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Fault Tolerant Control for Uncertain Networked Control Systems With Induced Delays and Actuator Saturation

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ABSTRACT The fault tolerant control problem is investigated for a class of uncertain networked control systems (NCSs) subject to actuator faults and actuator saturation in this paper. System parameter uncertainties are considered and a sector condition method is employed to deal with the saturation problem. Based on the network transmission environment, the NCSs can be modeled as a class of saturated discrete-time systems with time-varying delays and actuator faults. By using Lyapunov-Krasovskii (L-K) stability theory, a sufficient condition for the fault tolerant controller design of NCSs is derived in the form of linear matrix inequalities. Finally, simulation results are given to demonstrate the effectiveness of the proposed method.

INDEX TERMS Networked control systems, induced delays, actuator saturation, actuator faults, linear matrix inequalities.

I. INTRODUCTION

Networked control systems are one type of distributed control systems whose feedback paths are interconnected through wired or wireless communication network [1], [2]. NCSs have received increasing attention in recent years because of many advantages, such as low cost, easy maintenance, simplicity in installation and so on [3]. As a result, NCSs are widely applied in manufacturing plants, aircraft, vehicles and remote surgery [4]. The analysis and design of NCSs are complicated because of the use of a communication network shared among different devices. Due to the non-ideal behavior of the network, network-induced delays are usually encountered. Similarly, due to the measurement error, unmodeled dynamics and some other factors, NCSs often suffer uncertainties [5]. On account of physical limitations, actuator saturation frequently exists. Meanwhile, NCSs usually work in some poor environment, actuator fault is inevitable. Thus, investigations on fault tolerant control for uncertain NCSs with induced delays and actuator saturation are necessary.

In NCSs, uncertainties usually lead to system instability. Uncertain NCSs have been widely investigated in recent years. In [6], the stabilization problem was studied for uncertain networked control system with random but bounded delay. The robust stability and stabilization problem was investigated for uncertain networked flight control system with random time delays in [7]. The problem of $H\infty$ output tracking control was discussed for networked control systems with random time delays and system uncertainties in [8]. Except for system uncertainties, networked-induced delay is one of the typical characteristics of NCSs and also needs to be discussed.

Since networked control systems are usually connected by communication network, network-induced delays always exist and can affect system performance. So far, various achievements on induced delays have been obtained. In [9], the delay-dependent $H\infty$ controller design was studied for a class of uncertain networked control systems. In [10], a new delay system approach was presented for network-based control which was suit for a new time delay model. The problem of designing guaranteed cost controller was studied for a class of uncertain time-delay network control systems in [11]. In [12], a design technique was investigated for the observer-based output feedback controller of continuous networked control systems with sensor delays. The stochastic stability and stochastic stabilization problem was discussed for time-varying delay discrete-time singular Markov jump systems in [13]. In general, NCSs with time delays were widely investigated. However, as mentioned before, saturation problem in NCSs also can not be ignored in the control analysis of NCSs.

In recent years, saturation problems have been extensively investigated in some existing works. In [14], the problem of networked control was considered for delta operator systems with actuator saturation. The quantized $H\infty$ filtering problem was investigated for NCSs with multiple influencing factors such as randomly occurring nonlinearities and sensor saturation in [15]. In [16], a neural network state observer-based adaptive saturation compensation control was proposed for a class of time-varying delayed nonlinear systems with input constraints. Even though some recent works had considered both induced delays and saturation, actuator faults problem was seldom taken into account in these works.

In practical applications, because of poor work environment and complicated structure of NCSs, actuator faults frequently occur and have bad influence on system performance. Thus, it is of great importance to explore a suitable controller to guarantee the performance of NCSs with actuator faults. Fault tolerant control problem has been a hot topic in the past decades. In [17], the stabilization problem was studied for uncertain networked control systems with stochastic actuator failures. A study of fault tolerant control was given for wireless networked control systems in industrial automatic processes in [18]. Quantized $H\infty$ fault tolerant control was concerned for NCSs with partial actuator faults in [19]. Generally speaking, it is of great necessity to study the fault tolerant controller design problem for uncertain networked control systems with induced delays and actuator saturation.

Motivated by the discussions above, we aim to solve the fault tolerant controller design problem for uncertain networked control systems with induced delays and actuator saturation. By introducing a sector condition, the saturation can be transformed into a particular form. The closed-loop model of NCSs with actuator faults is modeled as discrete-time systems with unknown parameters which are related to the boundary values of actuator faluts. By employing the L-K functional approach, sufficient conditions are established for asymptotical stability analysis and stabilization of NCSs with system uncertainty, induced delays, actuator faults and saturation. The main contributions of this paper lie in:

1) System parameter uncertainties, time-varying network-induced delays, time-varying actuator faults and actuator saturation are considered together which may increase the difficulty of analysis and control.
2) By using sector condition, the actuator saturation can be transformed into a linear part and a nonlinear part satisfying the sector condition. By introducing some particular matrices, the NCSs with time-varying actuator faults can be modeled as systems with uncertain parameters.
3) By employing an appropriate L-K functional and using the discrete Jensen inequality, a fault tolerant control scheme is obtained for the NCSs with system uncertainty, induced delays, actuator faults and saturation.

The organization of this paper is organized as follows. Section 2 details the problem formulation. Stability criteria and controller design of NCSs without actuator faults are obtained in Section 3. Fault tolerant controller is designed in section 4. Simulation studies are presented in Section 5 to demonstrate the the effectiveness of our approach.

NOTATIONS

Throughout the paper, superscript $(\bullet)^T$ stands for the matrix transposition, R^n and $R^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. The notations $P > 0 (P < 0)$ mean that P is a real symmetric positive (negative) definite matrix. I represents the identity matrix. $diag\{\ldots\}$ stands for a block-diagonal matrix. The expression $A > B$ means that matrix $A - B$ is positive definite. The symbol $*$ within a matrix represents the symmetric part.

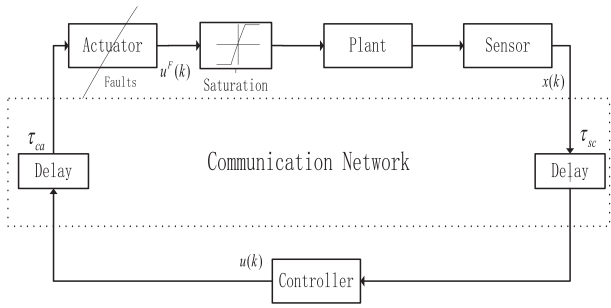

FIGURE 1. **NCSs model.**

II. PROBLEM FORMULATION

The structure of NCSs with actuator faults and actuator saturation is shown in Figure 1. The components of NCSs are connected by communication network. There exist induced delays in the communication links between sensor-to-controller and controller-to-actuator.

The discrete controlled plant of NCSs with actuator faults and actuator saturation shown in Figure 1 can be described as

$$x(k+1) = \overline{A}x(k) + \overline{B}sat(u^F(k)) \tag{1}$$

where $x(k) \in \Re^n$ is the state vector of the system and $u^F(k) \in \Re^m$ is the control input signal subject to actuator faults. $\overline{A}$ and $\overline{B}$ can be written as [5]

$$\overline{A} = A + \Delta A, \quad \overline{B} = B + \Delta B \tag{2}$$

where A and B are known constant matrices with appropriate dimensions; ΔA and ΔB are the time-varying parameter uncertainties which satisfy [5]

$$\begin{bmatrix} \Delta A & \Delta B \end{bmatrix} = D\Delta(k)\begin{bmatrix} E_1 & E_2 \end{bmatrix} \tag{3}$$

where D, E_1 and E_2 are known constant matrices and $\Delta(k)$ is a time-varying unknown matrix which satisfies $\Delta^T(k)\Delta(k) \leq I$.

The control input $u(k)$ is required to meet the saturation constraint and $sat(\cdot)$ is a saturation function defined as [20]

$$sat(u) = [sat(u_1),\ sat(u_2),\ \cdots,\ sat(u_m)]^T \tag{4}$$

with $sat(u_j) = sign(u_j)max\{u_{jmax}, \mid u_j \mid\}, j = 1, \ldots, m$ and a saturation boundary u_{jmax}.

