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Patient Flow Scheduling and Capacity Planning in a Smart Hospital Environment

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ABSTRACT Improving patient flow is a way to refine health services. An efficient patient flow can improve the quality of services and the utilization of resources. A smart environment could facilitate the experience of individuals within a physical space, such as a hospital. Meanwhile, a smart healthcare environment could improve patient flow through an efficient scheduling policy and the utilization of healthcare resources by an optimized capacity plan. This paper, first, explores a dynamic scheduling policy to improve the patient flow, and an efficient capacity scheme based on the varying patient flow. This scheduling policy and the capacity scheme can be built in a smart hospital environment through wireless sensor networks and smart healthcare systems. The research applies a formal modeling approach that can provide a quantitative analysis of systems. This approach, performance evaluation process algebra, can give strict definitions for the patient flow in order to model the dynamic scheduling policy and the capacity scheme; moreover, it provides a scalable performance analysis by the fluid flow approximation. Finally, this paper is concerned with how formal method might be used to model and analyze the scheduling policy and the capacity plan on improving the healthcare service before deployment.

INDEX TERMS Patient flow, scheduling, capacity planning, smart environment, PEPA.

I. INTRODUCTION

Smart environments can improve people's experience and behaviours in a physical space. They achieve the improvement through knowing enough information of individuals with the use of new technologies, such as wireless sensor network, smart systems, cloud technologies and some kinds of smart devices [1], [2]. These technologies help to collect useful information to support a decision making process provided by a smart healthcare system [3]. The smart environment can infer action from people's context and then influence collective behaviour of individuals in the environment. The practice conference of a well designed smart environment can improve individuals' experience by providing people's information in the environment and manage the healthcare resources intelligently [4], [5].

Today, quality of services (QoS) in national healthcare has become a conspicuous issue due to the growing pressure to improve quality and heightening patient [6]. The QoS in healthcare generally includes two aspects: quality of serving patients and healthcare management. In England, National Health Service (NHS) provides free healthcare services for all UK citizens. Thus, UK governments concern much for the utilization of healthcare resources and the quality of healthcare service. Therefore, research and investigations are funded to get the issues solved in UK. This paper generates a research under such background to improve the quality and efficiency of the healthcare service through a smart hospital environment.

In this paper, we proposes a dynamic scheduling policy which has more efficient scheduling ability to avoid long waiting in the hospital. The dynamic policy could improve serving efficiency by scheduling patients in terms of the real time status. Furthermore, the research also investigates an efficient capacity plan which the allocation of patients for each position in the department.

Model scenario is based on patient flow of a rheumatology department in UK. The patient flow is a systematic process of attending to patients, from the time they walk into a medical facility to the time they check out for discharge. It includes patient's medical activities and behaviours in the hospital. In this paper, the patient flow is modelled with a dynamic scheduling function in order to generate a performance analysis by measuring patient waiting queue in the department. On the basis of analysis, an efficient workflow scheme is proposed improve the patient flow further. The initial patient flow model is generated from the current workflow of the rheumatology department. The main goal of this paper is to improve the scheduling process and help the department to refine work procedure for better service quality and efficiency.

In the hospital, a smart environment can be designed to facilitate people's experience by modifying collective behaviours. The smart environment includes a set of devices and many intelligent supporting techniques, just as our dynamic scheduling policy and capacity planning scheme. However, to engineer a design efficiently that satisfies users, techniques are required early in the design to model and evaluate the effects of design. The goal of early evaluation aims to predict the impact of systems before deployment and provide useful information for later refinements or versions of design. This smart environment scenario has some novel features. Firstly, individual interactions are always behaved as implicit activities with in the smart environment. Secondly, individual behaviours are always affected by the environment or other individuals. These novel features bring a new perspective to the usability engineering.

This paper is concerned to model the patient flow using a formal method to discuss these novel features. A limitation of formal method for the analysis of collective behaviours is how to solve the state space explosion problem that caused by the model solving to complex models with multiple instances required to define the collective behaviour. Performance Evaluation Process Algebra (PEPA) is an expressive formal language for modelling distributed systems. PEPA provides a scalable model-based technique – Fluid Flow Analysis that supports analysis of many replicated instances with synchronised behaviours that could cause state explosion. This technique is built on the process algebra and add new techniques for quantitative analysis.

Section 2 gives a brief review of the previous work on evaluating some scheduling schemes. Section 3 describes the details of patient flow and specifies the current problems in the scenario. Section 4 introduces PEPA syntax and the fluid flow analysis. Section 5 includes the initial model based on current patient flow and performance evaluation of the dynamic scheduling policy; furthermore, an optimized capacity plan is proposed in terms of the analysis. In section 6, the patient flow model is evolved to a new scheme to improve the efficiency. Section 7 validates the results of formal models using another modelling technique. Section 8 draws conclusions and gives an outline of future research.

II. RELATED WORK AND CONTRIBUTIONS

In previous work, the patient flow is modelled as a closed network. All incoming patients return to the start point after completing a series of activities in the hospital. The closed network model can reach a steady state after running for a period of time. However, in fact, people always go to the hospital during the working hours except some emergencies. Thus, the number of patients always vary with time elapse. As a result, the patient flow model no longer reaches a steady state finally. This paper will explore this real world situation by creating an open network in the patient flow model. This research uses a scenario that directly comes from a hospital department. The patient flow model will be built in terms of department workflow.

Improving patient flow has always been a topic in the research of healthcare. Recently, some related study is conducted with the development of mainstream technologies, such as Internet of things, big data, smart environment, cloud computing, and so on. In [7], Chong proposes a system dynamic approach which is a modelling technique used for complex behaviours of organizational and social systems. Chong's study aims to examine the trade-offs of various safety and quality outcomes in an Emergency Department in order to evaluate the efficiency of healthcare systems. Similarly, our research has a final goal to improve efficiency and QoS of the healthcare system. However, a solution is provided by applying a smart environment in the physical space to support people's experience and healthcare management.

In Shao's research [8], the workflow of surgical operations is explored to reduce the disruption by creating a continuous time Markov chain (CTMC) model and then solving the Markov chain to conduct the related performance analysis. CTMC model can predict system status in future on the basis of properties in Markov process. This paper directly generates the model using CTMC. The drawback of such direct modelling approach comes from the state space explosion problem when a system has complex behaviours or massive instances. For this reason, PEPA is used to model the patient flow. PEPA is based on a underlying CTMC and implements scalable analysis using the fluid flow analysis to avoid the state space explosion.

Nikakhtar's [9] research aims to examine the relationship between patient flow in a social network and the corresponding network characteristics. In this research, a simulation model is developed to describe the patient flow and generate performance measure. Simulation techniques are widely used for the study of patient flow. Nevertheless, simulation also has some drawbacks in time-consuming and debugging especially when the target model has complex behaviours and interactions, or a large number of instances. Consequently, formal method could preserve high efficiency in building models and generating scalable analysis. Hence, our research adopts a formal method – PEPA to define the patient flow model and conduct a performance analysis.

