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# Adaptive Fractional Fuzzy Sliding Mode Control for Three-Phase Active Power Filter

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**ABSTRACT** This paper extends a conventional adaptive fuzzy sliding mode controller with integer order to fractional ones for a three-phase active power filter (APF) in order to track instruction current quickly and precisely and offset the harmonic current in the electrical grid. A fractional sliding surface is utilized to make the system work on the designed sliding surface stably. A fuzzy system with fractional adaptive laws is employed to precisely approximate the unknown parameters in the APF system. Simulations are given to demonstrate the excellent performance of the proposed controller. Moreover, the superiority of the proposed controller over the conventional adaptive fuzzy sliding mode controller is also illustrated by showing the smaller total harmonic distortion, which is a significant index to evaluate the current quality in the smart grid.

**INDEX TERMS** Sliding mode control, fuzzy system, fractional adaptive control.

## I. INTRODUCTION

Development of power technology has brought convenience to our lives, however, a lot of power quality problems have been generated. Harmonic current will decrease the energy efficiency in production, transmission, and utilization. APF is a new power electronic device which can produce compensating currents into the grid in order to offset the harmonic currents that may do great harm to the electric power system. Since APF performs better than passive filter, APF possesses broad prospects in application and develops rapidly.

Because of the widely application of APF in power systems over the past few years, many scholars have devoted a large quantity of efforts to make APF work in a more efficient way by applying intelligent control methods and they have gotten great achievements. Abdeslam *et al.* [1] introduced a neural method based on Adalines for the online extra extraction of the voltage components to recover a balanced and equilibrated voltage system. Wang *et al.* [2] presented comprehensive analysis and design for one-cycle controlled DC side APF on the basis of analyzing its circuit topology and basic principle. Fei and Hou [3] pointed out that the unknown parameters in the system can be approximated with a fuzzy controller and designed an adaptive controller with fuzzy control to adjust parameters online to track the instruction current. Rahmani *et al.* [4] derived a

proportional-integral control law by linearizing the inherently nonlinear shunt active power filter (SAPF) system so that the tasks of current dynamics and DC capacitor voltage dynamics became decoupled. Valiviita and Ovaska [5] introduced a multistage adaptive filtering system which generates the current reference delaylessly and accurately. To deal with the nonlinearity of APF, Hou and Fei [6] developed an adaptive fuzzy backstepping controller by combining the backingstepping method with adaptive fuzzy strategy in the design of current tracking control system. Shu *et al.* [7] took a new approach using filed-programmable gate array (FPGA) to implement a fully digital control algorithm of active power filter. Fei *et al.* [8] proposed an adaptive control technology and PI-fuzzy compound control technology to track instruction current and control DC voltage. Hua *et al.* [9] discussed a Lyapunov based control strategy to provide a general design framework for the model-based harmonic current elimination and reactive power compensation. Fang *et al.* [10] described a radical basis function (RBF) neural model reference adaptive control to control a single-phase APF and obtained excellent performance in tracking given instructional signal.

A common feature of the researches above is that they are based on the integer order. With the development of

engineering applications, fractional calculus becomes widespread concerned during these years, many scientists applied fractional order controllers in order to achieve better performance and have also obtained some progress. Shi *et al.* [11] extended the conventional model reference adaptive control (MRAC) systems to fractional ones by designing a fractional adaptation law for the fractional plant and fractional reference model. Luo and Liu [12] proposed a fractional order integration method for updating the parameters of fuzzy systems. Wang *et al.* [13] addressed the design of sliding mode controller (SMC) for an uncertain chaotic fractional order economic system. By driving the state trajectory to the switching surface and maintaining a sliding-mode condition in spite of the uncertainties, Chen *et al.* [14] proposed the method for designing sliding-mode controller for a class of fractional-order linear interval systems with the external disturbances. Yin *et al.* [15] designed an adaptive sliding mode controller for a novel class of fractional-order chaotic systems with uncertainty and external disturbance to realize chaos control. Lin and Lee [16] dealt with chaos synchronization between two different uncertain fractional order chaotic systems based on adaptive fuzzy sliding mode control. Ladaci and Charef [17] put forward a fractional model reference adaptive control algorithm for single input single output (SISO) plants to guarantee the closed loop stability with a satisfying level of performances and strong robustness. Takahiro and Ohmori [18] designed an adaptive law containing fractional order integrator in adaptive law to improve the response in the case of unsatisfying model matching condition. Odibat [19] presented an adaptive feedback control scheme for the synchronization of two coupled chaotic fractional order systems with different fractional orders. Tabatabaei [20] employed a fractional order MRAC method to control the angular velocity of the permanent magnet synchronous motor (PMSM).

