Received September 30, 2015, accepted November 10, 2015, date of publication December 7, 2015, date of current version December 14, 2015.

Digital Object Identifier 10.1109/ACCESS.2015.2506503

Cascading Failures in Smart Grid: Joint Effect of Load Propagation and Interdependence

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ABSTRACT The smart grid mainly suffers from two types of cascading failures: 1) interdependence cascading failure and 2) load propagation cascading failure. The former one happens due to the interdependence between power grid and communication networks. The latter one is caused by the load propagation in the single power grid. A tiny failure leads to the simultaneous occurrence of these two cascading failures. In this paper, we study the system robustness by considering the interdependence and load propagation. First, we develop a mathematical tool to analyze the load propagation in single network. Then, a percolation-based method is proposed to calculate the remaining fractions of survivals after the cascading failures stop. We estimate the node tolerance parameter T (the ratio of capacity to initial workload) threshold T_c', below which the entire system may suffer from the cascading failure. The effect of interdependence on T_c' is also studied, where lower T_c' is required for the less compact interdependence. We prove that the system performance approaches to the upper bound once the tolerance parameter $T \to \infty$. Our analysis indicates that the fraction of survivals in the power grid is always greater than that in communication network, although the initial failure occurs in the power grid. The extensive simulations validate our mathematical analysis, and demonstrate that the relation between the number of initial failures and tolerance parameter threshold is super-linear.

INDEX TERMS Smart grids, load propagation, interdependence cascading, percolation theory.

I. INTRODUCTION

As one of the emerging technologies, smart grid has been studied extensively in recent days. This new technology brings together three facets of a power grid, namely power generation, real-time monitoring and estimation of electricity consumption. Different technologies and resources are integrated in a smart power grid. There are two networks, a power grid and a communication/control network, which are strongly coupled. That is, the communication network functions depending on the electricity from the power grid. On the other hand, the power grid has to be controlled by the communication/control network, thus making these two networks interconnected and mutually dependent.

However, this growing interdependence leads to a more fragile smart grid than the conventional power grid [1]. A big issue the smart grid suffers from is that of cascading failure. The malfunction of a node may lead to sequential failures in the entire system. To address this issue, first we roughly classify cascading failures in smart grid into two types: *interdependence cascading failure* and *load propagation cascading failure*. The former is caused due to the interdependent nature of the relation between the power grid and the communication network [2], and an initial tiny failure in one network results in failures on the other network, which recursively lead to more failures back on the initial network. The latter type of cascading failure is caused due to the overloading of power node and the load propagation from node to node within the single power grid [9]. It is important that we understand these two different cascading failures.

Interdependence cascading failure is well studied recently by [2]–[5] and [17], where most of them are focusing on extracting interdependence models from real-world smart grid and estimating the robustness of their models. Buldyrev *et al.* [2] proposed a 'one-to-one' interdependence model and a percolation-based mathematical tool to calculate the fraction of survivals. Huang *et al.* [17] pointed out the different roles of nodes, and indicated a multiple-to-multiple interdependence model. Dong *et al.* [5] studied the robustness of n interdependent networks, with partial interdependence relationship. Random failure was applied in the works mentioned above, while a targeted attack was studied in [4]. Meanwhile, the work on load propagation cascading failures can be found in [8]–[11]. Hines *et al.* [10] introduced the backgrounds of power node overload cascading failures. Some load propagation schemes are proposed and discussed in [11]. Chen *et al.* [15] developed a new vulnerability assessment by combining attack methodology with DC flow.

The existing works on interdependence cascading failure have been performed using pure topological methods, which do not represent the real world smart grid properly, due to the lack of consideration of electricity characteristics [12]. Furthermore, the studies conducted on load propagation cascading failure, are restricted to the single power grid, and thus cannot be applied to smart grid because the key feature of interdependence is missing. More importantly, the two cascading failures occur simultaneously and affect each other.

In this article, by taking into account both the topological method and electricity characteristics, we aim to study the joint effect of the two cascading failures to be more closer to the real-world smart grid. We use the load propagation model in [11] where the workload of a removed node is equally redistributed among its neighbors. For the interdependence relation, we apply our previous 'k-n' model [17]. But, due to the capacity limit, the number of inter links each power node has, is restricted. So we propose a new modified *Balls and Bins* allocation scheme for the 'k-n' model. Assuming that both the power grid and the communication network are scale-free [16], in terms of random initial failure, we develop a method to estimate the node *tolerance parameter* threshold T_c (the ratio of capacity to initial workload) in single power grid, below which the grid suffers from load propagation cascading. A mathematical tool is proposed to calculate the fraction of survivals in power grid after the load propagation failure stops.

We devise a percolation-based mathematical method to simulate the process of interdependence cascading failure in the smart grid and obtain a set of transcendental function for the fractions of survivals in both power grid and communication network. Similar to the single power grid, we find a method to estimate the *tolerance parameter* threshold T_c' for smart grid. Our analysis demonstrates that given the same network topologies and initial removals, $T_c' > T_c$. We also find that the relation between T_c' and 'k-n' interdependence, is such that higher k indicates lower T_c'. The system robustness approaches the upper bound once tolerance parameter $T \rightarrow \infty$. We prove that this upper bound is the performance when the system suffers only from interdependence cascading failure.

The extensive simulations determine the values of T_c' and validate our mathematical analysis. In addition, we observe a super linear relation between the initial removals with T_c' in the specific system. Also, the system robustness of single power grid is better than smart grid, given the same parameters. The interesting finding is that, in smart grid, although the initial failure occurs in power grid, the fraction of survivals in power grid is always greater than that in communication network.

This paper is organized as follows: The background and related work are discussed in Section II. We introduce the load propagation model in Section III. The new Section *Balls and Bins* allocation scheme is proposed in Section IV. We develop mathematical tools as well as proofs in Section V. Extensive simulation results and comparisons are discussed in Section VI. Section VII deals with conclusions.

II. RELATED WORK

The recent studies on interdependent networks are focusing on building models and studying the transition phenomena. Percolation theory is widely used to reconstruct the process of the cascading failures due to the interdependence between networks. The authors presented a 'one-to-one' interdependence model with bidirectional inter relationship in [2], where each node operates relying on the unique node in the other network. Yagan *et al.* proposed the 'multiple-to-multiple' model in [3], where they proved that allocating each node with the same number of inter links is the optimal choice against random failures. Our previous study [17] pointed out that the nodes in the network may have different roles. We indicated an 'k-n' model for smart power grid, and studied the relation between the system robustness and control cost. As different types of nodes take on different responsibilities, the interdependence is also different for each node.

Similarly, Dong *et al.* [5] studied the system of n interdependent networks with partial interdependence relationship. A case of random failure is applied to the above work, whereas in [4], the authors focused on targeted attack on a network of networks (NON). Further, they investigated the system robustness of both fully interdependent and partially interdependent networks. Nguyen *et al.* [1] proposed an algorithm to identify the critical nodes, the removal of which, could lead to fatal destruction of the networks.