As presented in Figure 1, network-induced delays exist in both sensor-to-controller and controller-to-actuator. The induced delay generated from sensor to controller is denoted as $\tau_{sc}(k)$ and satisfies $0 < \tau_{sc}(k) \leq \bar{\tau}_{sc}$. Similarly, $\tau_{ca}(k)$ is the induced delay generated from controller to actuator which satisfies $0 < \tau_{ca}(k) \leq \bar{\tau}_{ca}$. $\bar{\tau}_{sc}$ and $\bar{\tau}_{ca}$ are positive integers. The computational delay τ_c of sensor, controller and actuator can be ignored because it is small enough compared with the other two kinds of delays. To design the fault tolerant controller, the following assumptions are needed:

Assumption 1 [20]: The system data of networked control system (1) is transmitted with a single packet and full state variables are available for measurements.

Assumption 2 [21]: For the networked control system (1), the sensor is time-driven. The controller and actuator are event-driven.

Assumption 3: There exists stabilization controller for the networked control system (1) subject to actuator saturation and actuator faults.

According to Assumption 1 and Assumption 3, a static state-feedback controller can be designed. The control signal $u(k)$ can be formulated as

$$u(k) = Kx(k - \tau_{sc}) \tag{5}$$

where $K \in R^{m\times n}$ is the control gain matrix which needs to be designed.

According to Figure 1, in the presence of actuator faults and induced delay τ_{ca}, we have

$$u^F(k) = M(k)u(k - \tau_{ca}) \tag{6}$$

where $M(k) = diag\{m_1, m_2 \ldots, m_m\}$ is actuator failure matrix and $m_i = 1$ represents that the i^{th} actuator is running without failure; $m_i = 0$ means that the i^{th} actuator can not work; $0 < m_i < 1$ represents that the i^{th}actuator has partial failure. For the actuator failure matrix $m(k)$, we have the following definitions.

Definition 1 [21]: The upper bound of fault matrix is defined as

$$m_u = diag\{m_{u1}, m_{u2}, \ldots m_{um}\}, 0 < m_{ui} \leq 1$$

while the lower bound of fault matrix is given by

$$m_l = diag\{m_{l1}, m_{l2}, \ldots, m_{lm}\}, 0 \leq m_{li} < 1$$

That is to say, $M(k) \in [m_l, m_u]$ which is time-varying. We define $M_0 = diag\{m_{01}, m_{02} \ldots, m_{0m}\}$, $m_{0i} = \frac{m_{ui}+m_{li}}{2}$. Furthermore, the following matrix is introduced:

$$G(k) = diag\{g_1(k), g_2(k), \ldots g_m(k)\}$$

with

$$g_i(k) = \frac{m_i(k) - m_{0i}}{m_{0i}}$$

Obviously, we have

$$\begin{aligned} -1 \leq \frac{m_{li} - m_{0i}}{m_{0i}} &\leq g_i(k) = \frac{m_i(k) - m_{0i}}{m_{0i}} \\ &\leq \frac{m_{ui} - m_{0i}}{m_{0i}} = \frac{m_{ui} - m_{li}}{m_{ui} + m_{li}} \leq 1 \end{aligned} \tag{7}$$

Based on (7), we have $-I \leq G(k) \leq I$. Naturally, we obtain

$$M(k) = M_0(I + G(k)) \tag{8}$$

According to (5) and (6), the two parts of networked-induced delays can be lumped together as $\tau(k) = \tau_{ca} + \tau_{sc}$ for analysis purpose, and then the fault tolerant controller is expressed as

$$u^F(k) = M(k)Kx(k - \tau(k)) \tag{9}$$

where $\tau(k) = \tau_{sc}(k) + \tau_{ca}$ which satisfies $0 < \tau(k) \leq \bar{\tau}$, $\bar{\tau} = \bar{\tau}_{sc} + \bar{\tau}_{ca}$. For simplicity, we use M represents for $M(k)$ in the later analysis. Then, the closed-loop system of NCSs can be written as

$$x(k + 1) = \bar{A}(k) + \bar{B}sat(MKx(k - \tau(k))) \tag{10}$$

To develop our results, the actuator saturation can be transformed into a particular form, and the following definition is needed to deal with the saturation problem.

Definition 2 [22]: A nonlinearity $\Psi(\bullet)$ is said to satisfy a sector condition if

$$(\Psi(v) - H_1 v)^T (\Psi(v) - H_2 v) \leq 0. \forall v \in R^r$$

for real matrices $H_1, H_2 \in R^{r\times r}$, where $H = H_2 - H_1$ is a positive definite symmetric matrix. In this case, we say that Ψ belongs to the sector $\left[H_1 \ H_2 \right]$.

As shown in [23] and [24], we assume that there exist diagonal matrices F_1 and F_2, such that $0 \leq F_1 \leq I \leq F_2$, then the saturation function $sat(u^F(k))$ can be expressed as

$$sat(u^F(k)) = F_1 u^F(k) + \Psi(u^F(k)) \tag{11}$$

where $F_1 u^F(k)$ is a linear function and $\Psi(u^F(k))$ is a non-linear function satisfying the sector condition with $H_1 = 0$, $H_2 = F$, which can be described as follow [23] [24]:

$$\Psi^T(u^F(k))(\Psi(u^F(k)) - Fu^F(k)) \leq 0 \tag{12}$$

where $F = F_2 - F_1$. As mentioned before, the closed-loop system can be written as

$$x(k + 1) = \overline{A}x(k) + \overline{B}F_1 MKx(k - \tau(k)) + \overline{B}\Psi(u^F(k)) \tag{13}$$

Before presenting the main results of this paper, we need to introduce the following lemmas which are crucial to develope our main results.

Lemma 1 [25]: For a given matrix $Z = Z^T > 0 \in R^{n\times n}$, two positive integers γ_1, γ_2 satisfying $\gamma_2 > \gamma_1 \geq 1$ and a vector function $\omega : [\gamma_1, \gamma_2] \rightarrow R^n$, then the following inequality hold:

$$\begin{aligned} &-(\gamma_2 - \gamma_1 + 1) \sum_{i=\gamma_1}^{\gamma_2} \omega^T(i) Z \omega(i) \\ &\leq -\left[\sum_{i=\gamma_1}^{\gamma_2} \omega(i) \right]^T Z \left[\sum_{i=\gamma_1}^{\gamma_2} \omega(i) \right] \end{aligned}$$

Lemma 2 [26]: Given the symmetric matrix $\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12}^T & \Theta_{22} \end{bmatrix}$, the following statements are equivalent:

1. $\Theta < 0$
2. $\Theta_{11} < 0, \Theta_{22} - \Theta_{12}^T \Theta_{11}^{-1} \Theta_{12} < 0$
3. $\Theta_{22} < 0, \Theta_{11} - \Theta_{12} \Theta_{22}^{-1} \Theta_{12}^T < 0$

Lemma 3 [27]: Given matrices Γ, Λ and symmetric matrix Υ with appropriate dimensions, then

$$\Upsilon + \Gamma F(k) \Lambda + \Lambda^T F^T(k) \Gamma^T < 0$$

holds for any $F^T(k)F(k) \leq I$, if and only if there exists a scalar $\alpha > 0$ such that

$$\Upsilon + \alpha \Gamma \Gamma^T + \alpha^{-1} \Lambda^T \Lambda < 0$$

III. STABILITY ANALYSIS AND CONTROLLER DESIGN WITHOUT ACTUATOR FAULTS

In this section, for the sake of designing the fault tolerant control scheme, we firstly assume that there are no actuator faults. A sufficient condition for the asymptotical stability of uncertain networked systems (1) and the control law without faults are developed.

According to the previous statements, no actuator faults happen means that $M(k) = I$ and $u^F(k)$ is replaced by $u(k)$. Thus, the closed-loop system can be written as follows:

$$x(k+1) = \bar{A}x(k) + \bar{B}F_1 Kx(k - \tau(k)) + \bar{B}\Psi(u(k)) \quad (14)$$

Then, the asymptotical stability criteria is given in Theorem 1.