The combination of intelligent controllers with fractional orders for APF can improve the tracking performance and stabilize the DC voltage in a short time. It is a very interesting work to combine fuzzy controller with adaptive fractional sliding mode control for the APF system together which has not been investigated so far in the literature. In this paper, an adaptive fractional fuzzy sliding mode control for three-phase active power filter is proposed. The proposed control strategy has the following advantages:

(1) The conventional adaptive sliding mode controller is extended to fractional ones for three-phase active power filter, in addition, a fuzzy controller is also applied to the APF system. By applying fractional modules to the sliding mode controller and adaptive controller, the system can obtain an extra degree of freedom apart from integer orders. That means the two fractional controllers access more adjusted parameters than integer order controllers so that the performance of the controller can be improved.

(2) Sliding mode controller is implemented to make the system work on the designed sliding surface stably. Fuzzy controller is proposed to approximate the unknown dynamic

model term. The whole control strategy with fractional orders is also derived in the sense of Lyapunov stability theory to guarantee the stability of the proposed system while improving total harmonic distortion (THD) performance and stabilizing DC voltage in a short time.

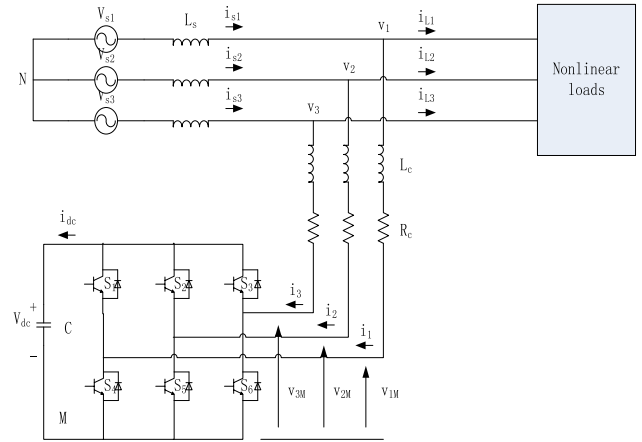


FIGURE 1. Block diagram of main circuit of APF.

## II. PRINCIPLE OF ACTIVE POWER FILTER

The investigation in this paper is based on the parallel-voltage type of APF. The three-phase alternating current can be seen almost everywhere, so the three-phase, three-wire system will be discussed. The block diagram of the three-phase three wire active power system is shown in Fig.1. The mathematical model of APF can be described next. According to circuit theory and Kirchhoff's theorem, we can get the following state equations:

$$\begin{cases} v_1 = L_c \frac{di_1}{dt} + R_c i_1 + v_{1M} + v_{MN} \\ v_2 = L_c \frac{di_2}{dt} + R_c i_2 + v_{2M} + v_{MN} \\ v_3 = L_c \frac{di_3}{dt} + R_c i_3 + v_{3M} + v_{MN} \end{cases} \quad (1)$$

The parameters of  $L_c$  and  $R_c$  are the inductance and resistance of the APF respectively,  $v_{MN}$  is the voltage between point M and N.

Taking the equations in (1) and combining with the absence of the zero-sequence in the three wire system currents yield:

$$v_{MN} = -\frac{1}{3} \sum_{m=1}^3 v_{mM} \quad (2)$$

In order to indicate the working status of insulated gate bipolar transistor (IGBT), we can define  $c_k$  as the switch function which goes as:

$$c_k = \begin{cases} 1, & \text{if } S_k \text{ is On and } S_{k+3} \text{ is Off,} \\ 0, & \text{if } S_k \text{ is Off and } S_{k+3} \text{ is On,} \end{cases} \quad k = 1, 2, 3 \quad (3)$$

Hence by writing  $v_{km} = c_k v_{dc}$ , then (1) becomes

$$\begin{cases} \frac{di_1}{dt} = -\frac{R_c}{L_c}i_1 + \frac{v_1}{L_c} - \frac{v_{dc}}{L_c}(c_1 - \frac{1}{3}\sum_{m=1}^3 c_m) \\ \frac{di_2}{dt} = -\frac{R_c}{L_c}i_2 + \frac{v_2}{L_c} - \frac{v_{dc}}{L_c}(c_2 - \frac{1}{3}\sum_{m=1}^3 c_m) \\ \frac{di_3}{dt} = -\frac{R_c}{L_c}i_3 + \frac{v_3}{L_c} - \frac{v_{dc}}{L_c}(c_3 - \frac{1}{3}\sum_{m=1}^3 c_m). \end{cases} \quad (4)$$

### III. ADAPTIVE FRACTIONAL FUZZY SLIDING MODE CONTROL

#### A. FRACTIONAL CALCULUS PRELIMINARIES

Define  ${}_aD_t^\alpha$  as the fundamental operator, where  $a$  and  $t$  are the bounds of the operation and  $\alpha$  is the order of fractional calculus. There are three commonly used definitions:

1. Grunwald–Letnikov definition

$${}_aD_t^\alpha f(t) = \lim_{h \rightarrow 0} h^{-\alpha} \sum_{j=0}^{\infty} (-1)^j \binom{\alpha}{j} (t - jh) \quad (5)$$

2. RL (Riemann-Liouville) definition

$${}_aD_t^\alpha f(t) = \frac{d^n}{dt^n} \left[ \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \right] \quad (6)$$

3. Caputo definition

$${}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (7)$$

In the following section, we denote  $D^\alpha$  instead of  ${}_aD_t^\alpha$ .

