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# A New Entropy Measure and COPRAS Method for Spherical Fuzzy Sets

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**ABSTRACT** One of the best ways to handle the ambiguity and unpredictable nature of decision-making is through fuzzy logic, and one of the most recent developments in this area is the concept of spherical fuzzy sets. Since the squared total of membership, non-membership, and hesitation degrees should be between 0 and 1, and each degree should be defined in [0, 1], the hesitation of the decision-maker(s) about an attribute can be conveyed more thoroughly. The ambiguity of a fuzzy set is computed with the help of an entropy measure, and the available entropy measures for spherical fuzzy sets have various limitations. So, in this study, we suggest an innovative entropy measurement for spherical fuzzy sets and demonstrate its capacity to satisfy the axiomatic requirements. We compared the proposed entropy metric and all currently available spherical fuzzy entropy metrics, considering different factors, including attribute weight computation, linguistic hedges, and ambiguity computation. With the help of the proposed entropy metric, we introduce the Complex Proportional Assessment method for spherical fuzzy sets and illustrate it with a numerical example.

**INDEX TERMS** Ambiguity, entropy measure, fuzzy set, linguistic hedges, multi-attribute decision-making, spherical fuzzy set.

## I. INTRODUCTION

By proposing the concept of fuzzy sets (FSs), Zadeh [1] awarded membership grades to elements of a set in the range [0, 1]. Zadeh's work in this area is noteworthy since it defines many of the set-theoretic features of crisp cases for FSs. Researchers became interested in FSs, and they found their use in "computer sciences", "communications", "intelligence sciences", "decision sciences", and "engineering". There are some recent studies on FSs and their many applications in the literature [2]–[5].

Atanassov [6] proposed the concept of intuitionistic fuzzy sets (IFSs) to generalize FSs. An intuitionistic fuzzy set (IFS) gives each of its elements a membership level $(\mu)$ and a non-membership level $(\nu)$ such that $\mu + \nu \leq 1, 0 \leq \mu, \nu \leq 1$. Various researchers have studied IFSs and applied them in many areas [7]–[11]; however, due to the constraint $\mu + \nu \leq 1$, IFSs are unable to handle situations in which $\mu + \nu > 1$. So, the concept of Pythagorean fuzzy sets (PYFSs) was developed by Yager [12] to answer the problems where $\mu + \nu > 1$. A Pythagorean fuzzy set (PYFS) gives each of its elements a membership level $(\mu)$ and a non-membership level $(\nu)$ such that $\mu^2 + \nu^2 \leq 1, 0 \leq \mu, \nu \leq 1$. To give more flexibility in assigning the membership levels, Yager [13] introduced the generalized version of IFSs and called them as q-rung orthopair fuzzy sets (q-ROPFSs). A q-rung orthopair fuzzy set (q-ROPFS) gives to each of its elements a membership level $(\mu)$ and a non-membership level $(\vartheta)$ such that $\mu^q + \nu^q \leq 1, 0 \leq \mu, \nu \leq 1, q \geq 1$. Many studies related to PYFSs and their extensions are available in the literature [14]–[22].

The FSs and IFSs cannot handle situations involving the concept of neutrality. For example, human voting, machine vision, feature selection, medical diagnosis, etc. To answer these issues, the idea of a picture fuzzy set (PIFS) was framed by Cuong and Kreinvoch [23]. A PIFS gives each of its elements a satisfaction level $(\mu)$, a non-satisfaction level $(\nu)$, and a neutrality level $(\eta)$ such that $\mu + \nu + \eta \leq 1, 0 \leq \mu, \nu, \eta \leq 1$. This new concept is very closer to human nature than the existing ones and is currently a trending research area now because of its applicability in image processing, decision-making, classification, etc. Some pioneer studies concerning PIFSs and their numerous applications are given in [24]–[31].

The PIFSs suggested by Cuong and Kreinvoch [23] are more efficient and reliable than the FSs and IFSs but, their scope is limited due to the constraint $\mu+\nu+\eta\leq 1$. So, the PIFSs were generalized, and a new concept known as spherical fuzzy sets (SFSs) was introduced by Ashraf et al. [32] . An spherical fuzzy set (SFS) gives each of its elements a satisfaction level $(\mu)$, a non-satisfaction level $(\nu)$, and a neutrality level $(\eta)$ such that $\mu^2+\nu^2+\eta^2\leq 1, 0\leq \mu,\nu,\eta\leq 1$. This means that FSs, IFSs, PYFSs, q-ROPFSs, and PIFSs are a part of the space of SFSs. So, SFSs are more robust and effective than all of these types of FSs. Some basic operations of SFSs were given by Mahmood et al. [33]. Kutlu and Kahraman [34] extended the TOPSIS method to the spherical fuzzy (SF) environment. The classical analytic hierarchy process (AHP) was extended to the SF area by Kutlu and Kahraman [35]. Integration of AHP and TOPSIS (the technique for order performance by similarity to ideal solution) in the SF environment with its applicability in manufacturing system selection was studied by Mathew et al. [36]. Shishavan et al. [37] proposed some novel SF similarity functions with their use in green supplier selection. Rafiq et al. [38] suggested some SF cosine similarity functions and applied them in decision-making. The SF VIKOR (Viekriterijumsko Kompromisno Rangiranje) method and its use in warehouse selection were given by Kutlu and Kahraman [39]. Wei et al. [40] developed some cosine function-based SF similarity metrics and their applications. Some SF metrics of similarity and distance with their use in the selection of mega projects were given by Khan et al. [41]. Jawad et al. [42] suggested a decision-making approach for portfolio selection in the SF environment. An integrated decision-making approach in the SF area was developed by Hoang et al. [43]. In the SF environment, Zhu et al. [44] combined the two decision-making techniques, namely DEMATEL (decision-making trial and evaluation laboratory) and MABAC (multi-attributive border approximation area comparison). The PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) method in the SF area was put forward by Akram et al. [45]. Pirbalouti et al. [46] suggested a new decision-making framework for interval-valued SFSs. Fetanat et al. [47] extended the TOPSIS method to a complex SF environment. Some other studies concerning SFSs and their diverse applications are given in [35], [46], [48]–[51]. There are many decision-making methods like RANCOM (RANking COMparison) [52], SPOTIS (Stable Preference Ordering Towards Ideal Solution) [53], COMET Characteristic Objects Method) [54], SIMUS (Sequential Interactive Method for Urban Systems) [55], etc. Some recent studies concerning the decision-making in the SF environment are available in the literature [56]–[59].

A fascinating question is how to calculate FS's level of uncertainty. The entropy measures the quantity of information produced by a random process. A higher entropy value indicates more information (uncertainty) in the process. Shannon [60] defines entropy as a theoretical assessment of the inherent uncertainty in information, which can be divided into three categories: fuzzy, non-specific, and contradictory. De Luca and Termini [61] established an axiomatic framework to define a measure of fuzzy entropy based on the concept of Shannon's [60] entropy, and Ebank [62] came up with a characterization outcome. Burillo and Bustince [63] suggested the first entropy measure for IFSs. Szmidt and Kacprzyk [64] proposed a non-probabilistic entropy measure for IFSs. A new entropy measure for IFSs known as "IF entropy of order $\alpha$ and type $\beta$" was developed by Verma and Merigo [65]. An entropy measure for PIFSs with its application in decision-making was proposed by Arya and Kumar [66]. Some other studies concerning the information measures for FSs and their extensions are available in the literature [2], [7], [16], [20], [67]–[72], [73]–[76], [77]–[85]. SFSs are an extension of PIFSs, and they also get over their drawbacks. Barukab et al. [86] proposed an SF entropy metric with its utility in group decision-making. Aydogdu and Gul [87] introduced an entropy metric for SFSs and utilized it for computing attribute weights in the SF-WASPAS (weighted aggregated sum product assessment) method. Li et al. [88] suggested a knowledge-based SF entropy metric with its applicability in the determination of expert weights in decision-making problems. However, these SF entropy metrics give unreasonable results in many situations; therefore, a new SF entropy metric is desirable.

The main motivating factors for this study are as follows:

- The existing SF entropy metric due to Barukab et al. [86] gives "0.5" as the ambiguity content for all those spherical fuzzy numbers in which membership and non-membership are equal i.e. $\mu=\nu$. This is not reasonable for computing the amount of uncertainty.
- The SF entropy metric due to Aydogdu and Gul [87] gives $E(C_1)=1$, when $C_1=(a,a,0.5), 0\leq a\leq 1$. This means that for different values of $a$, we get different SFSs but all have entropy equal to "1", which is totally irrational.
- All existing SF entropy metrics [86]–[88] lead to unreasonable results in the computation of ambiguity of different SFSs and the attribute weight computation.
- All of the available SF entropy metrics [86]–[88] are unable to handle the linguistic hedges properly.

So, because of the above factors, we introduce a novel SF entropy measure in this paper. The following are the study's main contributions:

- We offer a novel SF entropy metric based on all four membership levels and establish its validity.
- We contrast the offered SF entropy metric with all available SF entropy metrics through various examples related to ambiguity computation and attribute weight computation in the SF environment.

- We establish its superiority over the available SF entropy metrics through linguistic hedges.
- We use the proposed SF entropy metric to present the novel COPRAS method in the SF area.

The manuscript is organized as Section 2 is preliminary. A novel SF entropy metric with its properties is discussed in Section 3. Section 4 illustrates how the recommended SF entropy function compares numerically to the available SF entropy measurements in various ways. Section 5 introduces the novel Complex Proportional Assessment (COPRAS) method based on the developed SF entropy metric in the SF environment. Finally, the conclusion and future studies are discussed in Section 6.

## II. PRELIMINARY

Here $SFS(B)$ denote the collection of all SFSs in the universe $B=\{b_1,b_2,\ldots,b_p\}$.

***Definition 2.1*** [1] A fuzzy set $C_1$ in $B$ is given by

$$C_1=\left\{\left(b_t,\mu_{C_1}(b_t)\right),b_t\in B\right\},$$

where $0\le\mu_{C_1}(b_t)\le1$ is the grade of satisfaction of $b_t\in B$ in the set $C_1$.

***Definition 2.2*** [6] An intuitionistic fuzzy set $C_1$ in $B$ is given by

$$C_1=\left\{\left(b_t,\mu_{C_1}(b_t),\nu_{C_1}(b_t)\right),b_t\in B\right\},$$

where $0\le\mu_{C_1}(b_t)\le1$ and $0\le\nu_{C_1}(b_t)\le1$ are the grades of satisfaction and non-satisfaction respectively of $b_t\in B$ in the set $C_1$ such that $0\le\mu_{C_1}(b_t)+\nu_{C_1}(b_t)\le1$.