Theorem 1: The closed-loop system (14) with bounded induced delay $\tau(k) \in (0, \bar{\tau}]$ is asymptotically stable if there exist symmetric positive definite matrices P, Q, S and matrices L_1, L_2, L_3, K with appropriate dimensions satisfying

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} \\ * & * & \Omega_{33} & \Omega_{34} & \Omega_{35} \\ * & * & * & \Omega_{44} & \Omega_{45} \\ * & * & * & * & \Omega_{55} \end{bmatrix} < 0 \quad (15)$$

where

$$\begin{aligned}
\Omega_{11} &= L_1^T(\bar{A} - I) + (\bar{A} - I)^T L_1 + Q - S, \\
\Omega_{12} &= L_1^T \bar{B} F_1 K + (\bar{A} - I)^T L_2, \ \Omega_{13} = S, \\
\Omega_{14} &= L_1^T \bar{B}, \ \Omega_{15} = -L_1^T + (\bar{A} - I)^T L_3 + P, \\
\Omega_{22} &= L_2^T \bar{B} F_1 K + (\bar{B} F_1 K)^T L_2, \ \Omega_{23} = 0, \\
\Omega_{24} &= \frac{1}{2} K^T F^T + L_2^T \bar{B}, \\
\Omega_{33} &= -S - Q, \ \Omega_{25} = -L_2^T + (\bar{B} F_1 K)^T L_3, \\
\Omega_{34} &= 0, \ \Omega_{35} = 0, \ \Omega_{44} = -I, \ \Omega_{45} = \bar{B}^T L_3, \\
\Omega_{55} &= P + \bar{\tau}^2 S - L_3^T - L_3.
\end{aligned}$$

Proof: Let $y(k) = x(k+1) - x(k)$. From (14), we obtain

$$\begin{aligned}
&(\bar{A} - I)x(k) + \bar{B}F_1 Kx(k - \tau(k)) \\
&\quad + \bar{B}\Psi(u(k)) - y(k) = 0 \quad (16)
\end{aligned}$$

Construct a Lyapunov-Krasovskii functional candidate as

$$V(k) = V_1(k) + V_2(k) + V_3(k)$$

with

$$\begin{aligned}
V_1(k) &= x^T(k) P x(k) \\
V_2(k) &= \sum_{i=k-\bar{\tau}}^{k-1} x^T(i) Q x(i) \\
V_3(k) &= \sum_{i=-\bar{\tau}}^{-1} \sum_{j=k+i}^{k-1} \bar{\tau} y^T(i) S y(i) \quad (17)
\end{aligned}$$

The forward difference of $V_i(k)$, $i = 1, 2, 3$ can be written as follows:

$$\begin{aligned}
\Delta V_1(k) &= 2x^T(k) P y(k) + y^T(k) P y(k) \\
\Delta V_2(k) &= x^T(k) Q x(k) \\
&\quad - x^T(k - \bar{\tau}) Q x(k - \bar{\tau}) \\
\Delta V_3(k) &= V_3(k+1) - V_3(k) \\
&= \sum_{i=-\bar{\tau}}^{-1} \sum_{j=k+1+i}^{k} \bar{\tau} y^T(i) S y(i) \\
&\quad - \sum_{i=-\bar{\tau}}^{-1} \sum_{j=k+i}^{k-1} \bar{\tau} y^T(i) S y(i) \quad (18)
\end{aligned}$$

According to Lemma 1, we obtain

$$\begin{aligned}
\Delta V_3(k) &\leq \bar{\tau}^2 y^T(k) S y(k) \\
&\quad + \begin{bmatrix} x(k) \\ x(k - \bar{\tau}) \end{bmatrix}^T \bar{\Gamma} \begin{bmatrix} x(k) \\ x(k - \bar{\tau}) \end{bmatrix} \quad (19)
\end{aligned}$$

where

$$\bar{\Gamma} = \begin{bmatrix} -S & S \\ * & -S \end{bmatrix}$$

From (16), the following equation is true for matrices L_1, L_2, L_3 with appropriate dimensions:

$$\begin{aligned}
&2(x^T(k) L_1^T + x^T(k - \tau(k)) L_2^T + y^T(k) L_3^T) \\
&\quad \times \big[(\bar{A} - I)x(k) + \bar{B}F_1 Kx(k - \tau(k)) \\
&\quad + \bar{B}\Psi(u(k)) - y(k)\big] = 0 \quad (20)
\end{aligned}$$

Considering $M(k) = I$, $u(k) = Kx(k - \tau(k))$, and (12), we have

$$\Psi^T(u(k)) [\Psi(u(k)) - FKx(k - \tau(k))] \leq 0 \quad (21)$$

Then, we obtain

$$\begin{bmatrix} x(k - \tau(k)) \\ \Psi(u(k)) \end{bmatrix}^T \Xi \begin{bmatrix} x(k - \tau(k)) \\ \Psi(u(k)) \end{bmatrix} \geq 0 \quad (22)$$

where

$$\Xi = \begin{bmatrix} 0 & \frac{1}{2} K^T F^T \\ * & -I \end{bmatrix}$$

Denote the augmented state $\xi(k)$ as

$$\xi(k) = \begin{bmatrix} x(k) \\ x(k - \tau(k)) \\ x(k - \bar{\tau}) \\ \Psi(u(k)) \\ y(k) \end{bmatrix} \quad (23)$$

Combining (18), (19), (20) and (22), we can obtain that

$$\Delta V(k) \le \xi^T(k)\Omega\xi(k) \tag{24}$$

From the condition (15), we can conclude $\Delta V(k) < 0$, then the proof is completed.

Theorem 1 gives the sufficient condition for the stability of closed-loop system (14). However, it should be noted that the inequality in Theorem 1 is not LMI and not easy to deal with. Hence, in the next development, we present a LMI-based sufficient condition on the basis of Theorem 1 to design a state-feedback controller for the uncertain NCSs with induced delays and actuator saturation which is included in Theorem 2.

Theorem 2: For the given scalars $\bar{\tau} > 0, \omega_1, \omega_2$ and given matrices F, F_1, if there exist $\beta > 0$ and positive definite matrices $\hat{P}, \hat{Q}, \hat{S}$ and matrices $\hat{L}, Y$ with appropriate dimensions such that the following inequality holds:

$$\hat{\Omega} = \begin{bmatrix} \hat{\Omega}_{11} & \hat{\Omega}_{12} & \hat{\Omega}_{13} & \hat{\Omega}_{14} & \hat{\Omega}_{15} & \hat{\Omega}_{16} \\ * & \hat{\Omega}_{22} & \hat{\Omega}_{23} & \hat{\Omega}_{24} & \hat{\Omega}_{25} & \hat{\Omega}_{26} \\ * & * & \hat{\Omega}_{33} & \hat{\Omega}_{34} & \hat{\Omega}_{35} & \hat{\Omega}_{36} \\ * & * & * & \hat{\Omega}_{44} & \hat{\Omega}_{45} & \hat{\Omega}_{46} \\ * & * & * & * & \hat{\Omega}_{55} & \hat{\Omega}_{56} \\ * & * & * & * & * & \hat{\Omega}_{66} \end{bmatrix} < 0 \tag{25}$$

then the closed-loop system (14) is asymptotically stable and the controller gain is given as $K = \hat{L}Y^{-1}$, where

$$\begin{aligned}
\hat{\Omega}_{11} &= \omega_1(A-I)\hat{L} + \omega_1\hat{L}^T(A-I)^T + \hat{Q} - \hat{S} \\
&\quad + \beta\omega_1^2 DD^T, \\
\hat{\Omega}_{12} &= \omega_1 BF_1Y + \omega_2\hat{L}^T(A-I)^T + \beta\omega_1\omega_2 DD^T, \\
\hat{\Omega}_{13} &= \hat{S}, \hat{\Omega}_{14} = \omega_1 B, \\
\hat{\Omega}_{15} &= -\omega_1\hat{L}^T + \hat{L}^T(A-I) + \hat{P} + \beta\omega_1 DD^T, \\
\hat{\Omega}_{16} &= \hat{L}^T E_1^T \\
\hat{\Omega}_{22} &= \omega_2 BF_1Y + \omega_2 Y^T F_1^T B^T + \beta\omega_2^2 DD^T, \\
\hat{\Omega}_{23} &= 0, \hat{\Omega}_{24} = \frac{1}{2}Y^T F^T + \omega_2 B, \\
\hat{\Omega}_{25} &= -\omega_2\hat{L} + Y^T F_1^T B^T + \beta\omega_2 DD^T, \\
\hat{\Omega}_{26} &= \hat{L}^T E_2^T, \\
\hat{\Omega}_{33} &= -\hat{S} - \hat{Q}, \hat{\Omega}_{34} = \hat{\Omega}_{35} = \hat{\Omega}_{36} = 0, \\
\hat{\Omega}_{44} &= -I, \hat{\Omega}_{45} = B^T, \hat{\Omega}_{46} = E_2^T, \\
\hat{\Omega}_{55} &= \hat{P} + \bar{\tau}^2\hat{S} - \hat{L} - \hat{L}^T + \beta DD^T, \hat{\Omega}_{56} = 0, \\
\hat{\Omega}_{66} &= -\beta I.
\end{aligned}$$

