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Multi-Harmonic Currents Control Strategy for Five-Phase Permanent Magnet Machine with non-sinusoidal back-EMF

JIANYA ZHANG[1], S. J. He[2], KAI WANG[1], (Senior Member, IEEE)
[1]College of Automation, Nanjing University of Aeronautics and Astronautics, 29 Jiangjun Road, Nanjing, 211106, China
[2]Ningbo Haibo Group Co., Ltd, 1299 Songshan Road, Ningbo, 315800, China

Corresponding author: Kai Wang (e-mail: k_wang@ieee.org).

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ABSTRACT This paper describes an optimal torque per peak current control method for a five-phase permanent magnet (PM) machine considering both 3rd and 5th harmonic currents. These optimal ratios to the fundamental component are analytically derived to maximize the output torque. It is found that except for the 3rd harmonic current contributing to the output torque, the 5th harmonic current can also produce the additional positive torque. However, the 5th harmonic is zero sequence component for the five-phase machines, which does not exist in the phase windings. Hence, the neutral point is required to connect the middle point of the DC link capacitors for constructing flowing path. The conventional vector space decomposition (VSD) control is extended to zero sequence sub-plane, which can quantitatively control 5th harmonic current. For a prototype five-phase PM machine, the average torque can be increased by 21.4% with 3rd harmonic current injection. Meanwhile, 10.7% additional positive torque is achieved together with 3rd and 5th harmonic injection. The torque ripple remains similar to that without harmonics injection. Finally, the experiments are given to demonstrate the theoretical analysis.

INDEX TERMS Multi-phase, harmonics injection, output torque, zero sequence current

I. INTRODUCTION

Multiphase electric machines such as five-phase machines have numerous advantages over traditional three-phase machines, such as multi-degrees of freedom, low torque ripple, low power per phase and high reliability [1]-[4]. These outstanding merits make some unique characteristics for multiphase machines. One of them is to inject low-order harmonic components into phase currents to obtain high torque density [5], [6].

The combined sinusoidal plus 3rd harmonic currents was initially proposed to increase the output torque [7]. The effect of high order harmonics on the torque improvement was investigated. It was found that when the harmonic order over 7th, the torque improvement can be neglected [8]. Hence, most literature focuses on the 3rd harmonic injection to increase the torque capability [9]-[12]. In order to realize fundamental and harmonic currents decoupling control, the vector space decomposition (VSD) strategy is widely used, which introduces two orthogonal vector fundamental and 3rd harmonic sub-planes [13]-[15]. Hence, the fundamental and 3rd harmonic currents are concerted to DC component, achieving static error free tracking control [16]-[18]. Actually, the torque improvement can be divided into the enhancement of the fundamental current caused by 3rd harmonic injection and the torque generated by the 3rd harmonic components in back EMF and phase current [19]. Hence, maximum fundamental amplitude does not represent the output torque is optimal [20]. The key of harmonic current injection is to maximize the fundamental current for torque production with the same peak current [21]. Genetic algorithm is employed to calculate the optimal injection ratios of harmonic currents [22]. The relationship between the output torque and the 3rd harmonic current is established, which achieves the optimal value for maximizing the output torque [23].

As pointed out in [24], the 5th harmonic together with the 3rd harmonic has the possibility to increase the average torque of the five-phase PM machine. Although the 5th harmonic has little contribution to positive torque, it can increase the peaks of fundamental and 3rd harmonic currents to improve the total average torque. However, for five-phase PM machines, the 5th harmonic current belongs to zero sequence component. It would be eliminated in five-phase winding set. Hence injecting 5th harmonic to phase current is rather complicated. Beyond that, the current regulations of zero sequence sub-plane is not involved in the conventional VSD control strategy [25], [26].

Therefore, this paper is intended to control the 5th harmonic current effectively achieving the maximum output torque. The harmonic current ratios for optimal torque per peak current is analyzed in detail. The conventional dual-plane vector control is extended to zero sequence sub-plane, where a resonant controller is employed to regulate the 5th harmonic current. Compared to 3rd harmonic current injection, the proposed method can further improve the average torque and the torque ripple remains similar to that of the one without harmonic currents injection.