***Definition 2.3*** [23] A picture fuzzy set $C_1$ in $B$ is given by

$$C_1=\left\{\left(b_t,\mu_{C_1}(b_t),\nu_{C_1}(b_t),\eta_{C_1}(b_t)\right),b_t\in B\right\},$$

where $0\le\mu_{C_1}(b_t)\le1$, $0\le\nu_{C_1}(b_t)\le1$, and $0\le\eta_{C_1}(b_t)\le1$ are the grades of satisfaction, non-satisfaction, and neutrality respectively of $b_t\in B$ in the set $C_1$ such that $0\le\mu_{C_1}(b_t)+\nu_{C_1}(b_t)+\eta_{C_1}(b_t)\le1$. Also, $\omega_{C_1}(b_t)=1-\mu_{C_1}(b_t)-\nu_{C_1}(b_t)-\eta_{C_1}(b_t)$ is the refusal degree for the element $b_t\in B$ in the set $C_1$.

***Definition 2.4*** [33] A spherical fuzzy set $C_1$ in $B$ is given by

$$C_1=\left\{\left(b_t,\mu_{C_1}(b_t),\nu_{C_1}(b_t),\eta_{C_1}(b_t)\right),b_t\in B\right\},$$

where $0\le\mu_{C_1}(b_t)\le1$, $0\le\nu_{C_1}(b_t)\le1$, and $0\le\eta_{C_1}(b_t)\le1$ are the grades of satisfaction, non-satisfaction, and neutrality respectively of $b_t\in B$ in the set $C_1$ such that $0\le\mu_{C_1}^2(b_t)+\nu_{C_1}^2(b_t)+\eta_{C_1}^2(b_t)\le1$. Also, $\omega_{C_1}(b_t)=\sqrt{1-\mu_{C_1}^2(b_t)-\nu_{C_1}^2(b_t)-\eta_{C_1}^2(b_t)}$ is the refusal degree for the element $b_t\in B$ in the set $C_1$.

***Definition 2.5*** [32] For $C_1,C_2\in SFS(B)$, some operations are given below:

(1) $C_1\subseteq C_2$ iff $\mu_{C_1}(b_t)\le\mu_{C_2}(b_t),\nu_{C_1}(b_t)\ge\nu_{C_2}(b_t)$, and $\eta_{C_1}(b_t)\le\eta_{C_2}(b_t)\ \forall\ b_t$.

(2) $C_1=C_2$ iff $C_1\subseteq C_2$ and $C_1\supseteq C_2$.

(3) $$C_1\cup C_2=\left\{\left(\begin{array}{c}b_t,\\ max\left(\mu_{C_1}(b_t),\mu_{C_2}(b_t)\right),\\ min\left(\nu_{C_1}(b_t),\nu_{C_2}(b_t)\right),\\ max\left(\eta_{C_1}(b_t),\eta_{C_2}(b_t)\right)\end{array}\right),b_t\in B\right\}.$$

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(4) $C_1 \cap C_2 = \left\{ \begin{pmatrix} b_t, \\ min\left(\mu_{C_1}(b_t), \mu_{C_2}(b_t)\right), \\ max\left(\nu_{C_1}(b_t), \nu_{C_2}(b_t)\right), \\ min\left(\eta_{C_1}(b_t), \eta_{C_2}(b_t)\right) \end{pmatrix}, b_t \in B \right\}.$

(5) $(C_1)^c = \left\{\left(b_t, \nu_{C_1}(b_t), \mu_{C_1}(b_t), \eta_{C_1}(b_t)\right), b_t \in B\right\}$, where $c$ represents the compliment.

The following section suggests a novel SF entropy metric and its properties.

## III. A NEW SPHERICAL FUZZY ENTROPY MEASURE

The SFS concept expands upon the intuitionistic fuzzy set notion. An SFN $C_1$ is a quadruple $\left(\mu_{C_1}, \nu_{C_1}, \eta_{C_1}, \omega_{C_1}\right)$ such that $0 \le \mu_{C_1}, \nu_{C_1}, \eta_{C_1}, \omega_{C_1} \le 1$ and $\mu_{C_1}^2 + \nu_{C_1}^2 + \eta_{C_1}^2 + \omega_{C_1}^2 = 1$ is true. Entropy measurements should be greatest at one point, the same as probability measurements, when all member functions of the SFS are equal $\left(\mu_{C_1}^2 = \nu_{C_1}^2 = \eta_{C_1}^2 = \varphi_{C_1}^2 = \frac{1}{4}\right)$, and should be 0 when $C_1$ is a crisp set.
So, considering these facts, we introduce the axiomatic definition of an SF entropy measure.

***Definition 3.1*** A function $E: SFS(B) \to [0,1]$ is called an SF entropy metric if

(i) $E(C_1) = 0$ if and only if $C_1$ is a crisp set.
(ii) $E(C_1)$ attains its unique maximum when $\mu_{C_1}(b_t) = \nu_{C_1}(b_t) = \eta_{C_1}(b_t) = \omega_{C_1}(b_t) = \frac{1}{2}\ \forall\ b_t \in B$.
(iii) $E(C_1) \le E(C_2)$ when $C_1$ is crisper than $C_2$ i.e., $\mu_{C_1} \le \mu_{C_2}, \nu_{C_1} \le \nu_{C_2}, \eta_{C_1} \le \eta_{C_2}$ for $max\{\mu_{C_2}, \nu_{C_2}, \eta_{C_2}\} \le \frac{1}{2}$ or $\mu_{C_1} \ge \mu_{C_2}, \nu_{C_1} \ge \nu_{C_2}, \eta_{C_1} \ge \eta_{C_2}$ for $min\{\mu_{C_2}, \nu_{C_2}, \eta_{C_2}\} \ge \frac{1}{2}$.
(iv) $E(C_1) = E((C_1)^c)$, where $c$ represents the complement.

Now, we offer a novel SF entropy measure as given below.

$$E_{GD}(C_1) = \begin{cases} \frac{1}{4p}\sum_{t=1}^{p}\frac{1}{\alpha-1}\left[1-\begin{pmatrix}\left(\mu_{C_1}^2(b_t)\right)^{\alpha} \\ +\left(\nu_{C_1}^2(b_t)\right)^{\alpha} \\ +\left(\eta_{C_1}^2(b_t)\right)^{\alpha} \\ +\left(\omega_{C_1}^2(b_t)\right)^{\alpha}\end{pmatrix}\right], & \alpha \neq 1, \alpha > 0 \\ \frac{1}{4p}\sum_{t=1}^{p}-\begin{bmatrix}\mu_{C_1}^2(b_t)\log\mu_{C_1}^2(b_t) \\ +\nu_{C_1}^2(b_t)\log\nu_{C_1}^2(b_t) \\ +\eta_{C_1}^2(b_t)\log\eta_{C_1}^2(b_t) \\ +\omega_{C_1}^2(b_t)\log\omega_{C_1}^2(b_t)\end{bmatrix}, & \alpha = 1 \end{cases} \quad (1)$$

*Theorem 3.1* The function $E_{GD}$ is an SF entropy metric.
*Proof*. We will establish that $E_{GD}$ has the properties (i)-(iv) of Definition 3.1.
(i) Let $E_{GD}(C_1) = 0$, then

$$\frac{1}{4p}\sum_{t=1}^{p}\frac{1}{\alpha-1}\left[1-\begin{pmatrix}\left(\mu_{C_1}^2(b_t)\right)^{\alpha}+\left(\nu_{C_1}^2(b_t)\right)^{\alpha} \\ +\left(\eta_{C_1}^2(b_t)\right)^{\alpha}+\left(\omega_{C_1}^2(b_t)\right)^{\alpha}\end{pmatrix}\right] = 0,$$

$\Rightarrow$

$$\left(\mu_{C_1}^2(b_t)\right)^{\alpha}+\left(\nu_{C_1}^2(b_t)\right)^{\alpha}+\left(\eta_{C_1}^2(b_t)\right)^{\alpha}+\left(\omega_{C_1}^2(b_t)\right)^{\alpha} = 1.$$

Since $\alpha \neq 1$, so for the above equation to be true, we have the following four possibilities.

(a) $\mu_{C_1}^2(b_t) = 1, \nu_{C_1}^2(b_t) = \eta_{C_1}^2(b_t) = \omega_{C_1}^2(b_t) = 0\ \forall\ t$.
(b) $\nu_{C_1}^2(b_t) = 1, \mu_{C_1}^2(b_t) = \eta_{C_1}^2(b_t) = \omega_{C_1}^2(b_t) = 0\ \forall\ t$.
(c) $\eta_{C_1}^2(b_t) = 1, \mu_{C_1}^2(b_t) = \nu_{C_1}^2(b_t) = \omega_{C_1}^2(b_t) = 0\ \forall\ t$.
(d) $\omega_{C_1}^2(b_t) = 1, \mu_{C_1}^2(b_t) = \nu_{C_1}^2(b_t) = \eta_{C_1}^2(b_t) = 0\ \forall\ t$.

All of these possibilities indicate that $C_1$ is a crisp set.
Conversely, suppose that $C_1$ is a crisp set, we have four possibilities.

(a) $\mu_{C_1}^2(b_t) = 1, \nu_{C_1}^2(b_t) = \eta_{C_1}^2(b_t) = \omega_{C_1}^2(b_t) = 0\ \forall\ t$.
(b) $\nu_{C_1}^2(b_t) = 1, \mu_{C_1}^2(b_t) = \eta_{C_1}^2(b_t) = \omega_{C_1}^2(b_t) = 0\ \forall\ t$.
(c) $\eta_{C_1}^2(b_t) = 1, \mu_{C_1}^2(b_t) = \nu_{C_1}^2(b_t) = \omega_{C_1}^2(b_t) = 0\ \forall\ t$.
(d) $\omega_{C_1}^2(b_t) = 1, \mu_{C_1}^2(b_t) = \nu_{C_1}^2(b_t) = \eta_{C_1}^2(b_t) = 0\ \forall\ t$.

