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Statistical Robotics: Controlling Multi Robot Systems using Statistical-physics

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ABSTRACT In a multi-robot system consisting of numerous agents, it may be impractical to individually identify each agent. Consequently, issuing specific commands to each agent might not be feasible. We therefore, introduce the concept of a Statistical multi-robot system (SMRS). Such systems comprises a very large number of agents that cannot be identified or located individually. Moreover, Since it is impractical to track and communicate the complete configuration of an SMRS, we resort to statistical physics methods, specifically gas kinetic knowledge, to extract their distribution. But unlike in Thermodynamics, we employ the fact robotic agents can sense their environment, communicate their microscopic state, and change their local behavior to enable control.
The concept of an SMRS suggests that the comprising agents should be as simple as possible, for practical reasons. In this study, we demonstrate how an SMRS comprised of single-degree-of-freedom agents can be controlled by a global controller . Using the same rationale, we define a successful mission of an SMRS as one in which a sufficient portion of the agents accomplish the mission. To demonstrate the efficacy of our approach, we provide a motion planner and exemplify our formalism in both simulations and real-world experiments.

One-Sentence Summary: To control huge multi robotic systems we resort to the theory of statistical-physics and utilize its macroscopic properties.

INDEX TERMS Robotic Swarm, Multi Robot System , Statistical physics.

I. INTRODUCTION

In the field of robotics, one distinguishes between *multi-robot systems*, *robotic swarms* and *active-matter*. Multi-robot systems are generally characterized by their structured communication and coordination, often involving direct or centralized information exchange among more complex robots to execute specific tasks collaboratively. Conversely, robotic swarms typically embody decentralized, local communication strategies. In these systems, simpler robots interact based on basic signals and local cues, leading to emergent, collective behavior that emphasizes scalability and resilience to individual failures (see [1], [2]). Active matter differs significantly, involving self-driven entities that consume energy to create motion and forces.

A. RELATED WORK

active matter

Specifically, the field of active matter [3] focuses on the relationship between the swarm's macroscopic "thermodynamic" properties and certain motion traits that the agents exhibit. In the realm of active matter research, for example, the impact of Brownian motion on particles is commonly investigated through the development of stochastic differential equations [4], [5]. Active matter researchers typically investigate a theoretical framework wherein a collection of particles can actively move by acquiring kinetic energy from their surroundings. This closely mimics swarming, schooling, and other forms of collective behavior observed in various organisms, such as bacteria and animals [6], [7]. Numerous models have been developed for this aim, including those that implement an escape-and-pursuit strategy for individual agents [8], those that employ velocity alignment forces between two agents to achieve an average velocity [4], heuristic models, like the minimal Vicsek model [9] and the time-dependent propulsion model [10], which produce non-linear structure formation behavior (see also [11] for real-life implementation of these ideas demonstrating rudimentary control for confined superstructures). Note that in these studies, no mutual communication or global controller is presented. In our research, we shall use some communication abilities in order to expand the control abilities of such multi-agent systems.

robotic swarms

Robotic swarms - a more engineering-oriented topic has garnered increasing attention from the research community, as evidenced by recent studies [12]. The field of research related to robotic swarms primarily encompasses two main areas, the first of which pertains to goal-oriented tasks. These research domains focus on investigating the potential benefits of utilizing multiple agents, and employ various approaches and techniques [13]–[15]. However, despite the significant advances in swarm robotics research in this area, most methods require the controller to identify individual agents, which is impractical for very large swarms.

An alternative methodology aims to examine the relationship between the micro-behavior of individual agents and the macro-behavior of the entire swarm, by exploring the rules followed by each agent and their resulting collective behavior. For instance, to maintain the swarm in a single batch, one may design the swarm in a way that enables the uninterrupted transmission of information [16], [17]. Such an approach has demonstrated effectiveness, as in accomplishing self-assembly tasks [18].

A similar framework aims for minimal use of global information and relying mostly on local sensor data is introduced in [19] which treats agents as physical particles and includes features like discrete-time approximation for continuous behavior, frictional force for self-stabilization, and parameters like maximum force and velocity for each particle. Nevertheless, this approach does not address the option in which the particles have sensing capabilities, nor does it address the challenges related to motion planning.

It is important to note that the complexity of the agents involved and their communication capabilities inherently constrain the size of a swarm (as further clarified in the formal definition provided in Section II). Currently, examples of swarms comprising a large number of agents - referred to here as "hundreds or thousands" (or statistical multi robot systems) - are still sparse.

Rubenstein et al. [18] presented a swarm with thousands of agents that traverse via vibrations and are governed by virtual potential fields, limited to communication with nearby members. A parallel line of reasoning is provided in [20] where a set of microrobots controlled by a biaxial oscillating uniform magnetic field exhibit diverse behaviors with remarkable transitioning stability. Another example is provided by Xie et al. [21], which consists of a vast number of hematite colloidal particles controlled using alternating magnetic fields that form a virtual potential field. Their research demonstrates various collective behaviors, including liquid, chain, vortex, and ribbon-like movements. Miskin et al. [22] constructed a swarm with thousands of 70-micron-long agents using an electro-chemical actuator device that responds to ultra-low electric currents activated by laser beams.

The work by Li et al. [16], demonstrated a robotic swarm that meets the basic rational mentioned above. In this swarm, agents are only capable of uniform volumetric oscillation, lack individual identity or an addressable position, and are governed by a global signal. The agents in this swarm move via dilation alone while clinging to each other in a similar way to the push-me-pull-you mechanism in [23].

A similar approach was taken in [24], where a mechanical design rule enables the agents to act collectively in a collision-dominated environment.

It should be stated that the concept of local kinetic models had previously been explored primarily as a method for investigating and simulating the actions of robotic swarms[25],[26]. But applying these near field models for actual state estimation was never considered and so is an holistic statistical-mechanics view point for huge multi robot systems.

Multi-robot-systems

Multi-Robot Systems differ from swarm robotics in several key aspects. They typically exhibit higher levels of robot complexity and individual capabilities, concentrating on collaborative efforts to accomplish intricate tasks. In contrast to swarm robotics, where a large number of relatively simple robots execute tasks through emergent behavior, Multi-Robot Systems utilize the distinct strengths and capabilities of individual robots. These robots, possibly with unique roles, work in concert to achieve coordinated action [27].

A central challenge in Multi-Robot Systems is ensuring effective coordination among the robots. A prevalent approach to address this is the implementation of distributed algorithms. These algorithms enable robots to adapt to dynamic conditions and maintain coordination autonomously, without relying on centralized control [28],[29]. In the context of cooperative object transportation within Multi-Robot Systems, strategies like the leader-follower method are commonly employed [30]–[34]. Although a distributed approach minimizes computational and communication demands, it can constrain the flexibility of follower robots compared to methods where all agents play equivalent roles [35]. Innovative motion planning strategies that forego predefined trajectories have been explored, necessitating individual agent modeling [36], [37]. Additionally, decentralized control methods, including sliding mode controllers [38], and adaptive control techniques [39], are subjects of ongoing research. A fuzzy logic-based approach offers a simpler alternative, though its effectiveness has only been validated in scenarios with simplified kinematics and dynamics [40].

We posit that a unified framework is essential for bridging the research domains of active matter and robotics to produce truly efficient statistical multi robot system control. Section II will delve into the details of this connecting building block. This work deals with a broad problem of multi robot system control (spread and motion planning) but its main ideas can serve as a benchmark for more complex applicable scenarios.

Our objective is to expand the active matter approach by "closing the loop" on the system's macroscopic state, see Figure 1. We are motivated by the challenge inherent in identifying individual agents within a very large-scale multi-robot system (SMRS), where a multitude of identical robots operate. In such systems, it is impractical to track each robot's location through continuous communication, as this would be overly demanding in terms of communication resources. A vivid example of this can be seen in natural phenomena like murmurations of (identical) starlings or schools of (identical) fish.

Moreover, the economic feasibility of SMRS dictates that they be composed of inexpensive, simple robots, further limiting the use of sophisticated individual control mechanisms that are common in traditional multi-robot systems. This approach is particularly relevant in fields such as military operations and micro-agriculture, where the deployment of large numbers of low-cost, autonomous robots can offer significant strategic and operational advantages.