II. MATHEMATICAL MODEL OF FIVE-PHASE PM MACHINES WITH HARMONIC BACK EMFS

The voltage and flux linkage equations in fundamental and third sub-planes can be expressed as:

$$\begin{cases} u_{d1} = R_s i_{d1} + \dot{\psi}_{d1} - \omega_e \psi_{q1} \\ u_{q1} = R_s i_{q1} + \dot{\psi}_{q1} + \omega_e \psi_{d1} \\ u_{d3} = R_s i_{d3} + \dot{\psi}_{d3} - 3\omega_e \psi_{q3} \\ u_{q3} = R_s i_{q3} + \dot{\psi}_{q3} + 3\omega \psi_{d3} \end{cases} \tag{1}$$

$$\begin{cases} \psi_{d1} = \left(L_1 + 5L_{m1}/2\right) i_{d1} + \psi_{m1} \\ \psi_{q1} = \left(L_1 + 5L_{m1}/2\right) i_{q1} \\ \psi_{d3} = \left(L_1 + 5L_{m3}/2\right) i_{d3} + \psi_{m3} \\ \psi_{q3} = \left(L_1 + 5L_{m3}/2\right) i_{q3} \end{cases} \tag{2}$$

where u is the voltage; i is the current; R_s is the phase resistance; ψ is the flux; L_1 is the stator leakage inductance; L_{m1} is the fundamental inductance; L_{m3} is the third harmonic inductance; ω_e is the fundamental electrical angular frequency.

The average electromagnetic torque is obtained as:

$$T_e = \frac{5}{2} p_n \left(\psi_{d1} i_{q1} - \psi_{q1} i_{d1} + 3\psi_{d3} i_{q3} - 3\psi_{q3} i_{d3} \right) \tag{3}$$

$$\begin{cases} i_{d1} = 0 \\ i_{d3} = 0 \end{cases} \tag{4}$$

where p_n is the pole pairs.

It is obvious that the third harmonic current generates extra constant torque improving the average torque, which is written as:

$$T_{e3} = \frac{15}{2} p_n \left(\psi_{d3} i_{q3} - \psi_{q3} i_{d3} \right) \tag{5}$$

Besides, the 3rd harmonic current changes the profile of phase current. The fundamental peak current can be raised for a certain peak phase current, which means the torque generated by the fundamental back EMF and fundamental current is increased. Obviously, the conventional maximum torque per peak current (MTPPC) strategy is aimed at injecting 3rd harmonic to obtain as high as possible fundamental peak current. The optimal third harmonic ration is 1/6, the average torque can be increased by 15.4%, as shown in Fig. 1 [9].

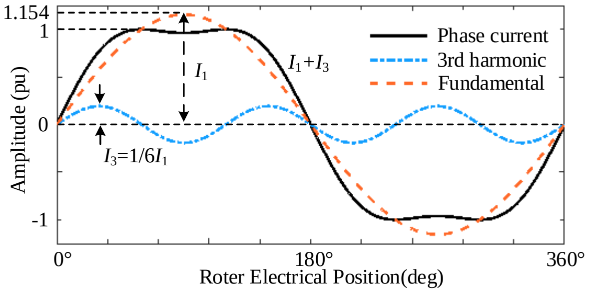

Fig. 1. Conventional optimal current with harmonic current injection.

For five-phase machines, the 7th harmonic current interacting with the 3rd harmonic back EMF induces the 12th torque ripple and the higher order harmonics have little effect. Besides the fundamental and 3rd harmonic components, the fifth harmonic should be also considered. Injecting 5th harmonic current can increase the both amplitude of fundamental and 3rd harmonic currents with a constant phase peak current, thereby further improving the average torque. Assuming the combined phase current with 3rd and 5th harmonics injection is expressed as:

$$\begin{cases} I\sin(\theta) = I_1 \sin(\theta) + I_3 \sin(3\theta) + I_5 \sin(5\theta) \\ I_3 = k_3 I_1 \\ I_5 = k_5 I_1 \end{cases} \tag{6}$$

where I is the phase peak current; I_1 means the fundamental peak current; I_3 is the 3rd harmonic peak current; I_5 symbolizes the 5th harmonic peak current; θ is the rotor position; k_3 and k_5 are the harmonic injection coefficients.

The gain of fundamental peak current can be expressed as:

$$\begin{cases} a = \dfrac{1}{\max\left[\sin(\theta) + k_3 \sin(3\theta) + k_5 \sin(5\theta)\right]} \\ k_3 \in [0,1], k_5 \in [0,1] \end{cases} \tag{7}$$

Thus, we have

$$I_1 = aI \tag{8}$$

The voltage and flux linkage equations for 5th harmonic can be written as:

$$u_0 = R_s i_0 + L_0 \frac{di_0}{dt} - \omega_e \psi_{m5} \sin(5\theta) \tag{9}$$

$$L_0 = L_1 + 5L_{m5}/2 \tag{10}$$

where L_{m5} is the 5th harmonic inductance; $i_0 = I_5 \sin(5\theta)$.