Proof: Let $\hat{L} = L_3^{-1}$, $L_1 = \omega_1 L_3 = \omega_1\hat{L}^{-1}$, $L_2 = \omega_2 L_3 = \omega_2\hat{L}^{-1}$, $\hat{P} = \hat{L}^T P\hat{L}$, $\hat{Q} = \hat{L}^T Q\hat{L}$, $\hat{S} = \hat{L}^T S\hat{L}$, $Y = K\hat{L}$ and ω_1, ω_2 are given parameters. Then pre- and post- multiply (15) by $diag\{\hat{L}^T\ \hat{L}^T\ \hat{L}^T\ I\ \hat{L}^T\}$ and $diag\{\hat{L}\ \hat{L}\ \hat{L}\ I\ \hat{L}\}$, we have

$$\bar{\Omega} = \begin{bmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13} & \bar{\Omega}_{14} & \bar{\Omega}_{15} \\ * & \bar{\Omega}_{22} & \bar{\Omega}_{23} & \bar{\Omega}_{24} & \bar{\Omega}_{25} \\ * & * & \bar{\Omega}_{33} & \bar{\Omega}_{34} & \bar{\Omega}_{35} \\ * & * & * & \bar{\Omega}_{44} & \bar{\Omega}_{45} \\ * & * & * & * & \bar{\Omega}_{55} \end{bmatrix} < 0 \tag{26}$$

where

$$\begin{aligned}
\bar{\Omega}_{11} &= \omega_1(\bar{A}-I)\hat{L} + \omega_1\hat{L}^T(\bar{A}-I)^T + \hat{Q} - \hat{S}, \\
\bar{\Omega}_{12} &= \omega_1\bar{B}F_1Y + \omega_2\hat{L}^T(\bar{A}-I)^T, \bar{\Omega}_{13} = \hat{S}, \\
\bar{\Omega}_{14} &= \omega_1\bar{B}, \bar{\Omega}_{15} = -\omega_1\hat{L} + \hat{L}^T(\bar{A}-I) + \hat{P}, \\
\bar{\Omega}_{22} &= \omega_2\bar{B}F_1Y + \omega_2 Y^T F_1^T\bar{B}^T, \bar{\Omega}_{23} = 0, \\
\bar{\Omega}_{24} &= \frac{1}{2}Y^T F^T + \omega_2\bar{B}, \bar{\Omega}_{25} = -\omega_2\hat{L} + Y^T F_1^T\bar{B}^T, \\
\bar{\Omega}_{33} &= -\hat{S} - \hat{Q}, \bar{\Omega}_{34} = \bar{\Omega}_{35} = 0, \\
\bar{\Omega}_{44} &= -I, \hat{\Omega}_{45} = \bar{B}^T, \bar{\Omega}_{55} = \hat{P} + \bar{\tau}^2\hat{S} - \hat{L} - \hat{L}^T.
\end{aligned}$$

According to (2) and (3), (26) can be rewritten as

$$\begin{aligned}
\bar{\Omega} = \tilde{\Omega} &+ \begin{bmatrix} \omega_1 D \\ \omega_2 D \\ 0 \\ 0 \\ D \end{bmatrix} \Delta(k) \begin{bmatrix} E_1\hat{L} & E_3\hat{L} & 0 & E_2 & 0 \end{bmatrix} \\
&+ \begin{bmatrix} \hat{L}^T E_1^T \\ \hat{L}^T E_2^T \\ 0 \\ E_2^T \\ 0 \end{bmatrix} \Delta^T(k) \begin{bmatrix} \omega_1 D^T & \omega_2 D^T & 0 & 0 & D^T \end{bmatrix} < 0
\end{aligned} \tag{27}$$

where

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & \tilde{\Omega}_{14} & \tilde{\Omega}_{15} \\ * & \tilde{\Omega}_{22} & \tilde{\Omega}_{23} & \tilde{\Omega}_{24} & \tilde{\Omega}_{25} \\ * & * & \tilde{\Omega}_{33} & \tilde{\Omega}_{34} & \tilde{\Omega}_{35} \\ * & * & * & \tilde{\Omega}_{44} & \tilde{\Omega}_{45} \\ * & * & * & * & \tilde{\Omega}_{55} \end{bmatrix}$$

$$\begin{aligned}
\tilde{\Omega}_{11} &= \omega_1(A-I)\hat{L} + \omega_1\hat{L}^T(A-I)^T + \hat{Q} - \hat{S}, \\
\tilde{\Omega}_{12} &= \omega_1 BF_1Y + \omega_2\hat{L}^T(A-I)^T, \tilde{\Omega}_{13} = \hat{S}, \\
\tilde{\Omega}_{14} &= \omega_1 B, \tilde{\Omega}_{15} = -\omega_1\hat{L} + \hat{L}^T(A-I) + \hat{P}, \\
\tilde{\Omega}_{22} &= \omega_2 BF_1Y + \omega_2 Y^T F_1^T B^T, \tilde{\Omega}_{23} = 0, \\
\tilde{\Omega}_{24} &= \frac{1}{2}Y^T F^T + \omega_2 B, \tilde{\Omega}_{25} = -\omega_2\hat{L} + Y^T F_1^T B^T, \\
\tilde{\Omega}_{33} &= -\hat{S} - \hat{Q}, \tilde{\Omega}_{34} = \tilde{\Omega}_{35} = 0, \\
\tilde{\Omega}_{44} &= -I, \tilde{\Omega}_{45} = B^T, \tilde{\Omega}_{55} = \hat{P} + \bar{\tau}^2\hat{S} - \hat{L} - \hat{L}^T.
\end{aligned} \tag{28}$$

From Lemma 3, if there exists a scalar $\beta > 0$ such that the following inequality (29) is true, then (27) holds.

$$\bar{\Omega} = \tilde{\Omega} + \beta \begin{bmatrix} \omega_1 D \\ \omega_2 D \\ 0 \\ 0 \\ D \end{bmatrix} \begin{bmatrix} \omega_1 D^T & \omega_1 D^T & 0 & 0 & D^T \end{bmatrix} + \beta^{-1} \begin{bmatrix} \hat{L}^T E_1^T \\ \hat{L}^T E_2^T \\ 0 \\ E_2^T \\ 0 \end{bmatrix} \begin{bmatrix} E_1\hat{L} & E_2\hat{L} & 0 & E_2 & 0 \end{bmatrix} < 0 \tag{29}$$

According to Lemma 2, if (25) is true, (29) is held. Then, $\Delta V(k) < 0$. Thus, the proof is completed.

Remark 1: In [28] and [29], one part of $\Delta V(k)$ was designed as $V_i(k) = \sum_{i=-\bar{\tau}}^{-1} \sum_{j=k+i}^{k-1} y^T(i)Sy(i)$. During the calculation of $\Delta V_i(k)$, a cross term like $-\sum_{i=k-\bar{\tau}}^{k-1} y^T(i)Sy(i)$ was produced. In the literatures, this term was simply enlarged to $-\sum_{i=k-\tau(k)}^{k-1} y^T(i)Sy(i)$, which may lead to considerable conservativeness. In this paper, we choose $V_i(k) = \sum_{i=-\bar{\tau}}^{-1} \sum_{j=k+i}^{k-1} y^T(i)\bar{\tau}Sy(i)$, which can lead to a more reasonable result.

Remark 2: If (15) holds, the sub-matrices of its main diagonal must be negative definite according to matrix theory. Thus, L_3 is non-singular which indicates that L_3 is invertible and assuming $\hat{L} = L_3^{-1}$ is reasonable.

IV. FAULT TOLERANT CONTROLLER DESIGN

In the previous section, sufficient conditions for the stability analysis and controller design for NCSs without faults are obtained. In this section, we will present a LMI-based sufficient condition on the basis of Theorem 1 and Theorem 2 to design a fault tolerant state-feedback controller for the uncertain NCSs with induced delays and actuator saturation.

By comparison to (14), taking time-varying actuator faults into consideration, the closed-loop system can be expressed as (13). Similar with the analysis in Theorem 1, we use $\bar{B}F_1MK$ to replace $\bar{B}F_1K$ and can easily obtained that if the following inequality is true, the closed-loop system (13) is asymptotically stable.

