IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received September 6, 2016, accepted September 13, 2016, date of publication September 23, 2016, date of current version October 15, 2016.

Digital Object Identifier 10.1109/ACCESS.2016.2613114

Fault Detection for Two-Dimensional Roesser Systems With Sensor Faults

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This work was supported in part by the National Natural Science Foundation of China under Grant 61473032, in part by the Program for New Century Excellent Talents in University under Grant NCET-13-0662, in part by the National Key Technologies Research and Development Program under Grant 2013BAB02B07, and in part by the Fundamental Research Funds for the Central Universities under Grant FRF-TP-14-010C1.

ABSTRACT This paper investigates the problem of fault detection for 2-D systems with lock-in-place sensor faults. Our attention is focused on detecting sensor faults in the presence of disturbances. To this end, a fault detection filter is designed, through which a residual signal is generated for both the fault-free and faulty cases. In light of the generalized Kalman–Yakubovich–Popov lemma for 2-D systems and matrix inequality techniques, convex filter design conditions are derived. Based on these conditions, an algorithm is proposed to calculate the parameters of a desired fault detection filter. Moreover, a residual evaluation function and a threshold are proposed for 2-D systems. Finally, an example is employed to illustrate the effectiveness of the proposed fault detection method.

INDEX TERMS Fault detection, 2-D systems, sensor faults.

I. INTRODUCTION

During the past decades, two-dimensional (2-D) systems [1]–[8] have received a large amount of attention due to their close relation to many engineering fields such as image processing, multi-dimensional signal processing, multi-dimensional digital filtering, thermal process, etc. 2-D system theory has been greatly developed and applied since the well-known Roesser model [1] and Fornasini & Marchesini (FM) model [2] were proposed. Recently, fault detection and isolation (FDI) of 2-D systems has been reported in the literature, e.g., [12]–[17]. An observer-based FDI method for 2-D systems was developed in [12] and [13]. A geometric FDI scheme for discrete-time 2-D Roesser systems was proposed in [14]. The generalized H_2 fault detection for 2-D Markovian jump systems was investigated in [15]. A Kalman-based fault detection algorithm was proposed for 2-D FM systems in [22]. It should be noticed that the FDI problems in [12]– [22] were investigated in full-frequency domain. By contrast, a finite-frequency fault detection method was developed for 2-D Roesser systems in [17] and [18].

Lock-in-place faults, or namely stuck faults usually occur in practical systems [9], [10], [19], [20]. However, fault detection of 2-D systems with stuck faults has not been well investigated in the literature. This motivates the present paper. To detect stuck faults, a fault detection filter is designed, through which a residual signal is generated. The residual signal directly reflects the fault information and can therefore be used to determine whether a fault occurs. By satisfying a set of performance indices simultaneously, the influence of disturbance on residual signal is minimized while the influence of fault on residual signal is maximized. It is worth noting that most faults including stuck faults have a property of finite frequency. By the generalized KYP lemma [11] and bounded real lemma, the fault detection filter design is formulated as a multiobjective optimization problem. Convex filter design conditions are derived by using matrix inequality techniques. Based on these conditions, an algorithm is proposed to calculate the parameters of the fault detection filter. Moreover, a residual evaluation function and a threshold are proposed for 2-D systems. Finally, an example is given to illustrate the effectiveness of the proposed method.

The rest of the paper is organized as follows. Section II gives the problem statement and preliminaries. Section III presents the main results of the paper. In this section, a fault detection filter is designed in detail. Moreover, a residual evaluation function and a threshold are given for 2-D systems. Section IV gives an example to illustrate the effectiveness of the proposed method. Finally, conclusions are given in Section V.

Notations. We use standard notations throughout this paper. For a matrix M, M^T, $M^\perp$ denote its transpose and orthogonal complement, respectively. $M > 0 (M < 0)$

means that M is positive definite (negative definite). The symbol $*$ will be used in some matrix expressions to induce a symmetric structure. The Hermitian part of a square matrix M is denoted by $He(M) := M + M^T$. $\sigma_{min}(G)$, $\sigma_{max}(G)$ denote the minimum singular value and the maximum singular value of the transfer matrix G, respectively. I denotes an identity matrix with appropriate dimension.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. SYSTEM MODEL

Consider the following 2-D discrete-time system described by the Roesser model:

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Bd(i,j)$$
$$y(i,j) = C \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Dd(i,j) \tag{1}$$

where $x^h(i,j) \in \mathbb{R}^{n_h}$, $x^v(i,j) \in \mathbb{R}^{n_v}$ are the horizontal state and the vertical state, respectively, $y(i,j) \in \mathbb{R}^{n_y}$ is the measured output, $d(i,j) \in \mathbb{R}^{n_d}$ is the disturbance, and A, B, C, D are known real matrices.