A. KEY CONTRIBUTION

Our approach distinguishes itself from current research directions in the following:

1) We identify the relevant gas kinetic equation that best describes the system's behavior and show how using it can be used to estimate the agents' spatial distribution. Intuitively, agents situated in dense areas are more likely to experience higher collision rates. To capitalize on this observation, we utilize a particle filter that does not require any knowledge of the agents' locations. This deviates from the conventional solutions like equipping each agent with a distance or bearing sensors (in cases where employing multiple distant sensor/camera views is not possible). Here, a collision/proximity sensor will suffice.
2) We restrict ourselves to issuing only global commands to all the agents at once. We show how this is sufficient for controlling such systems when a mission success is defined by means of distribution-moments. We demonstrate this with the extreme case of a system comprised of agents having only a single DOF.

It should be noted that the algorithms that we shall introduce lean heavily on the statistical-physics notion of a "statistical ensemble" where the control algorithm is designed according to a large set of states of the

II. CORE CONCEPTS

	Traditional swarm robotics	**Active Matter**	**Statistical Robotics**
Agents/Particles interactions	Controlled over each agent	Predetermined	May be controlled via global commands
Interactions rate	Measurable	Measurable	Measurable statistically
Measured quantities	individual agent state & swarm properties	Swarm properties & statistics of agents states	SMRS properties & statistics of agents states
Controllable quantities	Individual agents	The local behaviour	Manipulating random batches of agents
Goal	Controlling the agents to accomplish a mission	Global properties as functions of local ones	Controlling the SMRS distribution & position to accomplish a mission

TABLE 1: The main ideas of the available theoretical frameworks (naturally, entries may not cover all research work).

system.

B. STATISTICAL MULTI ROBOTIC SYSTEMS (SMRS)

This approach deviates from the standard decentralized control characteristic of a swarm, as it involves issuing global commands to the entire system, which is made up of simple agents—a feature usually associated with swarms. However, for clarity and accuracy, it's more appropriate to refer to this as a 'multi-robot system' rather than a 'robotic swarm', given the central command structure being employed.

Having in mind a multi robot system that includes hundreds or thousands of agents we formally define a statistical multi robot system as:

Definition 1: A multi robot system is called a "Statistical Multi Robotic System" (SMRS) if it has a large number of simple and indistinguishable agents for which, control relies only on statistical quantities and broadcast communication.

1) simplicity of agents

By 'simplicity', we are referring to the sensorial, communication, and mechanical capabilities of the agents, and 'control' refers to the SMRS's position and distribution. Considering this definition, it is natural to wonder *what level of simplicity is sufficient for agents to enable SMRS control?* It is important to highlight that our objective is to achieve full controlability over the SMRS, which includes the capabilities to steer it left or right and to alter its formation. This level of control is significantly more complex compared to the goal of synchronized motion seen in *Reynolds' boids* for example, where the agents are relatively simple due to the less demanding nature of their task.

Surprisingly, it turns out that the agents can possess an extremely rudimentary level of capabilities enabling distribution control.

In the design of multi robot systems, a critical challenge is defining the algorithms and capabilities that enable individual agents to enhance the overall system's performance. While statistical methods have scarcely been used in swarm algorithms [16], [42] they hold greater relevance in scenarios where the "law of large numbers" applies, such as in the case of SMRS's comprising a vast number of agents. We, therefore, propose the term *Statistical Robotics* to describe a distinct mathematical approach focused on measuring, estimating, predicting, and controlling the collective behavior and performance of an SMRS, based on limited measurements of its agents (see Figure 7).

2) Communcation and control

Given the previously stated rationale, it is logical to assume that, in many cases, a central controller would not be able to identify or communicate with each individual agent. Additionally, due to limitations in communication bandwidth, it is not feasible for all

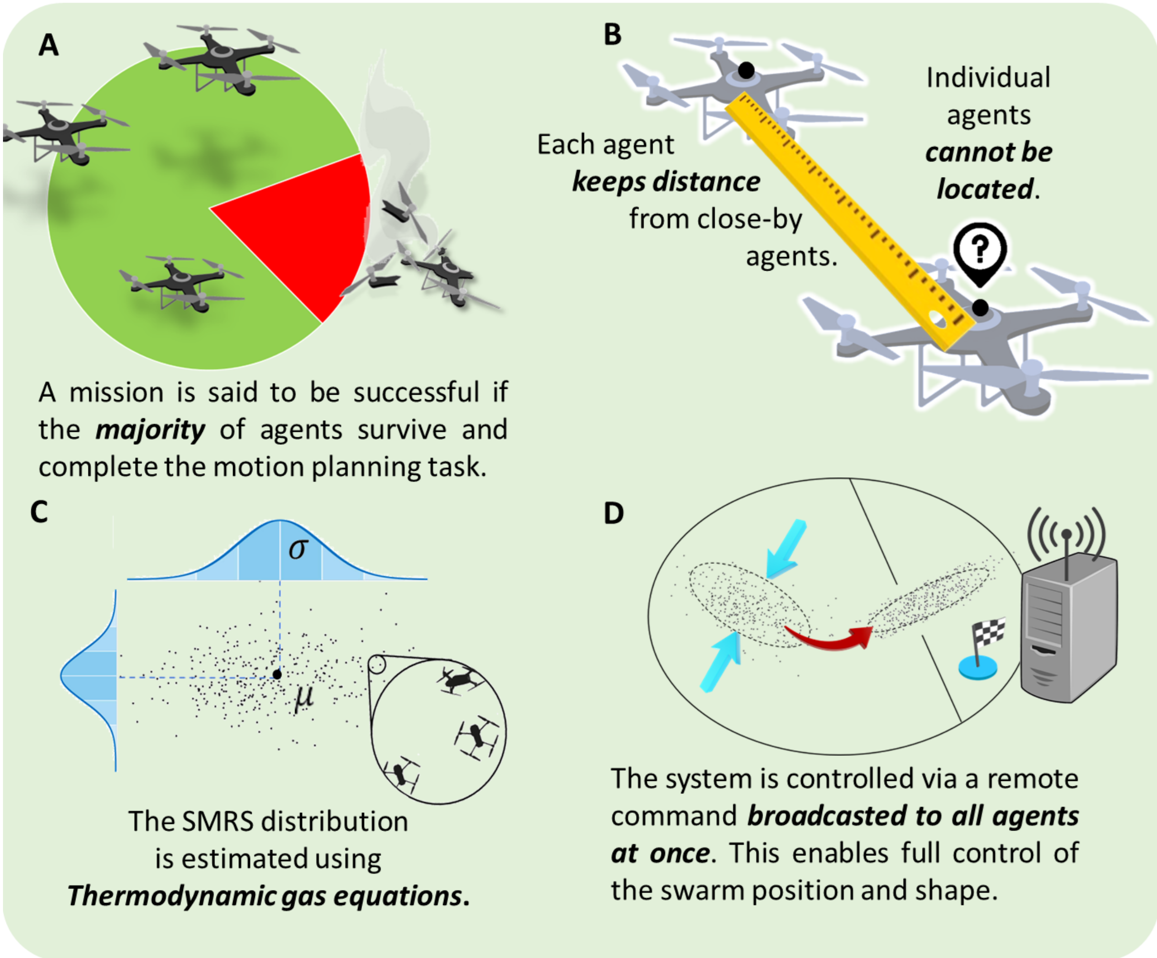

FIGURE 1: The fundamental reasoning that underpins the statistical approach to robotics: (A) Evaluating SMRS mission success through statistical analysis: setting a reasonable completion threshold for assigned task, rather than requiring every single agent to complete it; (B) Due to the vast number of agents involved, the agent mechanical, communication, and sensorial capabilities must be kept as simple as possible [41]. Agents keep distance from fellow agents and anonymously report their headings whenever they sense nearby agents; (C) The kinetic theory of gasses establishes a useful link between the swarm distribution and the expected collision rate which the agent experience. Any distribution can be estimated (see Figure 2); (D) The swarm is remotely controlled via global commands transmitted to all agents. The entire SMRS is viewed as a single entity with a probability distribution that represents the entity's "uncertain location".

agents to continuously transmit long sensor readings, as discussed in [43]. This is primarily due to the significant overhead traffic required to maintain real-time routing status for large-scale networks, along with limited communication bandwidth. Moreover, because individual agents in an SMRS cannot be uniquely identified, it is not possible to provide control signals for each agent. Instead, the transmission of control signals to all agents at once necessitates the use of a limited number of signals that are broadcasted to all agents (of which just a portion follow, as will be clarified below).