All of these possibilities lead us to $E_{GD}(C_1) = 0$.
(ii) To show the property (ii) of Definition 3.1, we use Langrage's multipliers for $E_{GD}(C_1)$ with

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$F\left(\mu_{C_1}^2, \nu_{C_1}^2, \eta_{C_1}^2, \omega_{C_1}^2\right) =$

$\frac{1}{4p}\sum_{t=1}^{p}\frac{1}{\alpha-1}\left[1-\left(\begin{matrix}\left(\mu_{C_1}^2(b_t)\right)^\alpha+\left(\nu_{C_1}^2(b_t)\right)^\alpha\\+\left(\eta_{C_1}^2(b_t)\right)^\alpha+\left(\omega_{C_1}^2(b_t)\right)^\alpha\end{matrix}\right)\right]$

$+\sum_{t=1}^{p}\lambda_t\left(\mu_{C_1}^2(b_t)+\nu_{C_1}^2(b_t)+\eta_{C_1}^2(b_t)+\omega_{C_1}^2(b_t)-1\right).$

Differentiate $F$ w.r.t. $\mu_{C_1}^2(b_t), \nu_{C_1}^2(b_t), \eta_{C_1}^2(b_t), \omega_{C_1}^2(b_t)$, and $\lambda_t$, we obtain

$$\frac{\partial F}{\partial \mu_{C_1}^2(b_t)} = -\frac{\alpha}{4p(\alpha-1)}\left(\mu_{C_1}^2(b_t)\right)^{\alpha-1}+\lambda_t$$
$$\frac{\partial F}{\partial \nu_{C_1}^2(b_t)} = -\frac{\alpha}{4p(\alpha-1)}\left(\nu_{C_1}^2(b_t)\right)^{\alpha-1}+\lambda_t$$
$$\frac{\partial F}{\partial \eta_{C_1}^2(b_t)} = -\frac{\alpha}{4p(\alpha-1)}\left(\eta_{C_1}^2(b_t)\right)^{\alpha-1}+\lambda_t$$
$$\frac{\partial F}{\partial \omega_{C_1}^2(b_t)} = -\frac{\alpha}{4p(\alpha-1)}\left(\omega_{C_1}^2(b_t)\right)^{\alpha-1}+\lambda_t$$
$$\frac{\partial F}{\partial \lambda_t} = \mu_{C_1}^2(b_t)+\nu_{C_1}^2(b_t)+\eta_{C_1}^2(b_t)+\omega_{C_1}^2(b_t)-1.$$

Setting all these derivatives equal to zero and solving them, we get

$\mu_{C_1}^2(b_t) = \left(\frac{4\lambda_t p(\alpha-1)}{\alpha}\right)^{\frac{1}{\alpha-1}}, \nu_{C_1}^2(b_t) = \left(\frac{4\lambda_t p(\alpha-1)}{\alpha}\right)^{\frac{1}{\alpha-1}},$
$\eta_{C_1}^2(b_t) = \left(\frac{4\lambda_t p(\alpha-1)}{\alpha}\right)^{\frac{1}{\alpha-1}}, \omega_{C_1}^2(b_t) = \left(\frac{4\lambda_t p(\alpha-1)}{\alpha}\right)^{\frac{1}{\alpha-1}},$ and $\mu_{C_1}^2(b_t)+\nu_{C_1}^2(b_t)+\eta_{C_1}^2(b_t)+\omega_{C_1}^2(b_t)=1.$

So,

$\left(\frac{4\lambda_t p(\alpha-1)}{\alpha}\right)^{\frac{1}{\alpha-1}}+\left(\frac{4\lambda_t p(\alpha-1)}{\alpha}\right)^{\frac{1}{\alpha-1}}+\left(\frac{4\lambda_t p(\alpha-1)}{\alpha}\right)^{\frac{1}{\alpha-1}}+\left(\frac{4\lambda_t p(\alpha-1)}{\alpha}\right)^{\frac{1}{\alpha-1}}=1.$

$\Rightarrow \left(\frac{4\lambda_t p(\alpha-1)}{\alpha}\right)^{\frac{1}{\alpha-1}} = \frac{1}{4}$ or $\lambda_t = \frac{\alpha}{4p(\alpha-1)}\left(\frac{1}{4}\right)^{\alpha-1}.$

Thus $\mu_{C_1}^2(b_t) = \left(\frac{\frac{4\alpha}{4p(\alpha-1)}\left(\frac{1}{4}\right)^{\alpha-1}p(\alpha-1)}{\alpha}\right)^{\frac{1}{\alpha-1}} = \frac{1}{4}$ or $\mu_{C_1}(b_t) = \frac{1}{2}.$

Similarly, we get

$\nu_{C_1}(b_t) = \frac{1}{2}, \eta_{C_1}(b_t) = \frac{1}{2}$, and $\omega_{C_1}(b_t) = \frac{1}{2}.$

Hence the required stationary point is $\mu_{C_1}(b_t) = \nu_{C_1}(b_t) = \eta_{C_1}(b_t) = \omega_{C_1}(b_t) = \frac{1}{2}$. Similarly, we can show that $\mu_{C_1}(b_t) = \nu_{C_1}(b_t) = \eta_{C_1}(b_t) = \omega_{C_1}(b_t) = \frac{1}{2}$ is also a stationary point when $\alpha = 1$.

Now, we will establish that the function $E_{GD}$ is a concave function. For this, we consider the function

$$g(z) = \begin{cases}\frac{1}{\alpha-1}(z-z^\alpha), & \alpha\neq 1, \alpha>0\\ -z\log z, & \alpha=1\end{cases}.$$

Since $g'(z) = \frac{1}{\alpha-1}(1-\alpha z^{\alpha-1})$ and $g''(z) = -\alpha z^{\alpha-2} < 0$ for $\alpha \neq 1, \alpha > 0$. Thus $g(z)$ is a strictly concave function of $z$.

Also, when $g(z) = -z\log z$, then $g'(z) = -1-\log z$ and $g''(z) = -\frac{1}{z} < 0$. So, $g(z)$ is a strictly concave function of $z$ for $\alpha = 1$ as well.

Since

$$E_{GD}(C_1) = \frac{1}{p}\sum_{t=1}^{p}\left[\begin{matrix}g\left(\mu_{C_1}^2(b_t)\right)+g\left(\nu_{C_1}^2(b_t)\right)\\+g\left(\eta_{C_1}^2(b_t)\right)+g\left(\omega_{C_1}^2(b_t)\right)\end{matrix}\right].$$

So $E_{GD}(C_1)$ is a concave function on the set $\{(\mu_{C_1}, \nu_{C_1}, \eta_{C_1}, \omega_{C_1}) | 0 < \mu_{C_1}, \nu_{C_1}, \eta_{C_1}, \omega_{C_1} < 1, \mu_{C_1}^2+\nu_{C_1}^2+\eta_{C_1}^2+\omega_{C_1}^2 = 1\}$.

Now, to show that the concave function $E_{GD}(C_1)$ attains its maximum at $\mu_{C_1}(b_t) = \nu_{C_1}(b_t) = \eta_{C_1}(b_t) = \omega_{C_1}(b_t) = \frac{1}{2}$, we will show that its Hessian matrix (HM) is negative semi-definite at the required stationary point.

The Hessian matrix of a function $f$ of four variables $z_1, z_2, z_3$, and $z_4$ is computed as

$$\mathrm{HM}(f) = \begin{bmatrix}\frac{\partial^2 f}{\partial z_1^2} & \frac{\partial^2 f}{\partial z_2 z_1} & \frac{\partial^2 f}{\partial z_3 z_1} & \frac{\partial^2 f}{\partial z_4 z_1}\\ \frac{\partial^2 f}{\partial z_1 z_2} & \frac{\partial^2 f}{\partial z_2^2} & \frac{\partial^2 f}{\partial z_3 z_2} & \frac{\partial^2 f}{\partial z_4 z_2}\\ \frac{\partial^2 f}{\partial z_1 z_3} & \frac{\partial^2 f}{\partial z_2 z_3} & \frac{\partial^2 f}{\partial z_3^2} & \frac{\partial^2 f}{\partial z_4 z_3}\\ \frac{\partial^2 f}{\partial z_1 z_4} & \frac{\partial^2 f}{\partial z_2 z_4} & \frac{\partial^2 f}{\partial z_3 z_4} & \frac{\partial^2 f}{\partial z_4^2}\end{bmatrix}.$$

So,

$$\mathrm{HM}\left(E_{GD}(C_1)\right) = \begin{bmatrix}-\frac{\alpha}{4p}\left(\mu_{C_1}^2(b_t)\right)^{\alpha-2} & 0 & 0 & 0\\ 0 & -\frac{\alpha}{4p}\left(\nu_{C_1}^2(b_t)\right)^{\alpha-2} & 0 & 0\\ 0 & 0 & -\frac{\alpha}{4p}\left(\eta_{C_1}^2(b_t)\right)^{\alpha-2} & 0\\ 0 & 0 & 0 & -\frac{\alpha}{4p}\left(\omega_{C_1}^2(b_t)\right)^{\alpha-2}\end{bmatrix}$$

$$= \frac{\alpha}{p4^{\alpha-1}}\begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{bmatrix}$$

at $\mu_{C_1}(b_t) = \nu_{C_1}(b_t) = \eta_{C_1}(b_t) = \omega_{C_1}(b_t) = \frac{1}{2}$, which is negative semi-definite for all $\alpha \neq 1, \alpha > 0$.

Similarly, we can show that for $\alpha = 1$, $\mathrm{HM}\left(E_{GD}(C_1)\right)$ is also negative semi-definite at $\mu_{C_1}(b_t) = \nu_{C_1}(b_t) = \eta_{C_1}(b_t) = \omega_{C_1}(b_t) = \frac{1}{2}$.

Thus $E_{GD}(C_1)$ attains its unique maximum at $\mu_{C_1}(b_t) = \nu_{C_1}(b_t) = \eta_{C_1}(b_t) = \omega_{C_1}(b_t) = \frac{1}{2}\ \forall\, t$.

(iii) Let $C_1$ be crisper than $C_2$ i.e., $\mu_{C_1} \leq \mu_{C_2}, \nu_{C_1} \leq \nu_{C_2}, \eta_{C_1} \leq \eta_{C_2}$ for $max\{\mu_{C_2}, \nu_{C_2}, \eta_{C_2}\} \leq \frac{1}{2}$ or $\mu_{C_1} \geq \mu_{C_2}, \nu_{C_1} \geq \nu_{C_2}, \eta_{C_1} \geq \eta_{C_2}$ for $min\{\mu_{C_2}, \nu_{C_2}, \eta_{C_2}\} \geq \frac{1}{2}$.