According to the reviewed literatures [10]–[15], current research has obtained many achievements especially in developing new solutions to improve the patient flow. In fact, there are still some gaps in this area. In our study, a real-life scenario with statistic figures is used to build patient flow models. Moreover, a formal approach (PEPA) is applied for modelling as it can provide efficient and strict model definitions. Finally, we adopt a novel analysing solution – Fluid Flow Approximation for the performance evaluation which can conduct a scalable analysis without the interference of state space explosion. The contributions of our work are summarized as follows:

- In this paper, we firstly complete an investigation and obtain the general workflow of the department in a hospital, and then collect massive figures based on a longterm statistics. These figures include the information of current schedule and staffing, the everyday arrivals for different kinds of patients, the duration of serving patients in each step of the workflow, and so on. This empirical research is conducted to learn the actual issues in the medical department and generate a real life model.
- The key issue in the department is to improve the quality of service by refining the scheduling process in the patient flow, and to make an efficient capacity plan based on the analysis of patient flow. This paper proposes a solution to fix the scheduling and capacity planing issues by building a smart environment in the department. Meanwhile, the research also develops a policy to conduct the dynamic scheduling and an approach to generate the efficient capacity plan. Finally, the research successfully helps the department refine current workflow and capacity scheme.
- Moreover, this research uses a formal method (PEPA) to implement the performance modelling and analysis. In formal models, functions are creatively applied in the model definition specified as action rates. Such functional rates could represent complex system behaviours, such as a decision making process, scheduling process, and so on. They actually expend the application range of formal method. In addition, the fluid flow approximation is chosen as the main analysing technique as it can prevent the state space explosion problem.

III. INITIAL PATIENT FLOW MODEL

The early research in the rheumatology department indicates that the scheduling process has a potential problem that is the inaccurate prediction of appointment time for each patient. In this department, the problem directly affects appointments for upcoming patients, which may cause a long waiting queue and the waste of healthcare resources. This paper proposes a solution to improve the patient flow and the department workflow through building a smart environment in the hospital. Smart environments could collect information, such as location, progress and even the vital signs of patients, from individuals in a local area. The information can be obtained through smart devices (e.g. smart phones), radio frequency identification devices (RFID), wireless sensor network (WSN), WIFI or 3G/4G network, and so on [1]–[3]. Individuals can be recognized and traced by connecting their own smart devices or issued smart cards to a smart system. Thereafter, their location can be traced in order to provide them the useful information, such as navigation tips, progress or predicted waiting time, and so forth. All these services can facilitate people's experience in the hospital, and also support healthcare management. On one hand of this research, the explored dynamic scheduling policy is based on such collected location information, which can dynamically generate prediction for following appointments and support the patient scheduling in the hospital. On the other hand, a capacity plan can be made in terms of the patient flow analysis.

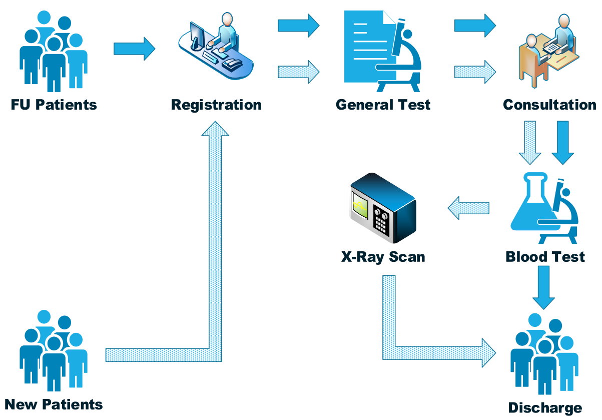

FIGURE 1. Patient flow of rheumatology department.

According to the investigation in hospital department, a general workflow is obtained which can represent the flow of most patients in this department. Figure 1 describes the detailed patient flow of the rheumatology department. As rheumatic disease is common in the old, rheumatology department has a relatively stable amount of patients. These patients can be divided into two categories: the first category includes new patients (NEW) that are the first time to the department. The new patients must take an X-Ray scan and a blood test as a part of routine practice. The other category are follow-up (FU) patients that come to the department for periodic reexamination or treatment. These patients do not need an X-Ray scan unless a special demand. Moreover, all patients must check in and have a general check-up for their physical conditions before meeting a consultant. Thereafter, the new patients will be arranged for the blood test and X-Ray scan; however, the follow-up patients just have a routine blood test. After completing examinations, patients are discharged until the results are available.

With a smart environment, patients need register their smart phones in system or be issued a smart card including a sensor. Their location can be traced by detectors deployed in the hospital environment. Then, the smart system is able to return related information to patients via their smart phones or deployed public displays. These information can notify patients what to do and where to go next. For the department, it is easy to know the progress of each incoming patient and estimate the time of completion so that appointment system can efficiently arrange up-coming patients. Furthermore, systems can schedule consultants for different patients based on the realtime information of their progress. Our research proposes a solution that can refine the current scheduling and capacity planing process based on such smart hospital environment. In addition, we also propose a new workflow for the department in order to improve the efficiency and quality of service.

IV. MODELLING AND ANALYSING APPROACHES

This section gives a brief introduction to PEPA language which is adopted as the main modelling technique in the work. PEPA, developed by Hillston in the 1990s [21], is a high-level model specification language for low-level stochastic models. Process algebra is designed as a formal modelling technique for concurrent systems that have subsystems interacting with each other. PEPA describes such systems as an interaction of the components that engage in activities. PEPA, as an extension of classical process algebra, is usually used for performance modelling and qualitative analysis of large and complex resource-sharing systems.

As a stochastic process algebra, PEPA has several attractive features wihch were not available in other performance modelling techniques. These features can be summarized as [21]:

- *Compositionality*, the ability to model a system as the interaction of subsystems.
- *Formality*, giving a precise meaning to all terms in the language.
- *Abstraction*, the ability to build up complex models from detailed components, disregarding the unnecessary details.

A. SYNTAX OF PEPA

PEPA is a compositional approach that decomposes a system into subsystems which are easier to model. PEPA models are constructed by the composition of components which perform individual activities or cooperate on shared ones. For each activity, PEPA defines a pair (α, r) where α is an action type and r is an action rate. PEPA language has only four combinators, which are *prefix, choice, cooperation* and *hiding* [21].

- *Prefix*: It is the basic building block of a sequential component: the process $(a, r).P$ performs action a at rate r and then evolves to component P.
- *Choice*: It generates a competition between two or more potential processes: $(a, r).P + (b, s).Q$ represents that either action a or b wins the race at the rate r or s and then behaves as P or Q respectively.
- *Cooperation*: Its operator joins two processes together, in which these two processes may share some actions: the process $P \bowtie_L Q$, $L = \{a, b\}$ denotes that processes P and Q must cooperate with the shared actions a and b. Any other action is performed independently. Additionally, $P \parallel Q$ in PEPA syntax means $P \bowtie_L Q, L = \phi$.
- *Hiding*: The process $P \mid \{a\}$ hides the action a from view and prevents other processes from joining with it.
- *Constant*: As assumed, a countable set of constants is defined. The constants are components with the meaning of defining an equation such as $A \stackrel{\text{def}}{=} P$ which gives the constant A the behaviour of the component P. This is the way of assigning names for behaviour components.

The syntax of PEPA is given as:

$$P ::= (a, \lambda).P \mid P + Q \mid P\langle L\rangle Q \mid P \mid Q \mid A$$

This PEPA statement involves all four combinators mentioned in the previous paragraph. The last part of this statement $P ::= A$ is to identify component P with A. When the rate of action is passive, we use the symbol $\top$.