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} \\ * & * & \Pi_{33} & \Pi_{34} & \Pi_{35} \\ * & * & * & \Pi_{44} & \Pi_{45} \\ * & * & * & * & \Pi_{55} \end{bmatrix} < 0 \tag{30}$$

where

$$\begin{aligned}
\Pi_{11} &= L_1^T(\bar{A} - I) + (\bar{A} - I)^T L_1 + Q - S, \\
\Pi_{12} &= L_1^T \bar{B}F_1MK + (\bar{A} - I)^T L_2, \ \Pi_{13} = S, \\
\Pi_{14} &= L_1^T \bar{B}, \ \Pi_{15} = -L_1^T + (\bar{A} - I)^T L_3 + P, \\
\Pi_{22} &= L_2^T \bar{B}F_1MK + (\bar{B}F_1MK)^T L_2, \ \Pi_{23} = 0, \\
\Pi_{24} &= \frac{1}{2}K^T M^T F^T + L_2^T \bar{B}, \\
\Pi_{25} &= -L_2^T + (\bar{B}F_1MK)^T L_3, \\
\Pi_{33} &= -S - Q, \ \Pi_{34} = 0, \ \Pi_{35} = 0, \\
\Pi_{44} &= -I, \ \Pi_{45} = \bar{B}^T L_3, \\
\Pi_{55} &= P + \bar{\tau}^2 S - L_3^T - L_3.
\end{aligned}$$

Obviously, (30) is also not LMI and not easy to deal with. Thus, we need to present a LMI-based criteria to design a fault tolerant state-feedback controller for the uncertain NCSs with induced delays and actuator saturation. The main result of the fault tolerant controller design of NCSs can be summarized as the following theorem.

Theorem 3: For the given scalars $\bar{\tau} > 0$, $\varepsilon_1, \varepsilon_2$ and given matrices F, F_1 if there exist $\alpha > 0$, $\lambda > 0$ and positive definite matrices $\tilde{P}, \tilde{Q}, \tilde{S}$ and matrices $\tilde{L}, Z$ with appropriate dimensions such that the following inequality hold:

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Phi}_{11} & \tilde{\Phi}_{12} & \tilde{\Phi}_{13} & \tilde{\Phi}_{14} & \tilde{\Phi}_{15} & \tilde{\Phi}_{16} & \tilde{\Phi}_{17} & \tilde{\Phi}_{18} \\ * & \tilde{\Phi}_{22} & \tilde{\Phi}_{23} & \tilde{\Phi}_{24} & \tilde{\Phi}_{25} & \tilde{\Phi}_{26} & \tilde{\Phi}_{27} & \tilde{\Phi}_{28} \\ * & * & \tilde{\Phi}_{33} & \tilde{\Phi}_{34} & \tilde{\Phi}_{35} & \tilde{\Phi}_{36} & \tilde{\Phi}_{37} & \tilde{\Phi}_{38} \\ * & * & * & \tilde{\Phi}_{44} & \tilde{\Phi}_{45} & \tilde{\Phi}_{46} & \tilde{\Phi}_{47} & \tilde{\Phi}_{48} \\ * & * & * & * & \tilde{\Phi}_{55} & \tilde{\Phi}_{56} & \tilde{\Phi}_{57} & \tilde{\Phi}_{58} \\ * & * & * & * & * & \tilde{\Phi}_{66} & \tilde{\Phi}_{67} & \tilde{\Phi}_{68} \\ * & * & * & * & * & * & \tilde{\Phi}_{77} & \tilde{\Phi}_{78} \\ * & * & * & * & * & * & * & \tilde{\Phi}_{88} \end{bmatrix} < 0 \tag{31}$$

then the NCSs (1) with actuator faults can be stabilized by fault tolerant controller $u(k) = Kx(k - \tau(k))$ with gain $K = Z\tilde{L}^{-1}$, where

$$\begin{aligned}
\tilde{\Phi}_{11} &= \varepsilon_1(A - I)\tilde{L} + \varepsilon_1\tilde{L}^T(A - I)^T \\
&\quad + \tilde{Q} - \tilde{S} + \lambda\varepsilon_1^2 DD^T, \\
\tilde{\Phi}_{12} &= \varepsilon_1 BF_1M_0Z + \varepsilon_2\tilde{L}^T(A - I)^T + \lambda\varepsilon_1\varepsilon_2 DD^T, \\
\tilde{\Phi}_{13} &= -\varepsilon_1\tilde{L} + \tilde{L}^T(A - I)^T + \tilde{P} + \lambda\varepsilon_1 DD^T, \\
\tilde{\Phi}_{16} &= 0, \ \tilde{\Phi}_{17} = \alpha\varepsilon_1 BF_1M_0, \ \tilde{\Phi}_{18} = (E_1\tilde{L})^T, \\
\tilde{\Phi}_{22} &= \varepsilon_2 BF_1M_0Z + \varepsilon_2(BF_1M_0Z)^T + \lambda\varepsilon_2^2 DD^T, \\
\tilde{\Phi}_{23} &= 0, \ \tilde{\Phi}_{24} = \frac{1}{2}Z^T M_0^T F^T + \varepsilon_2 B, \\
\tilde{\Phi}_{25} &= -\varepsilon_2\tilde{L} + Z^T F_1^T M_0^T B^T + \lambda\varepsilon_2 DD^T, \\
\tilde{\Phi}_{26} &= Z^T, \ \tilde{\Phi}_{27} = \alpha\varepsilon_2 BF_1M_0, \ \tilde{\Phi}_{28} = (E_2F_1M_0Z)^T, \\
\tilde{\Phi}_{33} &= -\tilde{S} - \tilde{Q}, \ \tilde{\Phi}_{34} = \tilde{\Phi}_{35} = \tilde{\Phi}_{36} = \tilde{\Phi}_{37} = \tilde{\Phi}_{38} = 0, \\
\tilde{\Phi}_{44} &= -I, \ \tilde{\Phi}_{45} = B^T, \ \tilde{\Phi}_{46} = 0, \ \tilde{\Phi}_{47} = \frac{1}{2}\alpha FM_0, \\
\tilde{\Phi}_{48} &= E_2^T, \ \tilde{\Phi}_{55} = \tilde{P} + \bar{\tau}^2\tilde{S} - \tilde{L} - \tilde{L}^T + \lambda DD^T, \\
\tilde{\Phi}_{56} &= 0, \ \tilde{\Phi}_{57} = 0, \ \tilde{\Phi}_{58} = 0, \ \tilde{\Phi}_{66} = -\alpha I, \ \tilde{\Phi}_{67} = 0, \\
\tilde{\Phi}_{68} &= 0, \ \tilde{\Phi}_{77} = -\alpha I, \ \tilde{\Phi}_{78} = (\alpha E_2F_1M_0)^T, \\
\tilde{\Phi}_{88} &= -\lambda I.
\end{aligned}$$

$$\Pi = \bar{\Pi} + \begin{bmatrix} 0 & L_1^T\bar{B}F_1M_0G(k)K & 0 & 0 & 0 \\ * & C & 0 & \frac{1}{2}K^TG^T(k){M_0}^TF^T & (\bar{B}F_1M_0G(k)K)^TL_3 \\ * & * & 0 & 0 & 0 \\ & & & 0 & 0 \\ & & & * & 0 \end{bmatrix}$$

$$= \Pi + \begin{bmatrix} 0 \\ K^T \\ 0 \\ 0 \\ 0 \end{bmatrix} G^T(k)$$

$$\begin{bmatrix} M_0^TF_1^T\bar{B}^TL_1 & M_0^TF_1^T\bar{B}^TL_2 & 0 & \frac{1}{2}M_0^TF^T & M_0^TF_1^T\bar{B}^TL_3 \end{bmatrix} + \begin{bmatrix} L_1^T\bar{B}F_1M_0 \\ L_2^T\bar{B}F_1M_0 \\ 0 \\ \frac{1}{2}FM_0 \\ L_3^T\bar{B}F_1M_0 \end{bmatrix} G(k)\begin{bmatrix} 0 & K & 0 & 0 & 0 \end{bmatrix}$$

Proof: According to (8) and (30), we have matrix Π, as shown at the top of this page, where

$$\bar{\Pi} = \begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13} & \bar{\Pi}_{14} & \bar{\Pi}_{15} \\ * & \bar{\Pi}_{22} & \bar{\Pi}_{23} & \bar{\Pi}_{24} & \bar{\Pi}_{25} \\ * & * & \bar{\Pi}_{33} & \bar{\Pi}_{34} & \bar{\Pi}_{35} \\ * & * & * & \bar{\Pi}_{44} & \bar{\Pi}_{45} \\ * & * & * & * & \bar{\Pi}_{55} \end{bmatrix}$$

$$\bar{\Pi}_{11} = L_1^T(\bar{A}-I) + (\bar{A}-I)^TL_1 + Q - S,$$
$$\bar{\Pi}_{12} = L_1^T\bar{B}F_1M_0K + (\bar{A}-I)^TL_2,\ \Pi_{13} = S,$$
$$\bar{\Pi}_{14} = L_1^T\bar{B},\ \Pi_{15} = -L_1^T + (\bar{A}-I)^TL_3 + P,$$
$$\bar{\Pi}_{22} = L_2^T\bar{B}F_1M_0K + (\bar{B}F_1M_0K)^TL_2,\ \bar{\Pi}_{23} = 0,$$
$$\bar{\Pi}_{24} = \frac{1}{2}K^T{M_0}^TF^T + L_2^T\bar{B},$$
$$\bar{\Pi}_{25} = -L_2^T + (\bar{B}F_1M_0K)^TL_3,\ \bar{\Pi}_{33} = -S - Q,$$
$$\bar{\Pi}_{34} = 0,\ \bar{\Pi}_{35} = 0,\ \bar{\Pi}_{44} = -I,\ \bar{\Pi}_{45} = \bar{B}^TL_3,$$
$$\bar{\Pi}_{55} = P + \bar{\tau}^2S - L_3^T - L_3,$$
$$C = L_2^T\bar{B}F_1M_0G(k)K + (\bar{B}F_1M_0G(k)K)^TL_2.$$

