interference and improve the EE of UDN.

INDEX TERMS Ultra-dense network, energy efficiency, clustering, game theory.

I. INTRODUCTION

With the rapid deployment of smart terminals and the exponential growth in network traffic volume, the traditional spectrum management and cell division techniques have been unable to meet the increasing traffic demand [1], [2]. In order to satisfy this ever-growing challenge and requirement, research activities on the fifth generation (5G) mobile networks have been widely developed. One of the 5G key technical objectives is to reach 1000 times higher mobile data volume compared to current long term evolution (LTE) system by 2020 [2]–[4], and ultra-dense network (UDN) is considered as one of the key techniques of 5G to achieve this goal. UDN generally consists of a large number of low power small base stations (BSs) with different types, whose deployment density is much higher than current mobile communication networks. These small BSs are usually deployed in hot spots and coverage holes, such as apartments, offices and shopping malls [4], [5]. However, the increase of small BSs and the decrease of cell size inevitably result in significant interference, which may degrade the performance (e.g., energy efficiency) of UDN [6].

Many resource management schemes have been developed to mitigate interference and improve system energy efficiency (EE) for cellular networks [7]–[12]. However, most of them cannot be applied in UDN directly due to great computational complexity. Recent years, some researches focus on reducing computational complexity before allocating resource for UDN. The clustering approach is considered as a promising way [13]–[16] to make the topology structures of UDN be simplified and reduce the complexity of resource allocation with minimal information exchange. Some existing works [13], [14] consider cluster-based resource allocation and interference management in femtocell networks. In [13], the authors use the max-sum algorithm to select cluster heads (CHs) and then cluster the femtocells based on CHs. The work in [14] clusters the femtocells based on a semi-defined programming algorithm. In order to mitigate the intra-cluster and inter-cluster interference, [13], [14] allocate resource based on centralized management. However, the same resource blocks (RBs) are not allowed to reuse in different femtocells in a cluster, which will lead to a low utilization of spectrum and cause a heavy burden for cluster heads to allocate resource in large-scale clusters. An algorithm which groups small cell UEs based on graph theory is proposed in [15]. It aims to minimize the total interference in each cluster and ensure that there is no strong interference between UEs. However, due to the large number of UEs, the interference graph which set up edges for any two UEs is very complex. In view of this, the authors in [16] cluster BSs according to their neighborhood relationships and group UEs in each BS-cluster based on the method of [15], but the imbalance of RBs allocation is not considered. Although these cluster-based resource allocation schemes can reduce computational complexity and can mitigate interference, they are not aimed at optimizing the EE of UDN.

Nevertheless, some other works focus on the EE problems of UDN. The authors in [17] and [18] have analyzed the relationship between the system EE and the density of BSs by using stochastic geometry. The work in [19] investigates how different BSs and antenna deployment strategies can impact EE for both indoor and outdoor UDNs. In [20], the authors explore the improvement of cell breathing for improving EE in UDN, which jointly consider the space- and time- variances of the traffic load. In [21], the joint power control and user scheduling problem is studied. These studies provide valuable insights of both EE gains and limitations of UDNs. However, they require a large amount of information interaction between BSs for EE optimization, which makes the computational complexity of each BS very high during resource allocation process.

Therefore, a novel cluster-based energy efficient resource allocation scheme is proposed in this paper to mitigate interference and improve EE for UDN with low complexity. First of all, we propose a modified K-means clustering algorithm, which combines subtractive clustering [22], [23] and K-means clustering algorithm. It can dynamically adjust the size (number) of BS-clusters according to the number and density of BSs. Then, we perform UE grouping with minimum intra-cluster interference in each BS-cluster. After clustering BSs and UEs, we develop a two-step RB assignment algorithm. The first step is a round robin RB assignment which is based on the size of UE-groups. The second step is the compensation phase for assigning remaining RBs to UEs. Finally, an iterative energy efficient power allocation algorithm is proposed based on a non-cooperative game for each BS-cluster.

Our main contributions in this paper can be summarized as follows: 1) We propose a modified clustering algorithm which can form clusters adaptively based on different network scenarios. 2) We design an effective two-step RB assignment algorithm according to the UE-groups to mitigate intra-cluster interference and balance the allocated RBs among UEs. 3) We develop a power allocation game and perform an iterative update algorithm to obtain convergence with a low complexity. 4) We theoretically analyze the complexity of our scheme and illustrate the advantage of BS-clustering before resource allocation.

The remainder of this paper is organized as follows. Section II describes the system model of UDN and formulates the energy efficient problem. Section III describes our two-stage cluster-based resource allocation scheme and implementation process. In Section IV, we analyze the complexity of our scheme. The simulation results are discussed in Section V and the concluding remarks are given in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a two-tier downlink UDN which consists of a picocell tier and a femtocell tier. As shown in Fig. 1, the triangle and the pentagram represent the pico BS (PBS) and femto BS (FBS), respectively. Each Voronoi cell is the coverage of a PBS and the area of a circle represents the coverage of a FBS.

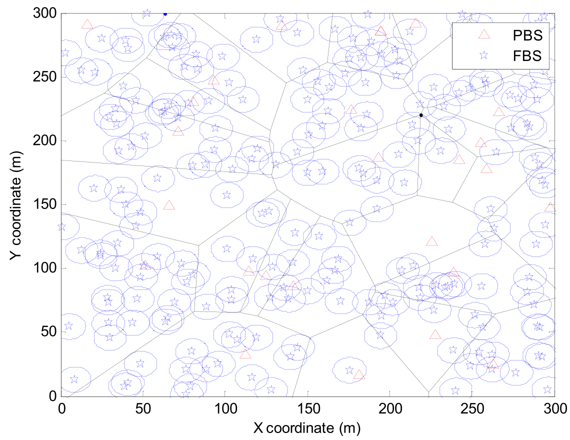

FIGURE 1. A two-tier UDN scenario in the area $\mathcal{A}$.

We model the spatial distribution of PBSs and FBSs as two independent homogeneous poisson point processes (HPPP) with density λ_p and λ_f in the two dimensional Euclidean plane $\mathcal{A}$. In this paper, there are two types of UEs, namely outside UEs and inside UEs, which are served by PBSs and FBSs, respectively. The outside UEs distribute randomly according to a HPPP with the density λ_{out} in $\mathcal{A}$, which connect with PBSs according to the closest distances. The inside UEs distribute according to a HPPP with the density λ_{in} in the coverage of each FBS with radius R_f.