B. PERFORMANCE MEASURE USING PEPA

To complete performance modelling and analysis of patient flow scheduling, an efficient modelling technique must be selected to fit the features of patient flow model. As mentioned before, the potential modelling techniques should be suitable for modelling a Markov process and avoiding the state space explosion problem. This subsection briefly introduces several often used techniques.

Currently, two popular modelling languages for Markov models are queuing networks and stochastic Petri nets (SPN). The queuing network is a directed graph in which each node represents a queue (also called service centre). Customers representing system jobs flow through the nodes and complete service. The arcs of the network represent the system topology and routing probabilities. The amount of customers currently occupying each service centre represents the current state of the system. Queuing network has the feature compositionality but it is informal.

In contrast, SPN is a formal mathematical modelling language. SPN is a directed graph with two kinds of node, places and transitions. The system state is represented through the number of tokens at each place in the network. SPN is an alternative mean of generating stochastic models for performance modelling. However, SPN has some restrictions in compositionality compared with queuing network. It is complex to represent the layered structure of systems. Now we choose PEPA as the main modelling language. PEPA, as a high-level modelling paradigm for CTMC, has a compositional structure. This compositionality can be exploited to reduce the state space of the CTMC [21]. Furthermore, this technique takes advantages of symmetries within the system, and may be formally defined based on the models PEPA description. Thus, a PEPA model can be applied with the underlying CTMC to define a Markov Reward Process (MRP) from which performance evaluation can be derived. PEPA's features are summarized at the beginning of section 4.

Queuing network provides compositionality but not formality; stochastic Petri nets offers formality but not compositionality; and neither gives abstraction mechanism [22].

C. FLUID FLOW APPROXIMATION

PEPA language offers compositional function for creating models towards large scale systems. Meanwhile, a novel performance analysis technique, fluid flow approximation, is provided for large scale models using PEPA.

Just as most discrete state-based modelling formalisms, process algebra easily suffer from the failure due to the generation of extremely large state space making the numerical solution via linear algebra costly or even intractable. Fluid flow approximation generates a set of coupled ordinary differential equations (ODEs) underlying continuous time Markov chains (CTMC). This approach is given two adjustments. Firstly, it does not calculate the probability distribution over the entire state space; instead, a more abstract state representation is chosen based on state variables [23]. Secondly, it is assumed that these state variables are subject to continuous rather than discrete change [23]. Based on these adjustments, fluid flow approximation offers an efficient solution with a set of ODEs, and lead to the evaluation of transient and steady state measures. This approach successfully avoids the state space explosion in the analysis via exploring ODEs.

The fluid flow analysis is on the basis of the vector form. The system inherently discrete with the entries within the numerical vector form always bing non-negative. With the change of the system state, the numerical vector form is incremented or decremented in steps of one. When each component type in the model is replicated a large number of times, these steps are relatively small. Thus, we can approximate the movement between states to be continuous, rather than occurring in discontinuous jumps. The objective of the fluid flow approximation is to replace the derivation graph of the PEPA model by a continuous model using a set of ordinary differential equations.

In the fluid flow approximation, we need specify the *exit activity* and *entry activity* of the local derivative of a sequential component. An activity (α, r) is an *exit activity* of D if D enables (α, r), such as $D \xrightarrow{(\alpha,r)} D'$. The set of exit activities of D is denoted by $Ex(D)$. The set of local derivatives for an exit activity (α, r) is denoted by $Ex(\alpha, r)$. Similarly, an activity (β, s) is an *entry activity* if a derivative D' enables (β, s), such as $D' \xrightarrow{(\beta,s)} D$. $En(D)$ denotes the set of entry activities of D.

After specifying the concepts of the *exit activity* and *entry activity*, the movement of the numerical state vector of the PEPA model are represented with these concepts. Here, we define $v_{i_j}(t) = N(C_{i_j}, t)$ for the jth entry of the ith subvector at time t; $N(C_{i_j}, t)$ denotes the number of instances of the jth local derivative of sequential component C_i. In a very short time slot δt, the change of the vector entry $v_{i_j}(t)$ can be denoted as:

$$\begin{aligned} N(C_{i_j}, t+\delta t) - N(C_{i_j}, t) &= \underbrace{-\sum_{(\alpha,r)\in Ex(C_{i_j})} r \times \min_{C_{k_l}\in Ex(\alpha,r)} (N(C_{k_l}, t))\,\delta t}_{exit\ activities} \\ &\quad + \underbrace{\sum_{(\alpha,r)\in En(C_{i_j})} r \times \min_{C_{k_l}\in Ex(\alpha,r)} (N(C_{k_l}, t))\,\delta t}_{entry\ activities} \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{dN(C_{i_j}, t)}{dt} &= \lim_{\delta t\to 0} \frac{N(C_{i_j}, t+\delta t) - N(C_{i_j}, t)}{\delta t} \\ &= -\sum_{(\alpha,r)\in Ex(C_{i_j})} r \times \min_{C_{k_l}\in Ex(\alpha,r)} (N(C_{k_l}, t)) \\ &\quad + \sum_{(\alpha,r)\in En(C_{i_j})} r \times \min_{C_{k_l}\in Ex(\alpha,r)} (N(C_{k_l}, t)) \end{aligned} \quad (2)$$

In formula (1), the first block represents the impact of exit activities; however, the second block records the impact of the entry activities. Now, we change the formula (1) based on dividing by δt and taking a limit. If $\delta t \to 0$, we obtain the formula (2). In the following analysis, a set of ODEs can be obtained from the PEPA model based on formula (2). The quantitative analysis is conducted through solving the ODEs.

V. CURRENT PATIENT FLOW MODEL AND ANALYSIS

This section briefly describes the model construction of current patient flow and related analysis. In this section, the dynamic scheduling policy has been developed in the previous work. This section mainly demonstrates the performance of a dynamic scheduling policy that is basis of the following work. Moreover, model parameters are introduced based on statistical data from the rheumatology department. The last subsection introduces the way of generating performance analysis with fluid flow approximation.

A. MODEL CONSTRUCTION

In previous work, the current patient flow (Figure 1) is modelled with two different flows for new patients and follow-up patients respectively. All activities in the flow are represented by a set of components such as: registration, general check-up, consultation, X-Ray scan and blood test.

The previous model are used to demonstrate how to construct a PEPA based patient flow model. According to PEPA syntax, it defines a series of sequential states for two types of patients (*New* and *FU*) in terms of the patient action flow. In PEPA model, each *State* is defined with an action type and a related rate (*actiontype*, *rate*), and then followed by the next *State*$'$ moving to, e.g. the expression $State \stackrel{\text{def}}{=} (actiontype, rate).State'$. Components for *New* and *FU* patients are modelled with a sequence of states based on the preceding expression, which can be found in the first and the second blocks in the following PEPA model. Activities (e.g. general test, blood test and X-ray scan) are defined as some components with a corresponding action which is displayed in the third block of the following PEPA model. These activity components have cooperation with patient components representing behaviours of patients through the last block of the model. The detailed PEPA model is displayed as follow:

$$\begin{aligned} \boldsymbol{New} &\stackrel{\text{def}}{=} (arriveNew, r_{arriveNew}).New\_reg \\ New\_reg &\stackrel{\text{def}}{=} (register, r_{register}).New\_test \\ New\_test &\stackrel{\text{def}}{=} (test, r_{test}).New\_blood \\ New\_con &\stackrel{\text{def}}{=} (newCon, r_{newCon}).New\_blood \end{aligned}$$