#### B. FUZZY LOGIC SYSTEMS

A fuzzy logic system is composed of some fuzzy rules, fuzzy and defuzzification strategies. A fuzzy logic system is usually constructed by series of fuzzy IF-THEN rules as:

$$R^l : \text{If } x_1 \text{ is } A_1^l \text{ and } \dots x_n \text{ is } A_n^l, \text{ then } y \text{ is } B^l$$

where  $A_i^l$  and  $B^l$  respectively represent the fuzzy sets of  $x_i$  and  $y$ , and  $l = 1, \dots, n$ ,  $n$  is the number of fuzzy rules.  $x_i$  and  $y$  follow the Gaussian membership functions:

$$\mu_{A_i^l}(x_i) = \exp\left(-\frac{(x_i - \psi_i)^2}{2\sigma_i^2}\right) \quad (8)$$

where  $\psi_i$  is the centre of the fuzzy set  $A_i^l$  and  $\sigma_i$  is the width of the fuzzy set.

The output of the system can be expressed using center-average defuzzifier, product inference and singleton fuzzifier.

$$y(x) = \frac{\sum_{l=1}^M h_l \left( \prod_{i=1}^n \mu_{A_i^l}(x_i) \right)}{\sum_{l=1}^M \left( \prod_{i=1}^n \mu_{A_i^l}(x_i) \right)} = \theta^T \xi(x)^T \quad (9)$$

where  $\mu_{A_i^l}(x_i)$  is the membership function of  $x_i$ ,  $\theta = (h_1, h_2, \dots, h_M)^T$  is adaptive parameter vector,

$\xi(x) = (\xi_1(x), \xi_2(x), \dots, \xi_M(x))$  is the vector of the fuzzy basis functions.

#### C. ADAPTIVE FRACTIONAL FUZZY SLIDING MODE CONTROLLER

The block diagram of adaptive fractional fuzzy sliding mode control system for APF is shown in Fig.2.

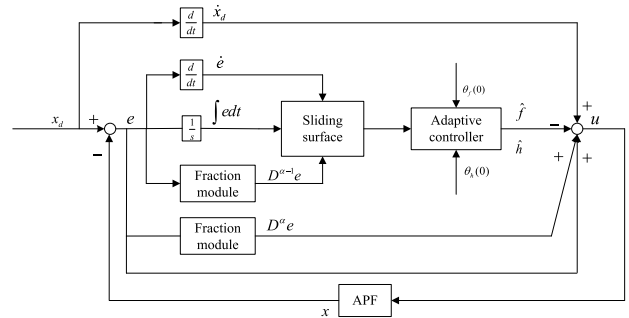


FIGURE 2. Block diagram of adaptive fractional fuzzy sliding mode control system for APF.

The three equations in (4) can be transformed into the following form:

$$\dot{x} = f(x) + bu + d \quad (10)$$

where  $x = [i_1, i_2, i_3]^T$ ,  $f(x) = -\frac{R_c}{L_c}i_k + \frac{v_k}{L_c}$ ,  $b = -\frac{v_{dc}}{L_c}$ , and  $u = \frac{v_{dc}}{L_c}(c_k - \frac{1}{3}\sum_{m=1}^3 c_m)$ ,  $k = 1, 2, 3$ .  $d$  is an unknown disturbance bounded such that  $\rho - |d| > \sigma_1$ , where  $\rho$  is a positive number and  $\sigma_1$  is not only a positive number but also tiny enough.

The tracking error is defined as  $e = x_d - x$ . Define a fractional order sliding surface as

$$s = -\lambda_1 e - \lambda_2 \int e - \lambda_3 D^{\alpha-1} e \quad (11)$$

where  $\lambda_1, \lambda_2, \lambda_3$  are designed and adjustable parameters.  $\alpha - 1$  is the fractional order of fractional derivate operation.