Take privacy into account, we focus on closed-femto access mode [12] in this paper. For convenience of description, we define PBSs and FBSs sets as $\mathcal{C} = (1, 2, \ldots, C)$ and $\mathcal{F} = (1, 2, \ldots, F)$, respectively. PBSs and FBSs reuse the same spectrum which can be evenly divided into L RBs. The set of RBs is denoted by $\mathcal{L} = \{1, 2, \ldots, L\}$. It's assumed that one RB is exclusively assigned to at most one UE. For clarity, we utilize subscripts "p" and "f" to distinguish the parameters which are associated with PBSs or FBSs. For example, $P^l_{p,i}$ denotes the transmit power of PBS $i (i \in \mathcal{C})$

on RB l. $P^l_{f,j}$ is the transmit power assigned on RB l by FBS j ($j \in \mathcal{F}$). P_p and P_f are the maximum transmit power of a PBS and a FBS. We denote by $M_{p,i}$ the number of UEs associated with PBS i, and $\mathcal{M}_{p,i}$ the set of UEs associated with PBS i. Likewise, the number of UEs and the set of UEs associated with FBS j are denoted by $M_{f,j}$ and $\mathcal{M}_{f,j}$. Moreover, let $M_p = \sum_{i=1}^{C} M_{p,i}$, $M_f = \sum_{j=1}^{F} M_{f,j}$, and $M = M_p + M_f$.

The signal to interference plus noise ratio (SINR) received by picocell UE m over RB l can be written as

$$SINR^l_{p,i,m} = \frac{P^l_{p,i}G^l_{p,i,m}}{\sum_{t\neq i,t\in\mathcal{C}}^{C} P^l_{p,t}G^l_{p,t,m}+\sum_{j\in\mathcal{F}}^{F} P^l_{f,j}G^l_{f,j,m}+\sigma^2}, \tag{1}$$

where $G^l_{p,i,m}$ is the channel gain from PBS i to UE m on RB l, $G^l_{f,j,m}$ is the channel gain from FBS j to UE m on RB l, and σ^2 is the variance of additive white gaussian noise (AWGN). For a femtocell UE, the corresponding SINR is

$$SINR^l_{f,j,m} = \frac{P^l_{f,j}G^l_{f,j,m}}{\sum_{i\in\mathcal{C}}^{C} P^l_{p,i}G^l_{p,i,m}+\sum_{t\neq j,t\in\mathcal{F}}^{F} P^l_{f,t}G^l_{f,t,m}+\sigma^2}. \tag{2}$$

Accordingly, we can get the rates of picocell UEs and femtocell UEs on RB l as

$$R^l_{p,i,m} = W\log_2(1+\Gamma\cdot SINR^l_{p,i,m}) \tag{3}$$

and

$$R^l_{f,j,m} = W\log_2(1+\Gamma\cdot SINR^l_{f,j,m}), \tag{4}$$

where $\Gamma = -\ln(5BER)/1.5$ is the SINR gap [7] under a given bit error rate. The binary indicator variables $\chi^l_{p,i,m}$ and $\chi^l_{f,j,m}$ describe whether RB l is allocated to UE m by PBS i and FBS j. For example, $\chi^l_{f,j,m} = 1$ or $\chi^l_{f,j,m} = 0$ represents RB l is assigned to UE m by FBS j or not. We denote by $\boldsymbol{\chi}_m = (\chi^1_m, \chi^2_m, \ldots, \chi^L_m)$ the vector indication variables for RB allocation of UE m, and denote by $\boldsymbol{\chi} = \{\boldsymbol{\chi}_1, \boldsymbol{\chi}_2, \ldots, \boldsymbol{\chi}_M\}$ the vector indication set for RB allocation of all UEs. The rate of an UE associated with PBS i or FBS j can be expressed as

$$R_{p,i,m} = \sum_{l=1}^{L}\chi^l_{p,i,m}W\log_2(1+\Gamma\cdot SINR^l_{p,i,m}), \tag{5}$$

$$R_{f,j,m} = \sum_{l=1}^{L}\chi^l_{f,j,m}W\log_2(1+\Gamma\cdot SINR^l_{f,j,m}). \tag{6}$$

The total throughput of the system is

$$R(\mathcal{P}) = \sum_{i\in\mathcal{C}}\sum_{m\in\mathcal{M}_{c,i}}\sum_{l\in\mathcal{L}}\chi^l_{p,i,m}R^l_{p,i,m} + \sum_{i\in\mathcal{F}}\sum_{m\in\mathcal{M}_{f,j}}\sum_{l\in\mathcal{L}}\chi^l_{f,j,m}R^l_{f,j,m}, \tag{7}$$

where $\mathcal{P}$ is the power feasible region of each BS.

As the energy consumption at each BS includes two parts: transmit power and circuit power [11], in the downlink transmission, the total power consumption can be defined as

$$P_t(\mathcal{P}) = \sum_{i\in\mathcal{C}}\sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}_{p,i}}\chi^l_{p,i,m}\xi_p P^l_{p,i} + C\cdot P_{C,p} + \sum_{j\in\mathcal{F}}\sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}_{f,j}}\chi^l_{f,j,m}\xi_f P^l_{f,j} + F\cdot P_{C,f}, \tag{8}$$

where $\xi_p \geq 1$ and $\xi_f \geq 1$ denote the reciprocal of drain efficiency of power amplifier of a PBS and a FBS, respectively. $P_{C,p}$ and $P_{C,f}$ represent the circuit power of a PBS and a FBS, respectively.

Hence, in this paper, the energy efficiency is defined as the ratio of the total network throughput over the total network power consumption with the units of *bits/joule*. The optimal RB and power allocation problem is to maximize EE in the overall network and can be formulated as

$$\begin{aligned}
\max_{\mathcal{P},\boldsymbol{\chi}}\ \eta &= \frac{R(\mathcal{P},\boldsymbol{\chi})}{P_t(\mathcal{P},\boldsymbol{\chi})}\\
s.t.\ C1:&\ \chi^l_{p,i,m}\in\{0,1\},\quad i\in\mathcal{C},\ m\in\mathcal{M}_{p,i},\ l\in\mathcal{L},\\
C2:&\ \chi^l_{f,j,m}\in\{0,1\},\quad j\in\mathcal{F},\ m\in\mathcal{M}_{f,j},\ l\in\mathcal{L},\\
C3:&\ P^l_{p,i,m} > 0,\quad m\in\mathcal{M}_{p,i},\ l\in\mathcal{L},\\
C4:&\ P^l_{f,j,m} > 0,\quad m\in\mathcal{M}_{f,j},\ l\in\mathcal{L},\\
C5:&\ \sum_{l\in\mathcal{L}}P_{p,i,l}\leq P_p,\quad \sum_{l\in\mathcal{L}}P_{f,j,l}\leq P_f,\ i\in\mathcal{C},\ j\in\mathcal{F},\\
C6:&\ \sum_{m\in\mathcal{M}_{p,i}}\chi^l_{p,i,m}\leq 1,\quad \sum_{m\in\mathcal{M}_{f,j}}\chi^l_{f,j,m}\leq 1,\ l\in\mathcal{L}.
\end{aligned} \tag{9}$$

Constrains $C1$, $C2$, $C6$ imply that each RB can only be assigned to one UE. Constraints $C3$ and $C4$ ensure the transmit power to be positive. $C5$ is the total transmission power constraints for BS.