$$
\begin{aligned}
New\_blood &\stackrel{\text{def}}{=} (blood, r_{blood}).New\_xray \\
New\_xray &\stackrel{\text{def}}{=} (xray, r_{xray}).New\_depart \\
New\_depart &\stackrel{\text{def}}{=} (depart, r_{depart}).Stop \\
\boldsymbol{FU} &\stackrel{\text{def}}{=} (arriveFU, r_{arriveFU}).FU\_reg \\
FU\_reg &\stackrel{\text{def}}{=} (register, r_{register}).FU\_test \\
FU\_test &\stackrel{\text{def}}{=} (test, r_{test}).FU\_con \\
FU\_con &\stackrel{\text{def}}{=} (fuCon, r_{fuCon}).FU\_blood \\
FU\_blood &\stackrel{\text{def}}{=} (blood, r_{blood}).FU\_depart \\
FU\_depart &\stackrel{\text{def}}{=} (depart, r_{depart}).Stop \\
Register &\stackrel{\text{def}}{=} (register, r_{register}).Register \\
Test &\stackrel{\text{def}}{=} (test, r_{test}).Test \\
Consultant\_New &\stackrel{\text{def}}{=} (newCon, r_{newCon}).Consultant\_New \\
Consultant\_FU &\stackrel{\text{def}}{=} (fuCon, r_{fuCon}).Consultant\_FU \\
Blood &\stackrel{\text{def}}{=} (blood, r_{blood}).Blood \\
Xray &\stackrel{\text{def}}{=} (xray, r_{xray}).Xray \\
Depart &\stackrel{\text{def}}{=} (depart, r_{depart}).Depart \\
Stop &\stackrel{\text{def}}{=} (stop, r_{stop}).Stop \\
Sys &\stackrel{\text{def}}{=} (New[p_1] \| FU[p_2]) \underset{L}{\bowtie} \\
&\quad (Register[n_1] \| Test[n_2] \| \\
&\quad Consultant\_New[n_3] \| \\
&\quad Consultant\_FU[n_4] \| Blood[n_5] \| \\
&\quad Xray[n_6] \| Depart[n_7] \| Stop[n_8]) \\
L &= \{register, test, newCon, fuCon, \\
&\quad blood, xray, depart, stop\}
\end{aligned}
$$

B. MODEL PARAMETERS

This research uses a set of statistic figures measured from the rheumatology department. All these figures are *cycle time* and *takt time* of each activity in the workflow, such as, registration, general test, consultation, blood test, and the discharge. The *cycle time* is a period required to complete one cycle of an operation; the *takt time* is the average unit production time needed to meet customer demand. The service rate $\mu_{service\ rate}$ used in the model can be calculated by taking the reciprocal of cycle time $t_{cycle\ time}$. Table 1 and Table 2 show the service rate and takt time of each component in the patient flow model.

TABLE 1. Service rate of components in the patient flow model.

Components	Cycle Time	Service Rate (patients/hour)
Registration	81 seconds	44
General Test	106 seconds	34
Consultation(FU)	15 minutes	4
Consultation(New)	30 minutes	2
Blood Test	252 seconds	14
Depart	243 seconds	14

TABLE 2. Takt time at components in the patient flow model.

Components	Takt Time
Registration	210 seconds
General Test	210 seconds
Consultation for Follow-up Patients	6 minutes
Consultation for New Patients	24 minutes
Blood Test	288 seconds
Depart	210 seconds

TABLE 3. Number of components used in the model.

N_{new}	N_{FU}	$N_{register}$	N_{test}	N_{conNew}
20	80	1	1	2
N_{conFU}	N_{blood}	N_{xray}	N_{depart}	
3	2	2	2	

In addition, a set of parameters are still required to analyse the capacity plan of patient flow. They are the number of patient components and other activity components showing in Table 3. All these parameters are calculated based on the cycle time and the takt time in Table 1 & 2.

C. PERFORMANCE ANALYSIS OF DYNAMIC SCHEDULING POLICY

This subsection demonstrates a performance analysis of the dynamic scheduling policy based on the preceding PEPA model. The fluid flow approach is used to solve PEPA model and generate performance analysis. In the analysis, the dynamic scheduling policy is compared with the currently used policy that is a static scheduling policy. Meanwhile, a capacity plan is also proposed for the workflow in terms of the statistical data, and a performance evaluation of the plan is conducted through the patient flow model.

The static scheduling policy can reduce the length of waiting queue to a stable low level. However, there are some potential problems that cause a delay of incoming patients being scheduled. This situation actually makes utilization of consulting process in a low level before the length of queue grows to a stable level. Thus, the dynamic scheduling policy is considered to improve scheduling process so that the queue length can reach the stable level faster. The dynamic policy uses a function rate to control the scheduling process based on different time slots. The function rate includes a dynamic factor which can alter with time flowing. Expression of the dynamic factor is:

$$Dynamic\ Factor = Max(Static\ Factor, 1 - aT) \quad (3)$$

This dynamic factor is a *Max* function involving a static factor and a formula $1 - aT$. T is the instant model run time, and a is a coefficient used for altering the value of formula to achieve the best situation in modelling. With this function, a dynamic factor obtains a value greater than the static factor and close to 1 at the start of run, namely the value of formula $1 - aT$. At the moment, patients are scheduled faster than the normal pace using the static scheduling. As time elapses, the value of formula $1 - aT$ decreases until less than the static factor. Now the dynamic factor has its value equaling to the static factor. The analysis results for the dynamic scheduling policy are shown in Figure 2.

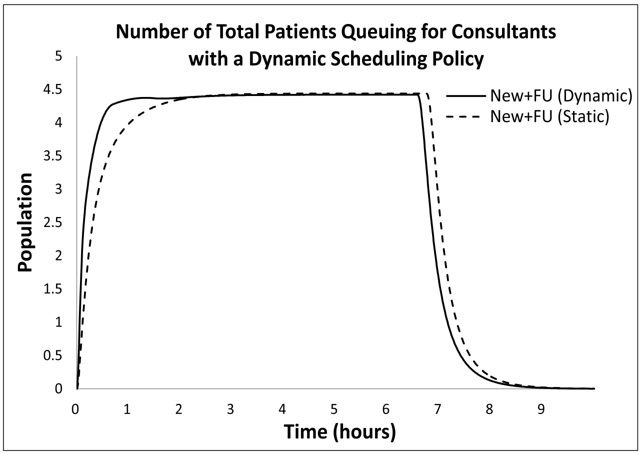

FIGURE 2. Number of total patients queuing for consultants with a dynamic scheduling policy.

Figure 2 demonstrates the difference between the dynamic scheduling policy and the static scheduling policy. In the figure, solid line exhibits the length of consulting queue with the dynamic scheduling policy. The dotted line indicates the length of queue using the static policy. As shown in the figure, it is clear that solid line has faster increase at the beginning in contrast to the dotted line, and it also terminates earlier that the dotted lines. This means that patients are scheduled to consultants in a shorter period by using the dynamic scheduling policy. In other words, the utilization of consulting service is improved during the beginning hours due to the fast scheduling process.