Now, when $\mu_{C_1} \leq \mu_{C_2}, \nu_{C_1} \leq \nu_{C_2}, \eta_{C_1} \leq \eta_{C_2}$ for $\{\mu_{C_2}, \nu_{C_2}, \eta_{C_2}\} \leq \frac{1}{2}$, then

$$\mu_{C_1} \leq \mu_{C_2} \leq \frac{1}{2}, \nu_{C_1} \leq \nu_{C_2} \leq \frac{1}{2}, \eta_{C_1} \leq \eta_{C_2} \leq \frac{1}{2}$$

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$$\text{or } 1-\mu_{C_1}^2-\nu_{C_1}^2-\eta_{C_1}^2 \geq 1-\mu_{C_2}^2-\nu_{C_2}^2-\eta_{C_2}^2 \geq \frac{1}{4}$$
$$\text{or } \omega_{C_1}^2 \geq \omega_{C_2}^2 \geq \frac{1}{4}$$
$$\text{or } \omega_{C_1} \geq \omega_{C_2} \geq \frac{1}{2}.$$

Also,
$\mu_{C_1}-\frac{1}{2} \leq \mu_{C_2}-\frac{1}{2} \leq 0, \nu_{C_1}-\frac{1}{2} \leq \nu_{C_2}-\frac{1}{2} \leq 0$, and $\eta_{C_1}-\frac{1}{2} \leq \eta_{C_2}-\frac{1}{2} \leq 0$.
So,
$\left(\mu_{C_1}-\frac{1}{2}\right)^2 \geq \left(\mu_{C_2}-\frac{1}{2}\right)^2, \left(\nu_{C_1}-\frac{1}{2}\right)^2 \geq \left(\nu_{C_2}-\frac{1}{2}\right)^2, \left(\eta_{C_1}-\frac{1}{2}\right)^2 \geq \left(\eta_{C_2}-\frac{1}{2}\right)^2$, and $\left(\omega_{C_1}-\frac{1}{2}\right)^2 \geq \left(\omega_{C_2}-\frac{1}{2}\right)^2$.
Thus,
$$\left(\mu_{C_1}-\frac{1}{2}\right)^2+\left(\nu_{C_1}-\frac{1}{2}\right)^2+\left(\eta_{C_1}-\frac{1}{2}\right)^2+\left(\omega_{C_1}-\frac{1}{2}\right)^2 \geq \left(\mu_{C_2}-\frac{1}{2}\right)^2+\left(\nu_{C_2}-\frac{1}{2}\right)^2+\left(\eta_{C_2}-\frac{1}{2}\right)^2+\left(\omega_{C_2}-\frac{1}{2}\right)^2.$$
This indicates that $C_1$ is far away from $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ compared to $C_2$. Since $E_{GD}(C_1)$ is a strictly concave function and attains its unique maximum at $\mu_{C_1}=\nu_{C_1}=\eta_{C_1}=\omega_{C_1}=\frac{1}{2}$. So, it follows that $E_{GD}(C_1) \leq E_{GD}(C_2)$. Similarly, for the other case i.e., $\mu_{C_1} \geq \mu_{C_2}, \nu_{C_1} \geq \nu_{C_2}, \eta_{C_1} \geq \eta_{C_2}$ for $min\{\mu_{C_2},\mu_{C_2},\eta_{C_2}\} \geq \frac{1}{2}$, we can show that $E_{GD}(C_1) \leq E_{GD}(C_2)$.
(iv) $E_{GD}((C_1)^c)=E_{GD}(C_1)$ follows from the expression of $E_{GD}(C_1)$.
Hence $E_{GD}(C_1)$ is a measure of entropy for SFSs.
The valuation characteristic of the proposed SF entropy metric is now discussed.
*Theorem 3.2* For any $C_1, C_2 \in SFS(B)$, we have
$$E_{GD}(C_1 \cup C_2)+E_{GD}(C_1 \cap C_2)=E_{GD}(C_1)+E_{GD}(C_2),$$
where ∪ and ∩ denote respectively the union and intersection of the SFSs.
*Proof 3.2* Let us divide the set $B$ into two sets $B_1$ and $B_2$ such that $B_1=\{b_t \in B, C_1 \subseteq C_2\}$ and $B_2=\{b_t \in B, C_1 \supseteq C_2\}$.
Thus $\forall\, b_t \in B_1$, we have $\mu_{C_1}(b_t) \leq \mu_{C_2}(b_t)$, $\nu_{C_1}(b_t) \geq \nu_{C_2}(b_t), \eta_{C_1}(b_t) \leq \eta_{C_2}(b_t)$, and $\forall\, b_t \in B_2$, we have $\mu_{C_1}(b_t) \geq \mu_{C_2}(b_t)$, $\nu_{C_1}(b_t) \leq \nu_{C_2}(b_t), \eta_{C_1}(b_t) \geq \eta_{C_2}(b_t)$.
So,
$$E_{GD}(C_1 \cup C_2)=\frac{1}{4p}\sum_{t=1}^{p}\frac{1}{\alpha-1}\left[1-\begin{pmatrix}\left(\mu_{C_1\cup C_2}^2(b_t)\right)^\alpha+\left(\nu_{C_1\cup C_2}^2(b_t)\right)^\alpha\\+\left(\eta_{C_1\cup C_2}^2(b_t)\right)^\alpha+\left(\omega_{C_1\cup C_2}^2(b_t)\right)^\alpha\end{pmatrix}\right]$$
$$=\frac{1}{4p}\sum_{B_1}\frac{1}{\alpha-1}\left[1-\begin{pmatrix}\left(\mu_{C_2}^2(b_t)\right)^\alpha+\left(\nu_{C_2}^2(b_t)\right)^\alpha\\+\left(\eta_{C_2}^2(b_t)\right)^\alpha+\left(\omega_{C_2}^2(b_t)\right)^\alpha\end{pmatrix}\right]$$
$$+\frac{1}{4p}\sum_{B_2}\frac{1}{\alpha-1}\left[1-\begin{pmatrix}\left(\mu_{C_1}^2(b_t)\right)^\alpha+\left(\nu_{C_1}^2(b_t)\right)^\alpha\\+\left(\eta_{C_1}^2(b_t)\right)^\alpha+\left(\omega_{C_1}^2(b_t)\right)^\alpha\end{pmatrix}\right].$$
$$E_{GD}(C_1 \cap C_2)=\frac{1}{4p}\sum_{t=1}^{p}\frac{1}{\alpha-1}\left[1-\begin{pmatrix}\left(\mu_{C_1\cap C_2}^2(b_t)\right)^\alpha+\left(\nu_{C_1\cap C_2}^2(b_t)\right)^\alpha\\+\left(\eta_{C_1\cap C_2}^2(b_t)\right)^\alpha+\left(\omega_{C_1\cap C_2}^2(b_t)\right)^\alpha\end{pmatrix}\right]$$
$$=\frac{1}{4p}\sum_{B_1}\frac{1}{\alpha-1}\left[1-\begin{pmatrix}\left(\mu_{C_1}^2(b_t)\right)^\alpha+\left(\nu_{C_1}^2(b_t)\right)^\alpha\\+\left(\eta_{C_1}^2(b_t)\right)^\alpha+\left(\omega_{C_1}^2(b_t)\right)^\alpha\end{pmatrix}\right]$$
$$+\frac{1}{4p}\sum_{B_2}\frac{1}{\alpha-1}\left[1-\begin{pmatrix}\left(\mu_{C_2}^2(b_t)\right)^\alpha+\left(\nu_{C_2}^2(b_t)\right)^\alpha\\+\left(\eta_{C_2}^2(b_t)\right)^\alpha+\left(\omega_{C_2}^2(b_t)\right)^\alpha\end{pmatrix}\right].$$
Thus
$$E_{GD}(C_1 \cup C_2)+E_{GD}(C_1 \cap C_2)=E_{GD}(C_1)+E_{GD}(C_2).$$

## IV. COMPARATIVE ANALYSIS AND MONOTONIC BEHAVIOR

Here, we compare how the proposed SF entropy metric performs against the existing information metrics through various aspects such as ambiguity computation, linguistic hedges, and attribute weight computation. We begin by listing the SF entropy measurements that are currently used in the literature.
Aydogdu and Gul [87]
$$E_{AG}(C_1)=\frac{1}{p}\sum_{t=1}^{p}\left(1-\frac{4}{5}\begin{bmatrix}\left|\mu_{C_1}^2(b_t)-\nu_{C_1}^2(b_t)\right|\\+\left|\eta_{C_1}^2(b_t)-0.25\right|\end{bmatrix}\right).$$
Barukab et al. [86]
$$E_{BAAAK}(C_1)=\frac{1}{2p}\sum_{t=1}^{p}\begin{pmatrix}\left(1-\left|\mu_{C_1}^2(b_t)-\nu_{C_1}^2(b_t)\right|\right)\\\times\begin{pmatrix}2-\mu_{C_1}^2(b_t)-\\\nu_{C_1}^2(b_t)-\eta_{C_1}^2(b_t)\end{pmatrix}\end{pmatrix}.$$
Li et al. [88]
$$E_{LLMY}(C_1)=\frac{1}{p}\sum_{t=1}^{p}\left(1-\frac{1}{\sqrt{2}}\sqrt{\begin{matrix}\left(\mu_{C_1}^2(b_t)\right)^2\\+\left(\nu_{C_1}^2(b_t)\right)^2\\+\left(\eta_{C_1}^2(b_t)\right)^2\\+\left(\omega_{C_1}^2(b_t)-1\right)^2\end{matrix}}\right).$$

### A. AMBIGUITY COMPUTATION

Here, we use the suggested SF entropy metric for determining the ambiguity content of SFSs and will contrast the results with the existing SF entropy metrics. After computing the ambiguity, we will compute the accuracy of the SF entropy metrics by using the following formula due to Zhang et al. [89].
$$Accuracy=\frac{Number\ of\ SFSs\ with\ distinct\ ambiguity}{Total\ number\ of\ SFSs.}\times 100.$$
An entropy metric with higher precision accuracy is better and more reliable.
***Example 4.1*** Consider five SFSs $C_1, C_2, C_3, C_4$, and $C_5$ in $B=\{b_1,b_2,b_3\}$ as shown below
$$C_1=\left\{\begin{matrix}(b_1,0.2,0.3,0.2),(b_2,0.1,0.5,0.4),\\(b_3,0.2,0.8,0)\end{matrix}\right\},$$

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$$C_2 = \left\{ \begin{matrix} (b_1, 0.10, 0.48, 0.41), (b_2, 0.2, 0.3, 0.5), \\ (b_3, 0.4, 0.1, 0.1) \end{matrix} \right\},$$
$$C_3 = \{(b_1, 0.5, 0.3, 0.1), (b_2, 0.4, 0.2, 0.2), (b_3, 0.3, 0.5, 0)\},$$
$$C_4 = \left\{ \begin{matrix} (b_1, 0.4, 0, 0.4), (b_2, 0.5, 0.4, 0.1), \\ (b_3, 0.4, 0.21, 0.04) \end{matrix} \right\},$$
$$C_5 = \{(b_1, 0.8, 0, 0.1), (b_2, 0.3, 0.1, 0.1), (b_3, 0.4, 0.4, 0.1)\}.$$

The ambiguous content of these five SFSs is given in Table.