III. THE CLUSTER-BASED ENERGY-EFFICIENT RESOURCE MANAGEMENT SCHEME AND IMPLEMENTATION

Note that problem (9) is in general NP-hard for obtaining the optimal solution [10], [11]. In order to obtain solutions with reasonable complexity, the resource allocation can be decomposed into RB allocation and power allocation. Even so, there is still a heavy burden for each BS to collect and analyze the strategic information from other BSs in UDN with numerous BSs. Therefore, we propose a distributed cluster-based energy-efficient resource allocation scheme. This scheme has two stages, which are clustering stage and resource allocation stage.

The clustering stage has two parts: BS-clustering and UE-grouping. We first divide all BSs in the network into several BS-clusters (sub-networks) based on a modified K-means algorithm. As a result, each BS only needs to analyze the strategic information from the other BSs within the same BS-cluster. This will greatly reduce the computational complexity of each BS. Besides, in order to further mitigate the intra-cluster interference, we assign UEs associated with different cells into UE-groups with low relative interference effects in each BS-cluster.

After clustering BSs and UEs, the resource allocation stage is performed. In the RB allocation part, we first allocate RBs iteratively to UE-groups, and then assign the remaining RBs of each BS to their UEs based on the channel gain. In the power allocation part, a game based iterative energy efficient power allocation algorithm is proposed. Each BS as a player allocates power selfishly to maximize its own energy efficiency. If no BS is able to improve its utility with a unilateral deviation, the processes of energy efficiency optimization are accomplished.

In the following sub-sections, we specifically describe the four parts of the proposed scheme, and discuss its implementation process in practical systems.

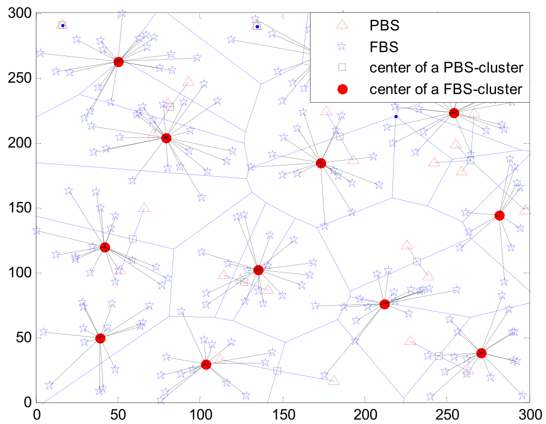

FIGURE 2. An example of clustering results.

A. BS-CLUSTERING

We obtain BS-clusters base on a modified K-means algorithm which combines with subtractive clustering and K-means clustering algorithm. The traditional K-means is constrained by the initial random selection of the cluster centers and the number of clusters K needs to be predetermined. In order to increase the flexibility of our algorithm, we firstly utilize the subtractive clustering technique, which can reasonably adjust the number of BS-clusters according to the BS density, to generate the centers and the number of BS-clusters. Then, we use K-means algorithm to obtain the final set of BS-clusters according to the centers and the number of BS-clusters which are generated by subtractive clustering technique. As different kinds of BSs have different coverages and transmit power, we cluster PBSs and FBSs respectively. An example of clustering results is shown in Fig 2, where the blue square and red points represent the initial clustering centers of PBSs and FBSs. In order to avoid repeated description, we just describe the process of FBSs clustering in Algorithm 1. In this algorithm, $\mathcal{SF} = \{sf_1, sf_2, \ldots, sf_F\}$ is the set of FBSs' coordinate, $\mathcal{FZ}$ is the set of FBS-cluster centers, and $\mathcal{FC}$ is the set of FBS clusters. δ and r_a are used to determine the number of BS-clusters in subtractive clustering.

Algorithm 1 FBS Clustering

Input: $\mathcal{SF}$, δ, r_a

Output: K, $\mathcal{FC}$, $\mathcal{FZ}$

1 Initialization: Let $\delta = 0.5$ and $r_a = 0.5 \min_x\{\max_y\{sf_x - sf_y\}\}$.

2 Perform subtractive clustering to get the FBS-cluster center set $\mathcal{FZ}$ and K FBS-clusters based on $\mathcal{SF}$.

3 Perform K-means algorithm to get the final FBS clustering set $\mathcal{FC} = \{\mathcal{FC}_1, \mathcal{FC}_2, \ldots, \mathcal{FC}_K\}$ with input parameters $\mathcal{FZ}$ and K. In particular, $\mathcal{FC}_k (k = 1, \ldots, K)$ is the set of FBSs in cluster i.

Similarly, we can cluster PBSs and obtain the set of PBS clusters $\mathcal{PC} = \{\mathcal{PC}_1, \mathcal{PC}_2, \ldots, \mathcal{PC}_K\}$ by using Algorithm 1.

B. UE-GROUPING

Because of the reuse of frequency, ultra-dense and heterogeneous BSs may introduce serious interference. In order to mitigate the intra-cluster interference, we adopt a modified UE grouping algorithm. Firstly, we build an interference graph $\mathbb{G}(\mathbf{V}, \mathbf{E})$. The vertex set $\mathbf{V}$ corresponds to the UEs and edge set $\mathbf{E}$ represents the downlink interference conditions between UEs in each FBS-cluster or PBS-cluster. We use relative channel loss metrics $\beta_{p,i,m}$ and $\beta_{f,j,m}$ to describe the impact of total interference on the UE from non-serving BSs in each PBS-cluster or FBS-cluster, which are defined as

$$\beta_{p,i,m} = \frac{\sum_{t \neq i \in \mathcal{PC}_k} PL_{p,t,m} + \sigma^2}{PL_{p,i,m}}, \quad m \in \mathcal{M}_{\mathcal{PC}_k}$$
$$\beta_{f,j,m} = \frac{\sum_{t \neq j \in \mathcal{FC}_k} PL_{f,t,m} + \sigma^2}{PL_{f,j,m}}, \quad m \in \mathcal{M}_{\mathcal{FC}_k}, \tag{10}$$

where $\mathcal{M}_{\mathcal{PC}_k}$ and $\mathcal{M}_{\mathcal{FC}_k}$ are the UE sets associated with PBS cluster $\mathcal{PC}_k$ ($\mathcal{PC}_k \in \mathcal{PC}$) and FBS-cluster $\mathcal{FC}_k$ ($\mathcal{FC}_k \in \mathcal{FC}$), respectively. $PL_{p,i,m}$ is the path loss of PBS i to UE m, $PL_{f,j,m}$ is the path loss of FBS j to UE m.