B. FAULT MODEL

In this paper, we consider a common type of sensor faults, i.e., lock-in-place faults or namely stuck faults.

Definition 1 (Lock-In-Place Sensor Fault Model): When lock-in-place sensor faults occur, the output signal is given by

$$y^F(i,j) = F_p y(i,j) + (I - F_p) f_p(i,j), \ p = 1, \dots, N_s \tag{2}$$

where p denotes the pth fault mode, N_s is the total number of possible fault modes, and

$$f_p = [f_{p1}, f_{p2}, \cdots, f_{pn_y}]^T \tag{3}$$

$f_{pk}(k = 1, 2, \cdots, n_y)$ can be zero or certain unknown non-zero constant which denotes the stuck value of the kth sensor. The diagonal matrix F_p is defined as

$$F_p = \mathrm{diag}\{F_{p1}, F_{p2}, \cdots, F_{pn_y}\}, \tag{4}$$

with $F_{pk} = 0$ or 1, $k = 1, 2, \cdots, n_y$.

Remark 1: Obviously, $F_{pk} = 0$, for $1 \le k \le n_y$, denotes that the kth sensor is faulty, whereas $F_{pk} = 1$ denotes that the kth sensor is fault-free.

C. PROBLEM FORMULATION

With lock-in-place sensor faults, 2-D system (1) becomes

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Bd(i,j)$$
$$y^F(i,j) = F_p C \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + F_p Dd(i,j) + f(i,j) \tag{5}$$

where $f(i,j) = (I - F_p) f_p(i,j)$.

The main purpose of this paper is to construct a fault detection filter

$$\begin{bmatrix} \hat{x}^h(i+1,j) \\ \hat{x}^v(i,j+1) \end{bmatrix} = \hat{A} \begin{bmatrix} \hat{x}^h(i,j) \\ \hat{x}^v(i,j) \end{bmatrix} + \hat{B} y^F(i,j)$$
$$\hat{y}(i,j) = \hat{C} \begin{bmatrix} \hat{x}^h(i,j) \\ \hat{x}^v(i,j) \end{bmatrix} \tag{6}$$

to detect whether such sensor faults occur, where $\hat{x}^h(i,j) \in \mathbb{R}^{n_h}$ and $\hat{x}^v(i,j) \in \mathbb{R}^{n_v}$ are the horizontal and vertical state of the filter, respectively, $\hat{y}^F(i,j) \in \mathbb{R}^{n_y}$ is the sensor output, $\hat{y}(i,j) \in \mathbb{R}^{n_y}$ is the output estimation, and matrices $\hat{A}$, $\hat{B}$, $\hat{C}$ are filter parameters to be determined later.

Define $\xi^h(i,j) := \begin{bmatrix} x^h(i,j) \\ \hat{x}^h(i,j) \end{bmatrix}$, $\xi^v(i,j) := \begin{bmatrix} x^v(i,j) \\ \hat{x}^v(i,j) \end{bmatrix}$, and define a residual signal $r(i,j) := y^F(i,j) - \hat{y}(i,j)$. Combining system (5) and filter (6), we have the following augmented system

$$\begin{bmatrix} \xi^h(i+1,j) \\ \xi^v(i,j+1) \end{bmatrix} = \bar{A} \begin{bmatrix} \xi^h(i,j) \\ \xi^v(i,j) \end{bmatrix} + \bar{B}_d d(i,j) + \bar{B}_f(i,j)$$
$$r(i,j) = \bar{C} \begin{bmatrix} \xi^h(i,j) \\ \xi^v(i,j) \end{bmatrix} + \bar{D}_d d(i,j) + \bar{D}_f f(i,j) \tag{7}$$

where

$$\bar{A} = \Sigma \begin{bmatrix} A & 0 \\ \hat{B}F_p C & \hat{A} \end{bmatrix} \Sigma^T, \ \bar{B}_d = \Sigma \begin{bmatrix} B \\ \hat{B}F_p D \end{bmatrix},$$
$$\bar{B}_f = \Sigma \begin{bmatrix} 0 \\ \hat{B} \end{bmatrix}, \bar{C} = \begin{bmatrix} F_p C & -\hat{C} \end{bmatrix} \Sigma^T,$$
$$\bar{D}_d = F_p D, \ \bar{D}_f = I.$$
$$\Sigma = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \tag{8}$$