There are extensive works on studying the vulnerability of power grid. An introduction of power cascading failure can be found in [10], where the authors also described two schemes to reduce the cost of blackout. Zio and Sansavini [11] analyzed three different cascading models, differing in load distribution schemes: equal redistribution among neighbors or redistribution based on the shortest paths. In [9], the authors proposed an extended topological model while taking into account the specific features of power systems, such as electrical distance and line flow limits. They developed two metrics *extended betweenness* and *net ability* to rank the criticality of network components. A new approach of vulnerability assessment for power grid was proposed in [15], where the authors combined a DC flow model with a traditional attack methodology.

Zhao *et al.* [8] investigated the phase transition phenomenon in terms of node capacity. They proved that for the capacity below the transition threshold, the system degrades due to the cascading failure. In [13], the authors tried to address the problem of how large of a capacity could make the network immune to cascading breakdowns.

III. LOAD PROPAGATION MODEL

We denote the single power grid by $G_p = (V_p, E_p)$, where V_p denotes the set of vertices and E_p denotes the set of edges. According to the previous studies [13]–[15], G_p is assumed to follow the scale-free network characteristics. Specifically, we denote the probability distribution of node degree by $\mathbf{P}_\mathrm{d}(z)$, and assume that $\mathbf{P}_\mathrm{d}(z) \propto z^{-\lambda}$, where z is node degree. The electricity is propogated from node to node through the power lines. The breakdown of few nodes may cause the load dynamic redistribution, and consequently lead to the breakdown of the remaining nodes. To study this cascading process, we introduce a load propagation model, as follows.

For a node i in scale-free network, assuming its initial workload is the function of its degree z, we have

$$L_i(z) = \alpha \cdot z^{\eta}, \tag{1}$$

where α and η are the pre-defined constants. It is reasonable to assume that each node has the limit of load, called *capacity*, which is proportional to the initial load

$$C_i(z) = T \cdot L_i(z), \quad T \geq 1 \tag{2}$$

where constant T is the *Tolerance parameter* [8], [13] that controls the tolerance of the power grid.

The breakdown or removal of a single node i would transfer its initial load to the remaining network. To simplify our study, this load is *uniformly* distributed to the neighbors. Therefore, the load of a neighbor j of the removed node i equals to the sum of its initial load plus the incremental load transferred from the removed node. The work load for node j is

$$L_j(z) = L_j(z) + \frac{L_i(z)}{z}. \tag{3}$$

Node j would be overloaded and out of work when $L_j(z) > C_j(z)$. Then the load redistribution process might continue to the neighbors of j. The overload cascading stops if one of the following situations is satisfied,

- The workload is less than the capacity for all of the remaining nodes,
- The entire nodes in V_p are removed.

IV. INTERDEPENDENT NETWORKS

We denote the communication network by $G_c = (V_c, E_c)$. In smart power grid, G_p and G_c are mutually dependent and connected. Several interdependence models have been studied in previous work. In this article, to study how interdependence affects the network robustness, we apply our 'k-n' model [17], where we briefly introduce as follows:

- The nodes in G_c are roughly divided into two based on their roles: *information relay node* and *control node*. The control node, or the *operation center*, is responsible for monitoring and operating the nodes in G_p. The control messages are transmitted through the information relay nodes, on the other hand, the nodes in V_p provide electricity to the nodes in G_c. To improve the system robustness, we let each node in V_p be under the control of k control nodes in V_c, which at the same time, could monitor all n nodes in V_p.
- *Control-Dependency* links (CD Link) and *Energy-Dependency* links (ED link), together form the inter links. Each control node has n Control-Dependency links to operate n nodes in G_p. While each node of G_p is under the control of k operation centers, and thus contains k Control-Dependency links. A *Balls and Bins* allocation procedure is applied to assign the Energy-Dependency links so that each node in V_c could obtain the energy from G_p.

Fig. 1 gives a sketch of 'k-n' interdependence model.

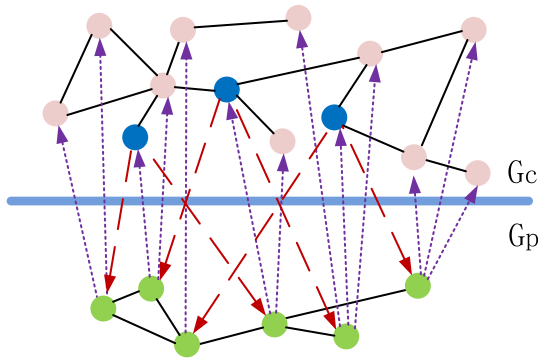

FIGURE 1. Each power node provides electricity to multiple communication nodes. 3 communication nodes are chosen as the operation centers (deep blue), where each of them operate 2 power nodes, thus $n = 2$. Each power station is controlled by 1 operation center, thus $k = 1$. ED link is from G_p to G_c, CD link is the opposite.

A. MODIFIED BALLS AND BINS ALLOCATION

After taking the initial workload and capacity into account, we believe there is a limit on the number of V_c nodes that a power node can support. Thus, the previous *Balls and Bins* allocation is no longer accurate, since it is based on the assumption that all power nodes are equal and have unlimited capacities.

For a power node i, we assume that the maximum number of communication nodes it can support is proportional to its initial load, thus we have

$$N_i(z) = \beta \cdot \alpha \cdot z^{\eta}. \tag{4}$$

The mean maximum number is given by

$$\langle N_i(z) \rangle = \sum_{z=0}^{\infty} \mathbf{P}_\mathrm{d}(z) \cdot N_i(z) = \beta \cdot \alpha \sum_{z=0}^{\infty} \mathbf{P}_\mathrm{d}(z) \cdot z^{\eta}. \tag{5}$$

We consider the nodes in G_p are bins and the nodes in G_c are balls. Assuming the sizes of G_p and G_c are S_p and S_c respectively, the total positions a ball can choose is $S_p \cdot \langle N_i(z) \rangle$.

Therefore, the probability for a ball to be allocated into the bins with initial load of $\alpha \cdot z^\eta$ equals

$$p = \frac{\mathbf{P}_\mathrm{d}(z) \cdot z}{S_p \cdot \langle N_i(z) \rangle}. \tag{6}$$

Define the number of balls in a bin with initial load of $\alpha \cdot z^\eta$ as a random variable ξ_z. Therefore, it holds that

$$\mathbf{P}(\xi_z = t) = \binom{z}{t}\binom{S_c}{t} \cdot p^t \cdot (1-p)^{S_c - t}, \quad t \le z \tag{7}$$

Furthermore, define the number of balls in a bin as a random variable ξ. It follows that

$$\mathbf{P}(\xi = t) = \sum_{z=0}^{\infty} \mathbf{P}(\xi_z = t) \cdot \mathbf{P}_\mathrm{d}(z), \tag{8}$$

where $\mathbf{P}_\mathrm{d}(z)$ is the degree distribution.

V. MATHEMATICAL ANALYSIS

In this section, we estimate the robustness of interdependent smart power grid by calculating the size of functioning giant component, and discuss how the 'k-n' interdependence model and load propagation affect the system. We study the system robustness by applying a random failure of fraction of $1-\phi$ on G_p. The definitions and notations are listed in Table 1.