The dynamic scheduling policy improves workflow efficiency and resource utilization. However, there are still some problems in completing time. From Figure 2, it is clear that patients usually need over 8 hours to complete their service. Factually, department expects a completion during 8 hours working time. Thus, the preceding capacity plan must be adjusted to reduce the total service time by increasing the number of consultants. Hence, next subsection will explore a new capacity plan with an altered number of patients.

D. OPTIMIZATION OF CAPACITY PLAN

In this step, the preceding capacity plan is changed to increase serving ability by adding more consultants in the workflow. Based on the analysis results, it can be found that the initial capacity plan causes a timeout which means the patient waiting time is too long. Thus, the new capacity plan should reduce the total waiting time after increasing the number of consultants. The initial capacity plan arranges 2 consultants for the new patients and 3 consultants for the follow-up patients based on the statistical figures. As the number of follow-up patients is much greater than the new patients in terms of our investigation, the new capacity plan tries to add

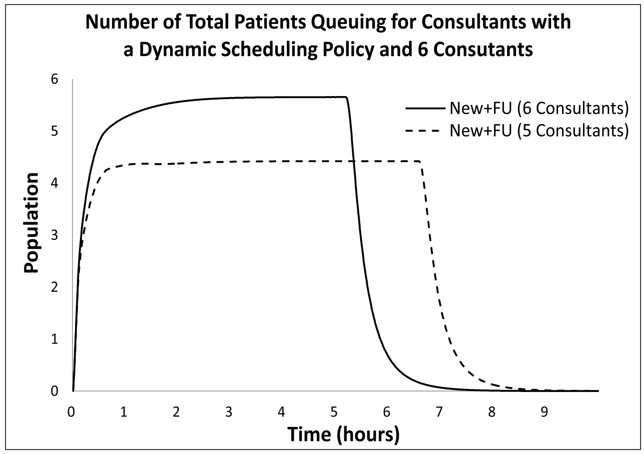

FIGURE 3. Number of patients queuing for consultants with a dynamic scheduling policy based on varying consultants.

one more consultant for the follow-up patients. Based on the plan, we obtain the following Figure 3.

In Figure 3, the solid line represents the queue length in the department for all patients based on the new capacity plan. It is clear that the solid line completes faster than the dotted line within 8 hours due to one more consultant working for the follow-up patients. Furthermore, from the solid line, the stable queue length is quite close to 6 which means that each consultant has only one patient in queue which is our expected length of waiting queue. The other patients are scheduled to come later in order to avoid unnecessary waiting. Hence, the new capacity plan using 6 consultants in the work flow is a better solution as it can efficiently complete all patients in 8 hours.

E. SUMMARY

This section compares the performance of dynamic scheduling policy with the currently used static scheduling policy. According to the analysis results, it is clear that the dynamic scheduling policy could improve the workflow efficiency and the resource utilization especially in the beginning hours. Moreover, the new capacity plan can meet the requirements about completing all services within 8 hours.

VI. EVOLVED PATIENT FLOW MODEL AND ANALYSIS

The preceding section explores the performance of the dynamic scheduling policy based on the current workflow of department. This section will consider to evolve the current workflow in order to reduce waiting queue further. The evolved workflow has a changed activity flow and a modified capacity plan.

A. EVOLVED PATIENT FLOW MODEL

According to our investigation, there is a potential problem in the current workflow. Due to the requirement of diagnosis, new patients must take an X-Ray scan for examination when they come to the department first time. Moreover, blood test is another necessary examination for both new patients and follow-up patients. In current workflow, both blood test and X-Ray scan are arranged after meeting consultants; thereafter patients leave for taking those two examinations and then come back to see consultants. Thus, patients need to see consultants twice; furthermore, blood test and X-Ray scan must be taken in other departments. As a result, it is a waste of time to arrange the patients coming at the first time to see consultants before taking blood test and X-Ray scan. As both blood test and X-Ray scan are required for diagnosis, it would be more efficient to take these examinations before seeing consultants.

In contrast to the initial workflow, the new workflow changes the activity order, which is to bring forward the X-Ray inspection and the blood test before the general test. The activity flow of follow-up patients remains the same. In addition, some consultants arranged for the new patients also serve the follow-up patients; however, some others arranged for the follow-up patients only serve the follow-up rather than the both. This is because the new patients must have the blood test and the X-Ray scan in another department. Hence, time is saved through bringing forward these examinations before seeing a consultant so that more patients can be served and then the efficiency can be improved.

Furthermore, when the new patients are doing their blood tests and X-Ray scan, the idle consultants for the new patients can be arranged to serve the follow-up patients rather than just waiting there. Such evolved workflow ideally improves the efficiency. Thus, even if there are just limited number of consultants, the total amount of patients being served can be improved without increasing waiting queue. The detailed changes of the evolved workflow are shown in Figure 4.

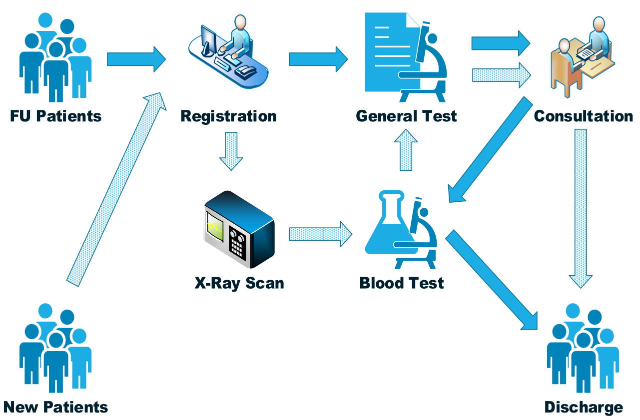

FIGURE 4. Evolved patient flow of rheumatoloty department.

In Figure 4, the dotted arrows clearly depict the activity flow of new patients. As X-Ray inspection must be handled in another department, it must be more efficient for the new to complete X-Ray scan before entering the department rather than leaving for X-Ray scan halfway and coming back.

Additionally, bringing forward the X-Ray scan and blood test for the new patients aims to arrange more consultants from the new patients to the follow-up patients, when the new patients are leaving for their blood tests and X-Ray scan. Thus, in the evolved patient flow model, consultants previously only working for the new patients now must work for the both. When the new patients leave for blood test or X-Ray scan, these consultants serve the follow-up patients; once the new patients complete, these consultants should come back for them.

B. EVOLVED PEPA MODEL

The evolved patient flow model is similar with the initial patient flow model. Most components are defined the same, such as *register*, *general test*, *blood test*, *X-ray scan* and *depart*. However, the model definition for activities of new patients is modified in terms of the evolved workflow. Moreover, the definition for follow-up patients also has changed activity flow. Thus, we can obtain the detailed PEPA model as:

$$\begin{aligned}
\boldsymbol{New} &\stackrel{\text{def}}{=} (arriveNew, r_{arriveNew}).New_reg\\
New_reg &\stackrel{\text{def}}{=} (register, r_{register}).New_xray\\
New_xray &\stackrel{\text{def}}{=} (xray, r_{xray}).New_blood\\
New_blood &\stackrel{\text{def}}{=} (blood, r_{blood}).New_test\\
New_test &\stackrel{\text{def}}{=} (test, r_{test}).New_con\\
New_con &\stackrel{\text{def}}{=} (newCon, r_{newCon}).New_depart\\
New_depart &\stackrel{\text{def}}{=} (depart, r_{depart}).Stop\\
\boldsymbol{FU} &\stackrel{\text{def}}{=} (arriveFU, r_{arriveFU}).FU_reg\\
FU_reg &\stackrel{\text{def}}{=} (register, r_{register}).FU_test\\
FU_test &\stackrel{\text{def}}{=} (test, r_{test}).FU_con\\
FU_con &\stackrel{\text{def}}{=} (fuCon, r_{fuCon}).FU_blood\\
&\quad + (newCon_fu, r_{fuCon}).FU_blood\\
FU_blood &\stackrel{\text{def}}{=} (blood, r_{blood}).FU_depart\\
FU_depart &\stackrel{\text{def}}{=} (depart, r_{depart}).Stop\\
Register &\stackrel{\text{def}}{=} (register, r_{register}).Register\\
Test &\stackrel{\text{def}}{=} (test, r_{test}).Test\\
Consultant_New &\stackrel{\text{def}}{=} (newCon, r_1).Consultant_New\\
&\quad + (newCon_fu, r_2).Consultant_New\\
Consultant_FU &\stackrel{\text{def}}{=} (fuCon, r_{fuCon}).Consultant_FU\\
Blood &\stackrel{\text{def}}{=} (blood, r_{blood}).Blood\\
Xray &\stackrel{\text{def}}{=} (xray, r_{xray}).Xray\\
Depart &\stackrel{\text{def}}{=} (depart, r_{depart}).Depart\\
Stop &\stackrel{\text{def}}{=} (stop, r_{stop}).Stop
\end{aligned}$$

Firstly from above PEPA model, it is can be seen that the activity flow for the new patients is altered as that in the first block: registration, X-Ray scan, blood test, general test, consulation and departure.

Secondly, consultants for the new patients are defined with two actions that are consulation for the new patients, namely *newCon*, and consulation for the follow-up patients, namely *newCon_fu*, such as:

$$Consultant\_New \stackrel{\text{def}}{=} (newCon,\ r_1).Consultant\_New + (newCon\_fu,\ r_2).Consultant\_New$$

Thirdly, for the follow-up patients, the activity flow remains the same except the state FU_con. Two actions appear there, standing for the process in which the follow-up patients are served by two groups of consultants.

Finally, the whole patient flow model can be defined as:

$$\begin{aligned} Sys &\stackrel{\text{def}}{=} (New[p_1]\|FU[p_2]) \bowtie_{L} \\ &\quad (Register[n_1]\|Test[n_2]\|Consultant\_New[n_3]\| \\ &\quad Consultant\_FU[n_4]\|Blood[n_5]\| \\ &\quad Xray[n_6]\|Depart[n_7]\|Stop[n_8]) \\ L &= \{register, test, newCon, newCon\_fu, fuCon, \\ &\quad blood, xray, depart, stop\} \end{aligned}$$

It is worth mentioning that a function rate must be applied in PEPA model on the basis of its rules when two or more actions are defined for a single component, such as the above PEPA statement, *Consultant_New*. Hence, the action rates for component *Consultant_New* is specified as below:

$$r_1 = \frac{New\_con}{New\_con + FU\_con} \times r_{newCon} \times \min(New\_con + FU\_con, Consultant\_New) \quad (4)$$

$$r_2 = \frac{FU\_con}{New\_con + FU\_con} \times r_{fuCon} \times \min(New\_con + FU\_con, Consultant\_New) \quad (5)$$

In the expression (4) and (5), New_con and FU_con represent the population of patient instance in this model state. The formal proof of using such function rate can be found in our previous work [30], [31].

C. PERFORMANCE ANALYSIS

The evolved patient flow model also applies the dynamic scheduling policy that has been explored in the preceding section. Analysis in this section uses the same technique that is the fluid flow approximation and the same configurations and parameters introduced in Section 5.2. However, the capacity plan is changed based on the evolved workflow. In the new plan, the number of consultants are changed to: 4 consultants arranged for the new patients and 2 consultants for the follow-up patients. This is because that the consultants serving the new also serve the follow-up patients when the new patients leave for examinations in another department. Thus, to ensure all incoming patients completed on time, one more consultant should be arranged for the new patients in contrast to the previous case.

Figure 5 indicates the number of patients queuing for consulting service without a scheduling policy. In the figure, solid lines represent queue length of the new patients and follow-up patients in the evolved workflow. The dotted lines stand for queue length based on the initial workflow. For the evolved model, it is clear that the queue length of follow-up patients has a dramatic decrease due to the new workflow; however, the queue length of new patients has a bit increase because consultants for the new patients also serve the follow-up patients. In any case, the evolved workflow has an overall reduced queue length compared with the initial workflow.

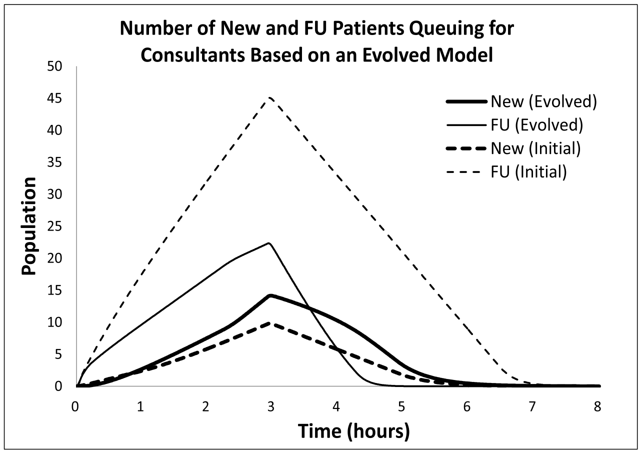

FIGURE 5. Number of new and FU patients queuing for consultants based on an evolved model.

In order to compare with the initial model, the following analysis uses the same parameters except the capacity scheme for consultants. The fluid flow approximation ais applied to run the analysis with the same condition setting. Analysis is conducted in the model without a scheduling policy in order to show the improvement of the evolved workflow. Thereafter, as the dynamic scheduling policy is the most efficient policy compared with the static policy, the analysis is then conducted based on the case using the dynamic scheduling policy.

In Figure 6 and 7, the preceding dynamic scheduling is applied to generate a performance analysis in the evolved

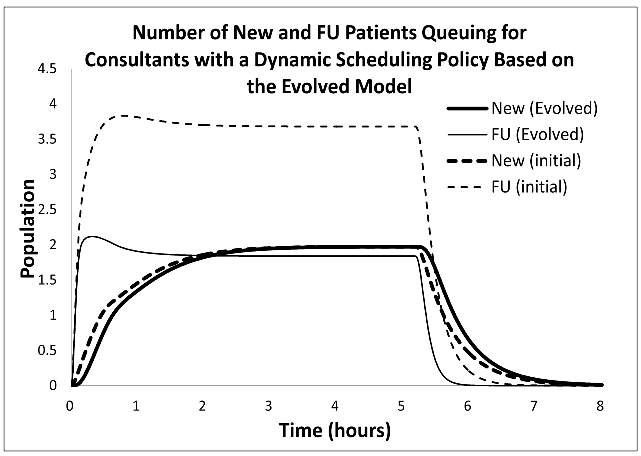

FIGURE 6. Number of new and FU patients queuing for consultants with a dynamic scheduling policy based on an evolved model.