The fault detection problem to be addressed can be formulated as follows: For a given 2-D system (5) with lock-in-place sensor faults, design a fault detection filter (6) such that the augmented system (7) is asymptotically stable and satisfies the following performance indices

$$\sigma_{min}(G_{rf}(\omega_h, \omega_v)) > \beta, \ \forall |\omega_h| \le \varpi_h, |\omega_v| \le \varpi_v \tag{9}$$
$$\sigma_{max}(G_{rd}(\omega_h, \omega_v)) < \gamma, \quad F_p = 0 \tag{10}$$
$$\sigma_{max}(G_{rd}(\omega_h, \omega_v)) < \gamma, \quad F_p = 1 \tag{11}$$

where β, γ are positive scalars, ϖ_h, ϖ_v are the frequency bounds of faults ($\varpi_h = \varpi_v = 0$ for lock-in-place faults), $G_{rf}(\omega_h, \omega_v) := \bar{C}(\mathrm{diag}\{e^{J\omega_h}I, e^{J\omega_v}I\} - \bar{A})^{-1}\bar{B}_f + \bar{D}_f$ is the transfer function from fault f to residual signal r, and $G_{rd}(\omega_h, \omega_v) := \bar{C}(\mathrm{diag}\{e^{J\omega_h}I, e^{J\omega_v}I\} - \bar{A})^{-1}\bar{B}_d + \bar{D}_d$ is the transfer function from disturbance d to residual signal r.

Remark 2: To detect sensor faults effectively in the presence of disturbances, a finite-frequency H_- performance index (9) is introduced to maximize the effect of sensor fault on residual signal. Moreover, two H_∞ indices (10) and (11) are used to describe the effect of disturbance on residual signal in faulty and fault-free cases, simultaneously.

D. SOME USEFUL LEMMAS

The following lemmas will be used in the sequel.

Lemma 1 (Finsler's Lemma [21]): Let $\eta \in \mathbb{R}^n$, $\mathcal{P} = \mathcal{P}^T \in \mathbb{R}^{n\times n}$ and $\mathcal{H} \in \mathbb{R}^{n\times m}$. Let $\mathcal{H}^\perp$ be any matrix such that $\mathcal{H}^\perp \mathcal{H} = 0$. The following statements are equivalent:

1) $\eta^T \mathcal{P}\eta < 0, \forall \mathcal{H}^T\eta = 0, \eta \neq 0$;
2) $\exists \mathcal{X} \in \mathbb{R}^{m\times n} : \mathcal{P} + \mathcal{H}\mathcal{X} + \mathcal{X}^T\mathcal{H}^T < 0$;

Lemma 2 (Generalized KYP Lemma [23]): Given system matrices (A, B, C, D) and a symmetric matrix Π, the following statements are equivalent:

1) *The finite frequency inequality*

$$\begin{bmatrix} G^T(\omega_h, \omega_v) & I \end{bmatrix} \Pi \begin{bmatrix} G(\omega_h, \omega_v) \\ I \end{bmatrix} < 0, \tag{12}$$

holds for all $|\omega_h| < \varpi_h, |\omega_v| < \varpi_v$, where $G(\omega_h, \omega_v) = C(\mathrm{diag}\{e^{j\omega_h}I, e^{j\omega_v}I\} - A)^{-1}B + D$;

2) *There exist Hermitian matrices $P = \mathrm{diag}\{P_h, P_v\}$, $Q = \mathrm{diag}\{Q_h, Q_v\}$ satisfying $Q > 0$, and*

$$\begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^T \begin{bmatrix} -P & Q \\ Q & P-2WQ \end{bmatrix} \begin{bmatrix} A & B \\ I & 0 \end{bmatrix} + \begin{bmatrix} C & D \\ 0 & I \end{bmatrix}^T \Pi \begin{bmatrix} C & D \\ 0 & I \end{bmatrix} < 0 \tag{13}$$

where $W = \mathrm{diag}\{\cos(\varpi_h)I, \cos(\varpi_v)I\}$.

III. MAIN RESULTS

In this section, we first derive the fault sensitivity condition (9) and the disturbance attenuation conditions (10) and (11). Based on these conditions, an algorithm is proposed to construct a desired fault detection filter. Finally, a residual evaluation function and a threshold are given for 2-D systems.

