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A Commercial Video-Caching System for Small-Cell Cellular Networks Using Game Theory

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ABSTRACT Evidence indicates that requesting video clips on demand accounts for a dramatic increase in data traffic over cellular networks. Caching part of popular videos in the storage of small-cell base stations (SBS) in cellular networks is an efficient method to reduce transmission latency and mitigate redundant transmissions. In this paper, we propose a commercial caching system consisting of a video retailer (VR) and multiple network service providers (NSPs). Each NSP leases its SBSs, with some price, to the VR for the purpose of making profits, and the VR, after storing popular videos in the rented SBSs, can provide better local video services to the mobile users, thereby gaining more profits. We conceive this system within the framework of a Stackelberg game by treating the SBSs as a specific type of resources. Then, we establish the profit models for both the NSPs and the VR based on stochastic geometry. We further investigate the Stackelberg equilibrium by solving the optimization problems in two cases, i.e., whether or not the VR has a budget plan on renting the SBSs. Numerical results are provided for quantifying the proposed framework by showing its efficiency on pricing and resource allocation.

INDEX TERMS Wireless caching, content-centric communications, small-cell networks, stochastic geometry, Stackelberg game.

I. INTRODUCTION

Wireless data traffic is expected to increase exponentially in the next few years, driven by a dramatic growth of mobile users (MU) and their bandwidth-hungry applications. There is evidence that MUs' downloading of on-demand videos is the major reason for the data avalanche over cellular networks [1]. There are numerous repetitive requests on the same videos from the MUs, such as online movies, leading to redundant transmissions. Fortunately, this redundancy can be reduced by locally storing popular videos, known as caching, into the memory of intermediate network nodes, effectively forming a local caching system [1]–[5]. This local caching brings video content closer to the MUs and alleviates redundant data transmissions via

redirecting the downloading requests to the intermediate nodes.

Generally, wireless data caching consists of two stages: data placement and data delivery [6]. In the data placement stage, popular videos are cached into local storages during off-peak time, while in the data delivery stage, requested videos are delivered from the local caching system to the MUs. Recent works advance the caching technology in device-to-device (D2D) networks and wireless sensor networks [7]–[9]. Specifically in [7], a caching scheme is proposed for a D2D based cellular network on the MUs' caching of popular video content. In this scheme, the D2D cluster size is optimized for reducing the downloading delay. In [8] and [9], the authors propose novel caching schemes for

wireless sensor networks, where the protocol model of [10] is adopted.

As small-cell embedded architectures will be prevailing in future cellular networks [11]–[16], caching relying on small-cell base stations (SBS), namely, small-cell caching, is a promising trend. The advantages brought about by the small-cell caching are threefold. First, popular videos are pushed closer to the MUs when cached in SBSs, reducing the transmission latency. Second, redundant data transmissions over SBSs' back-haul channels are mitigated, which are usually expensive [17]. Third, the majority of video traffic is offloaded from macro-cell base stations to SBSs.

In [18], a small-cell caching scheme, called 'Femto-caching', is proposed for a cellular network embedded with SBSs, where the data placement at the SBSs is optimized in a centralized manner for reducing the transmission delay imposed. In [19], the small-cell caching is investigated in the context of stochastic networks. The average performance is developed via stochastic geometry [20], [21], where the distribution of network nodes are modeled by Poisson point process (PPP).

From above discussions, current studies on wireless caching usually consider a specific aspect of caching, e.g., data placement optimization. However, a practical caching system should be coupled with many issues other than data placement. From a commercial or financial perspective, it will be more interesting to consider a comprehensive system, including the issues such as pricing on video streaming, renting local storages, and so on. Such a system may consist of video retailers (VR), network service providers (NSP) and MUs. The VRs, e.g., Youtube, purchase copyrights from video producers and publish the videos on their web-sites. The NSPs are typically operators of cellular networks, who are in charge of network facilities, such as macro-cell base stations and SBSs.

In such a commercial caching system, the VRs make profits by providing video streaming services to the MUs. As the central servers of the VRs, which store the popular videos, are usually located at backbone networks and far away from the MUs, an efficient solution is to locally cache these videos for reducing the transmission latency, thereby attracting more customers. These local-caching demands raised by the VRs offer the NSPs profitable opportunities from leasing their resources, i.e., the SBSs. In this sense, both the VRs and NSPs are the beneficiaries from the local caching system. However, each entity is selfish and wishes to maximize its own benefit, raising a competition and optimization problem, which can be effectively solved under the game theory framework.

We note that game theory has been successfully applied to wireless communications for solving competition problems on network resources [22], [23]. Stackelberg game is a commonly-used strategic game that consists of leaders and followers competing with each other for certain resources [24]. The leader moves first and the followers move subsequently.

In this paper, we will develop a commercial caching prototype, consisting of a video retailer (VR) and multiple network service providers (NSP), drawing upon a Stackelberg game framework. We consider the NSPs to be the leaders and the VR the follower, while treat the SBSs in the charge of the NSPs as a specific type of resources. The NSPs set the prices of leasing their SBSs, while the VR decides on renting a fraction of the SBSs based on the prices charged.

Specifically, we first follow the theory of stochastic geometry [20], [21], and model the MUs and SBSs in the network as two different ties of a Poisson point process (PPP) [25]. Under this network model, we define the concept of a successful video downloading event when an MU obtains the requested video directly from the storage of an SBS. Then we derive the probability of this event with a closed-form expression. Next, we establish the profit models for both the NSPs and the VR based on the probability derived. Furthermore, we investigate the Stackelberg equilibrium by solving the optimization problems in two cases, i.e., whether or not the VR has a deliberate budget plan on spending how much money in renting the SBSs. Numerical results are finally provided to quantify the proposed framework by showing its efficiency on pricing and resource allocation.

The rest of this paper is organized as follows. We describe the system model in Section II and investigate the caching procedure in Section III. The profit models and a Stackelberg game framework are formulated in Section IV. In Section V and VI, we investigate Stackelberg equilibriums for the two cases via solving a series of optimization problems. Our numerical results are detailed in Section VII, while our conclusions are provided in Section VIII.

II. SYSTEM MODEL

We focus our attention on a commercial small-cell caching system consisting of L NSPs, one VR, and multiple MUs, where each NSP is in charge of a number of SBSs. Denote by \mathcal{V} the VR and by $\mathcal{L} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_L\}$ the set of the NSPs.

A. NETWORK MODEL

Let us consider a small-cell network composed of SBSs owned by the L NSPs, where the SBSs owned by the l -th NSP \mathcal{L}_l , $l = 1, \dots, L$, equipped with a transmission power P_l , are spatially distributed as a homogeneous PPP (HPPP) Φ_l of intensity λ_l . Here, the intensity represents the average number of SBSs in per unit area. The VR \mathcal{V} rents a fraction $\tau_l \in [0, 1]$ of SBSs from \mathcal{L}_l for the purpose of caching its video files. We assume that the rented SBSs of \mathcal{L}_l are uniformly selected by the VR, and thus the distribution of these SBSs can be modeled as a "thinned" HPPP with the intensity of $\tau_l \lambda_l$. Each MU is affiliated with an NSP and connects to one of its SBSs for accessing network services. We model the distribution of the MUs that are affiliated with \mathcal{L}_l as an independent HPPP Ψ_l of intensity ζ_l , $\forall l$.

The wireless down-link channels spanning from the SBSs to the MUs are independent and identically distributed (*i.i.d.*),

and modeled as the combination of path-loss and Rayleigh fading. Without a loss of generality, we conduct our analysis on a typical MU \mathcal{M} located at the origin. The path-loss between \mathcal{M} and an SBS located at x is denoted by $\|x\|^{-\alpha}$, where α is the path-loss exponent. The channel power of the Rayleigh fading between them is denoted by h_x , where $h_x \sim \exp(1)$. The noise at each MU is Gaussian distributed with variance σ^2 .

We assume that the SBSs from the same NSP transmit on the same channel, causing mutual interferences. At the same time, different NSPs are allocated with orthogonal channels. Hence, there are no interferences across NSPs. To simplify the notation, we utilize the location x of a rented SBS from \mathcal{L}_l to represent a point in the HPPP $\Phi_l, \forall l$. The received signal-to-interference-plus-noise ratio (SINR) at a typical MU \mathcal{M} in Ψ_l from an SBS \mathcal{B} in Φ_l , located at x_l , can be expressed as

$$\rho(x_l) = \frac{P_l h_{x_l} \|x_l\|^{-\alpha}}{\sum_{x \in \Phi_l \setminus x_l} P_l h_x \|x\|^{-\alpha} + \sigma^2}, \quad (1)$$

where $\sum_{x \in \Phi_l \setminus x_l} P_l h_x \|x\|^{-\alpha}$ represents the interference from the SBSs of Φ_l except for \mathcal{B} .