TABLE 1. Notations for the analysis.

k	number of operation centers that control a specific power station
n	number of power stations each operation center operates
T	tolerance parameter
u	the probability a node can not reach to the giant component through a specific neighbor
$1-\phi_c$	the initial failure threshold above which the entire system collapse [2], [17]
$\lambda_\mathrm{P}, \lambda_\mathrm{c}$	power law exponents of networks G_p and G_c.
$F(\phi, G_p), F(\phi, G_c)$	the fraction of giant component in the subnetwork which occupies the fraction ϕ of the nodes in the entire network G_p and G_c

A. CALCULATION OF GIANT COMPONENT

We introduce a notation $F(\phi, G_p)$ [17] to denote the expected fraction of giant component in the subnetwork which occupies the proportion ϕ of nodes in network G_p. Accordingly, $F(\phi, G_c)$ is for network G_c. $F(\phi, G_p)$ is estimated by [7]. That is,

$$\begin{cases} F(\phi, G_p) = 1 - \sum_0^\infty \mathbf{P}_\mathrm{d}(z) \cdot u^z, \\ u = \sum_0^\infty \mathbf{Q}(z) \cdot (1 - \phi + \phi \cdot u^z), \end{cases} \tag{9}$$

where $\mathbf{Q}(z)$ is the number of edges connected with the node other than the edge we arrived with, called *excess degree distribution* [7], [17], and given by

$$\mathbf{Q}(z) = \frac{(z+1) \cdot \mathbf{P}_\mathrm{d}(z+1)}{\langle z \rangle}, \tag{10}$$

where $\langle z \rangle = \sum_0^\infty \mathbf{P}_\mathrm{d}(z) \cdot z$.

B. THRESHOLD T_c IN SINGLE POWER GRID G_p

Once $1-\phi$ of G_p is removed, the proportion of remaining functioning network is approximated by $\phi \cdot F(\phi, G_p)$ [17]. After the removal, the load of removed nodes is distributed to the remaining part of the network. The total load that need to be shifted R is the summation of initial load of the removed nodes, which is given by $\sum_{i=1}^{S_p(1-\phi\cdot F(\phi,G_p))} L_i(z)$. As far as $S_p \to \infty$, we simplify it to

$$R = S_p \cdot (1 - \phi \cdot F(\phi, G_p)) \cdot \sum_{z=0}^{\infty} \mathbf{P}_\mathrm{d}(z) \cdot \alpha \cdot z^\eta. \tag{11}$$

To avoid the load cascading failure, the shifted load R can not exceed the remaining volume of the network. Therefore,

$$R \le \sum_{i=S_p(1-\phi\cdot F(\phi,G_p))}^{S_p} (C_i(z) - L_i(z))$$

should be satisfied. The threshold T_c is obtained when both sides are exactly equal. That is

$$R = S_p \cdot \phi \cdot F(\phi, G_p) \cdot (T_c - 1) \cdot \sum_0^\infty \mathbf{P}_\mathrm{d}(z) \cdot \alpha \cdot z^\eta. \tag{12}$$

Applying Eq. (11), we have

$$\begin{aligned} 1 - \phi \cdot F(\phi, G_p) &= \phi \cdot F(\phi, G_p) \cdot (T_c - 1), \\ 1 &= T_c \cdot \phi \cdot F(\phi, G_p), \\ T_c &= \frac{1}{\phi \cdot F(\phi, G_p)}. \end{aligned} \tag{13}$$

Smaller T would more likely cause load cascading failures in the entire network.

C. FAILURE DUE TO LOAD REDISTRIBUTION

For an arbitrary node i, we define two random variables and an event as follows.

- Random Variable Z_i: the number of node i's neighbors.
- Random Variable X_i: the number of neighbors that could destroy node i by shifting their load.
- Event $\mathbf{O}_i$: node i is destroyed due to the excessive load shifted from its neighbors.

From the homogeneity of the distribution of node degrees, we denote the sequences of Z_i, X_i, and $\mathbf{O}_i$ by Z, X, and $\mathbf{O}$, without the loss of generality.

For a node, to compute $\mathbf{P}(\mathbf{O}|Z = z)$, we need to know $\mathbf{P}(\mathbf{O}|X = x)$, which is the probability that the shifted load from x neighbors is greater than the remaining volume of the node. Thus, applying Eqs. (1) and (3), we have

$$\mathbf{P}(\mathbf{O}|X = x) = \mathbf{P}\left\{x \cdot \sum_{z=1}^{\infty} \frac{\alpha \cdot z^\eta}{z} + \alpha \cdot z^\eta > T \cdot \alpha \cdot z^\eta\right\} \tag{14}$$

As x, z, α, η and T are constants here, the distribution of $\mathbf{P}(\mathbf{O}|X = x)$ is

$$\mathbf{P}(\mathbf{O}|X = x) = \begin{cases} 1, & x > \dfrac{(T-1)\cdot\alpha\cdot z^{\eta}}{\sum_0^{\infty}\frac{\alpha\cdot z^{\eta}}{z}}, \\ 0, & \text{Otherwise.} \end{cases} \tag{15}$$

The probability to choose x neighbors among total z neighbors $\mathbf{P}(X = x|Z = z)$ is given by

$$\mathbf{P}(X = x|Z = z) = \binom{z}{x} / 2^z.$$

Therefore, $\mathbf{P}(\mathbf{O}|Z = z)$ could be calculated by

$$\begin{aligned} \mathbf{P}(\mathbf{O}|Z = z) &= \sum_{x=0}^{z} \mathbf{P}(\mathbf{O}|X = x)\cdot\mathbf{P}(X = x|Z = z), \\ &= \sum_{\frac{(T-1)\cdot\alpha\cdot z^{\eta}}{\sum_0^{\infty}\frac{\alpha\cdot z^{\eta}}{z}}}^{z} \frac{\binom{z}{x}}{2^z}. \end{aligned} \tag{16}$$

D. LOAD PROPAGATION IN G_p

We denote u is the probability that a node cannot reach the giant component through a specific neighbor. After the initial removal of fraction $1-\phi$, for a node i with degree of z, there are three possible ways not belonging to the giant component

- It is removed
- Or it is present but not a member of the giant component
- Or the shifted load from neighbors destroys itself, i.e., i is overloaded.

The first case happens with probability of $1-\phi$. The second case happens if i can not reach to giant component through all its neighbors. The probability that none of the neighbors connects to giant component is $\phi\cdot u^z$. $\mathbf{P}(\mathbf{O}|Z = z)$ is the probability of the third case, given the number of neighbors is z. Thus,

$$\mathbf{P}(\mathbf{E}(\phi)) = 1-\phi+\phi\cdot(u^z+(1-u^z)\cdot\mathbf{P}(\mathbf{O}|Z = z)), \tag{17}$$

where $\mathbf{E}(\phi)$ is the event that the considered node disconnects with the giant component.