A. FAULT SENSITIVITY CONDITION

Theorem 1: For system (7) and given scalars $\beta > 0$, $\alpha > 0$, the finite-frequency index (9) is satisfied if there exist symmetric matrices $P_1 = \mathrm{diag}\{P_{1h}, P_{1v}\}$, $P_3 = \mathrm{diag}\{P_{3h}, P_{3v}\}$, $Q_1 = \mathrm{diag}\{Q_{1h}, Q_{1v}\}$, $Q_3 = \mathrm{diag}\{Q_{3h}, Q_{3v}\}$, matrices $P_2 = \mathrm{diag}\{P_{2h}, P_{2v}\}$, $Q_2 = \mathrm{diag}\{Q_{2h}, Q_{2v}\}$, $X_1, X_2, X_3, Y_1, Y_2, Z_1, \tilde{A}, \tilde{B}, \hat{C}$ satisfying the following inequalities:

$$Q = \Sigma \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \Sigma^T > 0 \tag{14}$$

$$\alpha^2 - \hat{C}T < 0 \tag{15}$$

$$\begin{bmatrix} -P_1 - X_1 - X_1^T & -P_2 - X_2^T - X_3 & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} \\ * & -P_3 - X_3 - X_3^T & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} \\ * & * & \Gamma_{33} & \Gamma_{34} & \Gamma_{35} \\ * & * & * & \Gamma_{44} & \Gamma_{45} \\ * & * & * & * & \Gamma_{55} \end{bmatrix} < 0 \tag{16}$$

where

$$\begin{aligned} &\Gamma_{13} = Q_1 - Y_1^T + X_1A, \ \Gamma_{14} = Q_2 - Y_2^T + \tilde{A}, \\ &\Gamma_{15} = -Z_1^T + \tilde{B}, \Gamma_{23} = Q_2^T - X_3^T + X_2A, \\ &\Gamma_{24} = Q_3 - X_3^T + \tilde{A}, \ \Gamma_{25} = \tilde{B} \\ &\Gamma_{33} = P_1 - 2Q_1 + Y_1A + (Y_1A)^T, \\ &\Gamma_{34} = P_2 - 2Q_2 + \tilde{A} + (Y_2A)^T, \\ &\Gamma_{35} = \tilde{B} + (Z_1A)^T, \Gamma_{44} = P_3 - 2Q_3 - \alpha^2 I + \tilde{A} + \tilde{A}^T, \\ &\Gamma_{45} = \hat{C}^T + \tilde{B}, \ \Gamma_{55} = \beta^2 I - I. \end{aligned} \tag{17}$$

Proof: In Lemma 2, let $\Pi = \begin{bmatrix} -I & 0 \\ 0 & \beta^2 I \end{bmatrix}$ and $\varpi_h = 0, \varpi_v = 0$, then the inequality (12) becomes

$$G_{rf}^T(\omega_h, \omega_v)G_{rf}(\omega_h, \omega_v) > \beta^2 I \tag{18}$$

which is further equivalent to (9). By Lemma 2, the H_- performance index (9) is satisfied if there exist symmetric matrices P, Q such that inequality (13) holds. Inequality (13) is equivalent to

$$\begin{bmatrix} \bar{A} & \bar{B}_f \\ I & 0 \\ 0 & I \end{bmatrix}^T \left(\begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -P & Q \\ Q & P-2Q \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix}^T + \begin{bmatrix} 0 & 0 \\ \bar{C}^T & 0 \\ \bar{D}_f^T & I \end{bmatrix} \Pi \begin{bmatrix} 0 & 0 \\ \bar{C}^T & 0 \\ \bar{D}_f^T & I \end{bmatrix}^T \right) \begin{bmatrix} \bar{A} & \bar{B}_f \\ I & 0 \\ 0 & I \end{bmatrix} < 0. \tag{19}$$

Inequality (19) can be further rewritten as

$$\Lambda^T \mathcal{P} \Lambda < 0 \tag{20}$$

where

$$\Lambda = \begin{bmatrix} \bar{A} & \bar{B}_f \\ I & 0 \\ 0 & I \end{bmatrix}, \mathcal{P} = \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -P & Q \\ Q & P-2Q \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix}^T + \begin{bmatrix} 0 & 0 \\ \bar{C}^T & 0 \\ \bar{D}_f^T & I \end{bmatrix} \Pi \begin{bmatrix} 0 & 0 \\ \bar{C}^T & 0 \\ \bar{D}_f^T & I \end{bmatrix}^T \tag{21}$$