Considering $G^T(k)G(k) \leq I$ and by using Lemma 1 and Lemma 3, we have $\Pi < 0$ if there exists $\alpha > 0$ such that the following inequality holds:

$$\tilde{\Pi} = \begin{bmatrix} \tilde{\Pi}_{11} & \tilde{\Pi}_{12} & \tilde{\Pi}_{13} & \tilde{\Pi}_{14} & \tilde{\Pi}_{15} & 0 & L_1^T\bar{B}F_1M_0 \\ * & \tilde{\Pi}_{22} & \tilde{\Pi}_{23} & \tilde{\Pi}_{24} & \tilde{\Pi}_{25} & K^T & L_2^T\bar{B}F_1M_0 \\ * & * & \tilde{\Pi}_{33} & \tilde{\Pi}_{34} & \tilde{\Pi}_{35} & 0 & 0 \\ * & * & * & \tilde{\Pi}_{44} & \tilde{\Pi}_{45} & 0 & \frac{1}{2}FM_0 \\ * & * & * & * & \tilde{\Pi}_{55} & 0 & L_3^T\bar{B}F_1M_0 \\ * & * & * & * & * & -\alpha I & 0 \\ * & * & * & * & * & * & -\alpha^{-1}I \end{bmatrix} < 0$$

where $\tilde{\Pi}_{ij} = \bar{\Pi}_{ij}, i, j = 1, 2, \ldots, 5$. Let $\tilde{L} = L_3^{-1}$, $L_1 = \varepsilon_1L_3 = \varepsilon_1\tilde{L}^{-1}$, $L_2 = \varepsilon_2L_3 = \varepsilon_2\tilde{L}^{-1}$, $\tilde{P} = \tilde{L}^TP\tilde{L}, \tilde{Q} = \tilde{L}^TQ\tilde{L}, \tilde{S} = \tilde{L}^TS\tilde{L}, Z = K\tilde{L}$ and $\varepsilon_1, \varepsilon_2$ are given parameters. Then, pre- and post-multiplying $\tilde{\Pi}$ by $diag\{\hat{L}^T\ \hat{L}^T\ \hat{L}^T\ I\ \hat{L}^T\ I\ \alpha I\}$ and $diag\{\hat{L}\ \hat{L}\ \hat{L}\ I\ \hat{L}\ I\ \alpha I\}$, we have

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & \Phi_{17} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} & \Phi_{27} \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} & \Phi_{37} \\ * & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46} & \Phi_{47} \\ * & * & * & * & \Phi_{55} & \Phi_{56} & \Phi_{57} \\ * & * & * & * & * & \Phi_{66} & \Phi_{67} \\ * & * & * & * & * & * & \Phi_{77} \end{bmatrix} < 0$$

where

$$\Phi_{11} = \varepsilon_1(\bar{A}-I)\tilde{L} + \varepsilon_1\tilde{L}^T(\bar{A}-I)^T + \tilde{Q} - \tilde{S},$$
$$\Phi_{12} = \varepsilon_1\bar{B}F_1M_0Z + \varepsilon_2\tilde{L}^T(\bar{A}-I)^T,\ \Phi_{13} = \tilde{S},$$
$$\Phi_{14} = \varepsilon_1\bar{B},\ \Phi_{15} = -\varepsilon_1\tilde{L} + \tilde{L}^T(\bar{A}-I)^T + \tilde{P},$$
$$\Phi_{16} = 0,\ \Phi_{17} = \alpha\varepsilon_1\bar{B}F_1M_0,$$
$$\Phi_{22} = \varepsilon_2\bar{B}F_1M_0Z + \varepsilon_2(\bar{B}F_1M_0Z)^T,\ \Phi_{23} = 0,$$
$$\Phi_{24} = \frac{1}{2}Z^TM_0^TF^T + \varepsilon_2\bar{B},$$
$$\Phi_{25} = -\varepsilon_2\tilde{L} + Z^TF_1^TM_0^T\bar{B}^T,\ \Phi_{26} = Z^T,$$
$$\Phi_{27} = \alpha\varepsilon_2\bar{B}F_1M_0,\ \Phi_{33} = -\tilde{S} - \tilde{Q},$$
$$\Phi_{34} = \Phi_{35} = \Phi_{36} = \Phi_{37} = 0,$$
$$\Phi_{44} = -I,\ \Phi_{45} = \bar{B}^T,\ \Phi_{46} = 0,\ \Phi_{47} = \frac{1}{2}\alpha FM_0,$$
$$\Phi_{55} = \tilde{P} + \bar{\tau}^2\tilde{S} - \tilde{L} - \tilde{L}^T,\ \Phi_{56} = 0,$$
$$\Phi_{57} = \alpha\bar{B}F_1M_0,\ \Phi_{66} = -\alpha I,\ \Phi_{67} = 0,$$
$$\Phi_{77} = -\alpha I.$$

According to (2) and (3), $\Phi < 0$ can be rewritten as

$$\Phi = \hat{\Phi} + \begin{bmatrix} \varepsilon_1 D \\ \varepsilon_2 D \\ 0 \\ 0 \\ D \\ 0 \\ 0 \end{bmatrix} \Delta(k) \begin{bmatrix} E_1\hat{L} & E_2F_1M_0Z & 0 & E_2 & 0 & 0 & \alpha E_2F_1M_0 \end{bmatrix} + \begin{bmatrix} (E_1\tilde{L})^T \\ (E_2F_1M_0Z)^T \\ 0 \\ E_2^T \\ 0 \\ 0 \\ (\alpha E_2F_1M_0)^T \end{bmatrix} \Delta^T(k) \begin{bmatrix} \varepsilon_1 D^T & \varepsilon_2 D^T & 0 & 0 & D^T & 0 & 0 \end{bmatrix} < 0$$

where

$$\hat{\Phi} = \begin{bmatrix} \hat{\Phi}_{11} & \hat{\Phi}_{12} & \hat{\Phi}_{13} & \hat{\Phi}_{14} & \hat{\Phi}_{15} & \hat{\Phi}_{16} & \hat{\Phi}_{17} \\ * & \hat{\Phi}_{22} & \hat{\Phi}_{23} & \hat{\Phi}_{24} & \hat{\Phi}_{25} & \hat{\Phi}_{26} & \hat{\Phi}_{27} \\ * & * & \hat{\Phi}_{33} & \hat{\Phi}_{34} & \hat{\Phi}_{35} & \hat{\Phi}_{36} & \hat{\Phi}_{37} \\ * & * & * & \hat{\Phi}_{44} & \hat{\Phi}_{45} & \hat{\Phi}_{46} & \hat{\Phi}_{47} \\ * & * & * & * & \hat{\Phi}_{55} & \hat{\Phi}_{56} & \hat{\Phi}_{57} \\ * & * & * & * & * & \hat{\Phi}_{66} & \hat{\Phi}_{67} \\ * & * & * & * & * & * & \hat{\Phi}_{77} \end{bmatrix}$$