Definition 1 (Coverage): An MU is defined to be “covered” by an SBS located at x as long as $\rho(x)$ is no lower than a pre-set SINR threshold δ .

Generally, an MU can be covered by multiple SBSs.

B. VIDEO POPULARITY

We now model the popularity distribution, i.e., the distribution of request probabilities, among video files. Denote by $\mathcal{F} = \{\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_F\}$ a video set, which consists of F popular video files to be possibly cached in the SBSs and then requested frequently by the MUs. The popularity distribution among \mathcal{F} is represented by a vector $q = [q_1, q_2, \dots, q_F]$. That is, the MUs make independent requests of the f -th video file $\mathcal{F}_f, f = 1, \dots, F$, with the probability of q_f .

Generally, the vector q can be modeled by a Zipf distribution [26], i.e.,

$$q_f = \frac{1/f^\beta}{\sum_{i=1}^F 1/i^\beta}, \quad \forall f, \quad (2)$$

where the exponent β is a positive value, characterizing the file popularity. A larger β corresponds to a higher content reuse, i.e., the most popular files account for the majority of requests. From Eq. (2), we can see that \mathcal{F}_1 has the highest popularity, while \mathcal{F}_F has the lowest one.

III. CACHING PROCEDURE

There are three stages in our caching system. In the first stage, the VR \mathcal{V} purchases the copyrights of popular videos in \mathcal{F} from video producers. In the second stage, the VR negotiates with the L NSPs on renting their SBSs for caching its popular videos. In the third stage, the MUs connect to the SBSs to obtain their desired videos.

We now explain the last two stages with more details. In the second stage, upon obtaining popular videos, the VR

negotiates with the NSPs on renting their SBSs and then caches these video clips into the memories of the rented SBSs. We assume that the SBSs owned by \mathcal{L}_l are equipped with a specified storage of Q_l , i.e., each SBS of \mathcal{L}_l can store Q_l video files. Meanwhile, each SBS of \mathcal{L}_l is required to cache the most popular Q_l files $\mathcal{F}_1, \dots, \mathcal{F}_{Q_l}$. Based on the prices offered by the NSPs, the VR then decides to rent a fraction τ_l of SBSs from \mathcal{L}_l . In the third stage, the MUs start to download videos on demand. Generally, an MU will connect to the nearest SBS that caches the requested files. To evaluate the downloading performance, we first introduce the following definition.

Definition 2 (Successfully Download Event $\mathcal{E}_{l,f}$): When an MU \mathcal{M} affiliated with \mathcal{L}_l demands \mathcal{F}_f , the request will be redirected to the nearest SBS \mathcal{B} in Φ_l that caches \mathcal{F}_f . If \mathcal{M} is covered by \mathcal{B} , the requested video can be obtained directly from \mathcal{B} , and we define such an event by $\mathcal{E}_{l,f}$.

In the case that the event $\mathcal{E}_{l,f}$ does not occur, the MU \mathcal{M} will connect to the central server of \mathcal{V} , posited at backbone networks, for the requested video clip, causing a high transmission latency. Regarding the probability $\Pr(\mathcal{E}_{l,f})$ of the event $\mathcal{E}_{l,f}$, we have the following theorem based on the stochastic geometry theory.

Theorem 1: The probability $\Pr(\mathcal{E}_{l,f}), \forall l, f$, can be expressed as

$$\Pr(\mathcal{E}_{l,f}) = \begin{cases} \int_0^\infty \exp(-\pi(1-\tau_l)\lambda_l C(\delta, \alpha) z^2) \\ \pi \tau_l \lambda_l \exp(-\pi \tau_l \lambda_l A(\delta, \alpha) z^2) \\ \exp(-\frac{z^\alpha \delta}{P_l} \sigma^2) \exp(-\pi \tau_l \lambda_l z^2) dz^2 & f \leq Q_l, \\ 0 & F \geq f > Q_l, \end{cases} \quad (3)$$

where we have

$$A(\delta, \alpha) \triangleq \frac{2\delta}{\alpha-2} {}_2F_1\left(1, 1-\frac{2}{\alpha}; 2-\frac{2}{\alpha}; -\delta\right),$$

$$C(\delta, \alpha) \triangleq \frac{2}{\alpha} \delta^{\frac{2}{\alpha}} B\left(\frac{2}{\alpha}, 1-\frac{2}{\alpha}\right). \quad (4)$$

Furthermore, ${}_2F_1(\cdot)$ in the function $A(\delta, \alpha)$ is the hypergeometric function and the Beta function in $C(\delta, \alpha)$ is formulated as $B(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt$.

Proof: Please refer to Appendix A. ■

Generally, the power of interference in a network is much greater than that of the noises. By assuming that $\frac{\sigma^2}{P_l}$ goes to zero, we further simplify Eq. (3) in the following corollary.

Corollary 1: In the case that $\frac{\sigma^2}{P_l}$ goes to zero and $f \leq Q_l$, we have

$$\Pr(\mathcal{E}_{l,f}) = \frac{\tau_l}{(1-\tau_l)C(\delta, \alpha) + \tau_l A(\delta, \alpha) + \tau_l}. \quad (5)$$

Proof: Please refer to Appendix B. ■

Remark 1: According to *Corollary 1*, when interference is dominant, the probability $\Pr(\mathcal{E}_{l,f})$ is independent of both the transmission power P_l and the intensity λ_l .

IV. PROBLEM FORMULATION

In our commercial caching system, the VR \mathcal{V} intends to rent a fraction τ_l of SBSs from the NSP $\mathcal{L}_l, \forall l$, for caching its popular videos, since it can make profit from providing the MUs with faster downloading services and mitigating unnecessary data traffic caused by MUs' repetitive downloading requests. At the same time, the NSPs will also be able to gain profits from renting their SBSs. Both the VR and the NSPs try to maximize their gains.

We first model the profits of the VR and the NSPs obtained from the caching system. Average profit is developed based on the stochastic geometry in terms of per unit area and per unit period ($/UAP$), e.g., $/month \cdot km^2$. Then we present the Stackelberg game formulation for the caching system. The equilibrium of the proposed game will also be investigated.

A. PROFIT MODELING

1) AVERAGE PROFIT OF VR

For the VR \mathcal{V} , the revenue gained, with the help of local caching, is from \mathcal{V} 's providing fast downloading services as well as mitigating the traffic from its central server. We denote by s the profit acquired by the VR when an MU downloads a video clip from the local caching system, and denote by K the number of video requests from each MU on average within an unit period. Then the overall income/ UAP of \mathcal{V} is calculated as

$$S^{cache} = \sum_{l=1}^L \sum_{n=1}^{Q_l} K \zeta_l q_n \Pr(\mathcal{E}_{l,n}) s. \tag{6}$$

Meanwhile, the VR needs to pay for renting the SBSs. We denote by s_l the price for renting an SBS from \mathcal{L}_l during an unit period. Then the overall payment/ UAP made by the VR can be expressed as

$$S^{rent} = \sum_{l=1}^L \tau_l \lambda_l s_l. \tag{7}$$

We assume that the VR may make a budget on how much payment it will make in renting the SBSs. The budget is defined by S/UAP , i.e., $S = S^{rent}$.

Then the net profit/ UAP obtained by the VR is

$$S^{VR} = S^{cache} - S^{rent}. \tag{8}$$

2) AVERAGE PROFIT OF NSPs

The income of the NSPs comes from leasing their SBSs. Meanwhile, the NSPs need to pay for the cost of maintaining the local caching system. We denote by c_l such cost on each rented SBS of \mathcal{L}_l during a unit period. Then the net-profit of \mathcal{L}_l can be expressed as

$$S_l^{NSP} = (s_l - c_l) \tau_l \lambda_l. \tag{9}$$

B. STACKELBERG GAME FORMULATION

Stackelberg game is a strategic game consisting of leaders and followers competing with each other for certain

resources [24]. The leaders move first and the followers move subsequently. In our small-cell caching system, we model the NSPs as the leaders, and the VR as the follower. The NSP $\mathcal{L}_l, \forall l$, imposes a price s_l for leasing one of its SBSs to the VR during a unit period. We thus define the price vector $s \triangleq [s_1, s_2, \dots, s_L]$. After the vector s is set, the VR updates the fraction τ_l of the SBSs that it tends to rent from \mathcal{L}_l .

1) OPTIMIZATION FORMULATION OF THE LEADERS

Observe from the above game model that the objective of the NSP \mathcal{L}_l is to maximize its profit S_l^{NSP} formulated in Eq. (9). Note that the fraction τ_l is a function of the price s_l under the Stackelberg game formulation. This means that the fraction of the SBSs that the VR is willing to rent depends on the specific price charged for renting an SBS. If the price s_l is too high, the VR will choose not to rent any SBS from \mathcal{L}_l . At the same time, if s_l is set too low, \mathcal{L}_l cannot make any profit. Since the maximum payment made by the VR is S , the NSPs have to compete with each other on the price such that they can be selected by the VR whilst keep their profit maximized.