Therefore, $\eta^T\mathcal{P}\eta < 0$, where $\eta = \Lambda\zeta$, and ζ is any nonzero vector. Define $\mathcal{H}^T := [-I \ \bar{A} \ \bar{B}_f]$, then we have

$$\mathcal{H}^T\eta = \mathcal{H}\Lambda\zeta = 0. \tag{22}$$

By Lemma 1, (20) is equivalent to

$$\mathcal{P} + \mathcal{H}\mathcal{X} + \mathcal{X}^T\mathcal{H}^T < 0 \tag{23}$$

where $\mathcal{X}$ is a multiplier. To make the problem tractable, we choose $\mathcal{X} = \begin{bmatrix} X^T & Y^T & Z^T \end{bmatrix}$, where $X = \Sigma \begin{bmatrix} X_1 & X_3 \\ X_2 & X_3 \end{bmatrix} \Sigma^T$, $Y = \Sigma \begin{bmatrix} Y_1 & X_3 \\ Y_2 & X_3 \end{bmatrix} \Sigma^T$, $Z = \begin{bmatrix} Z_1 & 0 \end{bmatrix} \Sigma^T$. Meanwhile, introduce the following matrices

$$P = \Sigma \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} \Sigma^T, \ Q = \Sigma \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} \Sigma^T. \tag{24}$$

Combining (22) and (23), we have (25), as shown at the top of the next page. Pre- and post-multiplying (25) by

$$\begin{bmatrix} -P-X-X^T & Q-Y^T+X\bar{A} & -Z^T+X\bar{B}_f \\ * & P-2Q-\bar{C}^T\bar{C}+Y\bar{A}+(Y\bar{A})^T & -\bar{C}^T\bar{D}_f+Y\bar{B}_f+(Z\bar{A})^T \\ * & * & -\bar{D}_f^T\bar{D}_f+\beta^2 I+Z\bar{B}_f+(Z\bar{B}_f)^T \end{bmatrix} < 0. \quad (25)$$

diag$\{\Sigma, \Sigma, I\}$ and its transpose, and substituting the corresponding matrices into (25), we can obtain the following inequality:

$$\begin{bmatrix} -P_1-X_1-X_1^T & -P_2-X_2^T-X_3 & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} \\ * & -P_3-X_3-X_3^T & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} \\ * & * & \Gamma_{33} & \Gamma_{34} & \Gamma_{35} \\ * & * & * & \tilde{\Gamma}_{44} & \Gamma_{45} \\ * & * & * & * & \Gamma_{55} \end{bmatrix} < 0 \quad (26)$$

where $\tilde{\Gamma}_{44} = P_3 - 2Q_3 - \hat{C}^T\hat{C} + \tilde{A} + \tilde{A}^T$, and other parameters were given in (17). To linearize inequality (26), an upper bound can be imposed on the coupled term $\hat{C}^T\hat{C}$ in $\tilde{\Gamma}_{44}$. Therefore, (26) can be guaranteed by (16) and

$$\hat{C}^T\hat{C} > \alpha^2 I. \quad (27)$$

Note that (27) corresponds to the outer region of a ball of radius α in the objective variable $\hat{C}$ and hence the feasible set is non-convex. Here we adopt the linearized method in [24] and [25]. Since $\hat{C}$ is a row vector, $\hat{C}T - ||T||_2^2 = 0$ with $||T||_2 = \alpha$ is a hyperplane tangent to the ball (27), and the search of a convex feasible subset of (27) can be achieved by imposing the following constraint $\alpha^2 - \hat{C}T < 0$, i.e., the condition (15). The proof is completed. ■

B. DISTURBANCE ATTENUATION CONDITION

The H_∞ performance indices (10) and (11) are introduced to minimize the effect of disturbance on residual signal. In this subsection, we will derive convex conditions under which the performance indices (10) and (11) are guaranteed.

Lemma 3 [26]: Given system matrices (A, B, C, D) and a given scalar $\gamma > 0$, system matrix A is asymptotically stable and the disturbance attenuation condition

$$\sigma_{max}(G(\omega_h, \omega_v)) < \gamma \quad (28)$$

is satisfied, where $G(\omega_h, \omega_v) := C(\text{diag}\{e^{J\omega_h}I, e^{J\omega_v}I\} - A)^{-1}B + D$, if there exist a symmetric matrix $\bar{P} = \text{diag}\{\bar{P}_h, \bar{P}_v\} > 0$ and a matrix G satisfying the following inequality

$$\begin{bmatrix} -\bar{P} & 0 & \bar{P}A & \bar{P}B \\ * & -I & C & D \\ * & * & -\bar{P} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \quad (29)$$

Remark 3: When sensors are fault-free or get faulty, the effect of disturbance on residual signal is attenuated via the following two theorems. Theorem 2 considers the disturbance attenuation performance for the faulty case, whereas Theorem 3 considers the disturbance attenuation performance for the fault-free case.