The derivative of the sliding surface is

$$\dot{s} = -\lambda_1(\dot{x}_d - f(x) - bu - d) - \lambda_2 e - \lambda_3 D^\alpha e \quad (12)$$

The equivalent controller is derived by setting  $\dot{s} = 0$  as

$$u_{eq} = \frac{1}{b\lambda_1}[-\lambda_1 f(x) - \lambda_1 d + \lambda_1 \dot{x}_d + \lambda_2 e + \lambda_3 D^\alpha e] \quad (13)$$

The sliding mode control signal is proposed as

$$u_{eq} = \frac{1}{b\lambda_1}[-\lambda_1 f(x) - \lambda_1 \rho \text{sgn}(s) + \lambda_1 \dot{x}_d + \lambda_2 e + \lambda_3 D^\alpha e] \quad (14)$$

where  $u_{sw} = \lambda_1 \rho \text{sgn}(s)$  is the switching controller.

Choose a Lyapunov function as

$$V = \frac{1}{2} s^T s \quad (15)$$

Then

$$\dot{V} = s^T \lambda_1 (d - \rho \text{sgn}(s)) \leq |s^T| \lambda_1 (|d| - \rho) \leq 0 \quad (16)$$

The control law (14) can not be applied to the system directly when  $f(x)$  and  $d$  are unknown. We can utilize the fuzzy system to acquire  $\hat{f}(x_k)$  and  $\hat{h}(s_k)$  to approximate  $f(x_k)$  and  $\rho \text{sgn}(s_k)$ ,  $k = 1, 2, 3$ .

Then the controller can be transformed into the following form with fuzzy input:

$$u_k = \frac{1}{b\lambda_1} [-\lambda_1 \hat{f}(x_k) - \lambda_1 \hat{h}(s_k) + \lambda_1 \dot{x}_{dk} + \lambda_2 e_k + \lambda_3 D^\alpha e_k] \quad (17)$$

where  $\hat{f}(x|\theta_{fk}) = \theta_{fk}^T \xi(x_k)^T$ ,  $\hat{h}(s|\theta_{hk}) = \theta_{hk}^T \phi(s_k)^T$ ,  $\xi(x_k)$  and  $\phi(s_k)$  are fuzzy vectors.  $\theta_{fk}^T$  and  $\theta_{hk}^T$  vary along with the following adaptive laws:

$$\dot{\theta}_{fk} = \lambda_1 r_1 s_k \xi(x_k)^T \quad (18)$$

$$\dot{\theta}_{hk} = \lambda_1 r_2 s_k \phi(s_k)^T \quad (19)$$

where  $r_1$  and  $r_2$  are positive constants,  $k = 1, 2, 3$ .

**Theorem:** The controller and adaptive laws are designed to make sure that  $x$  can track a given signal  $x_d$ , and the parameters  $\theta_f(t)$  and  $\theta_h(t)$  can be globally stable.

*Proof:* Define the optimal parameters as

$$\theta_f^* = \arg \min_{\theta_f \in \Omega_f} [\sup_{x \in R^n} |\hat{f}(x|\theta_f) - f(x)|] \quad (20)$$

$$\theta_h^* = \arg \min_{\theta_h \in \Omega_h} [\sup_{x \in R^n} |\hat{h}(s|\theta_h) - \rho \text{sgn}(s)|] \quad (21)$$

where  $\Omega_f$  and  $\Omega_h$  are aggregations of  $\theta_f$  and  $\theta_h$ .  $\sup f(x)$  represents the minimum upper bound of  $f(x)$  and  $\arg \min f(x)$  is the set of value of  $x$  for which  $f(x)$  has the largest value.

Define the minimum approximation error as

$$\omega = f(x) - \hat{f}(x|\theta_f^*) \quad (22)$$

When  $\theta$  gets the optimal parameter  $\theta_h^*$ ,  $\hat{h}$  turns into:

$$\hat{h}(s|\theta_h^*) = (\rho + \eta) \text{sgn}(s) \quad (23)$$

where  $|\omega| \leq \omega_{\max}$ ,  $\eta \geq 0$ , and  $|d(t)| \leq \rho$ .

Then the derivative of the sliding surface becomes

$$\begin{aligned} \dot{s}_k &= -\lambda_1 [\dot{x}_{dk} - f(x_k) - bu_k - d_k] - \lambda_2 e_k - \lambda_3 D^\alpha e_k \\ &= \lambda_1 [f(x_k) - \hat{f}(x_k)] - \lambda_1 \hat{h}(s_k|\theta_{hk}) + \lambda_1 d_k \\ &= \lambda_1 [\hat{f}(x_k|\theta_{fk}^*) - \hat{f}(x_k)] + \lambda_1 [\hat{h}(s_k|\theta_{hk}^*) - \hat{h}(s_k|\theta_{hk})] \\ &\quad - \lambda_1 \hat{h}(s_k|\theta_{hk}^*) + \lambda_1 d_k + \lambda_1 \omega_k \\ &= \lambda_1 \varphi_{fk}^T \xi(x_k)^T + \lambda_1 \varphi_{hk}^T \phi(s_k)^T + \lambda_1 d_k \\ &\quad + \lambda_1 \omega_k - \lambda_1 \hat{h}(s_k|\theta_{hk}^*) \end{aligned} \quad (24)$$

where  $\varphi_{fk} = \theta_{fk}^* - \theta_{fk}$ ,  $\varphi_{hk} = \theta_{hk}^* - \theta_{hk}$ ,  $k = 1, 2, 3$ .