The electromagnetic torque generated by 5th harmonic component is derived as:

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$$T_{e5} = p_n I_{s5}^T \frac{\partial \varphi_{m5}}{\partial \theta} \tag{11}$$

where $I_{s5} = I_5 \left[\sin(5\theta) \quad \sin(5\theta) \quad \sin(5\theta) \quad \sin(5\theta) \quad \sin(5\theta) \right]$, $\varphi_{m5} = \psi_{m5} \left[-\cos(5\theta) \quad -\cos(5\theta) \quad -\cos(5\theta) \quad -\cos(5\theta) \quad -\cos(5\theta) \right]$.

Hence, the electromagnetic torque generated by 5th harmonic component can be rewritten as:

$$T_{e5} = \frac{25}{2} p_n \psi_{m5} I_5 \left[1 - \cos(10\theta) \right] \tag{12}$$

It is can be seen that the 5th harmonic current interacting with the 5th harmonic back EMF would induce 10th torque ripple.

The total average torque can be rewritten as:

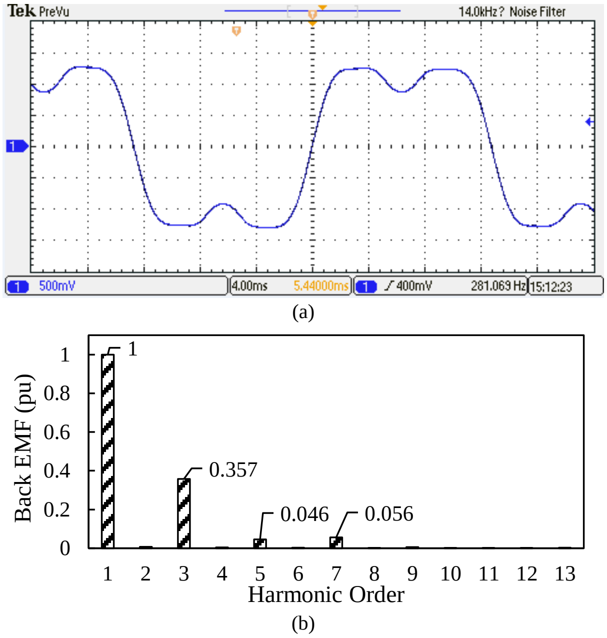
$$T_e = \frac{5}{2} p_n \left\{ \psi_{m1} i_{q1} + 3\psi_{m3} i_{q3} + 5\psi_{m5} I_5 \left[1 - \cos(10\theta) \right] \right\} \tag{13}$$

Substituting (6) and (8) into (13), we have

$$T_e = \frac{5}{2} p_n \left\{ \begin{array}{l} \psi_{m1} aI \\ +3\psi_{m3} k_3 aI \\ +5\psi_{m5} k_5 aI \left[1 - \cos(10\theta) \right] \end{array} \right\} \tag{14}$$

For the prototype given in TABLE II. , the measured phase back EMF with respect to time is shown in Fig. 2 (a). The corresponding harmonic analysis is shown in Fig. 2 (b). It is obvious that the 3rd harmonic is 35.7% of the fundamental one, the 5th harmonic accounts for 4.6%. Therefore, the relation between harmonic flux linkages can be obtained as:

$$\psi_{m1} : 3\psi_{m3} : 5\psi_{m5} = 1 : 0.357 : 0.046 \tag{15}$$



(a)

(b)

Fig. 2. Experimental result showing phase back EMF of machine operating at 600r/min. (a) Measured phase back EMF: e_A (0.5V/div), Horizontal: Time (4 ms/div), (b) Harmonics analysis of phase back EMF.