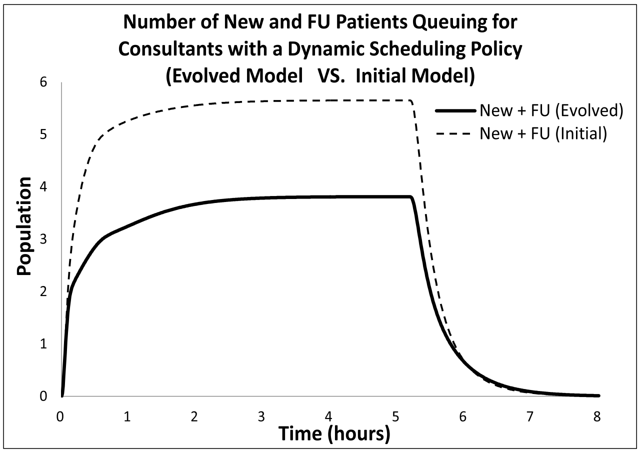

FIGURE 7. Number of new and FU patients queuing for consultants with a dynamic scheduling policy (Evolved model vs.initial model).

workflow model. Figure 6 represents the queue length of new patients and follow-up patients respectively based on the evolved workflow. The figure clearly shows that the stable queue length of follow-up patients is reduced from around 4.0 to 2.0. Such dramatic drop is caused by the evolved workflow in which consultants for the new patients can also serve the follow-up patients when they are idle. Once there are some new patients coming to the department, these consultants serve the new patients in priority. Moreover, the queue length for the new patients remains almost the same as that in the initial workflow. This means that the evolved workflow has no negative effect on the new patients compared with the initial workflow; meanwhile it improves the efficiency and utilization of consulting service through reducing the number of queuing follow-up patients.

Figure 7 displays the queue length of overall patients in the evolved workflow. In the figure, solid line represents the queue length in the evolved workflow which has a stale value between 3 and 4. However, in the initial workflow, the queue length represented by the dotted line is between 5 and 6. The figure clearly demonstrates that the evolved workflow can improve service efficiency for the overall patients. This improvement causes a dramatic decrease of waiting queue in the patient flow.

D. SUMMARY

In this section, an evolved workflow is developed to improve the efficiency and utilization based on the current workflow of the rheumatology department. According to the analysis results, the evolved workflow can successfully reduce the waiting queue by changing the activity flow of follow-up patients and reallocating consultants for two groups of patients. This means that the evolved workflow can obviously refine the patient flow through improving the efficiency and utilization of healthcare service.

VII. MODEL VALIDATION

This paper mainly uses a formal method to model the patient flow of a department and analyses the performance of related scheduling and capacity schemes by solving the underlying CTMC models with an ODE based fluid flow approach. According to the literature review, most preceding research using formal method for a Markov model creates models as a closed network or with infinite incoming events, which could bring the system to a steady state. However, this patient flow model is designed as an open network; meanwhile, the number of incoming patients is finite in each run of model in order to model the real situation of patient flow.

To ensure the accuracy of formal models, it is necessary to validate the preceding formal models with another modelling technique that has completely different modelling and analysing process. In this section, the validation of PEPA models is conducted through discrete event simulation (DES) that models the operation of a system as a discrete sequence of events in time and continuously tracks the system dynamics over time. In DES, time is broken up into small time slices and the system state is updated according to the set of activities happening in the time slice [32]. The simulation tool used for the validation is OMNeT++ that is an extensible, modular and C++ based simulation framework for modelling networks such as, queuing network and communication network. OMNeT++ is chosen as the main tool due to its powerful functions in simulating and analysing queue network. The patient flow model, in fact, is a kind of queue network.

A. DES TOOL: OMNeT++

According to the definition in the OMNeT++ community, OMNeT++ is an extensible, modular, component-based C++ simulation library and framework, primarily for building network simulators. OMNeT++ is developed to solve different kinds of network models, such as wired and wireless communication networks, queueing networks, and distributed networks, and so on. To facilitate simulation, OMNeT++ offers an Eclipse-based IDE, a graphical runtime environment, and a host of other tools. Therefore, the simulation is easy to generate and execute. OMNeT++ IDE extends Eclipse platform with some new functional modules: an editor with both graphical view and command lines (an NED file), a functional module for parameter setup and simulation configuration (an INI file) and an analysis module for the simulation output (an ANF file). More details of OMNeT++ and OMNeT++ IDE can be found in the documentation of OMNet++ which is available on the OMNeT++ community site.

B. SIMULATION MODEL

Simulation model is created in terms of PEPA model and related parameters. Thus, this model should be completely the same as that built with PEPA. Regarding these conditions, a queue network in the simulation should be built as a multi-server network, and the number of servers equals the number of components used in PEPA model.

Figure 8 is a simulation design scheme based on the initial patient flow model using PEPA. In the diagram, rectangles represent all defined components in the PEPA model and

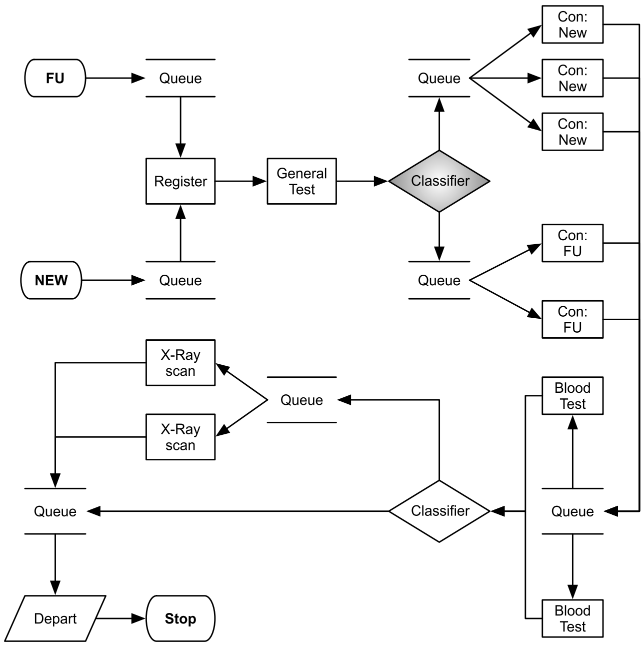

FIGURE 8. Simulation design scheme of patient flow model.

parallel lines represent queues of components. Arrows stand for the flow of patients in the department. The shaded diamond connected to the component *General Test* is defined as a scheduler in which scheduling policies are simulated. In the simulation model, all components and actions are completely equivalent to PEPA model as well as all parameters. This is to ensure the correctness of validation using discrete event simulation.

C. SIMULATION MEASUREMENT AND VALIDATION

In order to validate PEPA models, simulation measurements should be conducted under the same parameter conditions. Thus, all parameters applied in simulation are those figures displayed in Table 1 & 3. As the aim of this section is to validate PEPA models, we only simulate the initial patient flow and validate the related PEPA model with a no-scheduling scheme and models with two kinds of scheduling policies. Figure 9, 10 and 11 represent the simulation results of the initial patient flow with the no-scheduling scheme, the static scheduling policy and the dynamic scheduling policy respectively.

In Figure 9, dashed lines for the simulation results plot close to the solid lines representing results of PEPA model. Hence, we can argue that results from PEPA models using fluid flow analysis have reliable accuracy when the patient flow model does not include a scheduling policy. The simulation in this step is to validate the prototype of patient flow model so as to conduct the following comparison with scheduling policies.