Then, we need to build relationships (create edges) between any two UEs in each BS cluster. It's assumed that β_{m_1} and β_{m_2} represent the relative channel loss of two UEs (m_1 and m_2) belonging to the same BS cluster, and the weight of an edge between UEs can be defined as

$$E(m_1, m_2) = E(m_2, m_1) = \begin{cases} \beta_{m_1} + \beta_{m_2}, & m_1 \text{ and } m_2 \text{ are in different cells} \\ E_{th}, & \text{otherwise.} \end{cases} \tag{11}$$

The edge weight between two UEs logically represents the level of signal degradation to UEs used the same RBs. The smaller weight of the edge between m_1 and m_2 indicates that the level of signal degradation to m_1 and m_2 is smaller. So our object is to minimize the sum weight of all UE-groups in different clusters. As there is an upper bound of weight E_{th} to constrain the number of UEs in a UE-group, we set E_{th}

large enough to enable more UEs in different cells to be classified into the same group. In this case, the total interference can be mitigated effectively. We randomly select a vertex as the starting point from the interference graph $\mathbb{G}(\mathbf{V}, \mathbf{E})$ of each BS cluster, and traverse the graph iteratively by adding the vertices that minimize the weight of current UE-group. Once the sum weight of edges in a UE-group reaches the upper bound E_{th}, the UE-group is extracted and the new UE-group begins to form from the remaining vertices.

Because the process of UE grouping in PBS-cluster and FBS-cluster is similar, we only take the UE grouping of FBS-clusters as an example which is given in Algorithm 2. We assume $\mathbb{G} = \{\mathbb{G}_1, \mathbb{G}_2, \cdots, \mathbb{G}_K\}$ represents the interference graphs of K FBS-clusters. $\mathbf{V} = \{\mathbf{V}_1, \mathbf{V}_2, \cdots, \mathbf{V}_K\}$ and $\mathbf{E} = \{\mathbf{E}_1, \mathbf{E}_2, \cdots, \mathbf{E}_K\}$ are the sets of candidate vertices and edges, respectively. At the beginning, $\mathbf{V}$ holds all vertices in the interference graph of all FBS-clusters. ς is the current set of UE-group which is extending. ξ is the current candidate list which doesn't include the assigned vertices. $\mathbb{GU} = \{\mathbb{GU}_1, \mathbb{GU}_2, \cdots, \mathbb{GU}_K\}$ denotes the set of UE-group of K FBS-clusters and each element in $\mathbb{GU}$ represents a set of UE-group of a FBS-cluster.

Algorithm 2 UE Grouping

```
Input: 𝔾, V, E
Output: 𝔾𝕌
Initialization: ς = ∅, V contains all vertices of all
  FBS-clusters.
for k = 1 : K
   𝔾𝕌_k = ∅.
end
for k = 1 : K
   while V_k ≠ ∅
      Randomly select a vertex v from V_k and let ς = {v}.
      Create a candidate list ξ, ξ ← V_k\ς.
      while (the sum weight of ς is less than E_th)
         Select a vertex as v' = arg min_{v'∈ξ}(Σ_{v∈ς} E(v, v')).
         Add the selected vertex v' into ς as ς ← ς∪{v'}.
      end
      Remove set ς from set V_k as V_k ← V_k\ς, and
      extract the vertices of ς as a new UE-group.
         Let 𝔾𝕌_k ← 𝔾𝕌_k ∪ {ς} and reset ς = ∅.
   end
end
```

C. RB ALLOCATION

Decomposing RB and power allocation can reduce the complexity of the optimization problem (5) effectively, so we solve the problem of RB allocation firstly. Because the number of UEs within each cell is not identical, unbalanced problem exists in UE grouping. We develop a two-step proportional fair scheduling algorithm to allocate RBs. We first allocate RBs iteratively to UE-groups based on the channel gains and then iteratively assign the remaining RBs of each BS to their UEs based on the channel gain. In order to depict simply, we only take the RB allocation of FBSs as an example, and the details are given in Algorithm 3. In this algorithm, $\mathcal{L}' = \{\mathcal{L}'_1, \mathcal{L}'_2, \cdots, \mathcal{L}'_F\}$ represents the set of remaining RBs of each FBS after the first step in RB allocation and $\mathbf{G}$ is the channel gain set of all UEs. The outcome of RB allocation is denoted by a set Π with dimension $F \times L$. Each element, which is the index of UE, records the object of each RB allocation for each FBS. φ is the current set of UE-group which is being allocated RBs. $|\mathcal{FC}_k|$ and $|\varphi|$ denote the index set of FBSs and UEs in $\mathcal{FC}_k$ and φ, respectively.

Algorithm 3 RB Allocation

```
Input: 𝔾𝕌, G, FC, M, L
Output: Π
Initialization: Sort all UE-groups in each UE-group set
  of 𝔾𝕌 in the descending order of the number of UEs as
  𝔾𝕌' = {𝔾𝕌'_1, 𝔾𝕌'_2, ··· , 𝔾𝕌'_K}.
for k = 1 : K
   while L ≠ ∅
      Schedule the UE-groups in 𝔾𝕌'_k with round robin
      manner, and calculate the channel gain for each RB
      of the scheduled UE-group φ.
      Let G^l_φ = Σ_{m∈φ} G^l_m (∀l ∈ L), and assign l* to
      UE-group φ with l* = arg max_l G^l_φ.
      Remove l* from L as L ← L\{l*}.
      Mark the l*th RB of FBS in FC_k as |φ|, and let
      Π(|FC_k|, l*) ← |φ|.
   end
end
for j = 1 : F
   while L'_j ≠ ∅
   Assign l* to the current scheduled UE m, with l* =
   arg max_l G^l_{f,j,m}, l ∈ L'_j.
      Remove l* from L'_j as L'_j ← L'_j\{l*}.
      Mark the l*th RB of FBS j as m, and let Π(j, l*) ←
      m
   end
end
```

D. POWER ALLOCATION

Due to the large number of BSs in UDN, centralized power allocation is impractical. In this case, we seek for a suboptimal solution with distributed power allocation which is formulated as a non-cooperative game. Each BS as a player allocates power selfishly to maximize its own energy efficiency.

We focus on the power allocation of FBS j ($j \in \mathcal{FC}_k$). Let $\mathbf{P}_{f,-j}$ denote the set of other FBSs' power allocation vectors in $\mathcal{FC}_k$, and the best response of power allocation of FBS j can be described as

$$\mathbf{P}^*_{f,j} = \arg \max_{P^l_{f,j} \in \mathcal{P}_f} \eta_{f,j}(\mathbf{P}_{f,j}, \mathbf{P}_{f,-j}), \tag{12}$$

where $\mathcal{P}_f$ is the feasible region of FBSs' power. The objective is to design a feasible power allocation scheme to find the best response $\mathbf{P}^*_{f,j}$ under total power constraints. So FBS j needs to solve the following power optimization problem.