Choose a Lyapunov function as

$$V_k = \frac{1}{2} (s_k^2 + \frac{1}{r_1} \varphi_{fk}^T \varphi_{fk} + \frac{1}{r_2} \varphi_{hk}^T \varphi_{hk}) \quad (25)$$

Then the derivative of  $V_k$  becomes

$$\begin{aligned} \dot{V}_k &= s_k \dot{s}_k + \frac{1}{r_1} \varphi_{fk}^T \dot{\varphi}_{fk} + \frac{1}{r_2} \varphi_{hk}^T \dot{\varphi}_{hk} \\ &= s_k [\lambda_1 \varphi_{fk}^T \xi(x_k)^T + \lambda_1 \varphi_{hk}^T \phi(s_k)^T + \lambda_1 d_k + \lambda_1 \omega_k \\ &\quad - \lambda_1 \hat{h}(s_k|\theta_{hk}^*)] + \frac{1}{r_1} \varphi_{fk}^T \dot{\varphi}_{fk} + \frac{1}{r_2} \varphi_{hk}^T \dot{\varphi}_{hk} \\ &= \frac{1}{r_1} \varphi_{fk}^T [\lambda_1 r_1 s_k \xi(x_k)^T + \dot{\varphi}_{fk}] + \frac{1}{r_2} \varphi_{hk}^T [\lambda_1 r_2 s_k \phi(s_k)^T \\ &\quad + \dot{\varphi}_{hk}] + \lambda_1 s_k \omega_k + \lambda_1 s_k d_k - \lambda_1 s_k \hat{h}(s_k|\theta_{hk}^*) \end{aligned} \quad (26)$$

Because  $\hat{h}(s_k|\theta_{hk}^*) = (\rho + \eta) \text{sgn}(s_k)$ ,  $\eta \geq 0$ ,  $|d(t)| \leq \rho$ , and  $\dot{\varphi}_{fk} = -\dot{\theta}_{fk}$ ,  $\dot{\varphi}_{hk} = -\dot{\theta}_{hk}$ , therefore

$$\begin{aligned} \dot{V}_k &= \lambda_1 s_k \omega_k + \lambda_1 s_k d_k - \lambda_1 s_k \hat{h}(s_k|\theta_{hk}^*) \\ &= \lambda_1 s_k \omega_k + \lambda_1 s_k d_k - \lambda_1 s_k [(\rho + \eta) \text{sgn}(s_k)] \\ &= \lambda_1 s_k \omega_k - \lambda_1 s_k \eta \text{sgn}(s_k) + \lambda_1 s_k d_k - \lambda_1 s_k \rho \text{sgn}(s_k) \\ &= \lambda_1 s_k \omega_k - \lambda_1 s_k \eta \text{sgn}(s_k) + \lambda_1 s_k [d_k - \rho \text{sgn}(s_k)] \\ &\leq \lambda_1 [s_k \omega_k - \eta |s_k|] \leq \lambda_1 [\omega_k |s_k| - \eta |s_k|] \\ &= -\lambda_1 |s_k| (\eta - |\omega_k|) \end{aligned} \quad (27)$$

Choose  $\eta \geq \omega_{k \max}$ , then  $\dot{V}_k \leq 0$  which is proved to be semi-negative definite. This implies that  $s(t)$  is bounded. Since  $V(0)$  is bounded and  $V(t)$  is bounded and non-increasing, we can conclude that  $\int_0^t |s| (|\omega| - \eta) d\tau \leq V(0) - V(t) \leq \infty$ . From (24), we can know  $\dot{s}(t)$  is bounded. Correspondingly  $s(t)$  is uniformly continuous. According to Barbalart lemma,  $s(t)$  will asymptotically converge to zero,  $\lim_{t \rightarrow \infty} s(t) = 0$ . It can be concluded that  $e$  will asymptotically converge to zero,  $\lim_{t \rightarrow \infty} e(t) = 0$ .

#### IV. SIMULATION STUDY

In this section, a simulation example is presented to testify the proposed adaptive fractional fuzzy sliding mode control and show the feasibility at the platform of Matlab/Simulink package with SimPower Toolbox.

We choose  $\mu = \exp\{-[x + 4 - 2(i - 1)]^2\}$ ,  $i = 1, \dots, 6$  as the membership functions, where  $i$  is the number of fuzzy rules. When  $i = 6$ , we can achieve 36 fuzzy rules to approximate the unknown system. The member function degree is shown in Fig.3.