While it is possible to attain self-organizing behavior to a certain extent by equipping each agent's local controller with simple laws (such as leader-following in [44]), there are situations where relying exclusively on local controllers is inadequate and a central controller is required. For example in some cases, using local controllers alone to achieve aggregation it is not sufficient to guarantee a unified cluster of the entire SMRS due to sensory constraints, leading to the formation of various distinct agent clusters [8], [45]–[47]. To address this issue, one can refer to Section III-B below, where we demonstrate control over the spread of the entire system. In other words, applying the ideas introduced here for a global controller enables the avoidance of the MSRS separating into several clusters.

3) Mission Success

For an SMRS, the criteria for a successful solution that meets the goals of the mission should be reestablished so as to be measurable by statistical quantities. For example, some of the agents may occasionally encounter an obstacle without considerably affecting the entire system, and we therefore release the need of completing the task with all agents, which here is the motion planning task.

4) State estimation

In addition, when dealing with systems consisting of a large number of agents, computational limitations become a challenge. It is impractical to establish the complete state of the system at each time point using a bottom-up approach. Instead, the field of statistical-physics provides a framework for relating macroscopic observations, such as the bulk properties of materials, to the microscopic properties of individual atoms and molecules. Similarly, for SMRSs, partial knowledge of the state space is satisfactory, both in the sense of the fullness of information and in the sense of its accuracy. Throughout the paper we shall consider these underlying ideas.

statistical-physics, *active matter*, and *swarm robotics* are distinct fields of study with their own unique characteristics and research questions. While statistical-physics is concerned with the behavior of large numbers of particles, to make predictions about their collective properties, active matter, focuses on the study of self-propelled particles, and swarm robotics is concerned with developing algorithms and technologies to enable groups of robots to work together in a coordinated manner. The apparent differences between these disciplines and that of Statistical Robotics field which we suggest here, are summarized in table 1

III. RESULTS

A. APPLYING THE KINETIC THEORY OF GASES TO SMRS

To ensure successful completion of most missions, a swarm may operate as a cohesive unit, as a singular entity or in several distinct batches while maneuvering through the free space. Achieving this requires a crucial step of "closing the loop" through continuous estimation of the swarm's density distribution (and mean position). This estimation process assumes that individual agents within the swarm cannot be identified or individually-communicated with. It's important to note that the procedure detailed below can also be utilized to estimate other bulk properties of the swarm, such as the flow rate of agents through a cluttered workspace and the artificial pressure exerted by the swarm. However, this paper's section will only focus on recovering the density distribution of the SMRS. A simple solution may involve a set if distant cameras and a cross-correlation procedure similar to that used in a *Photogrammetry* technique. Nevertheless, for cases where such a setup is out of hand we offer using a particle-filter coupled with the thermodynamic traits of a gas according to the following scheme:

1) At each time-step, the agents report a central controller their orientations and the collision rate f_{coll} each experiences as depicted in Figure 2.B (for many cases, a predefined proximity may be considered as a collision);
2) the density field $\rho = \rho(x)$ is then extracted by applying a particle filter and using a suitable kinetic theory $f_{coll}(x) = F(\rho(x))$ that formulate the connection between the density and the collision rate, where x is the position vector.

Remark: The following are noteworthy: (I) None of the agents are aware of their locations; and (II) as our experiments show, there is no need for all agents to report every collision; (III) Furthermore, as will be clarified below, we apply a particle filter for this purpose, the algorithm begins with an initial guess and refines it over time, meaning we do not assume any initial conditions.

The Statistical-Physics literature offers several such models $F(\rho)$ for the collision rate.

This analogy assists in comprehending how robotic agents interact within various environmental contexts.

By drawing upon the kinetic theory of gases for dilute gases [48], akin to environments with sparse robotic agents, we can determine the collision rate. This is achieved by considering all agents, except one, as stationary and counting the number of collisions within a specific time frame. As an agent moves through space, it swips out a *collisional cylinder* with a volume of $\pi d^2 u$. The collisional rate for a single agent is then the product of this volume and the density per unit time, given by $\pi d^2 u\rho(x)$. Here, we assume an ideal gas behaviour and that the agents behave like hard spheres upon collision.

Similarly, consider an agent traversing an unknown path in a plane, with other agents in its vicinity moving at an average velocity u. The collision rate, calculated in the same manner, is expected to be:

$$f_{\text{coll}}(x) = 2du\rho(x) \tag{1}$$

where d represents the diameter of the agent or its proximity threshold.

Note that Equation 1 holds only for the ideal gas and hard sphere model. For example for dense solutions, the rate becomes independent of the particle size [49].

As an example, we introduce a simple algorithm to recover the density field of a planar SMRS. We construct the workspace as a grid of cells $x_{n,m} \subset \mathbb{R}^2$ of equal size (this can naturally be extended to any dimension). We then initialize a set of p samples (particles) $\{\zeta_j\}_{j=1}^{p}$ for each agent. These represent the associated agent's location probability density function (the PDF) in the plane. That is, for each agent and each particle j, there are some n and m for which $\zeta_j \in x_{n,m}$. We further assume that a certain preset portion of the agents share the collisions rate they experience with a centralized computer together with their odometry measurements, which we use in the density propagation stage as follows:

1) Comparing the reported collision rates f_{coll} of the reporting agents provides a way to compute the particle filter's importance weights $\{\omega_j\}_{j=1}^{p}$ as the likelihood $\omega_j = \prod_i e^{-\frac{1}{2}\left(f(\hat{x}_i)-\hat{f}_j(\hat{x}_i)\right)^2}$, where the summation of each agent's weights sums up to one and where $\hat{x}_i$ is the estimated location of the i^{th} agent.
2) Equation 1 provides a way to estimate the density map $\hat{\rho}(x)$: The estimated local density at the estimated location $\hat{x}_i$ of the i^{th} agent is given by $\hat{\rho}_j(\hat{x}_i) = \frac{f_i}{2du}$ where f_i is the reported collision rate of the i^{th} agent.

Accordingly, we conducted a simulation to recover a continuous density field on a 50×50 grid, of a swarm with $N = 500$ agents. The agents' initial positions and velocities were generated along a uniform random distribution with their velocities bounded to a unit (see Figure 2.A). Each agent moved linearly and adjusted their movement direction upon approaching another agent within a quarter grid cell length. We set the maximum step size to half a unit length and simulated the agents' orientation measurements using the random vector $\Delta x = v\Delta t + \nu$, where $\nu \sim \mathcal{N}(\vec{0}, Q)$, is a zero-mean normally distributed noise with diagonal covariance matrix Q taken as 10% of the step size. Figure 2.C shows a snapshot of the simulation and the adjusted estimated density map.

The algorithm performs well, as evidenced by the convergence of the final mean estimation error to the cell's length after a mere 150 time steps (see Figure 3.

For comparison, we assess the efficacy of our proposed approach against the widely used Extended Kalman Filter (EKF) and Particle Filter (PF), typically employed in localization tasks (see for example [50]). Both approaches involve agents moving consistently. The EKF and the PF utilize agents' odometry data and range measurements to three landmarks positioned at distinct corners of the playground. To ensure a fair evaluation, we fix the range resolution at 5 length units, corresponding to the dimensions of the grid cells, and the PF uses the same number of particles as the proposed algorithm. The performance of the PF and EKF estimations are illustrated in Figure 5 (depicted by the red solid and dashed blue lines), representing the average error of all agents over time. A thorough comparison reveals that our approach performs comparably to the PF and EKF, requiring less infrastructure for range measurement implementation. While the traditional filters' convergence is faster.

Since our focus is not on micromanaging individual agents, the proposed approach becomes particularly valuable when the emphasis shifts toward assessing the density of SMRS locations. The results in Figure 4 illustrate the efficiency of estimating swarm density where only part of the agents communicate with the central controller.