The probability that the vertex does belong to the giant component after the removal of fraction $1-\phi$ is

$$\sum_0^{\infty}\mathbf{P}_{\mathrm{d}}(z)\cdot(1-\mathbf{P}(\mathbf{E}(\phi))).$$

In other words, this is the expected fraction of remaining giant component. We define a new notation $\mathbf{J}(\phi, G_p)$ which represents the expected fraction of giant component of the subnetwork which occupies the fraction ϕ of the original entire network G_p, given by

$$\begin{aligned} \mathbf{J}(\phi, G_p) &= \frac{\sum_0^{\infty}\mathbf{P}_{\mathrm{d}}(z)\cdot(1-\mathbf{P}(\mathbf{E}(\phi)))}{\phi}, \\ &= \sum_0^{\infty}\mathbf{P}_{\mathrm{d}}(z)\cdot(1-u^z)(1-\mathbf{P}(\mathbf{O}|Z = z)). \end{aligned} \tag{18}$$

The probability that i does not connect to giant component through a specific neighbor j equals to the probability that j does not connect to the giant component through its remaining neighbors, i.e., except i. Therefore, we obtain the following equation

$$u = \sum_{z=0}^{\infty}\mathbf{Q}(z)\cdot(1-\phi+\phi\cdot(u^z+(1-u^z)\cdot\mathbf{P}(\mathbf{O}|Z = z))), \tag{19}$$

recall $\mathbf{Q}(z)$ is the excess degree distribution.

Combining Eqs. (16) and (19), we can obtain the value of u. Then applying it to Eq. (17) and (18), finally we can calculate the remaining fraction of network after the random removal $1-\phi$, with the load cascading taken into account.

E. INTERDEPENDENCE CASCADING FAILURES

First of all, we would like to estimate the threshold T_c' below which the interdependent networks would suffer a cascading failure due to load redistribution. After the random removal of a fraction $1-\phi$ of nodes in G_p, the proportion of remaining network equals to $\mu_{p_1}' = \phi$. We assume that the nodes belong to giant component could operate properly. Thus, the remaining functioning fraction μ_{p_1} in G_p is given by

$$\mu_{p_1} = \mu_{p_1}'\cdot F(\mu_{p_1}', G_p). \tag{20}$$

After the fragmentation in network G_p, the Energy-Dependency inter links connected with these removed nodes are removed. The corresponding nodes in network G_c lose the inter links and thus stop operate. The number of removed Energy-Dependency inter links is $\sum_0^{(1-\mu_{p_1})\cdot S_p} t_i$, where t_i represents the number of Energy-Dependency inter links the node i connects with. As far as S_p goes infinite, it could be approximated to

$$(1-\mu_{p_1})\cdot S_p\cdot\langle t\rangle,$$

where $\langle t\rangle$ is the mean value, given by

$$\langle t\rangle = \sum_0^{\infty}\mathbf{P}(\xi = t)\cdot t,$$

recall $\mathbf{P}(\xi = t)$ is given by Eq. (8).

As the total number of ED inter links is $\langle t\rangle\cdot S_p$, the probability for one energy inter link to be removed is $1-\mu_{p_1}$. Consequently, the fraction of nodes in G_c that has the ED inter link is approximate to μ_{p_1}. Thus, we have $\mu_{c_2}' = \mu_{p_1}$. The operating giant component in G_c is calculated by

$$\mu_{c_2} = \mu_{c_2}'\cdot F(\mu_{c_2}', G_c). \tag{21}$$

Roughly, the fraction $1-\mu_{c_2}$ of control nodes are removed, given the network size S_c is large enough. As a result, this fragmentation may lead to further failures in G_p. The probability for each Control-Dependency inter link to be removed is approximated to $1-\mu_{c_2}$, because each control node has the same ability to operate n nodes. With this in mind, the probability for a node in G_p loses all its k control links is $(1-\mu_{c2})^k$. The remaining fraction of nodes in G_p with at least one CD inter link is

$$\mu'_{p3} = (1-(1-\mu_{c2})^k)\cdot\mu_{p1}.$$

By the transformation idea in [2] and [17], to calculate the giant component, we consider the effect of removing the fraction of $1-\mu_{p3}$ of nodes in μ_{p1} has the same effect as taking out the same fraction size from μ'_{p1}. Thereby, from initial network G_p to μ'_{p3}, we have the following equivalent removing process:

$$1-\phi+\phi\cdot(1-\mu_{c2})^k = 1-(\phi-\phi\cdot(1-\mu_{c2})^k).$$

Thus, the equivalent $\mu'_{p3} = \phi\cdot(1-(1-\mu_{c2})^k)$. The fraction of functioning giant component is

$$\mu_{p3} = \mu'_{p3}\cdot F(\mu'_{p3}, G_p). \tag{22}$$

Following the above steps and progressing, we can observe the pattern of these equations as follows:

$$\begin{aligned}
\mu'_{p1} &= \phi,\\
\mu_{p1} &= \mu'_{p1}\cdot F(\mu'_{p1}, G_p),\\
\mu'_{p3} &= \phi\cdot(1-(1-\mu_{c2})^k),\\
\mu_{p3} &= \mu'_{p3}\cdot F(\mu'_{p3}, G_p),\\
\cdots,\quad & \cdots\\
\mu'_{p2j+1} &= \phi\cdot(1-(1-\mu_{c2j})^k),\\
\mu_{p2j+1} &= \mu'_{p2j+1}\cdot F(\mu'_{p2j+1}, G_p),
\end{aligned}$$

and

$$\begin{aligned}
\mu'_{c2} &= \mu_{p1},\\
\mu_{c2} &= \mu'_{c2}\cdot F(\mu'_{c2}, G_c),\\
\mu'_{c4} &= \mu_{p3},\\
\mu_{c4} &= \mu'_{c4}\cdot F(\mu'_{c4}, G_c),\\
\cdots,\quad & \cdots\\
\mu'_{c2j} &= \mu_{p2j-1},\\
\mu_{c2j} &= \mu'_{c2j}\cdot F(\mu'_{c2j}, G_c).
\end{aligned}$$

Once the cascading failures stop, at the final steady state, the following equations hold

$$\begin{aligned}
\mu'_{p2j+1} &= \mu'_{p2j+3} = \mu'_{p2j-1},\\
\mu'_{c2j} &= \mu'_{c2j+2} = \mu'_{c2j-2}.
\end{aligned}$$

We let $\gamma = \mu'_{p2j+1} = \mu'_{p2j+3} = \mu'_{p2j-1}$ and $\delta = \mu'_{c2j} = \mu'_{c2j+2} = \mu'_{c2j-2}$, then we obtain the transcendental equations:

$$\begin{cases}\gamma = \phi\cdot(1-(1-\delta\cdot F(\delta, G_c))^k),\\ \delta = \gamma\cdot F(\gamma, G_p).\end{cases} \tag{23}$$

Thus, the fractions of operating giant components in the final steady state are given by

$$\begin{cases}\lim_{j\to\infty}\mu_{pj} = \mu_{p\infty} = \gamma\cdot F(\gamma, G_p),\\ \lim_{j\to\infty}\mu_{cj} = \mu_{c\infty} = \delta\cdot F(\delta, G_c).\end{cases} \tag{24}$$

F. LOAD PROPAGATION IN INTERDEPENDENT NETWORKS

Eq. (24) gives the estimated result on the size of functioning fractions of networks. It only considers the effect of interdependence cascading failures. While, to avoid the cascading failure of overload, the redistributed load R can not exceed the remaining volume of the network. The threshold T'_c is obtained when both sides are exactly equal, where we get

$$\begin{aligned}
1-\gamma\cdot F(\gamma, G_p) &= \gamma\cdot F(\gamma, G_p)\cdot(T'_c-1),\\
T'_c &= \frac{1}{\gamma\cdot F(\gamma, G_p)} = \frac{1}{\delta}.
\end{aligned} \tag{25}$$

That is the threshold below which the entire system would suffer from the cascading failure caused by load redistribution.