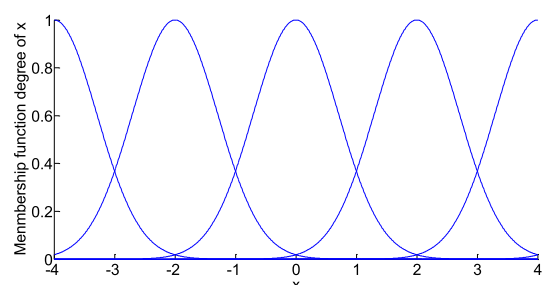


FIGURE 3. The member function degree of x.

Define the three membership functions as

$$\mu_{NM}(s) = \frac{1}{1 + \exp(5(s + 3))}, \quad \mu_{ZO}(s) = \exp(-s^2)$$

$$\mu_{PM}(s) = \frac{1}{1 + \exp(5(s - 3))}$$

We choose the parameters:  $\lambda_1 = 18, \lambda_2 = 5, \lambda_3 = 1$ , adaptive gain  $r_1 = 10000, r_2 = 500$ .

$$\theta_{hk} = [\theta_{hk1} \quad \theta_{hk2} \quad \theta_{hk3}]^T,$$

$$\theta_{fk} = [\theta_{fk1} \quad \theta_{fk2} \quad \theta_{fk3} \quad \theta_{fk4} \quad \theta_{fk5} \quad \theta_{fk6}]^T.$$

The inductance in the circuit of APF is  $10mH$  and the capacitance is  $100\mu F$ . Nonlinear load branch is rectifier bridge connecting parallel RC load, where  $R = 10 \Omega, L = 2mH$ . PI control is adopted for DC Voltage and the parameters are chosen as  $k_p = 0.03$  and  $k_i = 0$ .

At the time  $t = 0.04s$ , the switch of compensation circuit is closed and then the APF starts to work. The source currents of A phase before and after implementing adaptive fractional fuzzy sliding mode control are shown in Fig.4. We can clearly see that the source current is equal to the load current before connecting the APF, however, the source current tends to a steady state after a half cycle which is about  $0.01$  s. It is worth mentioning that the supply current is almost close to a sinusoidal wave which illustrating the good performance of APF in the steady state operation.

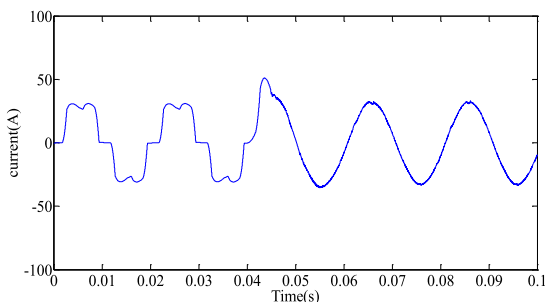
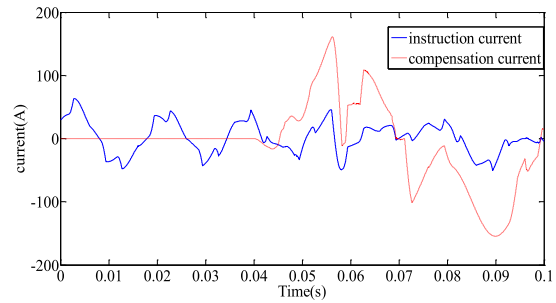
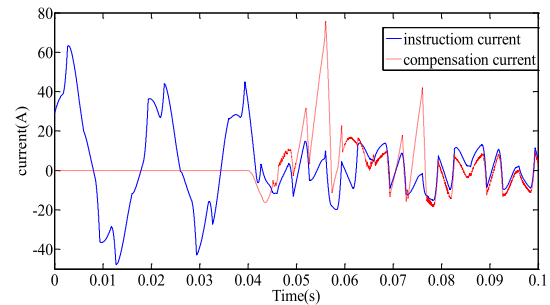


FIGURE 4. The improved source current of A phase.