**Table 1** Ambiguity content of different SFSs concerning Example 4.1

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | Accuracy (%) |
|---|---|---|---|---|---|---|
| $E_{AG}(C_t)$ | **-0.0507** | 0.1354 | 0.0293 | 0.0815 | **-0.0507** | 60 |
| $E_{BAAAK}(C_t)$ | 0.5102 | **0.6315** | **0.6315** | 0.6477 | 0.5836 | 60 |
| $E_{LLMY}(C_t)$ | 0.4911 | 0.5833 | **0.5958** | **0.5958** | 0.5501 | 60 |
| $E_{GD}(C_t)$ | 0.1576 | 0.1749 | 0.1773 | 0.1757 | 0.1406 | 100 |

(Bold values denote irrational results. $\alpha = 2$ in $E_{GD}$).

Table 1 provides the following observations:

(1) The SF entropy function $E_{AG}$ gives the ambiguous content of the two SFSs $C_1$ and $C_5$ to be negative, which is not rational.

(2) The SF entropy measure $E_{BAAAK}$ gives the ambiguous content of two different SFSs $C_2$ and $C_3$ to be the same i.e. 0.6315, which is not satisfactory.

(3) The SF entropy measure $E_{LLMY}$ gives the ambiguous content of two different SFSs $C_3$ and $C_4$ to be the same i.e. 0.5958, which is unreasonable.

(4) The suggested SF entropy measure $E_{GD}$ computes the ambiguous content of all five SFSs without any counterintuitive results. Also its accuracy is higher than the existing ones.

***Example 4.2*** Consider five SFSs $C_1, C_2, C_3, C_4$, and $C_5$ in $B = \{b_1, b_2, b_3\}$ as shown below

$$C_1 = \left\{ \begin{matrix} (b_1, 0.14, 0.38, 0.20), (b_2, 0.18, 0.43, 0.3), \\ (b_3, 0.4, 0.6, 0.2) \end{matrix} \right\},$$
$$C_2 = \left\{ \begin{matrix} (b_1, 0.4, 0.6, 0), (b_2, 0.48, 0.53, 0.14), \\ (b_3, 0.3, 0.3, 0.3) \end{matrix} \right\},$$
$$C_3 = \left\{ \begin{matrix} (b_1, 0.6, 0.37, 0.17), (b_2, 0.5, 0.3, 0), \\ (b_3, 0.4, 0.1, 0.1) \end{matrix} \right\},$$
$$C_4 = \left\{ \begin{matrix} (b_1, 0.42, 0.46, 0.45), (b_2, 0.66, 0.09, 0.16), \\ (b_3, 0.4, 0.3, 0.1) \end{matrix} \right\},$$
$$C_5 = \left\{ \begin{matrix} (b_1, 0.2, 0.5, 0.5), (b_2, 0, 0.47, 0.2), \\ (b_3, 0.5, 0.4, 0.3) \end{matrix} \right\}.$$

Table 2 shows the ambiguous content of these five SFSs.

**Table 2** Ambiguity content of distinct SFSs concerning Example 4.2

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | Accuracy (%) |
|---|---|---|---|---|---|---|
| $E_{AG}(C_t)$ | 0.0514 | **0.0958** | 0.0015 | 0.0548 | **0.0958** | 60 |
| $E_{BAAAK}(C_t)$ | 0.6065 | 0.6126 | 0.5965 | **0.5499** | **0.5499** | 60 |
| $E_{LLMY}(C_t)$ | **0.5628** | 0.4773 | **0.5628** | 0.4747 | 0.4855 | 60 |
| $E_{GD}(C_t)$ | 0.1757 | 0.1919 | 0.1718 | 0.1876 | 0.1934 | 100 |

(Bold values denote irrational results. $\alpha = 2$ in $E_{GD}$).

Table 2 provides the following observations:

(1) The SF entropy measure $E_{AG}$ gives the ambiguous content of the two different SFSs $C_2$ and $C_5$ to be 0.0958, which is not rational.

(2) The SF entropy measure $E_{BAAAK}$ gives 0.5499 as the ambiguity content of the two different SFSs $C_4$ and $C_5$, thereby considering them to be the same, which is not satisfactory.

(3) The SF entropy measure $E_{LLMY}$ gives the ambiguous content of two different SFSs $C_1$ and $C_3$ to be the same i.e. 0.5628, which is unreasonable.

(4) The suggested SF entropy measure $E_{GD}$ computes the ambiguous content of all five SFSs without any counterintuitive results. Also its accuracy is higher than the existing ones.

***Example 4.3*** Consider five SFSs $C_1, C_2, C_3, C_4$, and $C_5$ in $B = \{b_1, b_2, b_3\}$ as shown below

$$C_1 = \left\{ \begin{matrix} (b_1, 0.22, 0.52, 0.02), (b_2, 0.1, 0.4, 0.3), \\ (b_3, 0.6, 0.4, 0.1) \end{matrix} \right\},$$
$$C_2 = \left\{ \begin{matrix} (b_1, 0.17, 0.43, 0.52), (b_2, 0.2, 0.3, 0.5), \\ (b_3, 0.4, 0.3, 0.1) \end{matrix} \right\},$$
$$C_3 = \left\{ \begin{matrix} (b_1, 0.4, 0.4, 0.46), (b_2, 0.6, 0.4, 0), \\ (b_3, 0.4, 0.01, 0.2) \end{matrix} \right\},$$
$$C_4 = \left\{ \begin{matrix} (b_1, 0.4, 0.6, 0), (b_2, 0.3, 0.1, 0.5), \\ (b_3, 0.5, 0.4, 0.3) \end{matrix} \right\},$$
$$C_5 = \left\{ \begin{matrix} (b_1, 0.5, 0.4, 0), (b_2, 0.1, 0.7, 0.2), \\ (b_3, 0.19, 0.26, 0.3) \end{matrix} \right\}.$$

The ambiguous content of these five SFSs is given in Table 3.

**Table 3** Ambiguity content of different SFSs concerning Example 4.3

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | Accuracy (%) |
|---|---|---|---|---|---|---|
| $E_{AG}(C_t)$ | **0.0076** | 0.1903 | 0.1045 | 0.1253 | **0.0076** | 60 |
| $E_{BAAAK}(C_t)$ | **0.5718** | 0.6372 | 0.5969 | **0.5718** | 0.5758 | 60 |
| $E_{LLMY}(C_t)$ | **0.5370** | **0.5370** | 0.5049 | 0.4551 | 0.5251 | 60 |
| $E_{GD}(C_t)$ | 0.1810 | 0.1890 | 0.1843 | 0.1987 | 0.1723 | 100 |

(Bold values denote irrational results. $\alpha = 2$ in $E_{GD}$).

From Table 3, we have

(1) The ambiguous content of two different SFSs $C_1$ and $C_5$ computed by the SF entropy metric $E_{AG}$ is the same i.e. 0.0076, which is unreasonable.

(2) The SF entropy metric $E_{BAAAK}$ gives 0.5718 as the ambiguity content for two distinct SFSs $C_1$ and $C_4$, thereby leading to a counterintuitive situation.

(3) For two distinct SFSs $C_1$ and $C_2$, the ambiguity content present in them is 0.5370 as shown by the SF entropy metric $E_{LLMY}$, which is also not rational.

(4) The suggested SF entropy measure $E_{GD}$ computes the ambiguous content of all five SFSs without any

counterintuitive results. Also its accuracy is higher than the existing ones.

As a result of Examples 4.1–4.3, we conclude that the suggested entropy metric is superior to the current SF entropy measures reported in the literature regarding the ambiguous content of various SFSs.

### B. LINGUISTIC HEDGES

Here, we give an example to show the behavior of the suggested SF entropy measure. We offer an example incorporating linguistic hedges to make it mathematically sound and practically acceptable. By using a linguistic example, we will choose the best entropy in the SF environment by using several linguistic variables such as “LARGE”, “quite LARGE”, “very LARGE”, “quite very LARGE”, “very very LARGE”, etc. First, we recall the definition of the modifier $C_1^\delta$ of an SFS $C_1$.

***Definition 4.1*** [33] For any $C_1 = \left\{\left(b_t, \mu_{C_1}(b_t), \nu_{C_1}(b_t), \eta_{C_1}(b_t)\right);\ b_t \in B\right\} \in SFS(B), C_1^\delta, \delta > 0$ is defined as

$$C_1^\delta = \left\{\begin{pmatrix} b_t, \left(\mu_{C_1}(b_t) + \eta_{C_1}(b_t)\right)^\delta - \left(\eta_{C_1}(b_t)\right)^\delta, \\ \sqrt{1 - \left(1 - \left(\nu_{C_1}(b_t)\right)^2\right)^\delta}, \left(\eta_{C_1}(b_t)\right)^\delta \end{pmatrix};\ b_t \in B\right\}.$$

We provide an example using structured linguistic data to analyze and compare the suggested SF entropy.