Lemma 1: For a given power law exponent λ_p and initial removal $1-\phi$, $T'_c > T_c$.

Proof: Combining Eqs. (13) and (25), we have

$$\frac{T'_c}{T_c} = \frac{\phi\cdot F(\phi, G_p)}{\gamma\cdot F(\gamma, G_p)}. \tag{26}$$

According to Eq. (23), if γ and δ are nonzero, $F(\gamma, G_p)$ and $F(\delta, G_p)$ are nonzero too. Consequently, $(1-(1-\delta\cdot F(\delta, G_c))^k) < 1$. Thus, we have $\phi > \gamma$.

It is shown that the $F(\phi, G_p)$ is a monotone increasing function [7], thus $F(\gamma, G_p) < F(\phi, G_p)$. We have

$$\frac{T'_c}{T_c} = \frac{\phi\cdot F(\phi, G_p)}{\gamma\cdot F(\gamma, G_p)} > 1,$$

which completes the proof. □

Lemma 2: Recall a node in G_p is under the control of k nodes in G_c. For a given ϕ, for two different values of $k = k_1$ and $k = k_2$, where $k_1 < k_2$, the threshold $T'_c|_{k=k_1} > T'_c|_{k=k_2}$.

Proof: By excluding δ in Eq. (23), we obtain

$$\phi = \frac{\gamma}{1-(1-\gamma\cdot F(\gamma, G_p)\cdot F(\gamma\cdot F(\gamma, G_p), G_c))^k}.$$

The growth of k from k_1 to k_2 leads to the increase of denominator of the right side. As ϕ is fixed, γ is required to be improved to hold the equation. Once ϕ starts increasing, $F(\gamma, G_p)$ grows since it is a monotonically increasing function. Applying Eq. (25), we have $T'_c|_{k=k_1} > T'_c|_{k=k_2}$. Notice n is not involved in the equations, thus has no effect on T'_c. □

G. JOINT EFFECT OF INTERDEPENDENCE AND LOAD CASCADING

Now we estimate the system robustness by taking into account, the joint effect of interdependence and load propagation cascading failures. In power grid G_p, $\mathbf{J}(\phi, G_p)$ should be applied to replace $F(\phi, G_p)$, while in the communication network G_c, $F(\phi, G_c)$ is retained. Therefore, the Eqs. (23) and (24) come to

$$\begin{cases}\gamma = \phi\cdot(1-(1-\delta\cdot F(\delta, G_c))^k),\\ \delta = \gamma\cdot\mathbf{J}(\gamma, G_p),\end{cases} \tag{27}$$

and the final fractions of functioning sizes are

$$\begin{cases}\mu_{p\infty} = \gamma \cdot \mathbf{J}(\gamma, G_p), \\ \mu_{c\infty} = \delta \cdot F(\delta, G_c). \end{cases} \tag{28}$$

Theoretically, once solving γ and δ in Eq. (27), we can obtain the results. While, no analytical theory is yet available to give the closed-forms of $F(\cdot)$ and $\mathbf{J}(\cdot)$.

Lemma 3: Although the initial removal is upon G_p, the remaining fraction of survivals in G_p is still greater than that in G_c. That is, $\mu_{p\infty} \geq \mu_{c\infty}$.

Proof: Applying $\delta = \gamma \cdot \mathbf{J}(\gamma, G_p)$ in Eq. (27) to Eq. (28), we get

$$\begin{cases}\mu_{p\infty} = \delta, \\ \mu_{c\infty} = \mu_{p\infty} \cdot F(\delta, G_c). \end{cases} \tag{29}$$

Since $F(\delta, G_c)$ is not greater than 1, we have $\mu_{p\infty} \geq \mu_{c\infty}$. □

H. INFINITE TOLERANCE PARAMETER

Empirically, infinite value of tolerance parameter T means each power station has the infinite capacity, and thus never suffer from overload propagation failure. For neither a single power grid or interdependent smart grid, whether a node is to be removed or not depends completely on the topology effect. In other words, infinite T is equivalent to studying the cascading failures caused by the topology and interdependence.

Lemma 4: As the tolerance parameter increasing to infinite, i.e., $T \rightarrow \infty$, the fractions of survivals in both G_p and G_c meet the upper bounds, which are the results of pure 'k-n' interdependence model effect.

Proof: Recall that $\mathbf{P}(\mathbf{O}|Z = z)$ is the probability that a node i with degree z is destroyed by its neighbors the load shift. As the growing of T, we notice Eq. (16) is monotonically decreasing, which leads to the reduction of the right side of Eq. (19). A smaller u is required to hold this equation. Therefore, Eq. (18) is *monotonically increasing*.

Once tolerance parameter approaches infinity, Eq. (15) is always equal to 0, which consequently let Eq. (16) be 0. Thus Eq. (17) comes to

$$\mathbf{P}(\mathbf{E}(\phi)) = 1 - \phi + \phi \cdot u^z. \tag{30}$$

The probability that the vertex does belong to the giant component $\mathbf{J}(\phi, G_p)$ is

$$\begin{aligned}\mathbf{J}(\phi, G_p) &= \sum_0^\infty \mathbf{P}_\mathrm{d}(z) \cdot (1 - u^z), \\ &= 1 - \sum_0^\infty \mathbf{P}_\mathrm{d}(z) \cdot u^z. \end{aligned} \tag{31}$$

Eq. (19) now can be simplified to

$$u = \sum_0^\infty \mathbf{Q}(z) \cdot (1 - \phi + \phi \cdot u^z). \tag{32}$$

Combining Eqs. (31) and (32), and comparing with Eq. (9), we obtain

$$\mathbf{J}(\phi, G_p) = F(\phi, G_p). \tag{33}$$

Now, Eq. (24) and Eq. (28) are exactly the same. The upper bound exists for $\mathbf{J}(\phi, G_p)$, when the tolerance parameter T approaches infinity. □

VI. SIMULATION AND ANALYSIS

In this section, we couple two synthetic networks using 'k-n' interdependence model and apply the overload cascading model upon G_p. We evaluate how the system robustness is affected by these models and parameters. In all experiments the fraction $1 - \phi$ of nodes in G_p is removed randomly.

A. JOINT EFFECT OF INTERDEPENDENCE AND OVERLOAD CASCADING

First of all, we study the smart grid system on different network topologies by assigning different λ_p and λ_c. Fig. 2(a) and 2(b) illustrate the joint effect of interdependence and overload cascading. The system is fragile to random failures, where approximate 11% initial failure leads to the entire collapse when $\lambda_\mathrm{p} = \lambda_\mathrm{c} = 2.5$. As the number of removals increase, i.e., increasing of $1 - \phi$, the fractions of survivals $\mu_{p\infty}$ and $\mu_{c\infty}$ in both networks slightly decrease, followed by a rapid decline to zero. This performance drop is theoretically proved to be first-order discontinuous transition [18]. While in [17], we pointed out that in real life network and simulation, there exists a small transition interval due to the non-infinite sizes of networks. The system performance is heavily depending on the network topologies, where about 19% initial failure can destroy the system, given the power low exponents are 2.2 for both. Therefore, on the attack perspective, higher $1-\phi$ is required to knock down the system for lower λ_p and λ_c.