$$\hat{\Phi}_{11} = \varepsilon_1(A-I)\tilde{L} + \varepsilon_1\tilde{L}^T(A-I)^T + \tilde{Q} - \tilde{S},$$
$$\hat{\Phi}_{12} = \varepsilon_1 BF_1M_0Z + \varepsilon_2\tilde{L}^T(A-I)^T,\ \hat{\Phi}_{13} = \tilde{S},$$
$$\hat{\Phi}_{14} = \varepsilon_1 B,\ \hat{\Phi}_{15} = -\varepsilon_1\tilde{L} + \tilde{L}^T(\bar{A}-I)^T + \tilde{P},$$
$$\hat{\Phi}_{16} = 0,\ \hat{\Phi}_{17} = \alpha\varepsilon_1 BF_1M_0,$$
$$\hat{\Phi}_{22} = \varepsilon_2 BF_1M_0Z + \varepsilon_2(BF_1M_0Z)^T,\ \hat{\Phi}_{23} = 0,$$
$$\hat{\Phi}_{24} = \frac{1}{2}Z^TM_0^TF^T + \varepsilon_2 B,$$
$$\hat{\Phi}_{25} = -\varepsilon_2\tilde{L} + Z^TF_1^TM_0^TB^T,\ \hat{\Phi}_{26} = Z^T,$$
$$\hat{\Phi}_{27} = \alpha\varepsilon_2 BF_1M_0,\ \hat{\Phi}_{33} = -\tilde{S} - \tilde{Q},$$
$$\hat{\Phi}_{34} = \hat{\Phi}_{35} = \hat{\Phi}_{36} = \hat{\Phi}_{37} = 0,\ \hat{\Phi}_{44} = -I,$$
$$\hat{\Phi}_{45} = B^T,\ \hat{\Phi}_{46} = 0,\ \hat{\Phi}_{47} = \frac{1}{2}\alpha FM_0,$$
$$\hat{\Phi}_{55} = \tilde{P} + \bar{\tau}^2\tilde{S} - \tilde{L} - \tilde{L}^T,\ \hat{\Phi}_{56} = 0,$$
$$\hat{\Phi}_{57} = \alpha BF_1M_0,\ \hat{\Phi}_{66} = -\alpha I,\ \hat{\Phi}_{67} = 0,$$
$$\hat{\Phi}_{77} = -\alpha I.$$

According to Lemma 3, if there exists a scalar $\lambda > 0$ such that the following inequality (32) is true, then, $\Phi < 0$ holds.

$$\hat{\Phi} + \lambda \begin{bmatrix} \varepsilon_1 D \\ \varepsilon_2 D \\ 0 \\ 0 \\ D \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \varepsilon_1 D \\ \varepsilon_2 D \\ 0 \\ 0 \\ D \\ 0 \\ 0 \end{bmatrix}^T + \lambda^{-1} \begin{bmatrix} (E_1\tilde{L})^T \\ (E_2F_1M_0Z)^T \\ 0 \\ E_2^T \\ 0 \\ 0 \\ (\alpha E_2F_1M_0)^T \end{bmatrix} \begin{bmatrix} (E_1\tilde{L})^T \\ (E_2F_1M_0Z)^T \\ 0 \\ E_2^T \\ 0 \\ 0 \\ (\alpha E_2F_1M_0)^T \end{bmatrix}^T < 0 \quad (32)$$

According to Lemma 2, if (31) holds, (32) is held. Then $\Delta V(k) < 0$. Thus, the proof is completed.

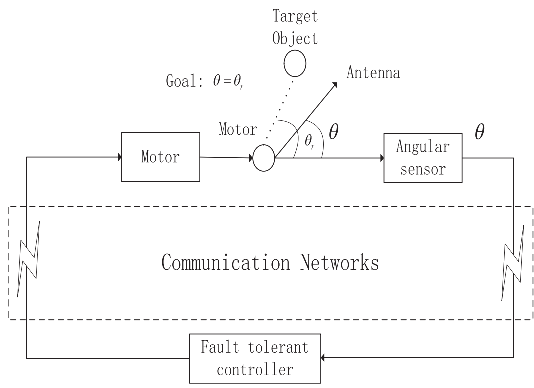

FIGURE 2. The angular positioning system.

V. NUMERICAL SIMULATION

In this section, to illustrate the effectiveness of the obtained results, a classical angular positioning system [30] is considered and presented in Fig 2, which consists of a rotating antenna at the origin of the plane and the antenna is driven by an electric motor. Assume that the angular position of the antenna θ, the angular position of the moving object θ_r and the angular velocity of the antenna $\dot{\theta}$ is measurable. The state variables are chosen as $x = \begin{bmatrix} \theta & \dot{\theta} \end{bmatrix}^T$.

The motion of the antenna can be described by the following discrete networked control system :

$$x(k+1) = \begin{bmatrix} \theta(k+1) \\ \dot{\theta}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0.1 \\ 0 & 0.99 \end{bmatrix} x(k) + \begin{bmatrix} -0.0787 & 0 \\ 0 & 0.0787 \end{bmatrix} sat(u(k))$$

We assume that the beforementioned system is an uncertain system with saturation level $u_{max} = 0.3$ and norm-bounded parameter uncertainties (3), where

$$D = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},\ \Delta(k) = \sin(k),$$
$$E_1 = \begin{bmatrix} 0 & 0.01 \end{bmatrix},\ E_2 = \begin{bmatrix} 0.01 & 0 \end{bmatrix}$$

The state response of open-loop system ($u(k) = 0$) is shown in Figure 3 and it is clear that the open-loop system is unstable.

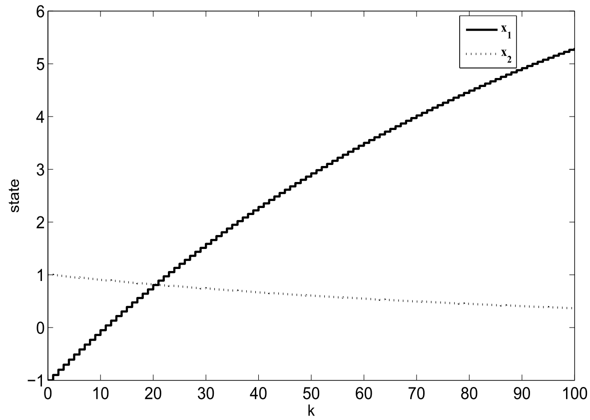

FIGURE 3. State trajectories of open-loop system.

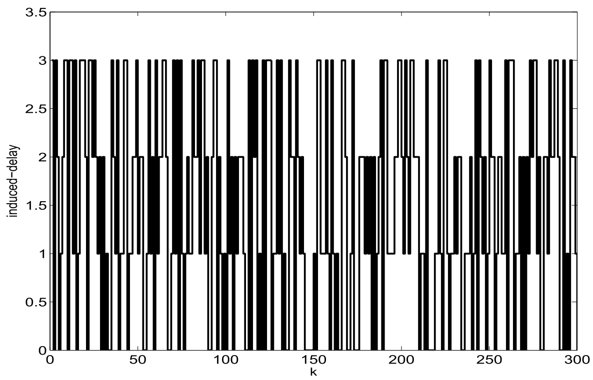

FIGURE 4. Induced delay.

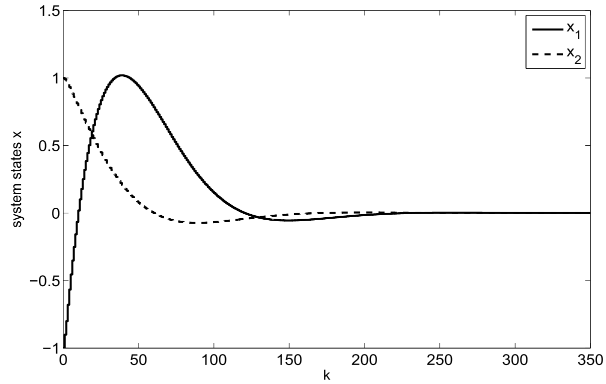

FIGURE 5. State trajectories of x without actuator faults.

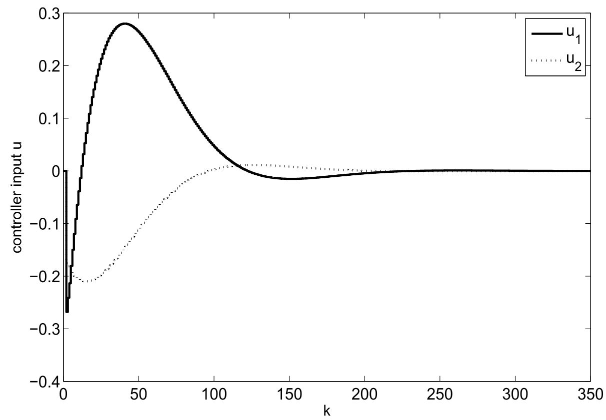

FIGURE 6. Trajectories of u without actuator faults.