The estimated density is derived by summing the collisions encountered by participant agents within a grid cell and dividing it by the total number of collisions. This computation yields an output matrix $\hat{\rho}$, which is then contrasted against the actual density matrix

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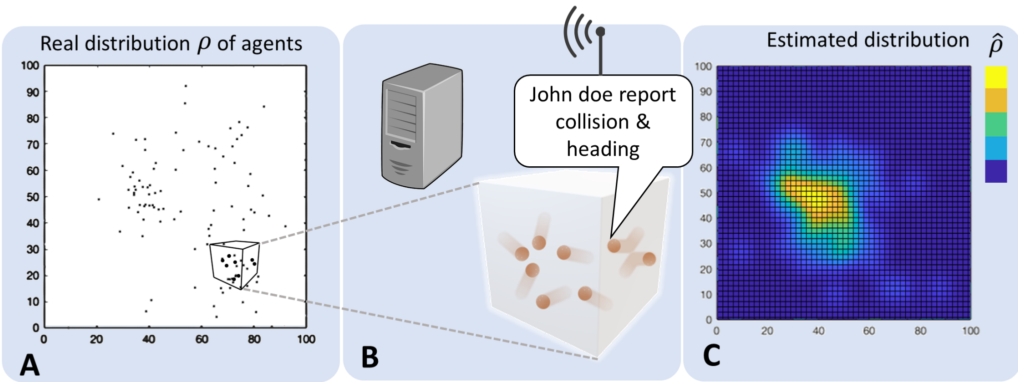

FIGURE 2: (A) A snapshot of an SMRS configuration with 500 agents; (B) At each time step, each agent reports the rate it senses other agents (i.e. with regards to a distance threshold) and its orientation; (C) The estimated swarm distribution. Dark cells correspond to low density and vice versa;

ρ. we conducted a series of experiments using 500 agents, with varying percentages of agents communicating their collision rates, ranging from 10% to 100%. Agents' motion and other considerations are left as in the previous demonstration (note that this is less than a proximity sensor)
Notably, the results revealed that the algorithm's performance was minimally affected even with a significant reduction in the number of communicating agents as 50%. Such a method can serve as a powerful tool for statistical control schemes as demonstrated in Sections III-B and III-C

B. CONTROLLING AN SMRS UNDER MINIMAL AGENT CAPABILITIES

A key task of the SMRS having $N \gg 1$ agents is to traverse a given path with an additional requirement that the swarm's spread should be bounded at every instant. This amounts to attaining a desired distribution $\rho_d(x)$, starting from an initial distribution $\rho_0(x)$. Although we shall assume normal distributions for simplicity, our discussion is not restricted to Gaussians. Recall that the agents are indistinguishable from one another, as stated in Definition 1, and assume that only global control commands are allowed. Surprisingly, under these assumptions, dual-direction single degree-of-freedom agents will do for the planar case (see the lower right illustration in Figure 6).
Remark: It is noteworthy to note that: (I) There is no degree of freedom for rotation available; (II) Since the agents possess limited sensorial and mechanical capabilities, their motion should be expected to be characterized by non-negligible random motion fluctuations. In other words the agents rotate as a side effect of the non-negligible error in their wheel alignment; Considering this, (III) an acceptable control scheme is limited to be of a binary nature, that is, the controller command is limited to 'move forward' or 'move backward' commands.

So, to translate the swarm to a desired position, one can toggle the agents movement (i.e., forward or backwards) in accordance with the resulting swarm movement. Explicitly, the central controller first commands all the agents to 'move forward' a small distance of ϵ units, it then estimates the resulting mean position movement. It is important to note that issuing such a command will result in different movement vectors due to the agents' random directions. If as a result the swarm translates away from the desired direction the controller's following command would be to move in the opposite direction, that is, a 'move backward' command. However, this naive control scheme for the SMRS translation $\Delta\mu$ which is based on fluctuations in the agents' headings will fail for $N \to \infty$, since the their headings will approach a uniform distribution.

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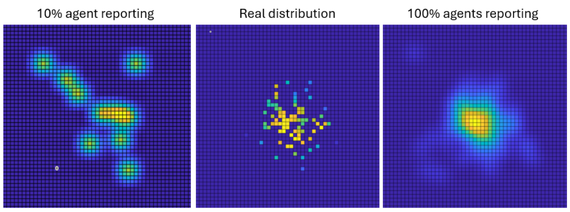

FIGURE 3: Density map estimations: The middle figure depicts the real agent distribution; the left map is the estimated density map when only 10% of the agents report their collisions' rate and their odometry; the right map is the estimated density map when all the agents report their collisions' rate and their odometry. Density maps are calculated according to Eq. 1.

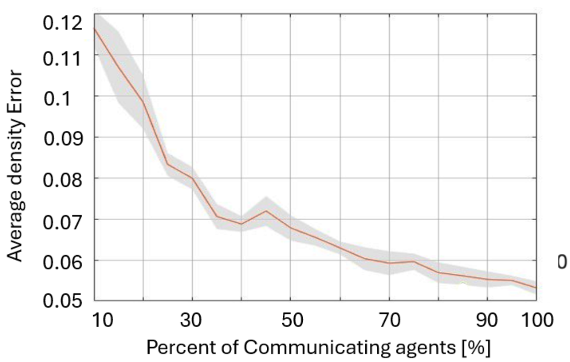

FIGURE 4: density estimation errors: the deviation of the estimated density maps and the actual density map across different proportions of communicating agents is provided. For this end we calculated the MSE between the real density map and the estimated one divided by the number of non-zero cells. This assessment is based on the average results from 10 Monte Carlo simulations for every communicating percentage. Each simulation ran for 150 steps. The red line presents the mean value while the gray region presents the STD.

To mitigate the symmetry that arises for large numbers, one can resort to activating only a subset of $n \leq N$ agents randomly, chosen at each time-step[1] (illustrated in Figure 6 as point D).

[1]In terms of thermodynamics standpoint we supposedly aim to reduce the Entropy of a 'gas' by means of uniform force (e.g., magnetic) which violates the second law of thermodynamics, much like *Maxwell's demon* setup [51]. In what follows we shall use Landauer's Principle to resolve this seeming contradiction, i.e., harness information bits for this end.

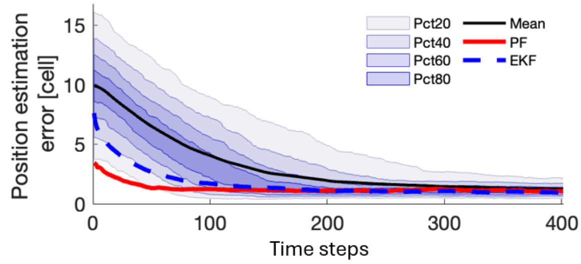

FIGURE 5: The fan plot below demonstrates the distribution of position estimations and its mean (solid black line), derived from collision rate data and a kinetic gas model. The solid red line represents the Root Mean Square error for all agents' position estimation errors using a partial filter. In contrast, the dashed blue line indicates the RMS error when employing an Extended Kalman Filter (EKF), with both methods utilizing data from three landmarks.

One possible approach to achieve this is by instructing each agent to generate a random number within the range of 1 to N at each time step. Subsequently, if the generated number is less than n, the agent should follow the instructions provided by the global controller. The number n can be broadcasted or predefined.

Formally, if the fluctuations in the agents' headings are the sole basis for control (i.e., non of the agent carry a compass) then the resulting mean translation of the entire SMRS towards a given direction at each time-step is:

$$\Delta\mu_{blind} \sim \frac{2\epsilon\sqrt{n}}{\pi N} \tag{2}$$

where ϵ is the small distance the agents travel upon receiving a 'move forward' command. The proof is rooted in the concept of a *random walk*, which is known to result in a mean displacement proportional to the square root of the number of steps taken [52]. To realize this one considers a sum of n planar random (step) vectors with arbitrary directions θ_j uniformly distributed in the interval $[0, 2\pi)$. The random walk end point is given by their sum $z = \sum_{j=1}^{n} e^{i\theta_j}$ whose absolute square can be easily reduced to:

$$|z|^2 = zz^* = n + \sum_{\substack{j,k=1 \\ k\neq j}}^{n} e^{i(\theta_j - \theta_k)}.$$

The expectation value of the absolute square is therfore:

$$\langle |z|^2 \rangle = n + \left\langle \sum_{\substack{j,k=1 \\ k\neq j}}^{n} e^{i(\theta_j - \theta_k)} \right\rangle .$$

Since each step has an equally likely chance of being in any direction, the average of the distribution, considering the equally likely positive and negative values, and thus the second term cancels out living $\langle |z|^2 \rangle = n$. Thus the resulting root-mean-square distance is $|z|_{\text{rms}}= \sqrt{n}$ as indicated.

A rigorous proof for Equation 2 is provided in the Methods Section V-A.

Note that for large swarms, the resulting mean translation $\Delta\mu_{blind}$ is very small. This is due to the fact that the individual agents lack information about their headings, and thus the global controller must rely solely on the *overall resultant translation* to 'close the loop', as illustrated in Figure 6 as point D.