***Example 4.4*** Consider an SFS $C_1 \in SFS(B), B = \{b_1, b_2, b_3, b_4, b_5\}$ given as

$$C_1 = \left\{\begin{matrix}(b_1, 0, 0, 0.1), (b_2, 0.1, 0.5, 0), (b_3, 0.2, 0.1, 0), \\ (b_4, 0.1, 0.5, 0.2), (b_5, 0, 0, 0)\end{matrix}\right\}.$$

With the help of Definition 4.1, we define the SFSs as More or less LARGE $= C_1^{\frac{1}{2}}$, LARGE $= C_1$, quite LARGE $= C_1^{\frac{3}{2}}$, very LARGE $= C_1^2$, quite very LARGE $= C_1^{\frac{5}{2}}$, very very LARGE $= C_1^3$ as below:

$$C_1^{\frac{1}{2}} = \left\{\begin{matrix}(b_1, 0, 0, 0.3162), (b_2, 0.3162, 0.3660, 0), \\ (b_3, 0.4472, 0.0708, 0), \\ (b_4, 0.1005, 0.3660, 0.4472), (b_5, 0, 0, 0)\end{matrix}\right\}.$$

$$C_1^{\frac{3}{2}} = \left\{\begin{matrix}(b_1, 0, 0, 0.0316), (b_2, 0.0316, 0.5920, 0), \\ (b_3, 0.0894, 0.1223, 0), \\ (b_4, 0.0749, 0.5920, 0.0894), (b_5, 0, 0, 0)\end{matrix}\right\}.$$

$$C_1^2 = \left\{\begin{matrix}(b_1, 0, 0, 0.0100), (b_2, 0.0100, 0.6614, 0), \\ (b_3, 0.0400, 0.1411, 0), \\ (b_4, 0.0500, 0.6614, 0.0400), (b_5, 0, 0, 0)\end{matrix}\right\}.$$

$$C_1^{\frac{5}{2}} = \left\{\begin{matrix}(b_1, 0, 0, 0.0032), (b_2, 0.0032, 0.7161, 0), \\ (b_3, 0.0179, 0.1575, 0), \\ (b_4, 0.0314, 0.7161, 0.0179), (b_5, 0, 0, 0)\end{matrix}\right\}.$$

$$C_1^3 = \left\{\begin{matrix}(b_1, 0, 0, 0.0010), (b_2, 0.0010, 0.7603, 0), \\ (b_3, 0.0080, 0.1723, 0), \\ (b_4, 0.0190, 0.7603, 0.0080), (b_5, 0, 0, 0)\end{matrix}\right\}.$$

We compare our suggested SF entropy function with the existing SF entropy functions for estimating the ambiguity content of these SFSs. After computing the ambiguity, we will compute the accuracy of the SF entropy metrics by using the following formula due to Zhang et al. [89].

$$Accuracy = \frac{Count(Right\ order\ of\ \ E(C_t))}{Total\ number\ of\ C_t.} \times 100.$$

An entropy metric with higher precision accuracy is better and more reliable. Fig. 1 and Table 4 show the results.

**Table 4** Ambiguity content of SFS regarding Example 4.4

| | $E_{AG}$ | $E_{BAAAK}$ | $E_{LLMY}$ | $E_{GD}$ |
|---|---|---|---|---|
| $C_1^{\frac{1}{2}}$ | -0.3634 | 0.7910 | 0.7599 | 0.1140 |
| $C_1$ | -0.3242 | 0.8134 | 0.8282 | 0.0778 |
| $C_1^{\frac{3}{2}}$ | -0.3162 | 0.7783 | 0.7993 | 0.0753 |
| $C_1^2$ | -0.3166 | 0.7403 | 0.7632 | 0.0752 |
| $C_1^{\frac{5}{2}}$ | -0.3088 | 0.7109 | 0.7320 | 0.0729 |
| $C_1^3$ | -0.2099 | 0.6878 | 0.7064 | 0.0691 |
| Right or wrong | Wrong | Wrong | Wrong | Right |
| No. of wrongs | 4 | 1 | 1 | 0 |
| Accuracy (%) | 33.3 | 83.3 | 83.3 | 100 |

($\alpha = 2$ in $E_{GD}$).

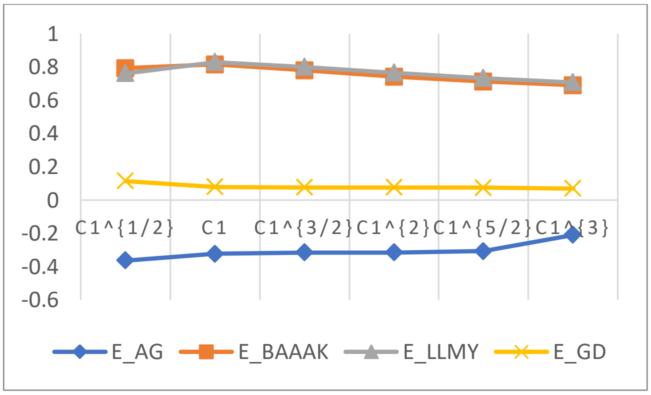


**Figure 1** Behavior of various SF entropy metrics concerning linguistic hedges

SF entropy metric $E$ should satisfy the following condition because of the linguistic hedges characterization.

$$E\left(C_1^{\frac{1}{2}}\right) > E(C_1) > E\left(C_1^{\frac{3}{2}}\right) > E(C_1^2) > E\left(C_1^{\frac{5}{2}}\right) > E(C_1^3).$$

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(2)

Table 4 provides the following observations:

(1) $E_{AG}\left(C_1^{\frac{1}{2}}\right) \ngtr E_{AG}(C_1) \ngtr E_{AG}\left(C_1^{\frac{3}{2}}\right) > E_{AG}(C_1^2) \ngtr E_{AG}\left(C_1^{\frac{5}{2}}\right) \ngtr E_{AG}(C_1^3)$.

(2) $E_{BAAAK}\left(C_1^{\frac{1}{2}}\right) \ngtr E_{BAAAK}(C_1) > E_{BAAAK}\left(C_1^{\frac{3}{2}}\right) > E_{BAAAK}(C_1^2) > E_{BAAAK}\left(C_1^{\frac{5}{2}}\right) > E_{BAAAK}(C_1^3)$.

(3) $E_{LLMY}\left(C_1^{\frac{1}{2}}\right) \ngtr E_{LLMY}(C_1) > E_{LLMY}\left(C_1^{\frac{3}{2}}\right) > E_{LLMY}(C_1^2) > E_{LLMY}\left(C_1^{\frac{5}{2}}\right) > E_{LLMY}(C_1^3)$.

(4) $E_{GD}\left(C_1^{\frac{1}{2}}\right) > E_{GD}(C_1) > E_{GD}\left(C_1^{\frac{3}{2}}\right) > E_{GD}(C_1^2) > E_{GD}\left(C_1^{\frac{5}{2}}\right) > E_{GD}(C_1^3)$.

Thus, we conclude that the suggested SF entropy function is more reasonable regarding linguistic variables because none of the existing SF entropy metrics exhibit the condition given in Eq. (2). Also, its accuracy is higher than the existing ones.

### C. ATTRIBUTE WEIGHT COMPUTATION

In multi-attribute decision-making (MADM) problems, the computation of attribute weights is a big issue. The attribute weights have a key role in selecting the best alternative. Here, we establish the utility of the suggested SF entropy metric in the computation of weights of attributes and also compare the outcome with the available SF entropy metrics. After computing the attribute weights, we will compute the accuracy of the SF entropy metrics by using the following formula due to Zhang et al. [89].

$$Accuracy = \frac{Number\ of\ attributes\ with\ distinct\ weights}{Total\ number\ of\ attributes} \times 100.$$

An entropy metric with higher precision accuracy is better and more reliable.

***Example 4.5*** Consider a MADM problem based on three alternatives $C_t, t = 1,2,3,$ and five attributes $D_s, s = 1,2,3,4,5$ in the form of an SF decision matrix as shown below.

$$E = \begin{bmatrix} (0.2,0.3,0.2) & (0.10,0.48,0.41) & (0.5,0.3,0.1) & (0.4,0,0.4) & (0.8\ 0,0.1) \\ (0.1,0.5,0.4) & (0.2,0.3,0.5) & (0.4,0.2,0.2) & (0.5,0.4,0.1) & (0.3,0.1,0.1) \\ (0.2,0.8,0) & (0.4,0.1,0.1) & (0.3,0.5,0) & (0.4,0.21,0.04) & (0.4,0.4,0.1) \end{bmatrix}$$

Now, we compute the weight of the attributes with the following entropy-based method.

$$w_s = W(D_s) = \frac{1-E(D_s)}{\sum_{s=1}^{5}(1-E_{GD}(D_s))}, s = 1,2,3,4,5. \quad (3)$$

Here $E$ is an SF entropy measure. The attribute weights computed by utilizing the available SF entropy metrics are presented in Table 5.

**Table 5** Values of attribute weights concerning Example 4.5

| | $E_{AG}$ | $E_{BAAAK}$ | $E_{LLMY}$ | $E_{GD}$ |
|---|---|---|---|---|
| $w_1$ | **0.2164** | 0.2455 | 0.2335 | 0.2018 |
| $w_2$ | 0.1781 | **0.1847** | 0.1889 | 0.1977 |
| $w_3$ | 0.1999 | **0.1847** | **0.1855** | 0.1971 |
| $w_4$ | 0.1892 | 0.1765 | **0.1855** | 0.1975 |
| $w_5$ | **0.2164** | 0.2087 | 0.2065 | 0.2059 |
| Accuracy (%) | 60 | 60 | 60 | 100 |

(Bold values denote irrational results. $\alpha = 2$ in $E_{GD}$).

From Table 5, we observe that the SF entropy metrics $E_{AG}$, $E_{BAAAK}$ and $E_{LLMY}$ give the same weight to two distinct attributes, which is not reasonable. However, the suggested entropy metric $E_{GD}$ gives proper attribute weights without unreasonable results and is highly accurate.

***Example 4.6*** Consider a MADM problem based on three alternatives $C_t, t = 1,2,3,$ and five attributes $D_s, s = 1,2,3,4,5$ in the form of an SF decision matrix as shown below.

$$E = \begin{bmatrix} (0.14,0.38,0.20) & (0.4,0.6,0) & (0.6,0.37,0.17) & (0.42,0.46,0.45) & (0.2,0.5,0.5) \\ (0.18,0.43,0.3) & (0.48,0.53,0.14) & (0.5,0.3,0) & (0.66,0.09,0.16) & (0,0.47,0.2) \\ (0.4,0.6,0.2) & (0.3\ 0.3,0.3) & (0.4,0.1,0.1) & (0.4,0.3,0.1) & (0.5,0.4,0.3) \end{bmatrix}$$

The attribute weights computed by utilizing the available SF entropy metrics and Eq. (3) are presented in Table 6.

**Table 6** Values of attribute weights concerning Example 4.6

| | $E_{AG}$ | $E_{BAAAK}$ | $E_{LLMY}$ | $E_{GD}$ |
|---|---|---|---|---|
| $w_1$ | 0.2018 | 0.1888 | **0.1794** | 0.2021 |
| $w_2$ | **0.1924** | 0.1858 | 0.2145 | 0.1981 |
| $w_3$ | 0.2124 | 0.1936 | **0.1794** | 0.2030 |
| $w_4$ | 0.2011 | **0.2159** | 0.2156 | 0.1991 |
| $w_5$ | **0.1924** | **0.2159** | 0.2111 | 0.1977 |
| Accuracy (%) | 60 | 60 | 60 | 100 |

(Bold values denote irrational results. $\alpha = 1.8$ in $E_{GD}$).