Figure 10 displays the queue length of the new patients and the follow-up patients based on the initial workflow with a static scheduling policy. Dashed lines represent the figures

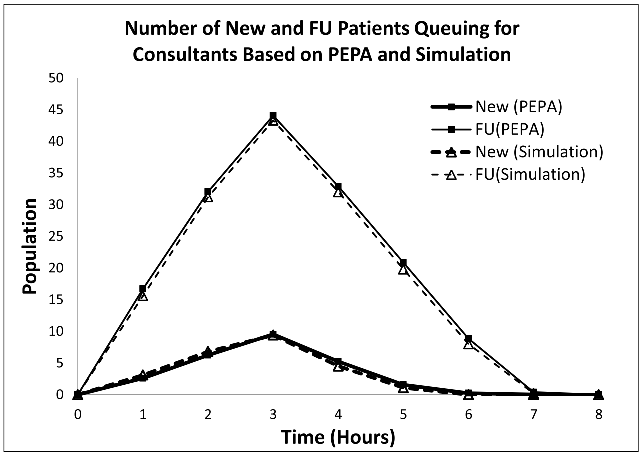

FIGURE 9. Number of new and FU patients queuing for consultants based on PEPA and simulation.

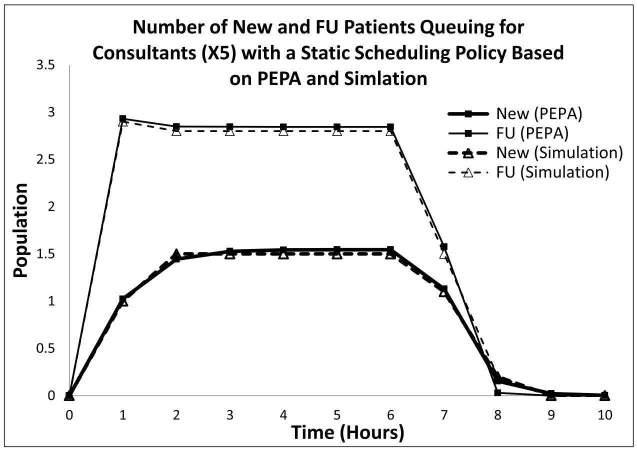

FIGURE 10. Number of new and FU patients queuing for consultants with a static scheduling policy based on PEPA and simulation.

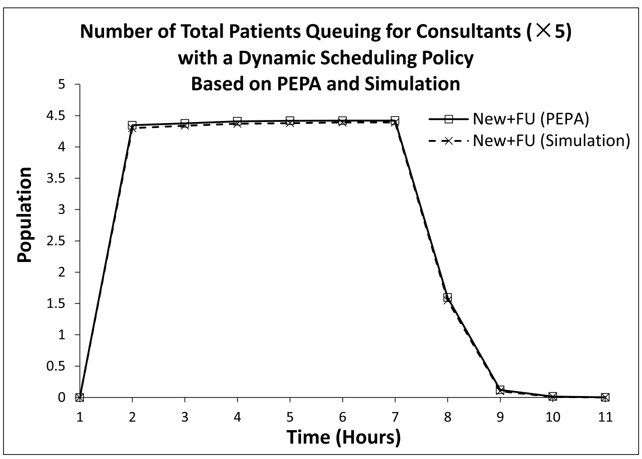

FIGURE 11. Number of new and FU patients queuing for consultants with a dynamic scheduling policy based on PEPA and simulation.

obtained from simulation that are around a value 1.5 for the queue length of new patients and a value 3.0 for the queue length of the follow-up patients in a stable phase. It is clear that both dashed lines for the simulation results approximate to solid lines that represent the results from PEPA model using fluid flow approximation. This step verifies that PEPA analysis based on the initial patient flow model with a static scheduling policy has accurate results by comparing with simulation results.

Figure 11 describes the results of PEPA model and simulation based on the patient flow model with the dynamic scheduling policy. Similarly, the simulation results denoted by the dashed line get very close to those results from PEPA model which are represented by a solid line. This proves the validity and high accuracy of PEPA model and corresponding analysis when the dynamic scheduling policy is applied in the patient flow model.

D. SUMMARY

In this section, validation using discrete event simulation is implemented to verify the accuracy of PEPA model and related analysis. As the simulation is a time-consuming work, we just complete the validation for some analysis results of PEPA model which are based on models of the initial workflow. PEPA models of two workflows (the initial and the evolved) have the same architecture and analysis technique – fluid flow approximation. Hence, simulation for one of the two workflows can provide adequate proof for the model validation. Here, the initial workflow is chosen for the validation. According to comparisons, we can get a conclusion that analysis based on PEPA models is highly accurate and reliable in contrast to the simulation results.

VIII. CONCLUSION

To improve the patient flow, a smart environment can be applied to support behaviours of patients and management of workflow in the hospital. Smart environment can be developed depending on technologies of IoT (Internet of Things) and CPS (Cyber Physical Systems). This paper aims to provide a solution applying a smart environment for the rheumatology department in order to obtain well improvement of its patient flow and workflow. This research, firstly, explores a dynamic scheduling policy that can improve the efficiency and utilization of consulting service and reduce patient waiting queue. Secondly, an evolved workflow and a new capacity plan are developed for the rheumatology department which can improve the efficiency and quality of service.

This paper has achievements in two areas. On one hand, the research accomplishes a performance modelling task for the patient flow of a department. The scheduling model is created on the basis of the department patient flow so as to refine the scheduling process by using a novel dynamic scheduling policy. The capacity model is initially built on the initial workflow; then the initial workflow is refined by giving a new action flow and capacity scheme, which can improve service efficiency greatly. On the other hand, this research adopts a formal method for performance modelling and analysis due to its features in compositionality and high efficiency. Another creative modelling technique is that function rates are used in the formal method to model complex system behaviours, such as the dynamic scheduling process. As a result, formal method has an immense scope of applications by using function rates.

According to the performance measurements, the dynamic scheduling policy has ability to improve the scheduling efficiency and service utilization due to its adaptable feature, especially when the number of incoming patients is varying. Based on the initial workflow of department, a evolved workflow is created in order to improve the service efficiency any further. From the analysis results, it is clear that the evolved workflow scheme greatly improves work efficiency by changing the order of activity flow and the consultant allocation scheme. Moreover, a new capacity plan is generated for the evolved workflow in terms of the statistical data and the performance results of the dynamic scheduling policy. Regrading the analysis, such new capacity plan can ensure that all scheduled patients can finish their activities within 8 working hours.

To extend this work, we will explore a patient flow model that is more close to the real situation further by considering the no-show and cancellation in the patient flow. This situation will takes more difficulties to the scheduling process. Thus, a specific and workable solution should be explored to improve the scheduling process. Furthermore, in fact, upcoming arrivals in the department usually do not arrive at a stable rate; and also for the serving activities of consultants, the service rate should be varying based on different consultants and unknown situations. In our future work, we will model the situation with unstable arrivals and varying consulting activities in order to promote a new scheduling policy.

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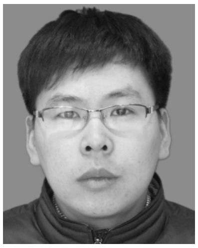

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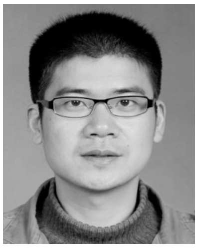

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