The optimization problem for each NSP can be summarized as follows.

Problem 1: The optimization problem of maximizing \mathcal{L}_l 's profit can be formulated as

$$\max_{s_l \geq 0} S_l^{NSP}(s_l, \tau_l), \quad \forall l. \tag{10}$$

2) OPTIMIZATION FORMULATION OF THE FOLLOWER

We adopt the expression of $\Pr(\mathcal{E}_{l,f})$ in Eq. (5). Then the profit gained by the VR in Eq. (8) can be further written as

$$\begin{aligned} S^{VR} &= \sum_{l=1}^L \sum_{f=1}^F K s \zeta_l q_f \Pr(\mathcal{E}_{l,f}) - \sum_{l=1}^L \tau_l \lambda_l s_l \\ &= \sum_{l=1}^L \frac{K s \zeta_l \tau_l \sum_{f=1}^{Q_l} q_f}{(1 - \tau_l) C(\delta, \alpha) + \tau_l A(\delta, \alpha) + \tau_l} - \sum_{l=1}^L \tau_l \lambda_l s_l. \end{aligned} \tag{11}$$

We can see from Eq. (11) that once the price vector s is fixed, the profit of \mathcal{V} depends on $\tau_l, \forall l$, i.e., the fraction of SBSs that are rented by \mathcal{V} . If \mathcal{V} increases the fraction τ_l , it will gain more profit, while at the same time, \mathcal{V} have to pay for renting more SBSs. Therefore, τ_l needs to be optimized for maximizing the profit of \mathcal{V} . By defining the fraction vector $\tau \triangleq [\tau_1, \dots, \tau_L]$, this optimization can be formulated as follows.

Problem 2: The optimization problem of maximizing \mathcal{V} 's profit can be written as

$$\max_{\tau \geq 0} S^{VR}(s, \tau). \tag{12}$$

Furthermore, in the case that the VR has a budget plan, there is the constraint $\sum_{l=1}^L \tau_l \lambda_l s_l = S$.

Problem 1 and *Problem 2* together form a Stackelberg game. The objective of this game is to find the Stackelberg Equilibrium (SE) points from which neither the leaders (NSPs) nor the follower (VR) have incentives to deviate.

In the following, we investigate the SE points for the proposed game.

C. STACKELBERG EQUILIBRIUM

For our Stackelberg game, the SE is defined as follows.

Definition 3: Let us define $s^* \triangleq [s_1^*, s_2^*, \dots, s_L^*]$, where $s_l^*, \forall l$, is a solution for *Problem 1*, and define $\tau^* \triangleq [\tau_1^*, \tau_2^*, \dots, \tau_L^*]$ a solution for *Problem 2*. Then the point (s^*, τ^*) is an SE for this Stackelberg game if for any (s, τ) with $s \geq 0$ and $\tau \geq 0$, the following conditions are satisfied:

$$\begin{aligned} S_l^{NSP}(s_l^*, \tau_l^*) &\geq S_l^{NSP}(s_l, \tau_l^*), \quad \forall l \\ S^{VR}(s^*, \tau^*) &\geq S^{VR}(s^*, \tau). \end{aligned} \quad (13)$$

Generally speaking, the SE of a Stackelberg game can be obtained by finding its perfect Nash Equilibrium (NE). In our proposed game, we can see that the NSPs strictly compete in a non-cooperative fashion. For a non-cooperative game, the NE is defined as the operating points at which no players can improve utility by changing its strategy unilaterally.

At the NSPs' side, the best response of each NSP is to solve *Problem 1*. To achieve this, we need to first find the best response function of the follower VR, based on which, we further solve the best response functions for the leaders. Therefore, in our game, we first solve *Problem 2* given a price vector s . Then with the obtained best response function τ^* of the VR, we solve *Problem 1* for the optimum price vector s^* .

In the following, we will have an in-depth investigation on this game-theoretic optimization. Specifically, we consider two cases in the optimization. In the first case, we assume that the VR does not have a concrete budget plan. That is, the VR will not take into account how much money it spends in renting SBSs. Thus the constraint $\sum_{l=1}^L \tau_l \lambda_l s_l = S$ is released during the optimization. In the second case, we assume that the VR has a budget by setting $\sum_{l=1}^L \tau_l \lambda_l s_l = S$ in the optimization.

V. OPTIMIZATION WITHOUT PAYMENT CONSTRAINT

In this section, we will solve the optimization problem in our game by assuming that the constraint $\sum_{l=1}^L \tau_l \lambda_l s_l = S$ is released. We first present the following lemma.

Lemma 1: Given a price vector $s = [s_1, \dots, s_L]$, the optimum solution of *Problem 2*, without considering the constraint $\sum_{l=1}^L \tau_l \lambda_l s_l = S$, can be expressed as

$$\tau_l^*(s_l) = \left[\frac{\sqrt{\frac{KsC(\delta, \alpha)\zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l s_l}} - C(\delta, \alpha)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} \right]^{\pm}, \quad \forall l, \quad (14)$$

where $[w]^{\pm}$ represents $0 \leq w \leq 1$.

Proof: The optimum solution τ_l^* can be obtained by deriving S^{VR} with respect to τ_l and solving $\frac{dS^{VR}}{d\tau_l} = 0$ under the constraint that $1 \geq \tau_l \geq 0$. ■

We can see from *Lemma 1* that $\tau_l^*(s_l) \propto s_l^{-\frac{1}{2}}$. If the price s_l is too high such that

$$s_l \geq s_l^{upper} \triangleq \frac{Ks\zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l C(\delta, \alpha)}, \quad (15)$$

where s_l^{upper} is calculated by letting $\tau_l^* = 0$, then there is $\tau_l^* \leq 0$. This means that, given the price $s_l \geq s_l^{upper}$, the VR will opt out renting any SBS from \mathcal{L}_l due to the high price charged. In this case, the NSP \mathcal{L}_l may need to lower the price s_l to some extent for ensuring $\tau_l^*(s_l) > 0$. Meanwhile, considering the cost c_l , \mathcal{L}_l will opt out leasing any SBS to the VR if there is $s_l \leq c_l$. Based on the above discussions, we have the following remark.

Remark 2: A necessary condition that the NSP \mathcal{L}_l participates in the game is $c_l < s_l < s_l^{upper}$.

On the other hand, if the price s_l given is too low such that

$$s_l \leq s_l^{lower} \triangleq \frac{KsC(\delta, \alpha)\zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l (A(\delta, \alpha) + 1)^2}, \quad (16)$$

where s_l^{lower} is calculated by letting $\tau_l^* = 1$, then there is $\tau_l^* \geq 1$. This means that the price given is too low such that the VR is willing to rent all the SBSs of \mathcal{L}_l . In this case, the NSP \mathcal{L}_l may need to increase the price s_l for acquiring more profit. Substitute $\tau_l^*(s_l)$ of *Lemma 1* into Eq. (9), and we have the following lemma regarding the optimum price s_l^* .

Lemma 2: Given the expression of τ_l^* in Eq. (14), the optimum solution of *Problem 1* can be expressed as

$$s_l^* = \max \left\{ \frac{1}{\left(x^{\frac{1}{3}} - \frac{1}{3c_l}x^{-\frac{1}{3}}\right)^2}, s_l^{lower} \right\}, \quad \forall l, \quad (17)$$

where

$$\begin{aligned} x &= \frac{C(\delta, \alpha)}{c_l U_l} + \sqrt{\left(\frac{C(\delta, \alpha)}{c_l U_l}\right)^2 + \frac{1}{27c_l^3}}, \\ U_l &\triangleq \sqrt{\frac{KsC(\delta, \alpha)\zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l}}. \end{aligned} \quad (18)$$

Proof: Please refer to Appendix C. ■

Remark 3: The optimum solution s^* in Eq. (18), combined with the solution of τ^* given by Eq. (14), constitutes the SE for the Stackelberg game.

From the above discussions, there exist no competitions among the NSPs when no payment constraint is imposed by the VR, since s_l^* and τ_l^* only depend on the parameters of \mathcal{L}_l .

VI. OPTIMIZATION UNDER BUDGET PLAN

We now focus our attention on the game theoretic optimization with the budget plan $\sum_{l=1}^L \tau_l \lambda_l s_l = S$. Usually, in this case, the VR does not have a sufficient S to rent enough SBSs. Thus, the money spent by the VR needs to be deliberately planned for renting the SBSs from those competent NSPs.