Problem 1:

$$\text{Max } \eta_{f,j} = \frac{\sum_{l=1}^{\mathcal{L}} R^l_{f,j}}{\sum_{l=1}^{\mathcal{L}} (\xi_f P^l_{f,j} + P^l_{f,C})}$$
$$s.t.\ C4, C5, \tag{13}$$

where $R^l_{f,j} = W\log_2(1+P^l_{f,j}G^l_{f,j}/(I^l_{f,j}+\sigma^2))$. $I^l_{f,j}$ denotes the total interference received by FBS j on l, which is expressed as

$$I^l_{f,j} = \sum\nolimits_{x\in\mathcal{FC}_k, x\neq j} P^l_{f,x}G^l_{f,x} + \sum\nolimits_{y\in\mathcal{FC}_{-k}} \frac{P_f}{LG^l_{f,y}} + \sum\nolimits_{i\in\mathcal{PC}} \frac{P_p}{LG^l_{p,j}}, \tag{14}$$

where $\sum_{y\in\mathcal{FC}_{-k}} P_f/LG^l_{f,y} + \sum_{i\in\mathcal{PC}} P_p/LG^l_{p,j}$ denotes the interference from other clusters with equal power allocation. $G^l_{f,x}$ is the channel gain from FBS x to the UE of FBS j on RB l. $P^l_{f,C}$ denotes the constant circuit power consumed by FBS j on RB l and $P_{f,C} = \sum_{l=1}^{\mathcal{L}} P^l_{f,C}$.

Problem 1 can be considered as a parameterized concave programming problem according to the *dinkelbach* algorithm [12], and thus, we have the following lemma.

Lemma 1:

$$\text{Max } F(\eta_{f,j}) = R_{f,j}(\mathbf{P}_{f,j}, \mathbf{P}_{f,-j}) - q_{f,j}P_t(\mathbf{P}_{f,j})$$
$$s.t.\ C4, C5, \tag{15}$$

where $R_{f,j} = \sum_{l=1}^{\mathcal{L}} R^l_{f,j}$, $P_t(\mathbf{P}_{f,j}) = \sum_{l=1}^{\mathcal{L}} (\xi_f P^l_{f,j} + P^l_{f,C})$ and $\mathbf{P}_{f,j}$ is the current power allocation strategy set of FBS j. The maximum EE of FBS j, $q^*_{f,j} = R_{f,j}(\mathbf{P}^*_{f,j})/P_t(\mathbf{P}^*_{f,j})$ can be obtained if and only if $F(q^*_{f,j}) = 0$.

Proof: The proof of the lemma and its detailed convergence analysis are described in [24].

According to the analyses above, we can design an equivalent optimal problem

$$\text{Max } U_{f,j}(\mathbf{P}_{f,j}, \mathbf{P}_{f,-j}) = R_{f,j}(\mathbf{P}_{f,j}, \mathbf{P}_{f,-j}) - q_{f,j}P_t(\mathbf{P}_{f,j})$$
$$s.t.\ C4, C5. \tag{16}$$

The optimal solution to (16) is equivalent to the solution to (13) at the optimal value $\mathbf{P}^*_{f,j}$, and solving the EE problem (13) can be realized by the optimal power allocation based on (16) for a given $q_{f,j}$ and then update $q_{f,j}$ until $F(q^*_{f,j}) = 0$. As a result, if we figure out the optimal problem (16), the EE problem can be solved.

In order to solve (16), we formulate a non-cooperative power allocation game as

$$\mathcal{G}_{\mathcal{FC}_k} = \{\mathcal{FC}_k, \mathcal{P}_{f,j}, \mathcal{U}_{f,j}\}, \tag{17}$$

where $\mathcal{FC}_k$ denotes the player set which includes $||\mathcal{FC}_k||$ FBSs and $\mathcal{P}_{f,j}$ refers to the strategy set. $\mathcal{U}_{f,j}$ is the utility function set which is given as

$$\mathcal{U}_{f,j}(\mathbf{P}_{f,j}, \mathbf{P}_{f,-j}) = \sum_{l=1}^{L}(W\log_2(1+P^l_{f,j}H^l_{f,j}) - q_{f,j}(\xi_f P^l_{f,j} + P^l_{f,C})), \tag{18}$$

where $H^l_{f,j} = G^l_{f,j}/(I^l_{f,j}+\sigma^2)$ and $\mathbf{P}_{f,j} = (P^1_{f,j}, P^2_{f,j}, \ldots, P^L_{f,j})$.

Definition 1: Formally, a transmit power allocation strategy $\mathbf{P}^* = (\mathbf{P}^*_{f,j}, \mathbf{P}^*_{f,-j})$ is a Nash equilibrium (NE) [25] of the game $\mathcal{G}_{\mathcal{FC}_k}$ if, for all players $j \in \mathcal{FC}_k$, we have that

$$\mathcal{U}_{f,j}(\mathbf{P}^*_{f,j}, \mathbf{P}^*_{f,-j}) \geq \mathcal{U}_{f,j}(\mathbf{P}_{f,j}, \mathbf{P}^*_{f,-j}), \tag{19}$$

where $\mathbf{P}_{f,j} \neq \mathbf{P}^*_{f,j}$.

The concept of NE offers a predictable outcome of a game in which multiple agents with conflicting interests compete through self-optimization and reach a stable point where no player wishes to deviate. The proofs for the existence and uniqueness of NE point in the power allocation game are as follows.

Theorem 1: Assuming the strategy space of power allocation is $a^l_{f,j} \leq P^l_{f,j} \leq b^l_{f,j}$, $\forall j \in \mathcal{FC}_k, l \in \mathcal{L}$, the Nash equilibrium exists and is unique in the game $\mathcal{G}_{\mathcal{FC}_k}$.

Proof: The Proof is presented in Appendix.

In the proposed game $\mathcal{G}_{\mathcal{FC}_k}$, we develop a distributed iterative power allocation algorithm to obtain the NE. Specifically, if the NE is $\mathbf{P}$, the power allocation can be updated as $\mathbf{P}(t+1) = \mathbf{f}(\mathbf{P}(t))$.

$$\mathbf{P}(t+1) = P^l_{f,j}(t+1) = \left[\frac{P^l_{f,j}(t)H^l_{f,j}(t)}{1+P^l_{f,j}(t)H^l_{f,j}(t)} \cdot \frac{W}{q_{f,j}\xi_f \ln 2}\right]^{b^l_{f,j}}_{a^l_{f,j}}, \tag{20}$$

where $[x]^b_a = \max\{a, \min\{x, b\}\}$. It has been proved that power allocation iteration converges to the fixed point (NE), if $\mathbf{f}(\mathbf{P})$ is a standard function which satisfies the following conditions [8]: 1) positivity $\mathbf{f}(\mathbf{P}) > 0$; 2) monotonicity $\mathbf{P}_1 > \mathbf{P}_2 \Rightarrow \mathbf{f}(\mathbf{P}_1) > \mathbf{f}(\mathbf{P}_2)$; 3) scalability $\mu\mathbf{f}(\mathbf{P}) > \mathbf{f}(\mu\mathbf{P}), \forall\mu > 1$.