Conversely, if the agents are able to sense their headings, such as by using a compass, each agent can move forward or backward individually to align with the desired trajectory (i.e., to toggle its direction of movement so that it will have a positive component towards the desired direction). In this scenario, as the headings are uniformly distributed, the expected value of the cosine component is $2/\pi$. Using the same rationale as in the previous proof (also provided in the Methods Section V-B), yields:

$$\Delta\mu_{compass} \sim \frac{2\epsilon n}{\pi N} \tag{3}$$

In this case, the resulting mean translation attains its maximum value when controlling the entire SMRS at each time-step ($n = N$) see point A in Figure 6.

However, it's worth noting that the SMRS is susceptible to dispersal as it moves, which needs to be addressed. Specifically, the variances σ^2 of the SMRS distribution increments in the 'blind' and 'with compass' scenarios as follows:

$$\Delta(\sigma^2)_{blind} \in \mathcal{N}\left(-\frac{n\epsilon^2}{N^2} + \frac{n\epsilon^2}{N}, \sigma\frac{2\epsilon\sqrt{n}}{N}\right) \tag{4}$$

$$\Delta(\sigma^2)_{compass} \in \mathcal{N}\left(-\frac{4\epsilon^2 n^2}{\pi^2 N^2} + \frac{n\epsilon^2}{N}, \sigma\frac{2\epsilon\sqrt{n}}{N}\right) \tag{5}$$

These expressions (points C and B respectively in Figure 6) are obtained following the mere definition of the standard deviation, together with the observation that the translation of n agents which is the average of the sum of their steps, can be viewed as a random-walk divided by n (proof is given in the Methods Section). Obviously, attempting to correct this by applying the same algorithm will not do. To resolve this, we resort to the 'blind' algorithm choosing a suitable $n \lesssim N$, as in point C in Figure 6. Of course, this results in an undesired translation occurring simultaneously (point D in Figure 6). Nevertheless, this effect is negligible. So the approximated overall translation in every time-step is that which is given in Equation 3 (point A in Figure 6) which means that for efficient motion control, it is preferable to incorporate a compass on each agent, and so we shall assume.

It should be noted that this algorithm can be further optimized by dynamically changing the step size, but this is out of this paper's scope. Furthermore, note that the proposed spread control scheme is bounded by the step size, but since one do not wish to set σ to zero this is not of concern to us and one may do with investigating only the incremental efficiency.

To demonstrate this rationale we conducted a set of experiments with a small sized swarm specially designed agents (see Figure 7). The robots were mechanically designed with the principles of a SMRS in mind, prioritizing simplicity and global command control. The agents, therefore, were limited to performing only a noisy forward and backward moves while a uniform command (F or B) was broadcasted to the entire swarm. Furthermore, the small number of agents indicate that the "law of large numbers" do not apply. So, to control such a swarm one may to follow the 'forward/backward' strategy without considering the subset scheme described in this section, but to speed up the algorithm this was applied as well. A video clip of such a typical experiment is provided in the paper's supplementary material. Applying the same rationale, one may also control the swarm's distribution axes, their direction, and length, to direct the swarm through narrow passages and so we shall do in the next Section.

C. MOTION PLANNING FOR AN SMRS

The classical motion planning mission for a planar swarm is the task of moving to a desired position x_d, avoiding a set of obstacles $\mathcal{O}$ while maintaining in a desired formation.

But recall that the agents of the SMRS can not be controlled individually, so a more natural aim for a motion planner, which follows a statistical-robotics point of view (see table in Section II), is to reduce

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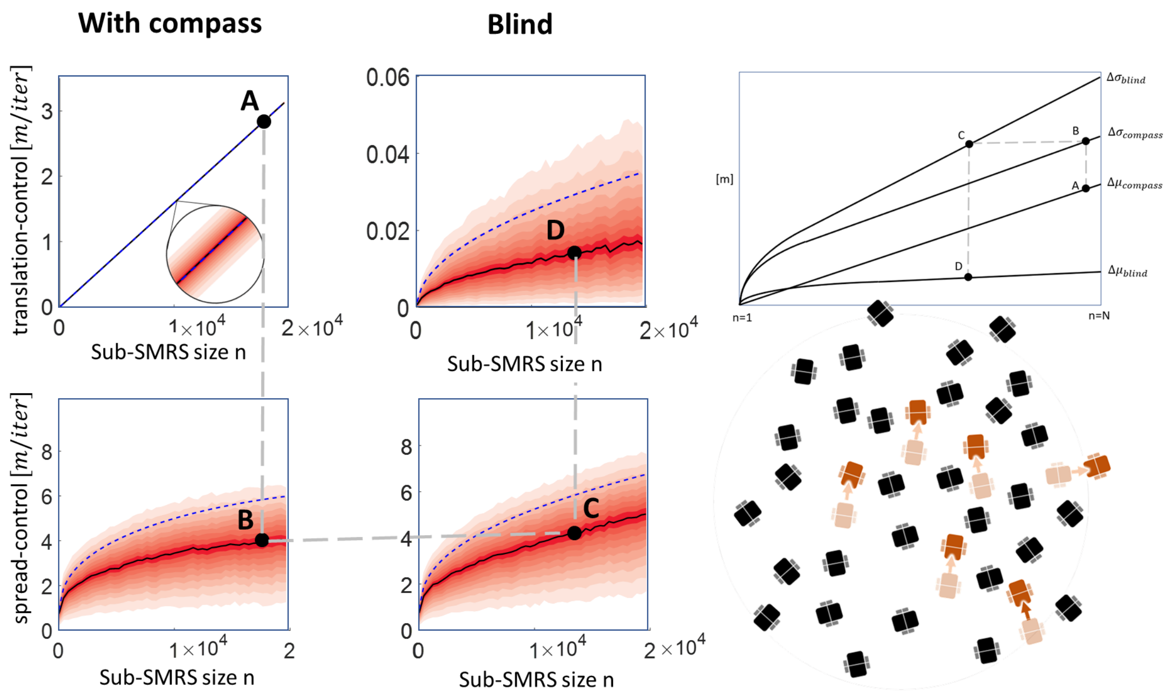

$\times 10^4$

FIGURE 6: (I) Lower-right: Controlling scheme for minimal agent capabilities. At each time step a subset of, or all, the agents are controlled (marked brown). Their headings are not directly controlled, instead, forward or backward movements of each are determined individually with accordance to a compass reading and the resulting SMRS position. The resulting change in the SMRS spread is addressed applying a 'blind' manoeuvre to a random subset of the swarm.

(II) Upper-right: The overall scheme rationale is given on the upper-right off-scale illustration where the x-axis is the controlled subset size $n \leq N$ and the y-axis represents the change in the position and spread (the variance) of the SMRS in a single time-step.

(III) The set of four fan charts illustrate the typical spread change and position change observed in a simulated experiment, with and without a compass. In each chart, the theoretical values are depicted as a dashed blue line. These were conducted with $\sigma = 300$ [m] and $N = 20,000$ agents where in each time step, agents translate $\epsilon = 5$ [units]. The simulation ran $2,000$ cases for each n. The translation control and spread control are correspondingly provided in the upper and lower charts. The controlling scheme which relies on heading sensors on each agent and the scheme which assumes no such sensor is available, are provided on left and on the right chats correspondingly. At their maximum values $\Delta\mu_{blind} << \Delta\mu_{compass}$. So, while traversing, it is better to use 'with compass' algorithm which results in the maximum of $\Delta\mu_{compass}$ (point A). This results in an undesired spread $\Delta\sigma_{compass}$ (point B). Implementing the 'blind' algorithm (point C) to reduce the spread, would result in a minor undesirable translation (point D).

the number of agents-obstacle collisions rather than preventing them altogether, setting a predetermined upper bound for the portion of failed agents (see Figure 7.A). Moreover, we think of the entire swarm distribution as a probability distribution of a single entity (the swarm), rather than as an aggregation of individual robots. Accordingly, the probability of the entity being in a unit of area is equivalent to the agents density located within that particular area.