We first solve the optimization problem at the VR and NSPs. Thereafter, a distributed price-updating mechanism is proposed and investigated.

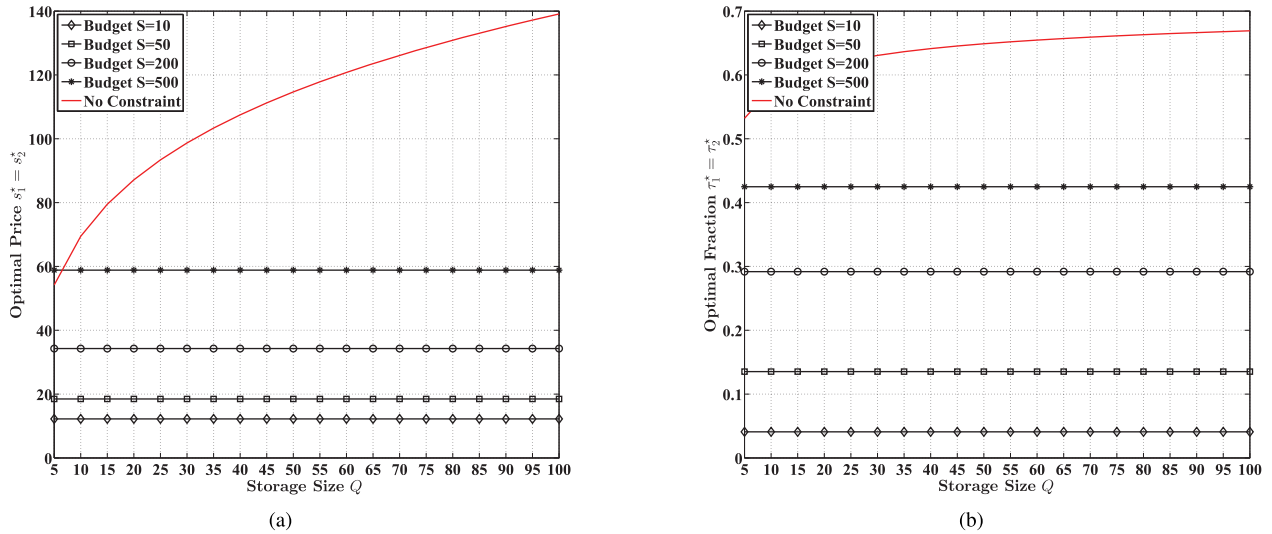


FIGURE 1. The optimal prices and fractions on each SBS versus the storage size Q in the symmetric 2-NSP scenario. In specific, sub-figure (a) depicts the prices change with Q , while sub-figure (b) depicts the fractions change with Q . We consider the budget plans $S = 10, 50, 200, 500$, and the case without budget.

A. OPTIMIZATION AT VR

Regarding the optimum solution at the VR’s side, we have the following theorem.

Theorem 2: Given a price vector $s = [s_1, \dots, s_L]$, the optimum solution $\tau_l^*, \forall l$, of *Problem 2* can be expressed as

$$\tau_l^*(s) = \begin{cases} 0 & \xi > \frac{Ks\zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l s_l C(\delta, \alpha)} - 1, \\ 1 & \xi < \frac{KsC(\delta, \alpha)\zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l s_l (A(\delta, \alpha) + 1)^2} - 1, \\ \sqrt{\frac{KsC(\delta, \alpha)\zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l s_l (1 + \xi)}} - C(\delta, \alpha) & \text{otherwise,} \\ \frac{\sqrt{KsC(\delta, \alpha)\zeta_l \sum_{f=1}^{Q_l} q_f}}{A(\delta, \alpha) - C(\delta, \alpha) + 1} & \end{cases} \quad (19)$$

where

$$\xi = \left(\frac{\sum_{j \in \mathcal{S}_2} \sqrt{KsC(\delta, \alpha)\lambda_j s_j \zeta_j \sum_{f=1}^{Q_j} q_f}}{(A - C + 1)(S - \sum_{j \in \mathcal{S}_1} \lambda_j s_j) + \sum_{j \in \mathcal{S}_2} \lambda_j s_j C(\delta, \alpha)} \right)^2 - 1, \quad (20)$$

where \mathcal{S}_1 and \mathcal{S}_2 are two sets of the subscripts of τ_l . For \mathcal{S}_1 , we have $\tau_j = 1, \forall j \in \mathcal{S}_1$. For \mathcal{S}_2 , we have $0 < \tau_j < 1, \forall j \in \mathcal{S}_2$.

Proof: Please refer to Appendix D. ■

The calculation of *Theorem 2* can be implemented in the following procedure. First, we utilize the conventional water-filling algorithm to optimize $\tau_l, \forall l$, without considering the constraint $\tau_l \leq 1$. Second, we check the optimization result. If, for example, there exists l' such that $\tau_{l'} > 1$, then we set $\tau_{l'} = 1$, and modify the original payment constraint to $\sum_{l=1, l \neq l'}^L \tau_l \lambda_l s_l = S - \lambda_{l'} s_{l'}$. Next, we execute a second round water-filling process on $\tau_l, \forall l, l \neq l'$ with the

updated constraint. By conducting this procedure iteratively, we finally achieve the optimum solution in *Theorem 2*.

Remark 4: From *Theorem 2* we can see that the optimum τ_l depends on the price vector s , which is contrasted by the result in *Lemma 1*, where the optimum τ_l only depends on s_l . This also means that VR’s budget plan will cause a competition among the NSPs.

B. OPTIMIZATION AT NSPs

Substituting Eq. (19) into Eq. (9), *Problem 1* can be rewritten as

$$\max_{s_l \geq 0} S_l^{NSP} = (s_l - c_l) \lambda_l \tau_l^*(s), \quad \forall l. \quad (21)$$

Note that Eq. (21) is a non-cooperative game by the NSPs. Due to the insufficient payment of the VR, the optimal pricing strategy of \mathcal{L}_l depends on other NSPs’ pricing strategies, causing competitions among the NSPs.

By taking the derivation of S_l^{NSP} to s_l and equating it to zero, we have

$$\frac{\partial S_l^{NSP}}{\partial s_l} = \lambda_l \tau_l^*(s) + (s_l - c_l) \lambda_l \frac{\partial \tau_l^*(s)}{\partial s_l} = 0, \quad \forall l. \quad (22)$$

Solving the above equations, we obtain the optimal price s_l^* . However, we can see that there is no closed-form for s_l^* , since each optimal price is related to other prices. One NSP needs to update its own price after the other NSPs change their prices. As such, the optimization process for the NSPs and the VR has to be conducted in an iterative manner.

C. ITERATIVE PROCESS

The iterative process can be summarized as follows. First, after rearranging (21), we have

$$s_l = \Gamma_l(s) \triangleq c_l - \frac{\tau_l^*(s)}{\partial \tau_l^*(s) / \partial s_l}. \quad (23)$$

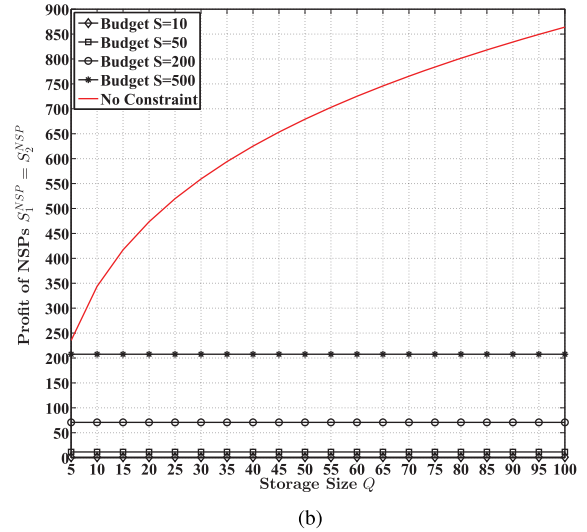
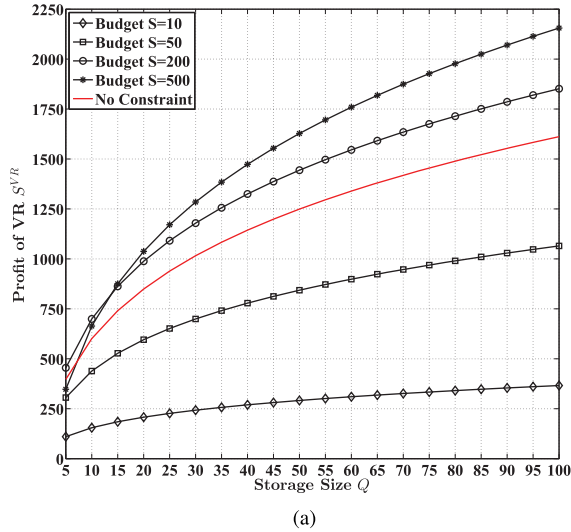


FIGURE 2. The various profits versus the storage size Q in the symmetric 2-NSP scenario. In specific, sub-figure (a) depicts the profit of the VR versus Q , while sub-figure (b) depicts the profit of each NSP versus Q . We consider the budget plans $S = 10, 50, 200, 500$, and the case without budget.