Theorem 2: For system (7) and a scalar $\gamma > 0$, assume that some sensors get faulty, which implies that $F_p = 0$, then the H_∞ performance index (10) is satisfied if there exist symmetric matrices $\bar{P}_{11} = \text{diag}\{\bar{P}_{11h}, \bar{P}_{11v}\}$, $\bar{P}_{13} = \text{diag}\{\bar{P}_{13h}, \bar{P}_{13v}\}$, matrices $\bar{P}_{12} = \text{diag}\{\bar{P}_{12h}, \bar{P}_{12v}\}$, G_1, G_2, X_3, $\tilde{A}$, $\tilde{B}$, $\hat{C}$ such that (30) and (31), shown at the top of the next page.

Proof: By using the slack variable techniques in [27], it is easy to prove that for system (7), system matrix $\bar{A}$ is asymptotically stable and the disturbance attenuation condition (10) is satisfied if there exist a symmetric matrix $\bar{P}_1 = \text{diag}\{\bar{P}_{1h}, \bar{P}_{1v}\}$ and a matrix G such that:

$$\begin{bmatrix} \bar{P}_1-G-G^T & 0 & G^T\bar{A} & G^T\bar{B}_d \\ * & -I & \bar{C} & \bar{D}_d \\ * & * & -\bar{P}_1 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (32)$$

Define $\bar{P}_1 = \Sigma \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ * & \bar{P}_{13} \end{bmatrix} \Sigma^T$, $\bar{P}_{1k} = \text{diag}\{\bar{P}_{1kh}, \bar{P}_{1kv}\}$, $k = 1, 2, 3$, and $G^T = \Sigma \begin{bmatrix} G_1 & X_3 \\ G_2 & X_3 \end{bmatrix} \Sigma^T$. Pre- and post-multiplying (32) by diag$\{\Sigma, I, \Sigma, I\}$ and its transpose, and substituting the corresponding matrices into the inequality, (31), is obtained. Therefore, (30) and (31) are sufficient conditions for the H_∞ performance index (10). The proof is completed. ■

Theorem 3: For system (7) and scalar $\gamma > 0$, assume that sensors are fault-free, which implies that $F_p = I$, then the H_∞ performance index (11) holds if there exist symmetric matrices $\bar{P}_{21} = \text{diag}\{\bar{P}_{21h}, \bar{P}_{21v}\}$, $\bar{P}_{23} = \text{diag}\{\bar{P}_{23h}, \bar{P}_{23v}\}$, matrices $\bar{P}_{22} = \text{diag}\{\bar{P}_{22h}, \bar{P}_{22v}\}$, $\tilde{A}$, $\tilde{B}$, H_1, H_2, X_3, $\hat{C}$ satisfying the inequalities (33) and (34), as shown at the top of the next page.

Proof: The proof is similar to that of Theorem 2. It is omitted here for brevity. ■

C. FAULT DETECTION FILTER DESIGN ALGORITHM

In the previous subsections, the fault detection problem has been formulated as a multi-objective optimization problem. One has to make trade-offs between the sensitivity and robustness performances. Based on Theorems 1-3, an algorithm, as shown on the next page provides a way to construct a desired fault detection filter.

D. RESIDUAL EVALUATION FUNCTION AND THRESHOLD

For model-based fault detection, a residual evaluation function is introduced to distinguish whether a fault has occurred [28]. Since the residual signal may be corrupted by disturbances, it is necessary to evaluate the residual over an

$$\bar{P}_1 = \Sigma \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ * & \bar{P}_{13} \end{bmatrix} \Sigma^T > 0 \tag{30}$$

$$\begin{bmatrix} \bar{P}_{11} - G_1 - G_1^T & \bar{P}_{12} - G_2^T - X_3 & 0 & G_1 A & \tilde{A} & G_1 B \\ * & \bar{P}_{13} - X_3 - X_3^T & 0 & G_2 A & \tilde{A} & G_2 B \\ * & * & -I & 0 & -\hat{C} & 0 \\ * & * & * & -\bar{P}_{11} & -\bar{P}_{12} & 0 \\ * & * & * & * & -\bar{P}_{13} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{31}$$