Fig. 2(c) and 2(d) show how system performs for various values of tolerance parameter. Accordingly, given the fixed removal, $\mu_{p\infty}$ and $\mu_{c\infty}$ grow once the tolerance parameter is increasing. The reason behind this phenomena is that the higher tolerance ability results in each power node having more vacant volume to adopt the shifted load from its neighbors, and thus becomes stronger in terms of overload propagation.

Comparing Fig. 2(a), 2(b), 2(c) and 2(d), we observe $\mu_{p\infty} \geq \mu_{c\infty}$, implying that the fraction of survivals in G_p is greater than that in G_c. This validates our Lemma 3.

Another finding from Fig. 2(e) and 2(f) is that the system robustness finally reaches to the upper bound as the growing of tolerance parameter. Given the initial failure of 0.05 (100 nodes out of 2000), $\mu_{p\infty}$ keeps stable on 0.91 when tolerance parameter exceeds 1.8. Same thing happens for the curve '150 removals'. We believe this upper bound is the system performance of single interdependence effect, where the load cascading effect is eliminated due to the huge vacant volume in power grid. Fig. 3 proves our finding, where it is the experiment that sets up T to infinite and only considers

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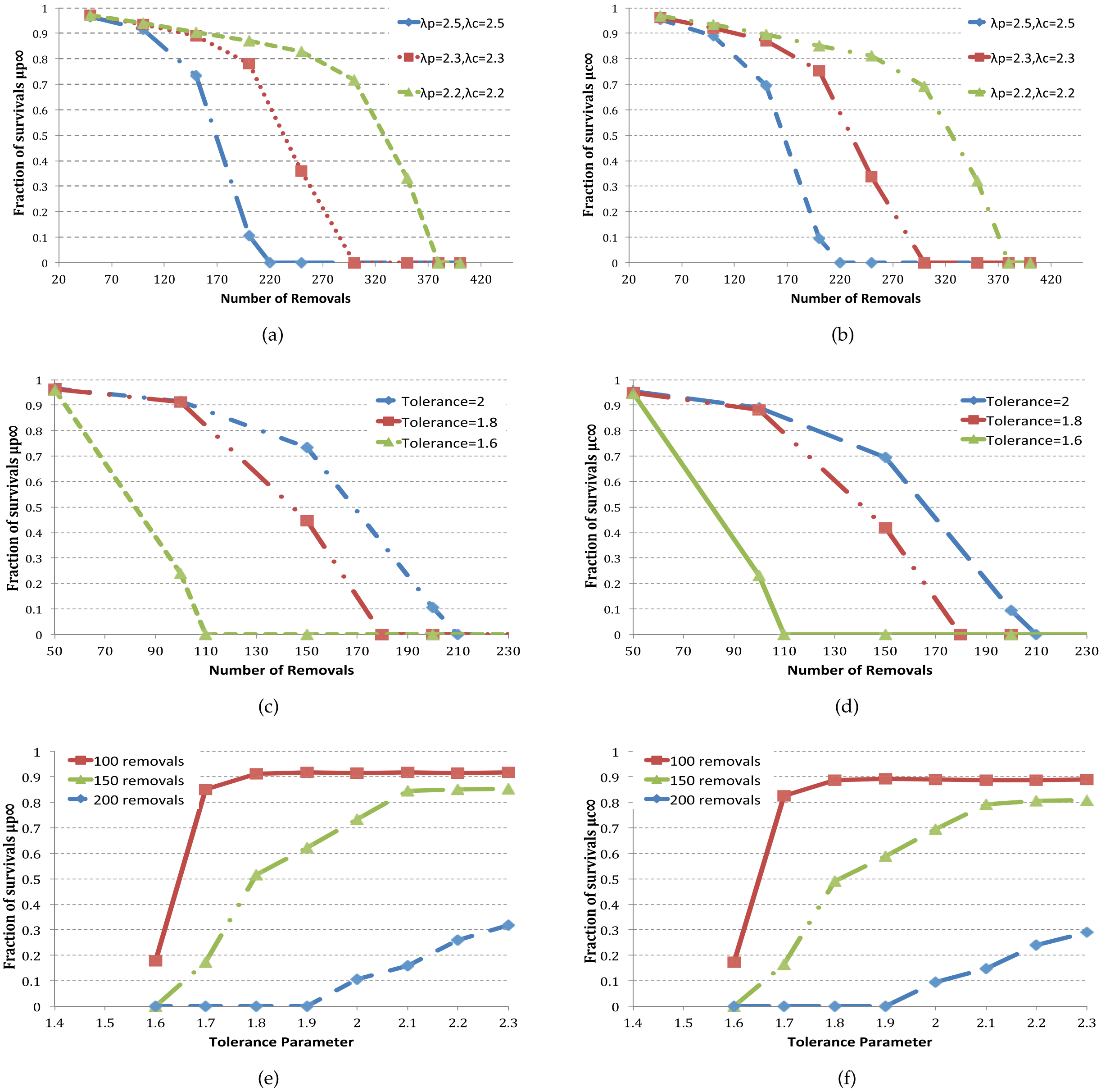

FIGURE 2. The initial size of G_p and G_c are 2000 and 7000 respectively. Random failure is applied on G_p. Both load propagation model and 'k-n' interdependence model are applied in the simulation. $k = 2$, $n = 5$. (a) and (b) compare the number of nodes left in different network topologies, with $T = 2$. In (c), (d), (e) and (f), we set up $\lambda_p = \lambda_c = 2.5$ and show how the system be affected by different tolerance parameter values and initial failures. Generally, a higher tolerance parameter T gives system a better performance, which reaches the upper bound once T keeps growing.

interdependence effect. For $\lambda_p = \lambda_c = 2.5$ and initial removal equals 100, μ_{p_∞} and μ_{c_∞} are 0.91 and 0.89 respectively, which are exactly the same values of upper bounds shown in Fig. 2(e) and 2(f). This validates our mathematical proof of Lemma 4.

We compare the tolerance threshold T_c' of various systems, as shown in Fig. 4. The value of T_c' for the system with lower λ_p and λ_c is also lower. To elaborate this, we firstly know that the scale-free network with power law of 2.2 has a longer fat tail than the network with power law of 2.5. The former one possesses more hubs, which, according to Eqs. (1) and (2), have more vacant space to absorb the shifted load from neighbors. Therefore, the network is firm to the load propagation. We also observe that the curve of $\lambda_p = \lambda_c = 2.5$ is cut out at the point that the number of removals is 200, where the curve is approaching to the interdependence threshold $1 - \phi_c$, as illustrated in Fig. 2(a). No matter how big the T_c' is, the system breaks down when it approaches to $1 - \phi_c$. Therefore, we conclude that the smart grid robustness depends on both T and $1 - \phi$. The system is totally knocked down whenever either T_c' or $1 - \phi_c$ is reached first.