Example 1: Firstly, we assume that the actuator faults do not happen. The parameters in (25) are given as

$$F = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}, \quad \omega_1 = \omega_2 = 0.1$$

Considering the induced delay bound $\bar{\tau} = 3$ and saturation level $u_{max} = 0.3$, we obtain the following controller gain according to Theorem 3:

$$K = \begin{bmatrix} 0.2735 & 0.0053 \\ -0.0877 & -0.2630 \end{bmatrix}$$

and

$$\hat{L} = \begin{bmatrix} 0.0060 & 0.0020 \\ -0.0000 & 0.0065 \end{bmatrix}, \quad Y = \begin{bmatrix} 0.0017 & -0.0005 \\ -0.0005 & -0.0015 \end{bmatrix}, \beta = 0.0016$$

In Figure 4, time-varying induced delays are shown. With a initial condition $x(0) = [-1; 1]$, simulation results are shown in Figure 5 and Figure 6. The state response of NCSs is shown in Figure 5 and the control input signal is shown in Figure 6. From which we can see that the system is asymptotically stable and the control signal satisfies the actuator saturation.

Example 2: When there exist time-varying actuator faults, we assume that the actuator failure matrix $M = diag\{m_1, m_2\}$, where $0.7 = m_1^{\min} \leq m_1 \leq m_1^{\max} = 1$, $0 = m_2^{\min} \leq m_2 \leq m_2^{\max} = 0.8$. Then, we have $M_0 = \begin{bmatrix} 0.85 & 0 \\ 0 & 0.4 \end{bmatrix}$. The parameters in Theorem 3 and the induced delay bound and u_{max} are the same as in Example 1. When we use the controller gain $K = \begin{bmatrix} 0.2735 & 0.0053 \\ -0.0877 & -0.2630 \end{bmatrix}$ obtained in Example 1, the induced delay is shown in Figure 7. Then, we can have that the states response $x(k)$ and control signal $u^F(k)$ are displayed in Figure 8 and Figure 9.

Then, from Theorem 3, we obtain a fault tolerant controller as

$$u^F(k) = \begin{bmatrix} 0.6216 & 0.1892 \\ -0.0314 & -0.6806 \end{bmatrix} x(k - \tau(k))$$

with $Z = \begin{bmatrix} 0.0091 & 0.0022 \\ 0.0022 & -0.0064 \end{bmatrix}$, $L = \begin{bmatrix} 0.0158 & 0.0008 \\ -0.0040 & 0.0093 \end{bmatrix}$, $\alpha = 2.9547, \lambda = 0.0392$. Induced delay is shown in Figure 10. The system state response and control signal obtained by the fault tolerant controller are given in Figure 11 and Figure 12.

From Figure 11 and Figure 12 we can see that by the fault tolerant controller obtained in Theorem 3, the system can

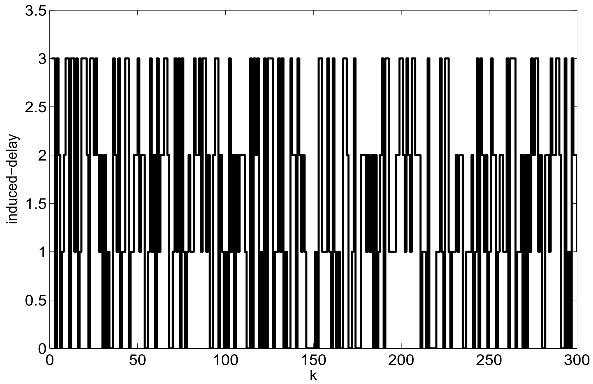

FIGURE 7. **Induced delay.**

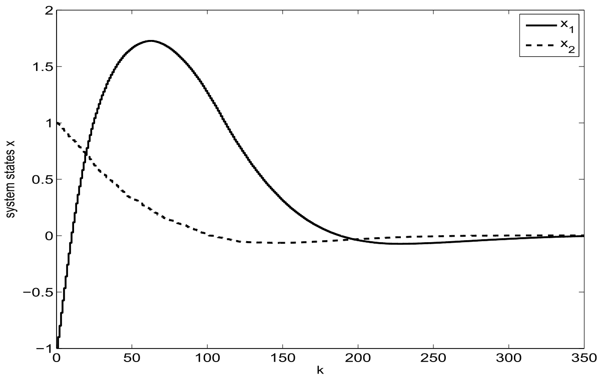

FIGURE 8. **State trajectories of x with actuator faults by the controller in Example 1.**

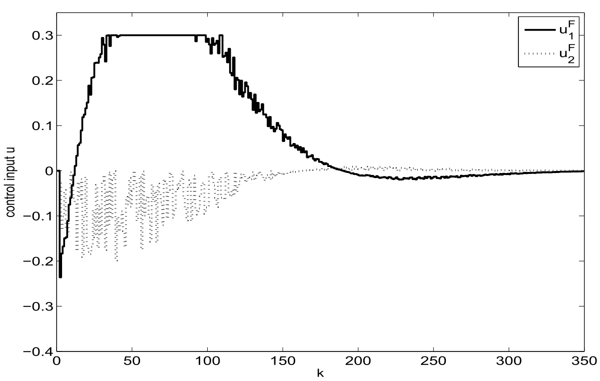

FIGURE 9. **Trajectories of u^F with actuator faults by the controller in Example 1.**

be asymptotically stable and the control signal satisfies the actuator saturation. In Figure 8 and Figure 9, we can see that the system become stable at about k=350, while in Figure 11 and Figure 12, the system become stable at about k=200, which means the fault tolerant controller can improve the system performance.

In conclusion, the performance of NCSs can be well maintained by the designed fault tolerant controller, which demonstrates the effectiveness and feasibility of the proposed method in this paper.

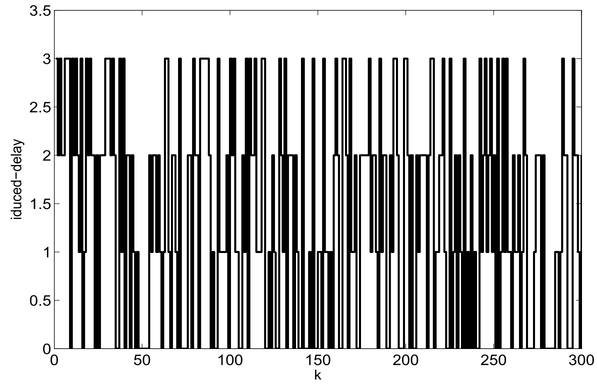

FIGURE 10. **Induced delay.**

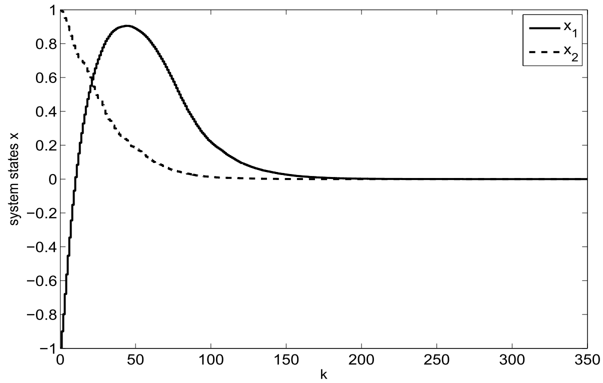

FIGURE 11. **State trajectory of x with actuator faults by the fault tolerant controller.**

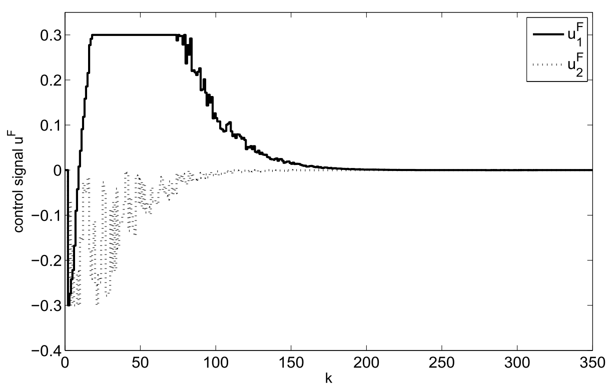

FIGURE 12. **Trajectory of u^F with actuator faults by the fault tolerant controller.**

VI. CONCLUSION

In this paper, the fault tolerant stability analysis and stabilization problem have been studied for uncertain NCSs with networked-induced delays and actuator saturation. We choose an appropriate L-K functional and sufficient conditions in terms of LMIs are derived to ensure the existence of a fault tolerant state feedback controller which can stabilize the system. Simulation examples are given to illustrate the effectiveness of the proposed approach. In the future, random packet dropout and communication constraint problem of NCSs can be further considered.

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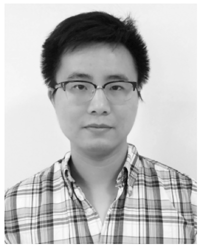

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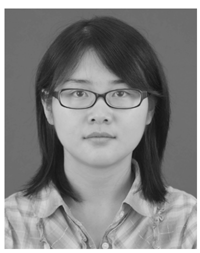

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