$$\bar{P}_2 = \Sigma \begin{bmatrix} \bar{P}_{21} & \bar{P}_{22} \\ * & \bar{P}_{23} \end{bmatrix} \Sigma^T > 0 \tag{33}$$

$$\begin{bmatrix} \bar{P}_{21} - H_1 - H_1^T & \bar{P}_{22} - H_2^T - X_3 & 0 & H_1 A + \tilde{B} C & \tilde{A} & H_1 B + \tilde{B} D \\ * & \bar{P}_{23} - X_3 - X_3^T & 0 & H_2 A + \tilde{B} C & \tilde{A} & H_2 B + \tilde{B} D \\ * & * & -I & C & -\hat{C} & D \\ * & * & * & -\bar{P}_{21} & -\bar{P}_{22} & 0 \\ * & * & * & * & -\bar{P}_{23} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{34}$$

evaluation window. For 2-D systems, an evaluation window should be defined by both horizontal and vertical directions to reflect the two-dimensional evolvement of variables [22]. Like in [22], the evaluation window is defined by a rectangular plane with the horizontal range from $i - s$ to i and the vertical range from $j - t$ to j. Then we define the residual evaluation function and the threshold as follows:

$$J_r(i,j) = \sqrt{\frac{\sum_{p=0}^{s} \sum_{q=0}^{t} r^T(i-p, j-q) r(i-p, j-q)}{(s+1)(t+1)}}$$
$$J_{th} = \sup_{f=0, d\neq 0} J_r(i,j) \tag{35}$$

Finally, the occurrence of faults can be detected according to the following logic rules:

$$J_r(i,j) > J_{th} \Rightarrow \text{with faults} \Rightarrow \text{alarm};$$
$$J_r(i,j) \leq J_{th} \Rightarrow \text{fault free} \Rightarrow \text{no alarm}. \tag{36}$$

IV. EXAMPLE

Consider the stationary random field in [29], which can be modeled by the following 2-D system

$$x(i+1, j+1) = a_1 x(i, j+1) + a_2 x(i+1, j) - a_1 a_2 x(i,j) + \omega(i,j) \tag{37}$$

where $x(i,j)$ is the state of the random field at spatial coordinate (i,j), $\omega(i,j)$ is a disturbance input, $a_1^2 < 1, a_2^2 < 1$, and a_1, a_2 are, respectively, the vertical and horizontal correlations of the random field.

Denote $x^h(i,j) := x(i,j+1) - a_2 x(i,j)$, $x^v(i,j) := x(i,j)$, and $d(i,j) := \omega(i,j)$. It is easy to see that the equation (37) can be described by a 2-D Roesser model (5) with $A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Let $a_1 = 0.3$, $a_2 = 0.2$, $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$, and $D = 1$. It is assumed that a lock-in-place sensor fault occurs in this system. The purpose here is to design a fault detection filter to detect the fault effectively in the presence of disturbances.

Algorithm 1

Step 1: Give a positive scalar α and a vector T satisfying $\|T\|_2 = \alpha$.

Step 2: Given β, solve the following optimization problem:

$$\begin{aligned} & min \quad \gamma \\ & s.t. \quad (14), (15), (16), (30), (31), (33), (33) \end{aligned} \tag{38}$$

If the above optimization problem is feasible, then the parameters of a desired fault detection filter can be obtained by

$$\hat{A} = X_3^{-1} \tilde{A}, \ \hat{B} = X_3^{-1} \tilde{B}, \ \hat{C} = \hat{C}. \tag{39}$$

Give $T = \begin{bmatrix} 0.1194 & 0.6073 \end{bmatrix}^T$, $\alpha = 0.6189$. To ensure that the augmented system (7) is sufficiently sensitive to faults, let $\beta = 0.8367$. By Algorithm 1, we can obtain the following parameters:

$$\hat{A} = \begin{bmatrix} 0.3550 & -0.2123 \\ 1.1347 & -0.0299 \end{bmatrix}, \hat{B} = \begin{bmatrix} -0.5866 \\ -0.2141 \end{bmatrix},$$
$$\hat{C} = \begin{bmatrix} 0.1079 & 0.6096 \end{bmatrix}$$

with the H_∞ performance index $\gamma = 1.5867$.