In the proposed game, as $P^l_{f,j} \in [a^l_{f,j}, b^l_{f,j}]$, $(j \in \mathcal{FC}_k, l \in \mathcal{L})$, condition 1 can be easily satisfied. If $P^l_{f,j} \geq \bar{P}^l_{f,j}$, $(j \in \mathcal{FC}_k, l \in \mathcal{L})$, we can obtain

$$\mathrm{f}(P^l_{f,j}) - \mathrm{f}(\bar{P}^l_{f,j}) = (P^l_{f,j} - \bar{P}^l_{f,j}) \cdot \left(\frac{H^l_{f,j}(t)}{1+P^l_{f,j}(t)H^l_{f,j}(t)} \cdot \frac{W}{q_{f,j}\xi_f \ln 2}\right) \geq 0, \tag{21}$$

which illustrates $\mathbf{f}(\mathbf{P})$ is a monotone function. Hence, conditions 2 can be satisfied. Besides, when $\forall\mu > 1$, and then

$$\mu\mathrm{f}(P^l_{f,j}) - \mathrm{f}(\mu P^l_{f,j}) = \frac{\mu W P^l_{f,j}(t)}{q_{f,j}\xi_f \ln 2}\left(\frac{H^l_{f,j}(t)}{1+P^l_{f,j}(t)H^l_{f,j}(t)} - \frac{H^l_{f,j}(t)}{1+\mu P^l_{f,j}(t)H^l_{f,j}(t)}\right) > 0, \tag{22}$$

which implies the function $\mathbf{f}(\mathbf{P})$ satisfy the scalability in conditions 3. These analyses indicate that the NE can be obtained by the iteration approach designed as (20).

In the process of iteration, if $P_{f,R}(P_{f,R} \geq 0)$ is the remaining power in the process of power allocation, then

$$b^l_{f,j} = P_{f,R} + P^l_{f,j}. \tag{23}$$

Note that $b^l_{f,j}$ should be update in each iteration and $a^l_{f,j} = 0$ in the game. We only take the optimal power allocation of a FBS-cluster as an example, which is described in Algorithm 4. $Q_{\mathcal{FC}_x} = \{\cdots, q_{f,j}, \cdots\}, j \in \mathcal{FC}_k$ is the input set of each iteration which represents the current EE set of each FBSs in $\mathcal{FC}_k$. $\mathbf{P}_{\mathcal{FC}_k}$ is the output power vectors of all FBSs in $\mathcal{FC}_k$. T and ε denote the maximum number of iterations and the maximum tolerance value, respectively.

Algorithm 4 Power Allocation

Input: $\mathcal{FC}_k$, $\mathbf{G}$, T
Output: $\mathbf{P}_{\mathcal{FC}_k}$
1 Initialization: Set the maximum number of iteration T and maximum tolerance value ε. Set iteration index $t = 0$ and initial EE $\eta_{\mathcal{FC}_k}(t) = 0$.
3 Let $P^l_{f,j} = P_f/L, j \in \mathcal{FC}_k, l \in L$ and $Q_{\mathcal{FC}_x} = R_{\mathcal{FC}_k}(\mathbf{P}_{\mathcal{FC}_x})/P_t(\mathbf{P}_{\mathcal{FC}_x})$.
4 Repeat:
5 Set $t = t + 1$, Update power strategies of each FBS in $\mathcal{FC}_k$ based on (20) and (23) and calculate the current EE $\eta_{\mathcal{FC}_k}(t)$.
6 if $\eta_{\mathcal{FC}_k}(t) - \eta_{\mathcal{FC}_k}(t-1) < \varepsilon$
7 Return $\mathbf{P}_{\mathcal{FC}_x}$ and $Q_{\mathcal{FC}_x}$.
8 else
9 Set $Q_{\mathcal{FC}_x} = R_{\mathcal{FC}_k}(\mathbf{P}_{\mathcal{FC}_x})/P_t(\mathbf{P}_{\mathcal{FC}_x})$.
10 until convergence indication is true or $t = T$.

E. IMPLEMENTATION

In this paper, we aim at developing a cluster-based energy efficient resource allocation scheme for UDN, and divide the scheme into four parts for implementation. According to the analysis above, the implementation details are as follows.

We first obtain the location information of PBSs and FBSs by core network and femto gateways (F-GW), respectively. And then according to the location information of BSs, we utilize the modified K-means clustering algorithm to get PBS-clusters and FBS-clusters, respectively. At the same time, each cluster needs to select a cluster head (CH) and only the CHs can get the location information of UEs and BSs from core network and F-GW. The selection of CHs needs to be based on the actual situation of the system, such as the load or location of BSs. Next, each CH implements the UE grouping and the first step RB allocation in each BS-cluster based on the proposed algorithms. Once CHs complete the process, they broadcast the results to each BS in their own BS-clusters. Then each BS accomplishes the second step RB allocation based on the received information. Because BSs are not required to report their information to other BSs and CHs, the whole process has a lower signaling overhead. After allocating RBs, the game based iterative energy efficient power allocation algorithm is applied. In this algorithm, each BS as a player selfishly allocates power to maximize its own energy efficiency. If no BS is able to improve its utility with a unilateral deviation, our scheme accomplishes all processes of energy efficiency optimization in UDN.

IV. COMPUTATIONAL COMPLEXITY

In this part, we analyze the complexity of our scheme based on different stages.

The complexity of clustering stage is composed by BS-clustering and UE-grouping. According to the algorithm complexity of subtractive clustering and K-means, the complexity of BS-clustering with K BS-clusters can be calculated as $O(d^2S^2) + O(SKt)$, where $O(d^2S^2)$ is the complexity of subtractive clustering with K clustering centers, and $O(SKt)$ is the complexity of k-means algorithm. Besides, S is the number of BSs, $d = 2$ represents the two dimensional coordinate of BSs, and t is number of iterations of K-means algorithm. If we assume the average numbers of BSs and UEs in each BS-cluster are N and M_c and assume the average number of UEs accessed to each BS is M_s, the complexity of UE-grouping is $(K(N-1)[M_s + (M_s - 1) + \cdots + 1])$, which can be rewritten as $O(S^2M^2)$ in the worst case.

The complexity of resource allocation stage is $O(KL^2 + KM_c \log(M_c) + M(L - LM_s/N_g - 1)^2) + O(SL(N-1)\bar{I})$, where $O(KL^2 + KM_c \log(M_c) + M(L - LM_s/N_g - 1)^2)$ and $O(SL(N-1)\bar{I})$ denote the complexity of RB allocation and power allocation, respectively. N_g, N and $\bar{I}$ are average number of UE-groups, BSs and iterations of power allocation in each BS-cluster.

The calculative complexity of all BSs in power allocation without BS-clustering is $O(SL(S-1))$ for each iteration. However, BS-clustering reduces it to $O(SL(N-1))$, where $S \gg N$. Although the processes of BS-clustering and UE-grouping in our scheme introduce some complexity, they effectively reduce the calculative complexity of each BS in power allocation and mitigate the interference efficiently in RB allocation.

V. SIMULATION RESULTS AND DISCUSSION

In this section, we conduct simulations to verify the effectiveness of the proposed scheme in mitigating interference and optimizing energy efficiency of UDN. The drain efficiencies of PBS' and FBS' power amplifier are set to 0.022 and 0.017, respectively. Other simulation parameters are listed in Table 1. We consider Reyleigh fading to model the channels between BSs to UEs, and the pathloss models for inside UEs and outside UEs are based on [12]. We compare our scheme with other three resource allocation schemes which are listed in Table 2.