We see from Table 6 that the existing SF entropy metrics $E_{AG}$, $E_{BAAAK}$ , and $E_{LLMY}$ give the same weight to two distinct attributes, which is not reasonable. However, the suggested SF entropy metric $E_{GD}$ gives proper attribute weights without unreasonable results and is highly accurate.

*Example 4.7* Consider a MADM problem based on three alternatives $C_t, t = 1,2,3,$ and five attributes $D_s, s = 1,2,3,4,5$ in the form of an SF decision matrix as shown below.

$$E = \begin{bmatrix} (0.22,0.52,0.02) & (0.17,0.43,0.52) & (0.4,0.4,0.46) & (0.4,0.6,0) & (0.5,0.4,0.0) \\ (0.1,0.4,0.3) & (0.2,0.3,0.5) & (0.6,0.4,0) & (0.3,0.1,0.5) & (0.1,0.7,0.2) \\ (0.6,0.4,0.1) & (0.4,0.3,0.1) & (0.4,0.01,0.2) & (0.5,0.4,0.3) & (0.19,0.26,0.3) \end{bmatrix}$$

The attribute weights computed by utilizing the available SF entropy metrics and Eq. (3) are presented in Table 7.

**Table 7** Values of attribute weights concerning Example 4.7

| | $E_{AG}$ | $E_{BAAAK}$ | $E_{LLMY}$ | $E_{GD}$ |
|---|---|---|---|---|

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| | | | | |
|---|---|---|---|---|
| $w_1$ | **0.2174** | **0.2092** | **01897** | 0.2010 |
| $w_2$ | 0.1774 | 0.1773 | **0.1897** | 0.1990 |
| $w_3$ | 0.1962 | 0.1970 | 0.2028 | 0.2002 |
| $w_4$ | 0.1916 | **0.2092** | 0.2233 | 0.1967 |
| $w_5$ | **0.2174** | 0.2073 | 0.1946 | 0.2031 |
| Accuracy (%) | 60 | 60 | 60 | 100 |

(Bold values denote irrational results. $\alpha = 2$ in $E_{GD}$).

From Table 7, we observe that the existing SF entropy metrics $E_{AG}$, $E_{BAAAK}$ , and $E_{LLMY}$ give the same weight to two distinct attributes, which is not reasonable. However, the suggested SF entropy metric $E_{GD}$ gives proper attribute weights without unreasonable results and is highly accurate. Thus, from Examples 4.5-4.7, we conclude that the suggested SF entropy metric is more robust and reasonable than the existing metrics regarding attribute weight computation.

### D. MONOTONIC BEHAVIOR

Here we study the monotonic behavior of the suggested entropy measure for various values of the parameter α. The ambiguity of a SFS is typically impacted by the effect of one or more external factors. Fuzzy entropy measures that exhibit a monotonic character with regard to parameters are useful for practical research including data mining or expert-based decision-making.

We consider the three different SFSs $\boldsymbol{C_1}, \boldsymbol{C_2}$, and $\boldsymbol{C_3}$ in $\boldsymbol{B} = \{\boldsymbol{b_1}, \boldsymbol{b_2}, \boldsymbol{b_3}\}$ as given below.

$$\boldsymbol{C_1} = \left\{ \begin{matrix} (\boldsymbol{b_1}, \mathbf{0.2}, \mathbf{0.3}, \mathbf{0.2}), (\boldsymbol{b_2}, 0.4, 0.5, 0.1), \\ (\boldsymbol{b_3}, \mathbf{0}, \mathbf{0.8}, \mathbf{0.2}) \end{matrix} \right\},$$

$$C_2 = \left\{ \begin{matrix} (b_1, 0.41, 0.48, 0.10), (b_2, 0.5, 0.3, 0.2), \\ (b_3, 0.1, 0.1, 0.4) \end{matrix} \right\},$$

$$C_3 = \{(b_1, 0.1, 0.3, 0.5), (b_2, 0.2, 0.2, 0.4), (b_3, 0, 0.5, 0.3)\}.$$

The ambiguity of these three SFSs $C_1, C_2$, and $C_3$ for various values of the parameter $\alpha$ is shown in Fig. 2.

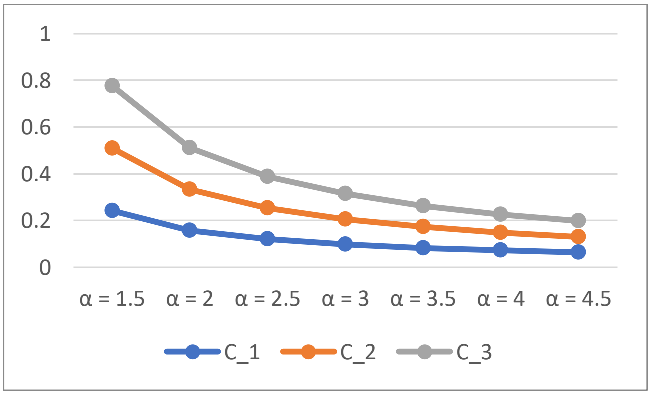


**FIGURE 2** Monotonic behavior of the proposed entropy metric.

From Fig. 2, we observe that the suggested entropy metric is monotonically decreasing with respect to the parameter $\alpha$ and is thus suitable for the situations where the external factors play a negative role.

Next, we will introduce a new decision-making method, i.e., COPRAS, in the SF environment.

## V. SPHERICAL FUZZY DECISION-MAKING BASED ON COPRAS

Here, we offer the COPRAS (Complex Proportional Assessment) method for SFSs, and this is based on the novel SF entropy function. Consider $Z = \{C_1, C_2, \dots, C_p\}$ to be the set of alternatives and $Y = \{D_1, D_2, \dots, D_q\}$ to be the set of attributes. We have to find out the most suitable alternative among all the alternatives $C_t, t = 1, 2, \dots, p$ by looking into the set of attributes. The alternative's information in accordance with attributes is given in the shape of SFNs in the decision matrix $E = [(\mu_{ts}, \nu_{ts}, \eta_{ts})]_{p\times q}$. The main steps of this method are as:

Step 1: Calculate the entropy of each attribute i.e., $E_{GD}(D_s), s = 1, 2, \dots, q$.

Step 2: Calculate the weight of each attribute $D_s, s = 1, 2, \dots, q$ by the following formula

$$w_s = W(D_s) = \frac{1 - E_{GD}(D_s)}{\sum_{s=1}^{q}(1 - E_{GD}(D_s))}, s = 1, 2, \dots, q.$$

Step 3: Formulate the weighted decision matrix $F = [(\mu'_{ts}, \nu'_{ts}, \eta'_{ts})]_{p\times q}$, where $(\mu'_{ts}, \nu'_{ts}, \eta'_{ts}) = (w_s\mu_{ts}, w_s\nu_{ts}, w_s\eta_{ts})$.

Step 4: Compute the score function $U((\mu'_{ts}, \nu'_{ts}, \eta'_{ts}))$ for all $t = 1, 2, \dots, p$ and $s = 1, 2, \dots, q$ by using the following formula given by [90]

$$U((\mu'_{ts}, \nu'_{ts}, \eta'_{ts})) = \frac{1}{3}(2 + \mu'_{ts} - \eta'_{ts} - \nu'_{ts}).$$

Step 5: Calculate $V_t = \frac{1}{|BA|}\sum_{s\in BA} U((\mu'_{ts}, \nu'_{ts}, \eta'_{ts}))$ and $W_t = \frac{1}{|CA|}\sum_{s\in BA} U((\mu'_{ts}, \nu'_{ts}, \eta'_{ts}))$, where $BA$ denotes the set of benefit attributes and $CA$ indicates the set of cost attributes, for all $t = 1, 2, \dots, p$.

Step 6: Calculate each alternative's relative weight $C_t, t = 1, 2, \dots, p$ by using the following formula

$$T_t = V_t + \frac{\sum_{t=1}^{p} e^{V_t}}{e^{V_t}\sum_{t=1}^{p}\frac{1}{e^{V_t}}}.$$

Step 7: Calculate the priority order $R_t, t = 1, 2, \dots, p$ by the following expression:

$$R_t = \frac{T_t}{\max T_t} \times 100.$$

Step 8: Rank the alternatives in descending order of the values of priority order, and the alternative with the highest priority order value is the most suitable alternative.

***Example 5.1*** We take the example from [36]. A manufacturer of tractor parts wants to update their production process. Using six evaluation criteria, $(D_1)$ Expandability, $(D_2)$ Adaptability, $(D_3)$ Competitiveness, $(D_4)$ Ease of use, $(D_5)$ Quality or results, $(D_6)$ Annual

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depreciation and maintenance data, they have assessed four different FMS (flexible manufacturing systems) $C_t, t = 1, 2, 3, 4$. The information about the four FMS is expressed in SFNs in the decision matrix $E$ below.

$$E = \begin{bmatrix} (0.59, 0.41, 0.41) & (0.59, 0.41, 0.41) & (0.95, 0.04, 0.04) & (0.59, 0.41, 0.41) & (0.95, 0.04, 0.04) & (0.66, 0.33, 0.33) \\ (0.59, 0.41, 0.41) & (0.59, 0.41, 0.41) & (0.59, 0.41, 0.41) & (0.50, 0.50, 0.50) & (0.59, 0.41, 0.41) & (0.13, 0.86, 0.13) \\ (0.50, 0.50, 0.50) & (0.95, 0.04, 0.04) & (0.59, 0.41, 0.41) & (0.50, 0.50, 0.50) & (0.50, 0.50, 0.50) & (0.41, 0.59, 0.41) \\ (0.50, 0.50, 0.50) & (0.59, 0.41, 0.41) & (0.59, 0.41, 0.41) & (0.50, 0.50, 0.50) & (0.33, 0.66, 0.33) & (0.25, 0.74, 0.25) \end{bmatrix}$$

Using the offered entropy function $E_{GD}(\alpha = 2)$, we compute the entropy of each attribute and obtain the following:

$$E_{GD}(D_1) = 0.1681, E_{GD}(D_2) = 0.1598,$$
$$E_{GD}(D_3) = 0.2024, E_{GD}(D_4) = 0.1628,$$
$$E_{GD}(D_5) = 0.1628, E_{GD}(D_6) = 0.2026.$$

Next, we form the weighted decision matrix by using (Step 3) as shown below

$$F = \begin{bmatrix} (0.04, 0.12, 0.04) & (0.05, 0.11, 0.05) & (0.08, 0.08, 0.08) & (0.10, 0.06, 0.06) & (0.10, 0.06, 0.06) & (0.08, 0.08, 0.08) \\ (0.06, 0.09, 0.06) & (0.08, 0.08, 0.08) & (0.08, 0.08, 0.08) & (0.10, 0.06, 0.06) & (0.16, 0.00, 0.00) & (0.08, 0.08, 0.08) \\ (0.02, 0.14, 0.02) & (0.10, 0.06, 0.06) & (0.08, 0.08, 0.08) & (0.10, 0.06, 0.06) & (0.10, 0.06, 0.06) & (0.09, 0.06, 0.06) \\ (0.11, 0.05, 0.05) & (0.16, 0.00, 0.00) & (0.09, 0.06, 0.06) & (0.16, 0.00, 0.00) & (0.10, 0.06, 0.06) & (0.09, 0.06, 0.06) \end{bmatrix}$$

Now, we compute the scores of all the SFNs given in the weighted decision matrix $F$ by using (Step 4) and these values are presented in Table 8.