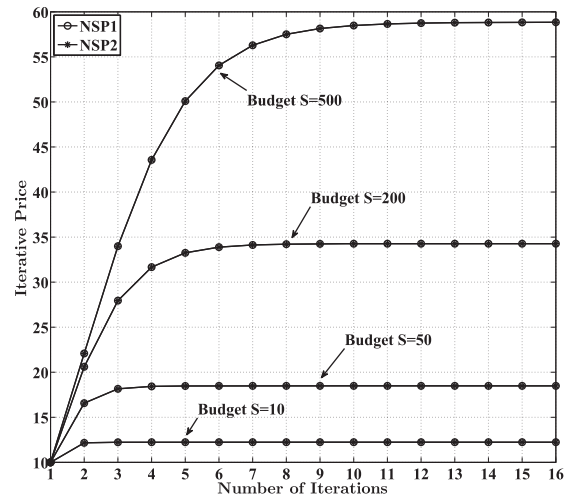
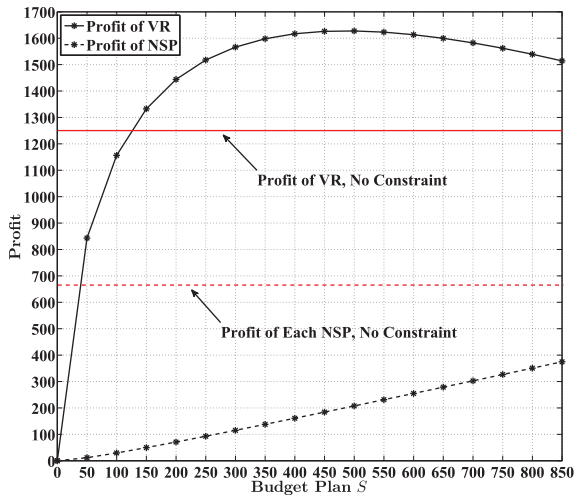


FIGURE 3. Profits of the VR and each NSP with and without budget plans. The storage size is set to $Q = 50$. Solid lines represents the profit of the VR, while dashed lines represents the profit of each NSP.

FIGURE 4. The updating of the prices for the two NSPs versus the number of iterations. The budget plans are set to $S = 10, 50, 200, 500$.

In order to calculate s_l in Eq. (23), each NSP communicates with the VR to obtain the values of $\tau_l^*(s)$ and $\partial \tau_l^*(s) / \partial s_l$. Then the updating of the NSPs' prices can be described by a vector of the form $s = \Gamma(s)$, where $\Gamma(s) = [\Gamma_1(s), \dots, \Gamma_L(s)]$. Consequently, an iterative method can be utilized to achieve the optimal solutions, expressed as

$$s^{(t+1)} = \Gamma^{(t)}(s^{(t)}), \quad (24)$$

where the superscription t represents the t -th iteration.

Note that $\Gamma^{(t)}$ may change in different iterations, since the two sets \mathcal{S}_1 and \mathcal{S}_2 can vary. To be specific in the optimization of τ at the VR in the t -th iteration, if, for example, there is $\tau_l > 1$, the value of τ_l will be set to one and s_l^* will be set to $s_l^{(t)}$. In this case, it is considered that the NSP \mathcal{L}_l

achieves its equilibrium and will quit the iteration for price update.

Remark 5: Once the iterative optimization process converges, the optimum solution s^* , combined with the solution of τ^* given by Eq. (19), constitutes the SE for the game.

VII. NUMERICAL RESULTS

In our numerical results, we investigate the system performance versus some key parameters. Although there are lots factors in our system, the storage size Q_l and budget constraint S are two important ones. Other factors, such as the Zipf parameter β and path-loss α , are usually not controlled by the game participants. Therefore, throughout this section, we set the $F = 500$, $\beta = 0.8$, $s = 1$, $K = 50$, $\alpha = 3$, and $\delta = 0.1$.

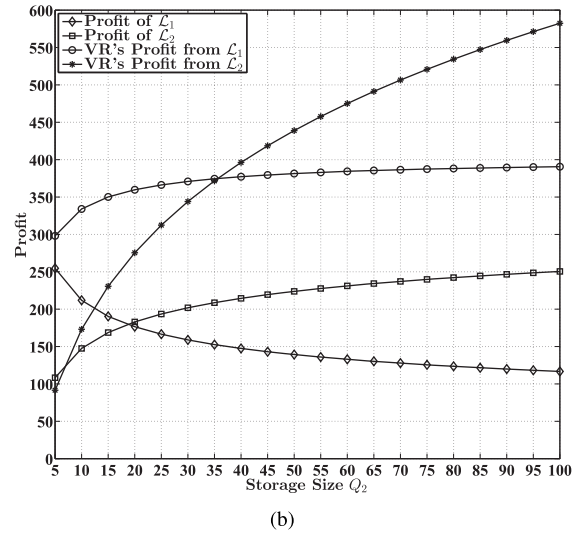
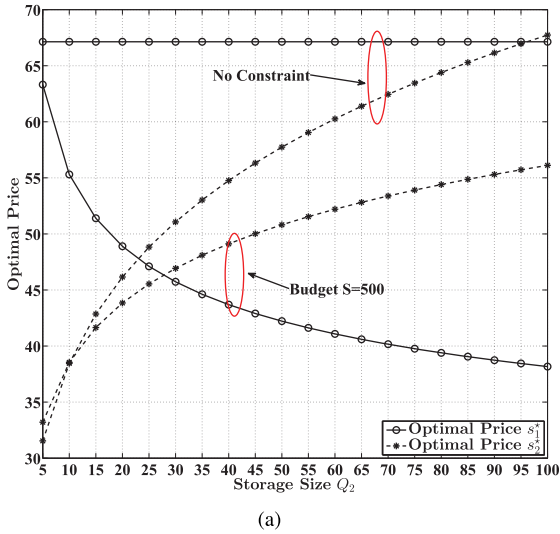


FIGURE 5. System performance versus the storage size Q_2 in the asymmetric 2-NSP scenario. In specific, sub-figure (a) depicts the optimal prices versus Q_2 without budget plans and with a budget $S = 500$, while sub-figure (b) depicts various profits versus Q_2 with the budget plan $S = 500$.

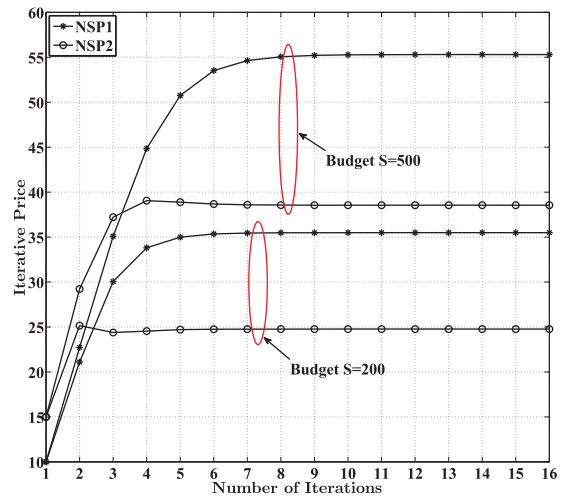
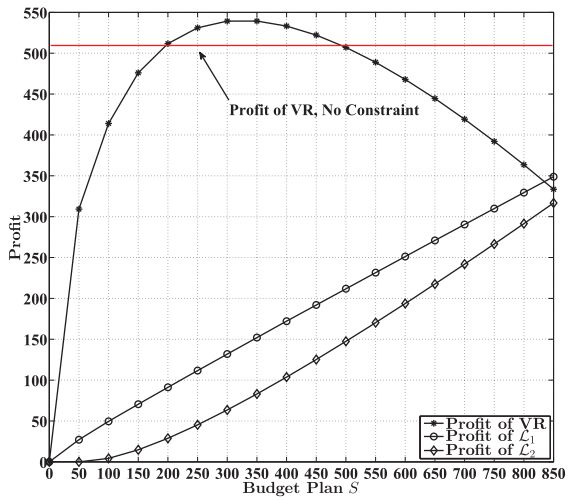


FIGURE 6. Profits of the VR and each NSP versus budget S . The storage size is set to $Q_1 = 30$ and $Q_2 = 10$.

FIGURE 7. The updating of the prices for the two NSPs versus the number of iterations. The budget plans are set to $S = 200, 500$.