Next, simulation is carried out to evaluate the effectiveness of the fault detection strategy for this system. In this

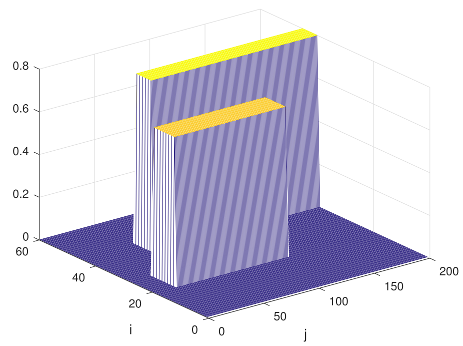

FIGURE 1. The fault $f(i,j)$.

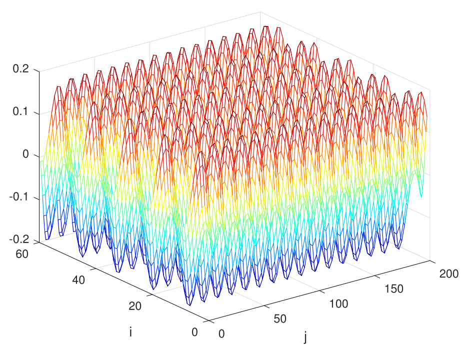

FIGURE 2. The disturbance $d(i,j)$.

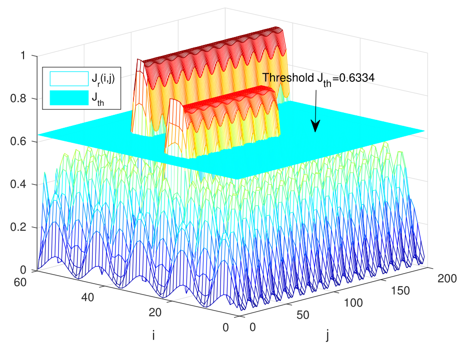

FIGURE 3. The residual evalution function $J_r(i,j)$ and the threshold J_{th} in three-dimensional space.

simulation, consider the following fault

$$f(i,j) = \begin{cases} 0.7, & 20 \le i \le 27, 20 \le j \le 120 \\ 0.8, & 40 \le i \le 45, j \ge 50 \\ 0, & otherwise \end{cases} \tag{40}$$

and a disturbance $d(i,j) = 0.1\sin(0.5i) + 0.1\cos(0.5j)$, which are shown in Figs. 1 and 2, respectively. Let the initial and boundary conditions be $x^h(i,0) = 0, x^v(0,j) = 0$, $\hat{x}^h(i,0) = 0, \hat{x}^v(0,j) = 0, \forall i,j$. The residual evaluation function is selected by (35). The threshold is chosen as $J_{th} = 0.6334$ by using [28, Algorithm 2] with $\delta = 0.1414$. The 3-D and 2-D plot of residual evaluation $J_r(i,j)$ and the threshold J_{th} are shown in Fig. 3 and Fig. 4, respectively.

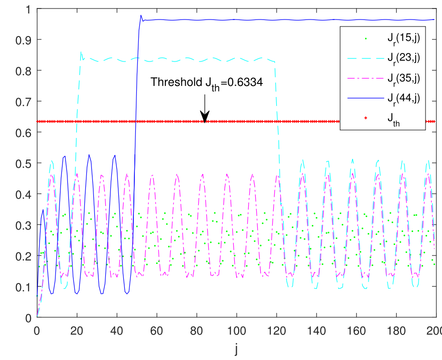

FIGURE 4. The residual evalution function $J_r(i,j)$ and the threshold J_{th} in two-dimensional space.

Fig. 3 shows that the residual evaluation is within the threshold limit when there is no fault and it exceeds the threshold value when the fault occurs. Variation of the residual evaluation $J_r(i,j)$ at several spatial points can be clearly examined in the 2-D plot as shown in Fig. 4. It can be easily seen from Fig. 4 that, at points $i = 15, 35$, the residual evaluation curve are below the threshold curve, which means that no fault occurs; at points $i = 23$ and $i = 44$, the residual curve are above the threshold curve when $20 \le j \le 120$ and $j \ge 50$ respectively, which means that the fault can be detected effectively.

V. CONCLUSIONS

This paper has investigated the problem of fault detection for 2-D Roesser system with stuck sensor faults. A fault detection filter has been designed to detect whether a sensor fault has occurred. This problem has been formulated as a multi-objective optimization problem, and an algorithm has been proposed to construct a desired fault detection filter. A residual evaluation function and a threshold have also been proposed for 2-D systems. The effectiveness of the proposed method has been shown by an example about stationary random field.

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