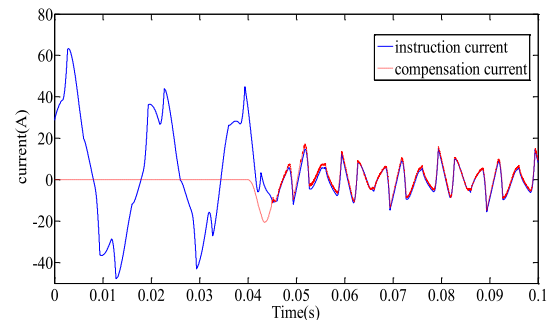
Instruction current and compensation current with different values of fractional order are given together in Fig.5. It can be seen that if  $\alpha$  is too small ( $\alpha = 0.3$ ), the tracking quality is terrible. While  $\alpha = 0.6$ , the compensation current can gradually track the instruction current, however, the effect is not excellent and the time spent in tracking the current is too long. If we choose  $\alpha = 0.8$ , it is obvious that the compensation current can quickly track the instruction current. So in the following simulation, we choose  $\alpha = 0.8$  in order to acquire better effects. Correspondingly, Fig.6 shows the compensation current tracking error while  $\alpha = 0.8$ , further verifying the effectiveness of the proposed adaptive fractional fuzzy sliding mode control in tracking compensation current. Fig.7 shows the unqualified THD at the start of simulation. In Fig.8, after APF works, we can see that THD is 1.68%, which is far less than the harmonic standard of



(a)



(b)



(c)

FIGURE 5. Instruction current and compensation current with different values of fractional order. (a)  $\alpha = 0.3$ . (b)  $\alpha = 0.6$ . (c)  $\alpha = 0.8$ .

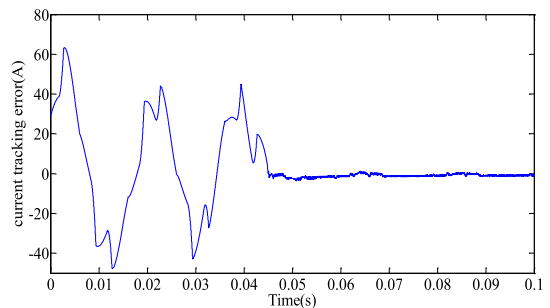


FIGURE 6. Compensation current tracking error.

IEEE of 5%. Adaptive parameters  $\theta_f$  and  $\theta_h$  are also shown in Fig.9 and Fig.10. It can be illustrated that the parameters of proposed adaptive fuzzy-sliding controller converge to stable constant values.

For the purpose of demonstrating that the adaptive fractional fuzzy sliding mode control has good robustness in

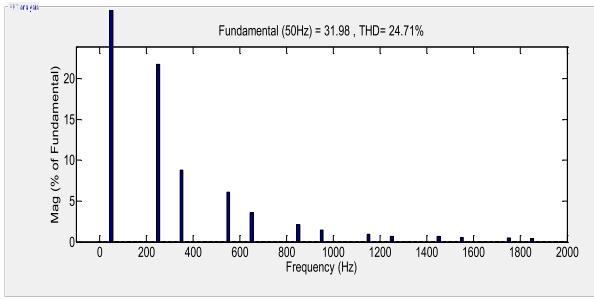


FIGURE 7. Harmonic spectrum of source current at 0s.

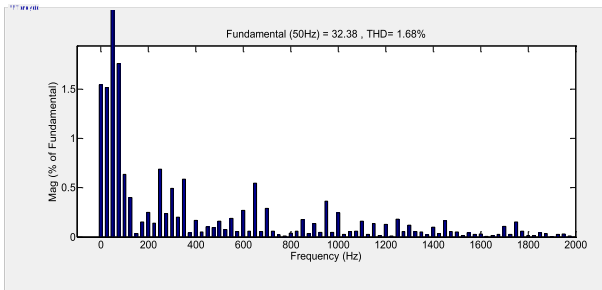


FIGURE 8. Harmonic spectrum of source current at 0.06s.

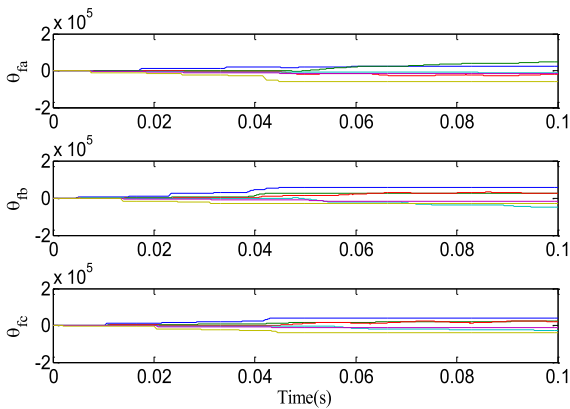


FIGURE 9. Adaptive law  $\theta_f$ .

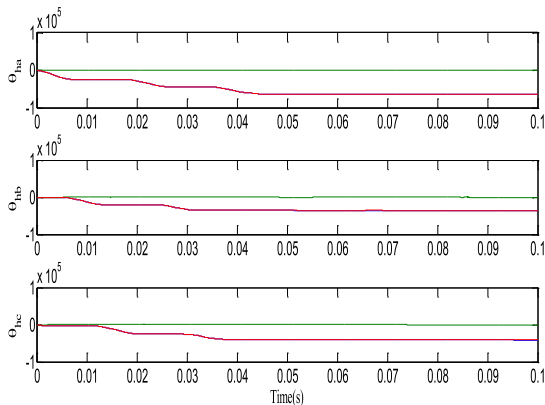


FIGURE 10. Adaptive law  $\theta_h$ .

the presence of load changes, we add the loads in a ladder-type increase. We add the same loads at the time 0.1s and 0.2s in order to make sure that the applied loads increase.