Keeping this in mind it is natural to generate failure-

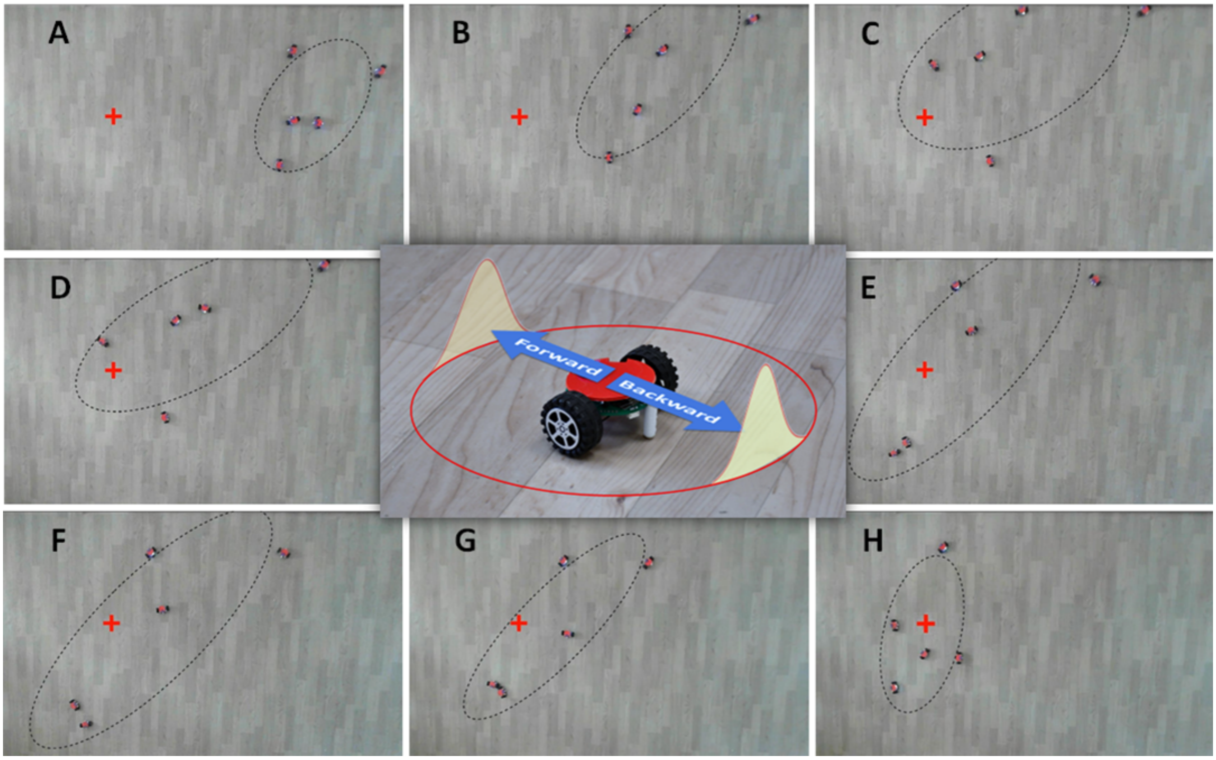

FIGURE 7: Controlling a real-life robot small-sized swarm via global commands relying solely on the swarm's statistics μ and Σ. Each robot can only move forward or backward, with an inherent directional error (middle). The swarm converges to the designated target with a desired spread (A-H).

probability topographic map $\mathcal{M}$. To do so, instead of employing the classical procedure of Minkowski sum between the robot's geometry and the obstacles binary function $\mathcal{O}(x)$, we convolve the normalized swarm distribution with the obstacles $\mathcal{M}(x) = \rho(x-\mu) * \mathcal{O}(x)$. This yields the percentage of expected agent loss as outlined in [53]. This is provided as contour lines in Figure 8.

This underlying concept can be applied to a variety of motion planning algorithms. For illustration purposes, we utilize a *road map* approach for solving the motion planning problem, where the waypoints were generated by *medial axis transform* [54] applied on the obstacle space. We then construct a weighted graph, where the weights are the maximal value of $\mathcal{M}$ along the path connecting a pair of points. The resulting path is calculated and is depicted as dashed blue line in Figure 8. As discussed in Section III-A, and Section III-B we assume that both the locations of the obstacles and the swarm's density distribution are known. We further assume that each agent is equipped with a compass. Consequently, to establish the SMRS motion with accordance to the path we apply a combination of two compliment controllers:

A local controller on all agents. It bases its decisions on an onboard sensor which aims to keep the agents in a given range from one another (in a somewhat similar manner as *Lennard-Jones potential* which governs the interactions between atoms).

A central controller: which considers the (macroscopic) distribution of the entire SMRS it capable of sending global commands to all agents at once. It employs the ideas established in Section III-A and Section III-B. Specifically:

The SMRS distribution and location are extracted by receiving the collision rates and headings, and comparing them to the appropriate gas Equation 1.

Shape control is obtained by first sending

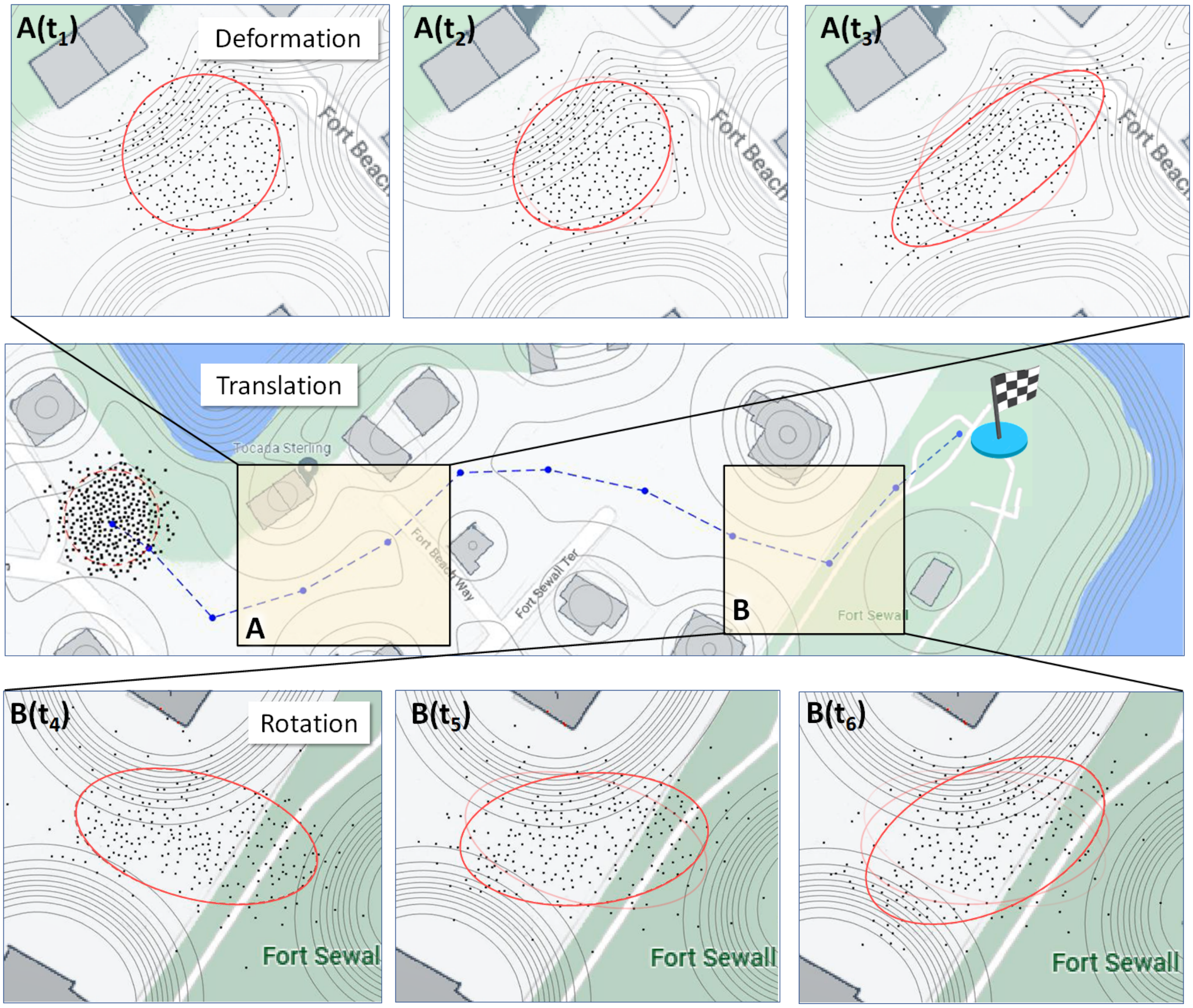

FIGURE 8: Controlling an SMRS by broadcasting a Forward/Backward command to all agents: One considers a "unified entity" probability distribution to represent the SMRS position distribution $\rho(x)$. The contour lines generated by convolving $\rho(x)$ with the obstacle space, represent the expected portion of failed agents at every swarm position; Subfigures $A(t_1)$, $A(t_2)$, and $A(t_3)$ show a sequence of screenshots that illustrate how the SMRS adapts to the narrow passage ahead, by deforming from a circular shape to an oval. This deformation is necessary to prevent the number of failed agents from exceeding a predefined acceptable limit; The lower figure series $B(t_4)$, $B(t_5)$ and $B(t_6)$ depicts a sequence of snapshots where the SMRS is rotated for the same end.

a blind 'forward' command to all agents. A fraction $n < N$ of them comply. The central controller, then, compares the desired shape it aims to achieve with the resulting distribution, and reverses its command accordingly. **To comply with the requirement for a predetermined portion limit of failed agents**, the controller reshapes, rotates and moves the SMRS to avoid contour lines that indicate over-fail. If a large portion of the SMRS is expected to collide with an obstacle while reshaping, the controller translates the SMRS forward along the path while reshaping it. Moreover, if a certain portion of the roadmap is narrow, the controller reshapes it to a thin Gaussian shape before moving it.