At first, we validate the convergence of our scheme by examining the evolution of energy efficiency in iterations. Fig. 3(a) shows the energy efficiency evolution of different

TABLE 1. **Simulation parameters.**

Parameters	Value
Area of UDN	300m∗300m
Carrier Frequency	2GHz
Total Bandwidth	3.6MHz
Bandwidth per RB	180KHz
Number of RBs	20
PBSs/FBSs TX power	30dBm/20dBm
PBSs/FBSs circuit power	0.5W/0.1W
Radius of FBSs	10m
Density of PBSs	0.0003 PBSs/ m^2
Density of FBSs	0.0005~0.0027 FBSs/ m^2
Density of outside UEs	0.003 UEs/ m^2
Density of inside UEs	0.1 UEs/m^2
Wall penetration loss	15dB
Rayleigh fading parameter	1
Thermal noise level	-174dBm/Hz
Bit error rate	10e-5

TABLE 2. **Related resources allocation schemes.**

Name	BS clustering	UE grouping	RB allocation	Power allocation
S1	Algorithm 1	Algorithm 2	Algorithm 3	Algorithm 4
S2	Algorithm 1	Algorithm 2	Algorithm 3	Avg. power allocation
S3	Algorithm 1	--	Random RB allocation	Algorithm 4
S4	--	--	Random RB allocation	Avg. power allocation

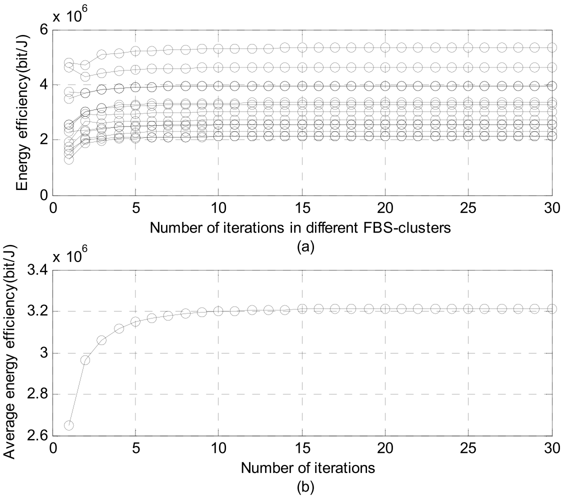

FIGURE 3. **Energy efficiency evolution in different FBS-clusters, (a) Energy efficiencies of different FBSs clusters. (b) Average energy efficiency of all FBS-clusters.**

FBS-clusters when the density of FBS is $\lambda_f = 0.0023$. We can see that the energy efficiency can reach stable state in less than 20 iterations. In other words, the power allocation game can obtain Nash equilibrium within 20 iterations in this example. Due to the number and the locations of FBSs in each FBS-cluster are different, the final convergent values of energy efficiency in different FBS-clusters are unequal. Fig. 3(b) is the evolution of average energy efficiency of all FBS-clusters in iterations. We can see that the average energy efficiency reaches stable state in about 15 iterations, which clearly illustrates the convergence of the power allocation game in this paper.

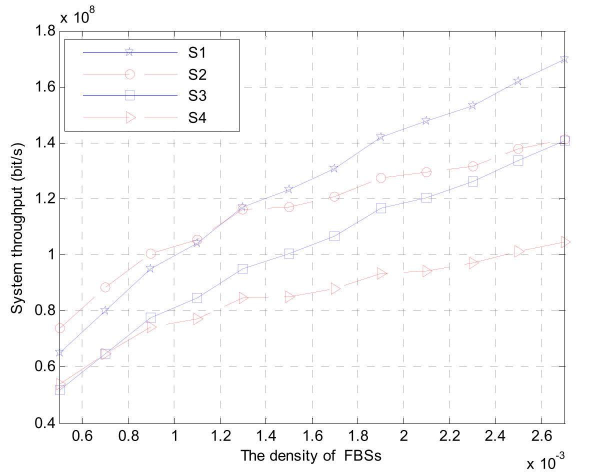

FIGURE 4. **System throughput under different densities of FBSs.**

The next, we investigate the impact of the FBSs' density on system throughput of UDN. Fig. 4 shows the system throughput with different density of FBSs for four schemes listed in Table 2. We can see that the system throughput increase with the density of FBSs in all schemes. The proposed scheme (S1) has higher throughput than other schemes when $\lambda_f > 0.0012$, and this superiority is more obvious if λ_f is further increased. Comparing S1 with S3 and comparing S2 with S4, we can find that the schemes which are based on Algorithm 2 and Algorithm 3 (S1 and S2) can obtain higher throughput. This is because that we assign UEs associated with different cells into UE-groups with low relative interference effects (Algorithm 2), and allocate the same RBs to UEs in a UE-group (Algorithm 3). In this way, the system interference can be mitigated effectively. On the other hand, comparing S1 with S2, S1 can obtain higher throughput. It indicates that the power allocation strategies based on Algorithm 4 (S1) can effectively mitigate interference.

In fact, the system throughput increases with the number of RBs. However, as the density of FBSs increases, the system will introduce severe inter-cell interference which makes the rate on each RB decrease greatly. As a result, the system spectral efficiency decreases with the density of FBSs for all schemes as shown in Fig. 5. It is noticeable that the proposed scheme S1 has the highest system spectral efficiency when $\lambda_f > 0.0012$.

Finally, we compare the system energy efficiency of our scheme with the other three schemes by varying λ_f. Unlike the variation curves of system throughput and the system spectral efficiency, the energy efficiency in Fig. 6 increases with λ_f to a peak and then start to decrease for an arbitrary curve. This is because when λ_f surpasses a certain threshold, continuing to increase λ_f will make more interfering FBSs close to UEs, which causes the gain of the system throughput cannot compensate the negative effects of the increased power consumption. As the density of FBSs increases, the