A. 2 SYMMETRIC NSPs

First, we study a symmetric 2-NSP scenario with the two NSPs \mathcal{L}_1 and \mathcal{L}_2 . In this symmetric scenario, we set $c_1 = c_2 = 10$, $\lambda_1 = \lambda_2 = 10/km^2$ and $\zeta_1 = \zeta_2 = 80/km^2$. Also, we assume the storage size $Q \triangleq Q_1 = Q_2$, i.e., the SBSs of the two NSPs always have the same storage size. Furthermore, we consider the cases with and without budget plans. Particularly in the case with budget plans, we set $S = 10, 50, 200, 500$. Due the symmetric settings, we have $s_1^* = s_2^*$, $\tau_1^* = \tau_2^*$, and $S_1^{NSP} = S_2^{NSP}$.

Fig. 1 depicts the prices and the fractions on the rented SBSs versus the storage size Q . From the two sub-figures, we can see that in the case that there are budget plans, the optimal price and optimal fraction of the resources keep constant to Q . That is, the NSPs are unable to claim a higher price for renting their SBSs in the budget-planning case by deploying

a higher storage volume in these SBSs. This is due to the competition between the two symmetric NSPs under a strict budget imposed by the VR. Also from the figure, a higher budget of the VR leads to a higher price charged by the NSPs and a higher fraction of rented SBSs by the VR.

On the other hand, in the non-budget case, the optimal price and fraction are increasing functions of Q . Obviously in this case, the price charged by the NSP and the rented fraction by VR are much higher than those in the case with budget plans. That is, without a compulsory constraint on the budget, the VR needs to pay a much higher rent, i.e., $S^{rent} > S = 500$, to achieve the Stackelberg Equilibrium. However, this does not imply that the VR can gain a higher profit compared to the budget-planning case, as discussed in the following.

Fig. 2(a) and Fig. 2(b) illustrate the profits gained by the VR and each NSP versus Q , respectively. It is interesting to

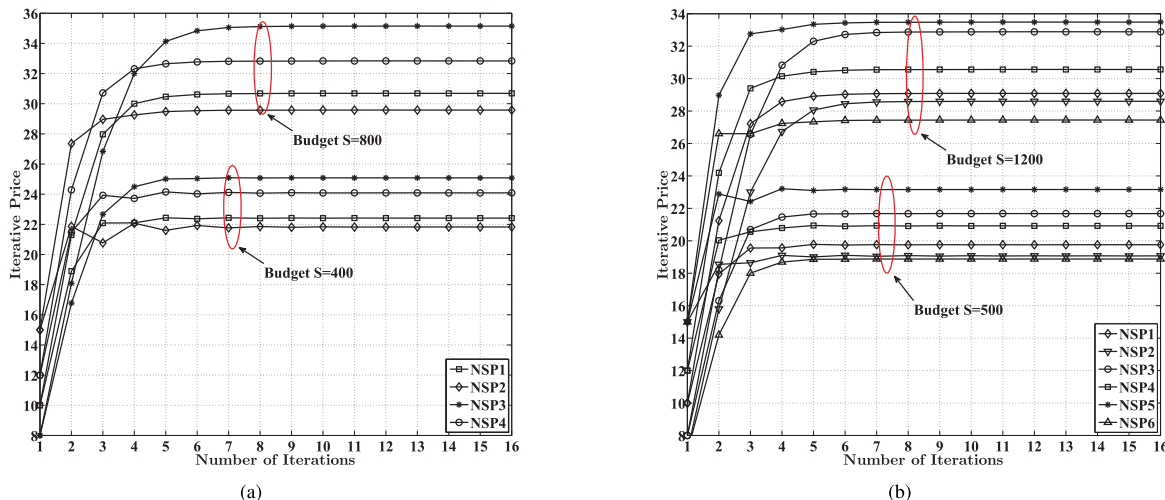


FIGURE 8. The updating of the prices in the 4-NSP scenario and the 6-NSP scenario versus the number of iterations.

observe that in the budget-planning case, the profit of each NSP keeps constant to Q , due to the constant s_1^* and τ_1^* , as shown in Fig. 1. Although each NSP’s profit increases with the growth of VR’s budget, they can gain a much higher profit in the non-budget case. As to the VR’s profit, we can see from the figure that when the VR has the budget plans $S = 200, 500$, it can gain a higher profit than that in the non-budget case, even though in the latter case the VR pays more rent $S^{rent} > S = 500$. In this sense, the VR may need to select an appropriate budget S to maximize its profit.

Fig. 3 shows the profits of the VR and each NSP with and without budget plans, where the storage size is fixed to $Q = 50$. We can see that when the budget $S = 500$, the VR can achieve the maximum profit around 1620, which is much higher than the non-budget case with the profit around 1250. Additionally, the profit of each NSP is almost linearly increase with S , while it is always lower than the one in the non-budget case. Since the VR and the NSPs are competing with each other, a proper budget S is generally helpful for the VR to maximize its profit.

Fig. 4 depicts the price updating versus the number of iterations for the two symmetric NSPs, where the storage size is set to $Q = 50$. It is observed that for a small budget, the iterative process converges quickly, generally around four or five iterations. In the case $S = 500$, it takes around 10 iterations before converging. Also, since the two NPSs are symmetric, the updated prices in each iteration for the two NSPs are the always the same.

B. 2 ASYMMETRIC NSPs

Then we focus on an asymmetric 2-NSP scenario, where $c_1 = 10, c_2 = 15, \lambda_1 = 10/km^2, \lambda_2 = 40/km^2, \zeta_1 = 50/km^2, \zeta_2 = 120/km^2$. Fig. 5 shows the optimal prices and various profits versus the storage size Q_2 with the fixed $Q_1 = 30$. In Fig. 5(a), we can see that for the non-budget case without a payment constraint, the optimal price s_1^* is constant to Q_2 , since there is no competitions

between the two NSPs, while s_2^* increases with the growth of Q_2 , meaning that increasing Q_2 will be beneficial to L_2 for charging a higher price. Furthermore, in the budget-planning case with $S = 500$, the optimal price s_1^* is decreasing but s_2^* is increasing with the growth of Q_2 . This is because the budget plan leads to competitions between the two NSPs: L_2 becomes more and more competent when increasing the storage size of its SBSs in contrast to the fixed storage Q_1 of L_1 .

In Fig. 5(b), we consider various profits with $S = 500$. The profit of L_1 is decreasing but the profit of L_2 is increasing with Q_2 . This is consistent with the trends of their prices with Q_2 in Fig. 5(a): A higher price charged by L_2 leads to its higher profit. We also decompose the VR’s profit into two parts, with one contributed by L_1 , and the other one contributed by L_2 . We can see that both parts of the profits increase with regard to Q_2 . However, the profit contributed by L_2 improved much more rapidly when Q_2 increases. Fig. 6 illustrates the profits of the VR and each NSP with and without budget plans, where the storage size is fixed to $Q_1 = 30$ and $Q_2 = 10$. We can see that when the budget is around $S = 300$, the VR can achieve the maximum profit 540, which is even higher than the non-budget case with the profit around 510. Additionally, the profit of each NSP is almost linearly increasing with S .

Fig. 7 demonstrates the price updating of the NSPs with budget plans $S = 200, 500$. From the figure, we can see that convergence generally occurs within about 6 iterations.

C. MULTIPLE ASYMMETRIC NSPs

To demonstrate the convergence of the iterative optimization process with budget plans, we investigate the price updating for a 4-NSP scenario and a 6-NSP scenario.

In the 4-NSP scenario, based on the settings of the above asymmetric 2-NSP scenario, we add two more NSPs, with $c_3 = 8, c_4 = 12, \lambda_3 = 15/km^2, \lambda_4 = 25/km^2, \zeta_3 = 80/km^2, \zeta_4 = 90/km^2$. Also, in the 6-NSP scenario, based on the

settings of the 4-NSP scenario, we further add two more NSPs, with $c_5 = 15$, $c_6 = 7$, $\lambda_5 = 20/\text{km}^2$, $\lambda_6 = 22/\text{km}^2$, $\zeta_5 = 70/\text{km}^2$, $\zeta_6 = 85/\text{km}^2$. The storage sizes for the four NSPs in the 4-NSP scenario are $Q_1 = 15$, $Q_2 = 20$, $Q_3 = 25$, $Q_4 = 30$, while for the 6-NSP scenario, we further add $Q_5 = 35$ and $Q_6 = 40$.

Fig. 8 shows the price updating process for the two scenarios. In specific, Fig. 8(a) depicts the price updating for the 4 NSPs in the 4-NSP scenario with budget $S = 400, 800$. Fig. 8(b) depicts the price updating for the 6 NSPs in the 6-NSP scenario with budget $S = 500, 1200$. In both scenarios, we can see that the iterative optimization between the NSPs and the VR can converge quickly within no more than 6 iterations. This demonstrates the effectiveness of the proposed optimization algorithm in multiple-NSP asymmetric scenarios with arbitrary chosen parameters.