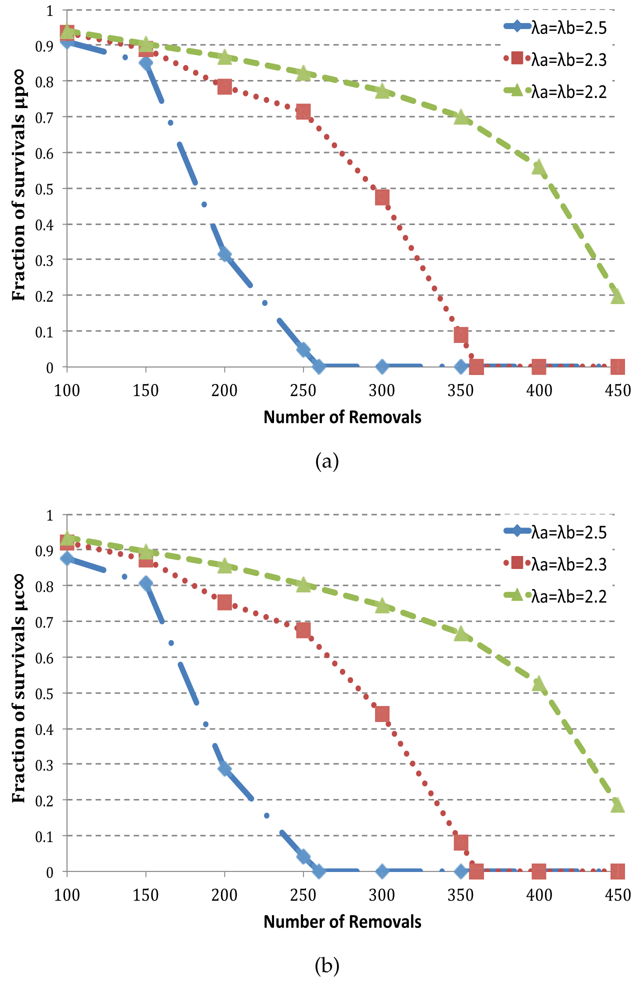

FIGURE 3. **The number of survivals in the system which only considers 'k-n' interdependence, with $k = 2$, $n = 5$. G_p and G_c have 2000 and 7000 nodes respectively. Compared with Fig. 2(e) and 2(f), we observe the performance of only considering 'k-n' interdependence effect is the upper bound.**

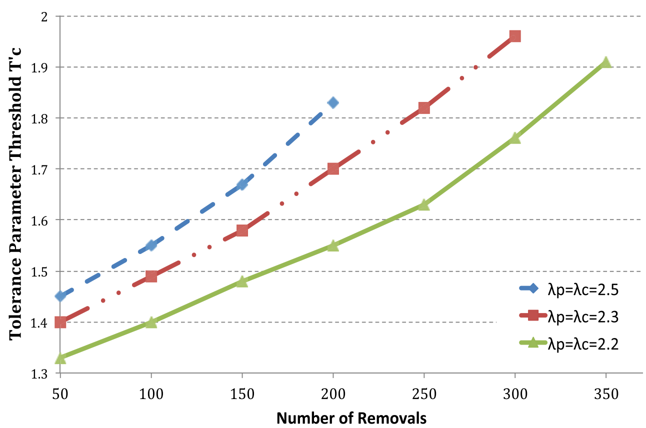

FIGURE 4. **Tolerance threshold T_c'. $G_p = 2000$ and $G_c = 7000$, $k = 2$, $n = 5$. The super linear relation between the initial removal and T_c' is demonstrated.**

Another observation from Fig. 4 is that the relation between the number of initial removals and tolerance parameter threshold is super linear.

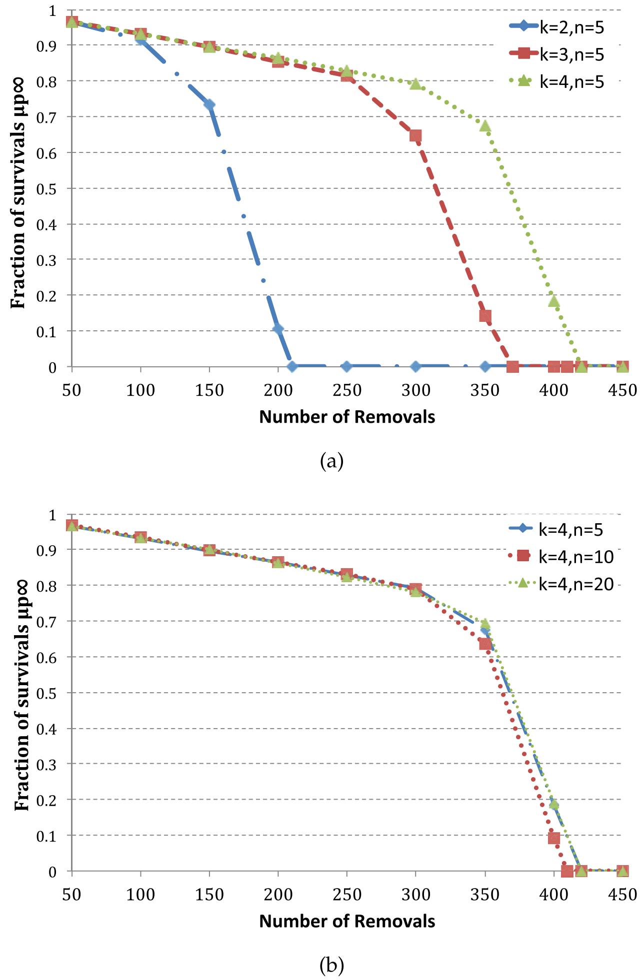

FIGURE 5. **System robustness versus k and n. $G_p = 2000$, $G_c = 7000$, $T = 2$. Higher k indicates higher robustness, while n only has limited effects.**

B. COMPARISON OF k AND n

The sub-linear relation between k and the fraction of survivals was demonstrated in our previous paper [17]. However, we would like to evaluate how the system robustness is affected by k and n in this new model by taking the load propagation into account. According to Fig. 5(a), both μ_{p_∞} and $1-\phi_c$ grow for the increasing k. With fixed $T = 2$, for $k = 4, n = 5$, the threshold $1 - \phi_c$ is 0.21, which is almost two times the curve $k = 2, n = 5$. The performance incremental quantity from $k = 2$ to $k = 3$ is much greater than that from $k = 3$ to $k = 4$. This validates our previous conclusion that the relation between k and robustness is sub-linear. Fig. 5(b) tells us another story that n has no effect on the system. It validates our Eqs. (27) and (28), where n is not involved. In other words, the system performance is independent from n. However, we notice that tiny fluctuations occur around threshold $1 - \phi_c$. Recall n is the number of power stations that each operation center could operate. Given fixed size of S_p and k, the number of operation centers is inversely proportional to n [17]. Therefore, for each simulation, whether the initial random removal is upon these

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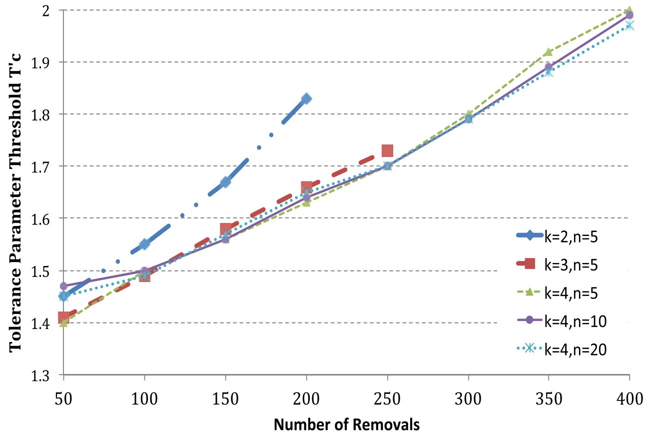