Communication between:

- **The central controller and the agents** occurs through broadcasting binary commands (forward/backward) to all agents, which some agents follow.
- **The agents and the central controller** involves a subset of the agents reporting collisions anonymously, without transmitting any additional information (specifically, not their locations).
- **Agent-to-agent communication** does not take place.

The sensors available to the agents include a collision sensor and a compass ("less than" a proximity sensor).

To demonstrate our scheme we set an SMRS of 300 disc shaped agents of radius 0.8 [meters] length, each. The agents' speed limit was set to 5 [meters] per time-step. The arena is a peninsula 400×160 [meters], in Boston city. The simulated agents were assumed to be equipped with a collision sensor that measures the distance to other agents ahead within line-of-sight up to a distance of 5 [meters]. Figure 8 depicts a typical simulation run.

IV. DISCUSSION

The control of huge robotic swarms presents a significant challenge in the field of swarm robotics. Existing approaches for achieving *goal-oriented tasks* often rely on the identification of individual agents, which becomes impractical and resource-intensive for swarms comprising a large number of agents. Employing an alternative methodology of *active matter* will not suit these needs as it focuses on exploring the relationship between the micro-behavior of individual agents and the resulting macro-behavior of the entire swarm. We propose a novel approach (Figure 9) that leverages statistical-physics to sense the swarm's state and control its macroscopic characteristics.

As with any robotic system, in order to effectively control an SMRS and enable it to function as a unified entity it is required to "close the loop". That is, continuously estimating the swarm's density distribution and mean position under the assumption that individual agents cannot be identified and under the assumption that external sensing is beyond reach.

To this end, we utilize the expression for the *collision rate* of gas particles, which is a fundamental quantity in classical statistical-physics that was first introduced in [55]. Collision rate establishes the connection between the gas temperature T, the rate at which the gas particles collide with each other, and and their density. Maxwell–Boltzmann distribution indicates that $\sqrt{T}$ is proportional to the particles' average velocity. Therefore, since the velocities of the agents are known (in this case, all velocities are set to a predefined value, apart from the momentarily fixed ones), the collision rate can be reduced to the relationship between the local density and local collision rate, as indicated by Equation 1. By utilizing the fact that an agent can report its headings and frequency of approaches by other agents, Equation 1 can be employed as an estimator for the density map.

We devised an experimental simulation that aimed to present a substantial challenge for the aforementioned estimator. Specifically, our experiment simulated a scenario in which the agents, confined within a closed area, demonstrated non-cohesive behavior reminiscent of Brownian random motion, rather than exhibiting a cohesive and predictable robotic-swarm dynamics. Our estimator successfully and continuously estimated the density function of the swarm, despite their seemingly disorderly and erratic movements. The algorithm converged within a reasonable time period, dynamically adjusting to the swarm's density map dynamics over time.

Furthermore, since the SMRS comprises thousands of agents by nature, the communication bandwidth is expected to be limited. Therefore, assuming that all the agents report their collisions at every instance is non-feasible. To examine the estimator under this limitation, we repeated the same experiment with only a fraction of the agents reporting their collisions. Our results show that even with a reduction in the number of communicating agents from 100% to as low as 10%, the algorithm's performance remained unaffected. This implies that our paradigm suits well for SMRS which expected to behave in a much more orderly manner.

From the same perspective, considering that individual communication with each agent in the SMRS is not feasible, the only viable approach for providing control signals to the swarm is to rely exclusively on broadcast communication. Our proposed approach involves employing an "advance towards" command, where all agents move in the desired direction based on possessing a compass. However, it is evident that this approach leads to agents spreading out while translat-

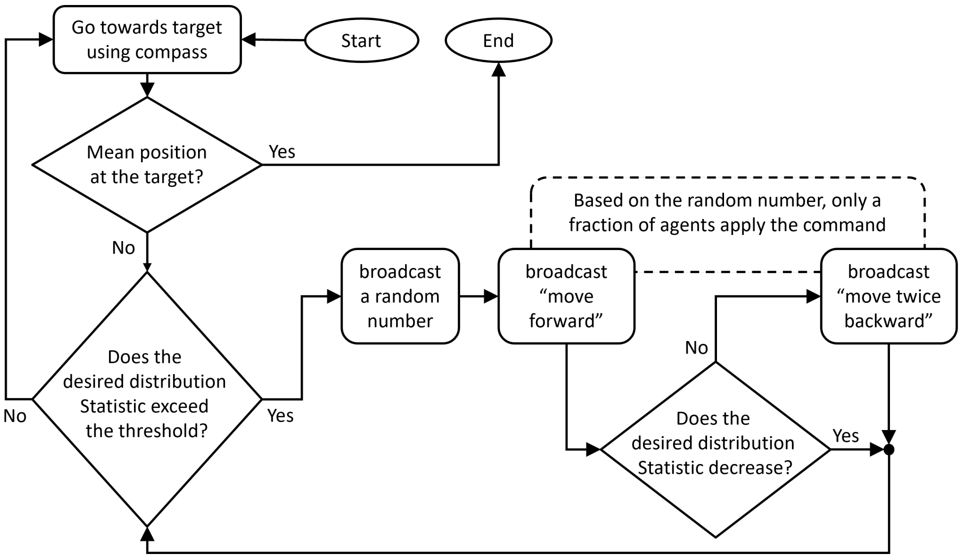

FIGURE 9: Control Scheme Overview: The dashed block encompase agent-side local decision-making and actions. The solid blocks illustrate the central controller's decision-making process, coordinating and overseeing the overall system operation. The scheme distribution-statistic goal function may be the spread of the swarm, its orientation or any other desired goal function which can be measured statistically.

ing to the target. As a naive solution, we considered communicating with the entire swarm using a global toggle command (forward/backwards), implementing a gradient descent scheme to reduce the spread of the swarm. However, the law of large numbers suggests that this approach is insufficient, given that the agents directions are uniformly distributed. Consequently, to maintain the swarm's spread, we investigated an alternative strategy involving communication with only a fraction of the agents at any given instance.

To showcase this, we performed a series of experiments using a small-sized real robot swarm. The objective assigned to the swarm was to achieve translation while maintaining a reasonable spread. Each individual agent was assumed to be equipped with a compass (in the real-world implementation, the heading was extracted using a top-view camera). The swarm successfully accomplished all the assigned tasks, as demonstrated in Figure 7 and the supplementary material, which includes a video showcasing the swarm's performance. Additionally, the convergence rate of the swarm met the expected rate, as depicted in Figure 6 and formulated in detail in subsection III-B and in the Methods Section.

In this particular experiment, each agent was only able to follow a forward and backward command, translating according to its current heading. The average direction uncertainty of 35° played a crucial role in enabling control over the swarm, which would otherwise be limited to endless back-and-forth movement.

This represents the epitome of being a "simple agent" possessing only one degree of freedom and being equipped with a compass sensor alone. As anticipated, this approach proved to be time-consuming. Therefore, we relaxed the simplicity requirement and assumed that each agent possesses two planar degrees of freedom, a compass, and the ability to measure the distance to neighboring agents in its vicinity. We conducted a second set of experiments via simulations involving SMRS with hundreds of such agents. The assigned tasks for the SMRS required navigating through narrow passages while moving towards a target following a calculated path (further discussed below). Throughout the simulations, the controller demonstrated the ability to maintain its spread, execute rotations and reshaping maneuvers (see Figure 8 and the supplementary material, which includes a video showcasing the swarm's performance). During the second set of experiments, we conceptualized the swarm as a unified entity, akin to a 'cloud,' rather than as a mere aggregation of individual agents. This perspective is crucial to adhere to the statistical approach in motion planning, which aims to minimize the number of failed agents while advancing the swarm towards a target. Specifically, we consider this entity to have an uncertain location, represented by a probability distribution that corresponds to the normalized estimated density map, similar to the approach discussed in [56]. This framework allows us to construct a path and shape for the swarm that align with the predefined threshold for failed agents.