VIII. CONCLUSIONS

In this paper, we have considered a commercial small-cell caching system consisting of multiple NSPs and one VR, where the NSPs lease their SBSs to the VR for gaining profits, while the VR, after storing popular videos to the rented SBSs, can provide faster transmissions to the MUs, hence gaining more profits. We proposed a Stackelberg game theoretic framework by viewing the SBSs as a type of resources. We first modeled the MUs and SBSs using two independent PPPs with the aid of stochastic geometry, and developed the probability expression of successful downloading. Then, based on the probability derived, we set up the profit models and formulated a Stackelberg game for maximizing the average profit of the NSPs as well as the VR. We also investigated the Stackelberg Equilibrium for the two cases by solving a series of optimization problems. Finally, we provided several numerical results for showing that the proposed schemes are effective in both pricing and SBSs allocation.

APPENDIX A

PROOF OF THEOREM 1

It is obvious that if $F \geq f > Q_l$, we have $\Pr(\mathcal{E}_{l,f}) = 0$, since each SBS of Φ_l only caches the most popular Q_l video files. Then we consider the case when $f \leq Q_l$. We assume that the SBSs of \mathcal{L}_l rented by \mathcal{V} and caching \mathcal{F}_f follow the HPPP Φ'_l having the intensity of $\tau_l \lambda_l$. We consider a typical MU \mathcal{M} affiliated with \mathcal{L}_l who tries to connect to the nearest SBS \mathcal{B} in Φ'_l . The event $\mathcal{E}_{l,f}$ represents that this SBS can support \mathcal{M} with an SINR no lower than δ .

We carry out the analysis on $\Pr(\mathcal{E}_{l,f})$ for the typical MU \mathcal{M} located at the origin. We denote by z the distance between \mathcal{M} and \mathcal{B} , by x_Z the location of \mathcal{B} , and by $\rho(x_Z)$ the received SINR at \mathcal{M} from \mathcal{B} . Then the average probability that \mathcal{M} can download the desired video from \mathcal{B} is

$$\Pr(\mathcal{E}_{l,f}) = \Pr(\rho(x_Z) \geq \delta) = \int_0^\infty \Pr\left(\frac{P_l h_{x_Z} z^{-\alpha}}{\sum_{x \in \Phi_l \setminus x_Z} P_l h_x \|x\|^{-\alpha} + \sigma^2} \geq \delta \mid z\right) f_Z(z) dz$$

$$= \int_0^\infty \Pr(h_{x_Z} \geq u(I + \sigma^2) | z) f_Z(z) dz = \int_0^\infty \mathbb{E}_I(\exp(-sI) \exp(-u\sigma^2)) f_Z(z) dz, \tag{25}$$

where we have

$$I \triangleq \sum_{x \in \Phi_l \setminus x_Z} P_l h_x \|x\|^{-\alpha}, \quad u \triangleq \frac{z^\alpha \delta}{P_l}, \tag{26}$$

and $f_Z(z)$ is the probability density function of z . The derivation of $f_Z(z)$ is based on the null probability of the HPPP Φ'_l with the intensity of $\tau_l \lambda_l$. More specifically in Φ'_l , since the number of the SBSs k in an area of A follows the Poisson distribution, the probability of the event that there is no SBS in the area with the radius of z can be calculated as $\Pr(k = 0 | A = \pi z^2) = e^{-A\tau_l \lambda_l} \frac{(A\tau_l \lambda_l)^k}{k!}$ [20].

By using the derivation of the above expression, we arrive at $f_Z(z) = 2\pi \tau_l \lambda_l z \exp(-\pi \tau_l \lambda_l z^2)$. Note that the interference I consists of two parts. The first part, denoted by I_1 , comes from the SBSs in Φ_l but not in Φ'_l , and the second part, denoted by I_2 , comes from the SBSs in Φ'_l excluding the nearest SBS \mathcal{B} . We have the expressions of the two interference as

$$I_1 = \sum_{x \in \Phi_l \setminus \Phi'_l} P_l h_x \|x\|^{-\alpha}, \quad I_2 = \sum_{x \in \Phi'_l \setminus x_Z} P_l h_x \|x\|^{-\alpha}. \tag{27}$$

Since the two parts of interference are independent, there is

$$\mathbb{E}_I(\exp(-uI)) = \prod_{i=1}^2 \mathbb{E}_{I_i}(\exp(-uI_i)). \tag{28}$$

For $\mathbb{E}_{I_1}(\exp(-uI_1))$, we have

$$\begin{aligned} & \mathbb{E}_{I_1}(\exp(-uI_1)) \\ & \stackrel{(a)}{=} \mathbb{E}\left(\prod_{x \in \Phi_l \setminus \Phi'_l} \int_0^\infty \exp(-uP_l h_x \|x\|^{-\alpha}) \exp(-h_x) dh_x\right) \\ & = \mathbb{E}\left(\prod_{x \in \Phi_l \setminus \Phi'_l} \frac{1}{1 + uP_l \|x\|^{-\alpha}}\right) \\ & \stackrel{(b)}{=} \exp\left(- (1 - \tau_l) \lambda_l \int_{\mathbb{R}^2} \left(1 - \frac{1}{1 + uP_l \|x\|^{-\alpha}}\right) d\|x\|\right) \\ & = \exp\left(-2\pi \frac{\delta^{\frac{2}{\alpha}}}{\alpha} (1 - \tau_l) \lambda_l B\left(\frac{2}{\alpha}, 1 - \frac{2}{\alpha}\right) z^2\right), \end{aligned} \tag{29}$$

where (a) is based on the independence of channel fading, while (b) follows from $\mathbb{E}\left(\prod_x v(x)\right) = \exp(-\lambda \int_{\mathbb{R}^2} (1 - v(x)) dx)$, where $x \in \Phi$ and Φ is an PPP in \mathbb{R}^2 with the intensity λ [25].

Furthermore, since the interference I_2 is imposed by the SBSs outside the circle with radius z . Then we arrive at

$$\begin{aligned} & \mathbb{E}_{I_2}(\exp(-uI_2)) \\ & = \mathbb{E}_{\Phi_l}\left(\prod_{x \in \Phi'_l \setminus x_Z} \int_0^\infty \exp(-uP_l h_x \|x\|^{-\alpha}) \exp(-h_x) dh_x\right) \end{aligned}$$

$$\begin{aligned}
 &= \mathbb{E}_{\Phi_l} \left(\prod_{x \in \Phi_l' \setminus x_Z} \frac{1}{1 + z^\alpha \delta \|x\|^{-\alpha}} \right) \\
 &= \exp \left(-\tau_l \lambda_l \int_z^\infty 2\pi \left(1 - \frac{1}{1 + z^\alpha \delta r^{-\alpha}} \right) r \, dr \right) \\
 &= \exp \left(-2\pi \frac{\delta}{\alpha - 2} \tau_l \lambda_l {}_2F_1 \left(1, 1 - \frac{2}{\alpha}; 2 - \frac{2}{\alpha}; -\delta \right) z^2 \right). \tag{30}
 \end{aligned}$$

Based on Eqs. (29) and (30), we obtain $\mathbb{E}_l(\exp(-uI))$ in Eq. (28). We further substitute Eq. (28) into (25). According to the definitions of the functions $A(\delta, \alpha)$ and $C(\delta, \alpha)$, we have

$$\begin{aligned}
 \Pr(\rho(x_Z) \geq \delta) &= \pi \tau_l \lambda_l \int_0^\infty \exp \left(-\pi(1 - \tau_l) \lambda_l C(\delta, \alpha) z^2 \right) \\
 &\quad \exp \left(-\pi \tau_l \lambda_l A(\delta, \alpha) z^2 \right) \exp(-u\sigma^2) \exp \left(-\pi \tau_l \lambda_l z^2 \right) \, dz^2. \tag{31}
 \end{aligned}$$

This completes the proof. ■

APPENDIX B

PROOF OF COROLLARY 1

In the case that $\frac{\sigma^2}{P_l}$ goes to zero, we have

$$\lim_{\frac{\sigma^2}{P_l} \rightarrow 0} \exp \left(-\frac{z^\alpha \delta \sigma^2}{P_l} \right) = 1. \tag{32}$$