**Table 8** Scores of the SFNs

| Alternatives | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $D_6$ |
|---|---|---|---|---|---|---|
| $C_1$ | 0.6249 | 0.6290 | 0.6398 | 0.6537 | 0.6537 | 0.6398 |
| $C_2$ | 0.6336 | 0.6383 | 0.6398 | 0.6537 | 0.7155 | 0.6398 |
| $C_3$ | 0.6181 | 0.6536 | 0.6398 | 0.6537 | 0.6537 | 0.6543 |
| $C_4$ | 0.6664 | 0.7157 | 0.6543 | 0.7155 | 0.6537 | 0.6543 |

Next, we compute $V_t, W_t$, relative weight $T_t$ and $R_t$ for all $t = 1, 2, 3, 4$ (Table 9). Finally, the ranking of FMSs in decreasing order of the values of $R_t, t = 1, 2, 3, 4$ is presented in Table 9.

**Table 9** Ranking of alternatives

| Alternatives | $V_t$ | $W_t$ | $T_t$ | $R_t$ | Ranking |
|---|---|---|---|---|---|
| $C_1$ | 0.6432 | 0.6249 | 3.8566 | 99.2397 | 2 |
| $C_2$ | 0.6574 | 0.6336 | 3.8430 | 98.8900 | 3 |
| $C_3$ | 0.6510 | 0.6181 | 3.8862 | 100 | 1 |
| $C_4$ | 0.6787 | 0.6664 | 3.7614 | 96.7904 | 4 |

From Table 9, we arrive at the result that the FMS $C_3$ is the most desirable.

We compare the outcomes of the offered decision-making technique with several available techniques, as shown in Table 10.

**Table 10** Ranking results by various available methods

| Method | Ranking results |
|---|---|
| Spherical fuzzy AHP-TOPSIS [36] | $C_3 > C_4 > C_1 > C_2$ |
| Interval-valued MCDM method [91] | $C_3 > C_4 > C_1 > C_2$ |
| MACBETH [92] | $C_3 > C_4 > C_1 > C_2$ |
| Preference selection index method [93] | $C_3 > C_4 > C_2 > C_1$ |
| Combinatorial mathematics-based decision-making method [94] | $C_3 > C_4 > C_2 > C_1$ |
| COPRAS method (This paper) | $C_3 > C_1 > C_2 > C_4$ |

We observe that the best alternative is $C_3$ as shown by all the methods including our suggested one. This establishes the validity and effectiveness of the suggested COPRAS method.

### E. ANALYSIS OF RANKING RESULTS

A critical consideration revolves around comparing the accuracy of two rankings. A straightforward technique involves examining their consistency or inconsistency, albeit this method proves inadequate, primarily applicable to only two or three basic rankings [95]. A more prevalent approach entails employing coefficients of monotonic dependence between two variables. In this method, the rankings derived for a set of alternatives under consideration serve as our variables. Various coefficients are available in the literature to measure the similarity of the two rankings. Some of them are given below:

Costa and Soares [96]

$$r_w = 1 - \frac{6\sum_{t=1}^{p}(x_t - y_t)^2\left((N - x_t + 1) + (N - y_t + 1)\right)}{N(N^3 + N^2 - N - 1)}.$$

Salabun and Urbaniak [97]

$$WS = 1 - \sum_{t=1}^{p}\left(2^{-x_t}\frac{|x_t - y_t|}{\max\{|1 - x_t|, |1 - y_t|\}}\right).$$

Kizielewicz et al. [98]

$$r_s = 1 - \frac{6\sum_{t=1}^{p}(x_t - y_t)^2}{N(N^2 - 1)}.$$

Blest [99]

$$\vartheta = 1 - \frac{12\sum_{t=1}^{p}(N + 1 - x_t)^2 y_t - N(N + 1)^2(N + 2)}{N(N + 1)^2(N - 1)}.$$

Here $x_t$ denotes the mean position in reference ranking, $y_t$ denotes the mean position in other ranking, and $N$ denotes the ranking length.

**Table 11** Correlations with reference ranking of Preference selection index method

| Coefficient | Spherical fuzzy AHP-TOPSIS [36] | Interval-valued MCDM method [91] | MACBETH [92] | COPRAS method (This paper) |
|---|---|---|---|---|
| $r_w$ | 0.7200 | 0.7200 | 0.7200 | 0.8000 |
| $WS$ | 0.7083 | 0.7083 | 0.7083 | 0.4375 |
| $r_s$ | 0.8000 | 0.8000 | 0.8000 | 0.8000 |
| $\vartheta$ | 0.7200 | 0.7200 | 0.7200 | 0.8000 |

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Table 11 displays the computed correlation coefficients for the Spherical fuzzy AHP-TOPSIS [36], Interval-valued MCDM method [91], MACBETH [92], and COPRAS method, along with the reference ranking for the Preference selection index method [93]. The suggested COPRAS method has high similarity values corresponding to the three coefficients $r_w, r_s, \vartheta$ and a lower value corresponding to the coefficient $WS$, whereas the existing three decision-making methods have lower similarity values corresponding to the three coefficients $r_w, r_s, \vartheta$ and a high value corresponding to the coefficient $WS$.

### *F. SENSITIVITY ANALYSIS*

Here we discuss the sensitivity analysis of the parameter $\boldsymbol{\alpha}$ on attribute weights and ranking of alternatives. For different values of $\boldsymbol{\alpha}$, we get different attribute weights (Example 5.1) as shown in Fig. 3.

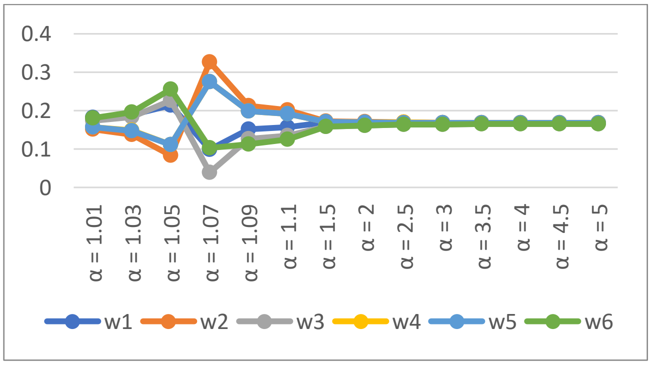


**FIGURE 3** Effect of $\boldsymbol{\alpha}$ on attribute weights

From Fig. 2, we observe that for smaller values of $\alpha$, the weights are more distinguishable and for large values of $\alpha$, the weights are less distinguishable.

For various values of the parameter $\alpha$, we obtain the ranking results (Example 5.1) as shown in Fig. 4.

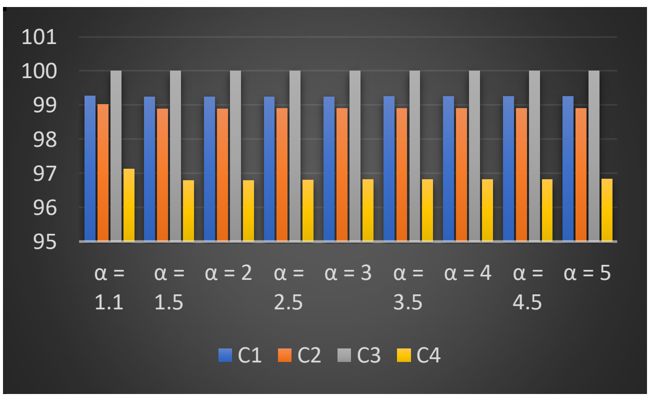


**FIGURE 4** Effect of $\boldsymbol{\alpha}$ on ranking of alternatives

From Fig. 3, we conclude that the ranking of alternatives is invariant with respect to the parameter $\alpha$.

## VI. CONCLUSION

Entropy measure in fuzzy/non-standard fuzzy environment is handy in the computation of ambiguity and mainly in attribute weight computation in a multi-attribute decision-making problem. The main contributions and results of this study are given below.

- A new set of fundamental conditions necessary for a function to be an SF entropy function has been given along with a novel SF entropy function. The suggested SF entropy metric has satisfied all the necessary axiomatic requirements. Furthermore, the valuation property of the proposed metric has also been discussed.
- The limitations of all available SF entropy functions have been highlighted, particularly in ambiguity computation, attribute weight computation, and linguistic hedges. These limitations have all been addressed by the suggested entropy function without any counter-intuitive results.
- A multi-attribute decision-making technique known as COPRAS in the SF area has been proposed with the suggested entropy measure. The newly created SF entropy metric has computed the weights of attributes in the proposed COPRAS method. The ranking of the alternatives by the COPRAS method has been compared with several available methods, and the results are satisfactory.

The main advantages of this study are given below.

- The suggested entropy metric is able to handle all those SFSs in which the membership and non-membership grades are equal.
- The proposed entropy metric is more reasonable and effective in computing the ambiguity of different SFSs and also in the determination of attribute weights in MCDM problems.
- The COPRAS method, proposed in this paper is more robust than some of the existing decision-making methods due to the high similarity values corresponding to the three coefficients $r_w, r_s, \vartheta$.
- Due to the presence of the parameter $\alpha$, the proposed entropy metric can be applied to those complex uncertain situations, where the existing ones lead to counterintuitive results.

One limitation of this study is that artificially generated data was used to calculate the numerical examples. Another limitation is that in some decision-making problems, the higher values of the parameter $\alpha$ may not give promising attribute weights.

The impact of this study can be increased by analyzing the suggested measures using actual data. Furthermore, because of the current study, applying the suggested entropy measure to obtain a class of similarity and dissimilarity measures between spherical fuzzy sets may be possible. Furthermore, future research may examine further theoretical relationships between different spherical fuzzy information measures. Also, the suggested method has promising future studies on different fuzzy sets.

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