The outcomes of these experiments demonstrate the effectiveness of this approach and underscore the essential role of compass-based direction sensing in SMRS control. The procedure outlined for SMRS sensing can be applied to estimate other collective properties of the swarm, such as the flow rate of agents in a cluttered workspace, as well as the artificial pressure exerted by the swarm. This can be achieved by utilizing a particle filter coupled with the thermodynamic characteristics of a gas.

V. METHODS

To prove Equations 2 and 3 assume all agents are not equipped with a compasses unless stated otherwise. At each time step, the controller computes the expected destination function value for a forward and for a backward signals, and chooses the one that minimizes the objective function.

A. TRANSLATION CONTROL

Let us consider, for now, the distance to the target point $\|\mu - \mu_d\|$ as the objective function. So, at each time step τ, the mean position incremental translation $\Delta\mu_n(\tau)$ of the complying subset ($n < N$), calculated as the root mean square $\sqrt{\langle \bullet^2 \rangle}$ with respect to infinite trials is:

$$\Delta\mu_n\,(\tau) = \frac{\epsilon}{n}\sum_{i=1}^{n} v_i(\tau)$$

where $v_i(\tau)$ are the direction vectors taken by each agent at time tau. It turns out that such a strategy will do for controlling the SMRS's spread but will fail for the translation mission. To acknowledge this, note that having the agents randomly chosen, the calculation of this subset-mean travel distance is equivalent to solving the *Pearson random walk* problem [52]. In other words, if the agents do not sense their heading direction (i.e., are not equipped with a compass) the norm of the translation at each time step $|\Delta\mu_n\,(\tau)|$ can be calculated as the random walk of n steps divided by n. The asymptotic probability distribution of a random walk is known to be a Rayleigh type with a root-mean-square-distance $\epsilon\sqrt{n}$. Accordingly, the center of mass of the entire SMRS translates $\sqrt{n}\epsilon/N$ at every time step. In every time step, the activated subset may translate in any given direction as long as it has a negative component towards the designated target. But since these directions are uniformly distributed with respect to the time-steps, the expected value of the cosine component is $2/\pi$ which together constitute the expected translation $\Delta\mu_N$ of the entire SMRS, given below.

$$\Delta\mu_N \sim \frac{2\epsilon\sqrt{n}}{\pi N}$$

For the case where a compass is incorporated on each agent, the step-wise translation is a result of the expected value of the cosine component is $2/\pi$ and therefore will simply amount to $2n\epsilon/N\pi$.

B. SPREAD CONTROL

To prove Equation 5 Set X and Y disjoint random vector-variables that $|X|= N$, $|Y|= n$ and define $Y' = Y \pm \Delta\mu_Y$, were $\Delta\mu_Y$ is the incremental translation of the mean of Y. The standard deviation of the combined (pooled) distribution of X and Y' is:

$$\sigma_{X\cup Y'} = \sqrt{\frac{\sum_X (x_i - \mu_{X\cup Y'})^2 + \sum_Y (y'_i - \mu_{X\cup Y'})^2}{N}}$$

but since

$$\mu_{X\cup Y'} = \mu_{X\cup Y} + \frac{n}{N}\Delta\mu_Y = \mu_{X\cup Y} \pm \alpha\,\Delta\mu_Y$$

we can write

$$\sigma_{X\cup Y'} = \sqrt{\frac{\sum_X (x_i - (\mu_{X\cup Y} \pm \alpha\,\Delta\mu_Y))^2 + \sum_Y ((y_i \pm \Delta_i) - (\mu_{X\cup Y} \pm \alpha\,\Delta\mu_Y))^2}{N}} \quad (6)$$

Squaring and recalling that $\sum_X (x_i - \mu_{X\cup Y}) + \sum_Y (y_i - \mu_{X\cup Y}) = 0$, one gets

$$\sigma^2_{X\cup Y'} = \sigma^2_{X\cup Y} + \frac{1}{N}\left(\sum_X \alpha^2 \Delta\mu_Y^2 + \sum_Y [(\Delta_i - \alpha\Delta\mu_Y)^2 \pm 2\Delta_i^\top (y_i - \mu_{X\cup Y})\right) \quad (7)$$

Expanding this:

$$\sigma^2_{X\cup Y'} = \sigma^2_{X\cup Y} + \frac{1}{N}\left(\sum_X \alpha^2\Delta\mu_Y^2 + \sum_Y [\Delta_i^2 + \alpha^2\Delta\mu_Y^2 - 2\alpha\Delta_i^\top\Delta\mu_Y \pm 2\Delta_i^\top(y_i - \mu_{X\cup Y})]\right) \tag{8}$$

Collecting

$$\sigma^2_{X\cup Y'} = \sigma^2_{X\cup Y} + \alpha^2\Delta\mu_Y^2 - 2\alpha^2\Delta\mu_Y^2 + \frac{\sum_Y [\Delta_i^2 \pm 2\Delta_i^\top(y_i - \mu_{X\cup Y})]}{N} \tag{9}$$

and

$$\sigma^2_{X\cup Y'} = \sigma^2_{X\cup Y} - \alpha^2\Delta\mu_Y^2 \mp 2\alpha\Delta\mu_Y^\top\mu_{X\cup Y} + \frac{\sum_Y \Delta_i^2 \pm 2\Delta_i^\top y_i}{N} \tag{10}$$

plugging $\|\Delta_i\|^2 = \epsilon$ yields:

$$\sigma^2_{X\cup Y'} = \sigma^2_{X\cup Y} - \alpha^2\Delta\mu_Y^2 \mp 2\alpha\Delta\mu_Y^\top\mu_{X\cup Y} + \alpha\epsilon^2 \pm 2\frac{\sum_Y \Delta_i^\top y_i}{N} \tag{11}$$

Note that:

(1) For the 'blind' algorithm $\Delta\mu_Y$ is simply the *two dimensional simple random n-walk* divided by n so

$$\sqrt{\langle\Delta\mu_Y^2\rangle_{blind}} = \frac{\epsilon}{\sqrt{n}}$$

For the 'with compass' algorithm where all agents traverse in the half-plane the term is:

$$\sqrt{\langle\Delta\mu_Y^2\rangle_{compass}} = \frac{2\epsilon}{\pi}$$

(2) Whether we use the 'blind' algorithm or the 'with compass' algorithm, the resulting inner products $\Delta_i^\top y_i$ are random variables taken from a one dimensional Gaussian distribution $\mathcal{N}(0, \sigma_{X\cup Y})$. So the variance of the nominator in the last term is simply the *one dimensional Gaussian random walk* provided as $\sigma_{X\cup Y}\epsilon\sqrt{n}$;

(3) One may set $\mu_{X\cup Y} = 0$.

Which yields:

$$\Delta(\sigma^2_{X\cup Y}) \in \mathcal{N}\left(-\alpha^2\Delta\mu_Y^2 + \alpha\epsilon^2, 2\epsilon\frac{\sigma_{X\cup Y}\sqrt{n}}{N}\right) \tag{12}$$

This defines a difference equation in which its solution is the *Lambert function* with an additional constraint that $\Delta(\sigma^2_{X\cup Y})$ is taken positive under our control law.

VI. CONCLUSIONS

The controllable and the observable parameters in the statistical-physics community and in the robotics community differ: Physicists study the relationships between macroscopic and microscopic parameters, given macroscopic observations. Extracting information experienced by individual particles is often out of reach. In the field of robotic swarms, agents may be equipped with sensors that enable one to estimate the swarm's macroscopic characters, which are often not observable for huge swarms. The framework developed in this study may enable physicists in the statistical-mechanics community to introduce new way of thinking and provide the robotics community with access to the mathematical formalism provided by statistical-physics (such as *Peculation theory*, the notion of *Pressure* etc.).

INCLUSION & ETHICS

The authors declare that they have no competing interests. All authors declare that no material within this manuscript has been previously published.

DATA AVAILABILITY

Accession codes will be available before publication.

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