Then we rewrite Eq. (3) when $f \leq Q_l$ as

$$\begin{aligned}
 &\lim_{\frac{\sigma^2}{P_l} \rightarrow 0} \Pr(\mathcal{E}_{l,f}) \\
 &= \int_0^\infty \exp \left(-\pi(1 - \tau_l) \lambda_l C(\delta, \alpha) z^2 \right) \\
 &\quad \exp \left(-\pi \tau_l \lambda_l A(\delta, \alpha) z^2 \right) \pi \tau_l \lambda_l \exp \left(-\pi \tau_l \lambda_l z^2 \right) \, dz^2 \\
 &= \frac{\tau_l}{(1 - \tau_l)C(\delta, \alpha) + \tau_l A(\delta, \alpha) + \tau_l}. \tag{33}
 \end{aligned}$$

This completes the proof. ■

APPENDIX C

PROOF OF LEMMA 2

The optimum solution s_l^* can be obtained by deriving S_l^{NSP} with respect to s_l and solving $\frac{ds_l^{NSP}}{ds_l} = 0$. We have

$$\begin{aligned}
 \frac{\partial S_l^{NSP}}{\partial s_l} &= \frac{\lambda_l}{A(\delta, \alpha) - C(\delta, \alpha) + 1} \\
 &\quad \left(\sqrt{\frac{KsC(\delta, \alpha)\zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l s_l}} - C(\delta, \alpha) \right. \\
 &\quad \left. - \frac{1}{2}(s_l - c_l) \sqrt{\frac{KsC(\delta, \alpha)\zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l}} s_l^{-\frac{3}{2}} \right). \tag{34}
 \end{aligned}$$

Since we have $U_l = \sqrt{\frac{KsC(\delta, \alpha)\zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l}}$, we shall obtain s_l^* from solving the following equation:

$$s_l^{-\frac{3}{2}} + \frac{1}{c_l} s_l^{-\frac{1}{2}} - \frac{2C(\delta, \alpha)}{c_l U_l} = 0. \tag{35}$$

Let $s_l^{-\frac{1}{2}} = x^{\frac{1}{3}} - \frac{1}{3c_l} x^{-\frac{1}{3}}$, and Eq. (35) can be converted to

$$x^2 - \frac{2C(\delta, \alpha)}{c_l U_l} x - \frac{1}{27c_l^3} = 0. \tag{36}$$

Adopt the positive x , and we arrive at

$$x = \frac{C(\delta, \alpha)}{c_l U_l} + \sqrt{\left(\frac{C(\delta, \alpha)}{c_l U_l} \right)^2 + \frac{1}{27c_l^3}}. \tag{37}$$

Substitute x to $s_l^{-\frac{1}{2}} = x^{\frac{1}{3}} - \frac{1}{3c_l} x^{-\frac{1}{3}}$, we obtain the optimum s_l . Also, since there is

$$\begin{aligned}
 \frac{\partial^2 S_l^{NSP}}{\partial s_l^2} &= \frac{\lambda_l}{A(\delta, \alpha) - C(\delta, \alpha) + 1} \\
 &\quad \times \left(-\frac{3}{4} c_l u_l s_l^{-\frac{5}{2}} - \frac{1}{4} u_l s_l^{-\frac{3}{2}} \right) < 0, \tag{38}
 \end{aligned}$$

the function S_l^{NSP} is a concave function of s_l . There exists a unique optimum price s_l^* that maximizes S_l^{NSP} . Furthermore, in the case of $s_l^* < s_l^{lower}$ which leads to $\tau_l > 1$, we adopt $s_l^* = s_l^{lower}$ to ensure $\tau_l = 1$. This completes the proof. ■

APPENDIX D

PROOF OF THEOREM 2

We obtain the optimum solution by proposing a modified water-filling algorithm with extra constrains. First, we rewrite *Problem 2* as

$$\begin{aligned}
 \max &\sum_{l=1}^L \frac{Ks\zeta_l \tau_l \sum_{f=1}^{Q_l} q_f}{(1 - \tau_l)C(\delta, \alpha) + \tau_l A(\delta, \alpha) + \tau_l} - \sum_{l=1}^L \tau_l \lambda_l s_l \\
 \text{s.t. } &\textcircled{1} \sum_{l=1}^L \tau_l \lambda_l s_l \leq S, \quad \textcircled{2} \tau_l \geq 0, \quad \textcircled{3} \tau_l \leq 1, \quad \forall l. \tag{39}
 \end{aligned}$$

Note that $\textcircled{1}$ and $\textcircled{2}$ are the traditional constrains for water-filling principle, while the constrains in $\textcircled{3}$ are particular to our model. Using the Lagrangian function

$$\begin{aligned}
 \mathbb{L}(\boldsymbol{\tau}, \boldsymbol{\mu}, \boldsymbol{\nu}, \kappa) &= -\sum_{l=1}^L \left(\frac{Ks\zeta_l \tau_l \sum_{f=1}^{Q_l} q_f}{(1 - \tau_l)C(\delta, \alpha) + \tau_l A(\delta, \alpha) + \tau_l} \right. \\
 &\quad \left. - \tau_l \lambda_l s_l + \mu_l \tau_l - \nu_l \tau_l - \xi \tau_l \lambda_l s_l \right), \tag{40}
 \end{aligned}$$

where $\boldsymbol{\mu} \triangleq [\mu_1, \dots, \mu_L]$, $\boldsymbol{v} \triangleq [v_1, \dots, v_L]$, and ξ are the Lagrangian multipliers. We obtain the necessary and sufficient Karush-Kuhn-Tucher (KKT) conditions as

$$\begin{aligned} \frac{\partial \mathbb{L}(\boldsymbol{\tau}, \boldsymbol{\mu}, \boldsymbol{v}, \kappa)}{\partial \tau_l} &= 0, \\ \mu_l &\geq 0, \\ \mu_l \tau_l &= 0, \\ v_l &\geq 0, \\ v_l(\tau_l - 1) &= 0, \quad \forall l. \end{aligned} \quad (41)$$

The condition $\mu_l \tau_l = 0$ results in $\tau_l = 0$ or

$$\tau_l = \frac{\left(\sqrt{\frac{KsC(\delta, \alpha) \zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l s_l (1+\xi) + v_l}} - C(\delta, \alpha) \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}, \quad (42)$$

while the condition $v_l(\tau_l - 1) = 0$ results in $\tau_l = 1$ or

$$\tau_l = \frac{\left(\sqrt{\frac{KsC(\delta, \alpha) \zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l s_l (1+\xi) - \mu_l}} - C(\delta, \alpha) \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}. \quad (43)$$

Then we arrive at

$$\tau_l = \begin{cases} 0 & \xi > \frac{Ks\zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l s_l C(\delta, \alpha)} - 1, \\ 1 & \xi < \frac{KsC(\delta, \alpha) \zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l s_l (A(\delta, \alpha) + 1)^2} - 1, \\ \sqrt{\frac{KsC(\delta, \alpha) \zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l s_l (1+\xi)}} - C(\delta, \alpha) & \text{otherwise,} \end{cases} \quad (44)$$

Finally, the unknown variable ξ is chosen such that the constraint $\sum_{l=1}^L \tau_l \lambda_l s_l \leq S$ is fulfilled. In the case $\xi = 0$, which means $\sum_{l=1}^L \tau_l \lambda_l s_l < S$, the problem degrades to that in Section V, i.e., S is sufficiently large such that this constraint can be neglected.

In the case $\xi > 0$, which means $\sum_{l=1}^L \tau_l \lambda_l s_l = S$, we first consider the two sets \mathcal{S}_1 and \mathcal{S}_2 . For \mathcal{S}_1 , we have $\tau_l = 1$, $\forall l \in \mathcal{S}_1$. For \mathcal{S}_2 , we have $0 < \tau_l < 1$, $\forall l \in \mathcal{S}_2$. Then we arrive at

$$\xi = \left(\frac{\sum_{j \in \mathcal{S}_2} \sqrt{KsC(\delta, \alpha) \lambda_j s_j \zeta_j \sum_{f=1}^{Q_j} q_f}}{(A - C + 1)(S - \sum_{j \in \mathcal{S}_1} \lambda_j s_j) + \sum_{j \in \mathcal{S}_2} \lambda_j s_j C(\delta, \alpha)} \right)^2 - 1, \quad (45)$$

Note that the sets \mathcal{S}_1 and \mathcal{S}_2 are determined such that

$$\frac{KsC(\delta, \alpha) \zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l s_l (A(\delta, \alpha) + 1)^2} - 1 < \xi < \frac{Ks\zeta_l \sum_{f=1}^{Q_l} q_f}{\lambda_l s_l C(\delta, \alpha)} - 1. \quad (46)$$

This completes the proof. \blacksquare

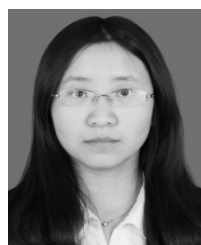
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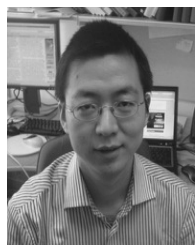
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