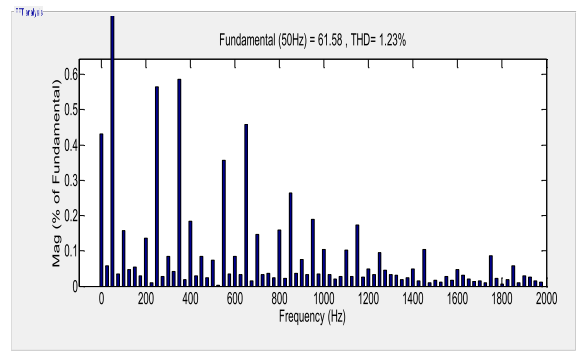


FIGURE 11. Harmonic spectrum of source current at 0.16s.

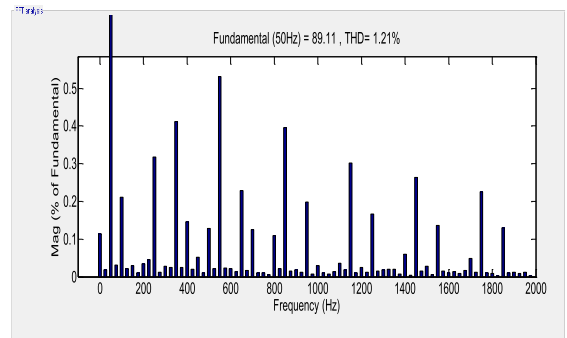


FIGURE 12. Harmonic spectrum of source current at 0.26s.

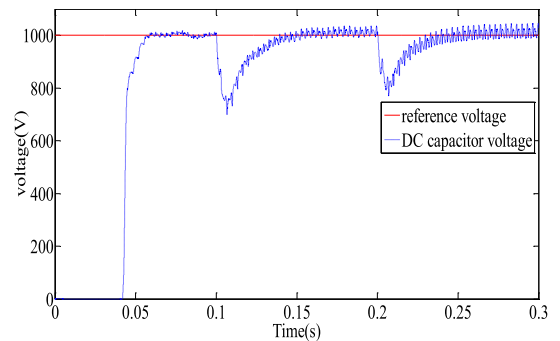


FIGURE 13. DC capacitor voltage with the load increasing.

In Fig.11 and Fig.12 we can see the harmonic spectrum of source current in different working situations and THD is still under 5% which means the system has strong robustness. In addition, the DC capacitor voltage also can tend to be stable by implementing the PI controller. It can be seen in Fig.13 that the DC capacitor voltage can also be adjusted to a stable status regardless of the changes of the applied load.

At last, comparison between adaptive fuzzy sliding control system with fractional control and without fractional control is also given so as to show the superiority of the proposed adaptive fractional fuzzy sliding mode control. We can clearly see the better THD performance with fractional control in Table.1. The performance of APF is obviously better than that of the conventional integral control methods. In the

**TABLE 1.** THD in proposed controller and normal controller.

Time	THD(%)	
	Adaptive fractional fuzzy sliding mode control	Conventional adaptive fuzzy sliding mode control
0	24.71%	24.71%
0.06s	1.68%	1.96%
0.16s	1.23%	1.49%
0.26s	1.21%	1.44%

simulation, the overheads of the two control strategies are all about 17 minutes, and it shows the computations caused by using the fractional order calculations will not increase the complexity in the simulation.

## V. CONCLUSION

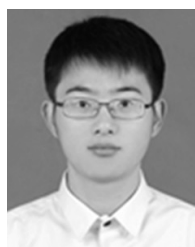
In this paper, an adaptive fractional fuzzy sliding mode control for three-phase active power filter has been put forward and verified. First of all, a fractional sliding surface is defined. The unknown parameters in the fractional sliding mode controller can be approached precisely by applying the fuzzy system with adaptive laws. The excellent dynamic performance, asymptotic stability and strong robustness are illustrated through the simulation result, showing the small tracking error, good THD performance and fast stable DC voltage compared with the conventional adaptive fuzzy sliding mode control.

In consideration of practical application, the biggest challenge is to conduct real-time experiment to verify the good performance of proposed strategy. The proposed adaptive fractional fuzzy sliding mode control can be realized with hardware such as digital signal processor (DSP). The experimental test bench is developing for the implement of the proposed control scheme in our laboratory. Other intelligent control strategies can also be investigated after the experimental test bench is completed.

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