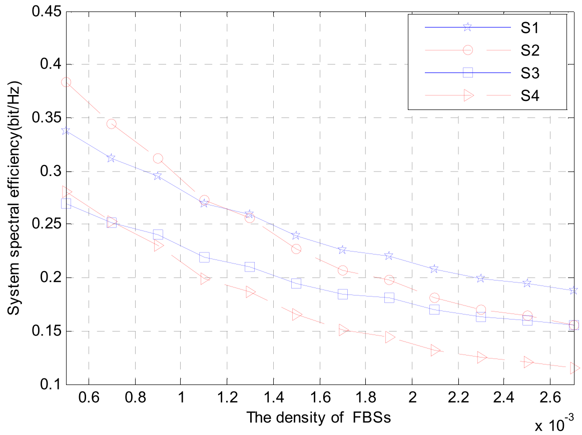

FIGURE 5. **System spectral efficiency under different densities of FBSs.**

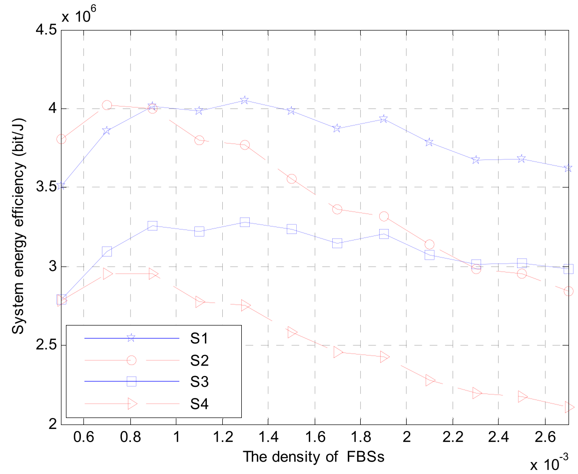

FIGURE 6. **System energy efficiency under different densities of FBSs.**

system energy efficiency of S1 and S3 decrease more slowly compared to S3 and S4. This indicates that our power allocation algorithm effectively improves energy efficiency. In particular, the proposed scheme S1 has the highest energy efficiency when $\lambda_f > 0.0009$, and S1 improves the energy efficiency by 71% compared to S4 when $\lambda_f = 0.0027$. This advantage will be more obvious with the further increase of λ_f.

VI. CONCLUSION

In this paper, we have designed a distributed cluster- based energy-efficient resource allocation scheme for UDN. This scheme has two stages, namely, clustering stage and resource allocation stage. In the clustering stage, we firstly separate BSs in the two-tier UDN into multiple BS-clusters based on a modified K-means algorithm, which is beneficial to reduce the computational complexity of power allocation. To mitigate the intra-cluster interference, we develop another clustering algorithm to group UEs in each BS-cluster into UE-groups with minimum intra-cluster interference. In the resource allocation stage, a two-step RB allocation algorithm and a game based iterative energy efficient power allocation algorithm have been proposed. System-level simulation results show that the proposed scheme can mitigate interference, meanwhile, it can improve the system throughput and energy efficiency efficiently, and this superiority is more obvious if the density of BSs is further increased in UDN.

APPENDIX

Theorem 2: Assuming the strategy space of power allocation is $a^l_{f,j} \le P^l_{f,j} \le b^l_{f,j}, \forall j \in \mathcal{FC}_k, l \in \mathcal{L}$, the Nash equilibrium exists and is unique in the game $\mathcal{G}_{\mathcal{FC}_k}$.

Proof: According to Nash theorem, if the following two conditions are satisfied, a Nash Equilibrium exists in the game $\mathcal{G}_{\mathcal{FC}_k}$.

1) $P^l_{f,j}$ is a nonempty, convex and compact subset in the Euclidean space.

2) $\mathcal{U}^l_{f,j}$ is continuous and quasi-concave in $P^l_{f,j}$.

As $P^l_{f,j} \in [a^l_{f,j}, b^l_{f,j}]$, condition 1 can be easily satisfied. For conditions 2, it is obvious that $\mathcal{U}^l_{f,j}$ is continuous with respect to $P^l_{f,j}$. So we only need to proof the quasi-concavity of $\mathcal{U}^l_{f,j}$ in $P^l_{f,j}$.

Note that

$$\frac{\partial \mathcal{U}^l_{f,j}(P^l_{f,j}, \mathbf{P}^l_{f,-j})}{\partial P^l_{f,j}} = \frac{SINR^l_{f,j}}{1 + SINR^l_{f,j}} \cdot \frac{W}{q_{f,j}\xi \ln 2}, \tag{24}$$

$$\frac{\partial^2 \mathcal{U}^l_{f,j}(P^l_{f,j}, \mathbf{P}^l_{f,-j})}{\partial (P^l_{f,j})^2} = -\frac{W}{\ln 2}\left(\frac{H^l_{f,j}}{1 + SINR^l_{f,j}}\right) < 0, \tag{25}$$

and $\partial^2 \mathcal{U}^l_{f,j}(P^l_{f,j}, \mathbf{P}^l_{f,-j})/\partial (P^l_{f,j})^2 < 0$, the utility function $\mathcal{U}^l_{f,j}(P^l_{f,j}, \mathbf{P}^l_{f,-j})$ is strictly concave with respect to $P^l_{f,j}$ with the given interference power. So, the Nash equilibrium of game $\mathcal{G}_{\mathcal{FC}_k}$ exists.

The optimal solution of $\mathcal{G}_{\mathcal{FC}_k}$ is $\arg\max \mathcal{U}^l_{f,j}(P^l_{f,j}, \mathbf{P}^l_{f,-j})$. For the continuous derivative function $\mathcal{U}^l_{f,j}$, the necessary condition for the first order derivative optimization is $\partial \mathcal{U}^l_{f,j}(P^l_{f,j}, \mathbf{P}^l_{f,-j})/\partial P^l_{f,j} = 0$. If $(P^l_{f,j})^*$ is local optimal in $[a^l_{f,j}, b^l_{f,j}]$, the Nash equilibrium can be obtained by

$$\frac{\partial \mathcal{U}^l_{f,j}(P^l_{f,j}, \mathbf{P}^l_{f,-j})}{\partial P^l_{f,j}} = 0$$
$$\Rightarrow (P^l_{f,j})^* = \frac{(SINR^l_{f,j})^*}{1 + (SINR^l_{f,j})^*} \cdot \frac{W}{q_{f,j}\xi \ln 2} \tag{26}$$

Due to the utility function $\mathcal{U}^l_{f,j}(P^l_{f,j}, \mathbf{P}^l_{f,-j})$ is strictly concave with respect to $P^l_{f,j}$, according to the sign of the derivative, it would monotonically increase or decrease with increasing $P^l_{f,j}$. If there is no local optimum in $[a^l_{f,j}, b^l_{f,j}]$, then the optimal value would be $b^l_{f,j}$, if $\partial \mathcal{U}^l_{f,j}(P^l_{f,j}, \mathbf{P}^l_{f,-j})/\partial P^l_{f,j} > 0$ in $[a^l_{f,j}, b^l_{f,j}]$ or $a^l_{f,j}$ otherwise. So the optimal solution of the game $\mathcal{G}_{\mathcal{FC}_k}$ is

$$(P^l_{f,j})^* = \begin{cases} a^l_{f,j}, & P^l_{f,j} \leq a^l_{f,j} \\ \dfrac{(SINR^l_{f,j})^*}{1+(SINR^l_{f,j})^*} \cdot \dfrac{W}{q_{f,j}\xi \ln 2}, & a^l_{f,j} \leq P^l_{f,j} \leq b^l_{f,j} \\ b^l_{f,j}, & P^l_{f,j} \geq b^l_{f,j}. \end{cases} \tag{27}$$

This completes the proof.

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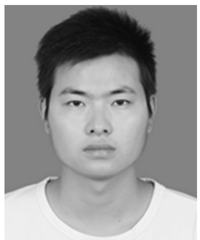

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