FIGURE 6. **Tolerance threshold T_c' versus k, n. The fluctuations occur around the start points of $k = 4$, $n = 5, 10, 20$. The reason is that higher n indicates fewer operation centers [17]. For each single simulation, whether the initial random removal is upon these operation centers or not leads to quite different results.**

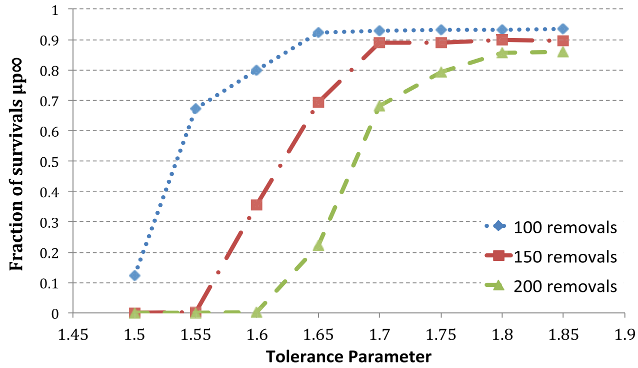
operation centers or not leads to quite different results. To weaken the fluctuation, we believe it is reasonable to repeat the simulation for a massive number of times.

Fig. 6 shows that T_c' also heavily depends on k rather than on n. For a given $1-\phi$, the tolerance threshold grows with the decreasing of k. For the case of $n = 5, 1-\phi = 0.1$, T_c' equals 1.83, 1.66, 1.63, respectively for $k = 2, 3, 4$. This validates our Lemma 2. The reason could be explained as the following: higher k indicates higher system robustness in terms of topology failure. Thus, the system has more nodes survived, consequently has more vacant space to deal with the load propagation. The pressure of adopting shifted load on each individual node is therefore reduced.

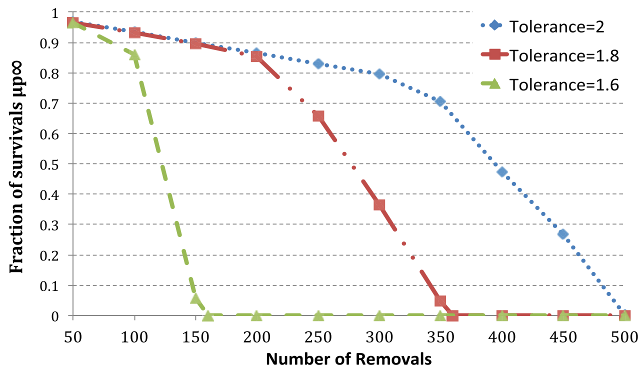
For the case of fixed k, our mathematical Eq. (25) indicates that the threshold is not affected by n. While in our simulation, the change of n does affect T_c' limitedly. Generally, the three curves in Fig. 6 for $n = 5$, $n = 10$ and $n = 20$ are very close to each other. While, given the initial removal is 50, the values of T_c' for $n = 10$ and 20 are obviously greater than $n = 5$. We believe the reason is similar with Fig. 5(b), that whether the initial removal upon the operation centers or not causes quite different consequences. The mean results of simulation of these three curves would be almost the same once we enlarge the number of samples.

C. SINGLE POWER GRID VERSUS INTERDEPENDENT NETWORKS

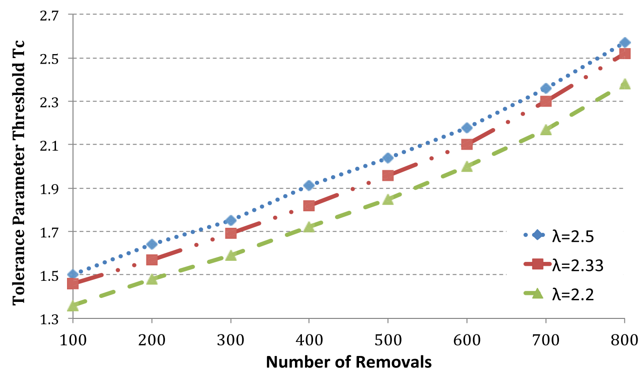
We compare the systems of single power grid and interdependent smart grid. Fig. 7 indicates how different tolerance parameter affects the single power grid. Comparing Fig. 7(a) with Fig. 2(e), we observe that given the same $1-\phi$ and network topology, the tolerance threshold for interdependent smart grid is higher than that for single power grid, e.g., $T_c' = 1.9$ and $T_c = 1.6$ if the initial removal is 200 nodes. Meanwhile, μ_{p_∞} in Fig. 7(a) is higher than that in Fig. 2(e), implying that the single power grid is stronger against random failures. Therefore, we conclude that the interdependence between the two networks decreases the system robustness dramatically. The comparison between Fig. 7(b) and Fig. 2(c) tells the same story that the number of survivals in single power grid is greater than that in coupled smart grid, thus the former one is much more reliable. We also notice that with the growth of T, the system performance is approaching the upper bound in Fig. 7(a).



(a)



(b)



(c)

FIGURE 7. **The system robustness and tolerance parameter in single power grid, with $G_p = 2000$. Similar to interdependent networks, growing k also leads to a upper bound performance, as shown in (a). Comparing (b) with Fig. 2(c), the number of survivals in single power grid is greater. (c) illustrates a super linear relation between initial removal and T_c.**

Fig. 7(c) draws the values of T_c of different network topologies. Generally, T_c grows with the increase of initial failures. We notice that the growth rate is increasing, e.g., for the curve of $\lambda = 2.2$, T_c increases from 1.35 to 1.6 when initial removal goes from 100 to 300, but expands from 2.0 to 2.37 when the removal grows from 600 to 800. This illustrates a super linear relation between initial removal and T_c.

According to Fig. 4, the T'_c for interdependent smart grid is 1.76 when $\lambda_p = \lambda_c = 2.2$ and initial removal is 300. Comparing with Fig. 7(c), $T_c = 1.6$ in the single power grid with same parameters. This validates our Lemma 1 that the required tolerance parameter for interdependent network is higher than that of a single power grid.

VII. CONCLUSIONS

In this paper, we classify two types of cascading failures that may occur in smart grid. Our aim is to study the joint effect of these two cascading failures to make the analysis be more closer to the real world smart grid. Furhtermore, mathematical tools are proposed to estimate the tolerance threshold, below which the entire system suffers from the load propagation cascading. A percolation-based method is devised to calculate the fractions of survivals in both power grid and communication network. We prove that even if the initial failure is on the power grid, the remaining part in it is still greater than that in the communication network. Using both mathematical methods and experimental analysis, we find the upper bound of system robustness. The simulation validates our analysis and demonstrates that the relation between the number of initial removals and the tolerance parameter threshold is super linear.

This work is helpful in understanding the cascading failures in smart grid. Finding more practical load propagation model is one of our future directions.

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