From (14) and (15)

$$T_e = \frac{5}{2} p_n \psi_{m1} aI \left\{ \begin{array}{l} 1 \\ +0.357 k_3 \\ +0.046 k_5 \left[1 - \cos(10\theta) \right] \end{array} \right\} \tag{16}$$

According to (6) and (16), with the aid of Mathcad and Matlab Optimization Toolbox in Appendix A, the optimal injection coefficients for the maximum torque per peak current are derived

$$\begin{cases} k_3 = 0.251 \\ k_5 = 0.082 \\ a = 1.202 \end{cases} \tag{17}$$

The optimal profile of the phase current is obtained as shown in Fig. 3. The fundamental peak current is increased by 20.2%

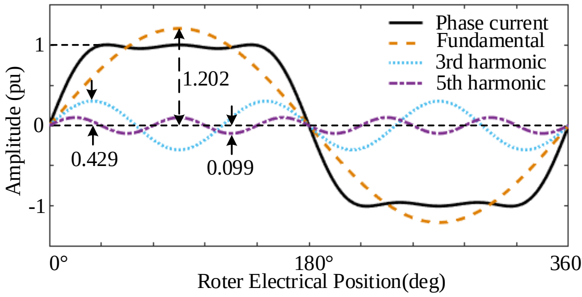


Fig. 3. Optimal phase current with 3rd and 5th harmonics injection.

From (17), the electrical torque (16) can be rewritten as:

$$\begin{cases} T_e = T_{e1} + T_{e3} + T_{e5} \\ T_{e1} = \frac{5}{2} p_n \psi_{m1} I * 1.202 \\ T_{e3} = \frac{5}{2} p_n \psi_{m1} I * 0.107 \\ T_{e5} = \frac{5}{2} p_n \psi_{m1} I * 0.005 \left[1 - \cos(10\theta) \right] \end{cases} \tag{18}$$

It is concluded that the electrical torque generated by 5th harmonic component is only 0.4% with respect to the fundamental one, which can be ignored. Thus, the torque ripple is almost the same to that without 5th harmonic current injection. The action of the 5th harmonic current improves the amplitude of fundamental and 3rd harmonic currents.

From (18), the average torque can be simplified as:

$$T_e = 1.309 * \frac{5}{2} p_n \psi_{m1} I \tag{19}$$

Compared with sinusoidal drive, injecting the 3rd and 5th harmonics into phase current increases the output torque by 31%.

The root mean square (RMS) of phase current can be calculated as:

$$I_{RMS} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \left(I_{phs}^2 \right) dt} \tag{20}$$

Substituting (6) and (17) into (20), the RMS current can be given

$$I_{RMS} = 0.879I \tag{21}$$

The RMS current is increased about 24%, compared with $0.707I$, which results in extra copper losses. The contrast performance indexes are given in TABLE I. For the other prototypes, the optimal 3rd and 5th harmonic injection coefficients can be obtained as Fig. 4.

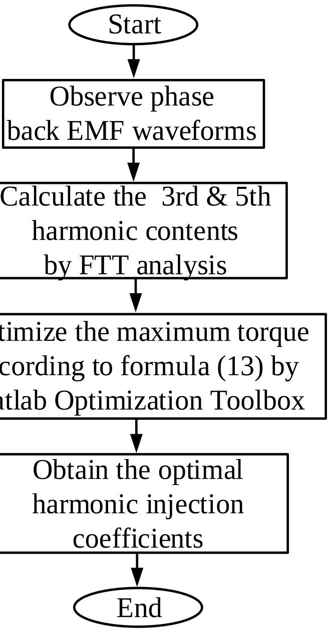


Fig. 4. Procedure of evaluating harmonic injection coefficients for optimal toque per peak current.

As a conclusion, injecting 3rd and 5th harmonic currents is an effective approach to improve the average torque within power devices for a short time. However, the harmonics increase copper loss leading to efficiency decline, which is the inherent shortcoming of the maximum torque per peak current control.

TABLE I. PERFORMANCE CONTRAST INDEXES IN DIFFERENT CONTROL OBJECTIVES

Performance Indexes (pu)	Sine	Sin+3rd	Sin+3rd+5th
Phase peak current	I	I	I
Fundamental	I	$1.154I$	$1.202I$
3rd harmonic	0	$0.192I$	$0.429I$
5th harmonic	0	0	$0.099I$
Average torque	T_e	$1.22T_e$	$1.31T_e$
RMS	$0.707I$	$0.827I$	$0.879I$
Copper loss	$2.5I^2R_s$	$3.42I^2R_s$	$3.86I^2R_s$
Increased torque	0	22%	31%

III. CONTROL STRATEGY OF HARMONIC CURRENTS IN FIVE-PHASE PM MACHINE

Based on VSD control, the fundamental current is mapped to $\alpha_1\beta_1$ sub-plane; 3rd harmonic current is mapped to $\alpha_3\beta_3$ sub-plane. The associated transformation matrices are briefly described in Appendix B. Many studies have verified the effectiveness of the fundamental and 3rd harmonic currents regulation [25], [26].

However, for five-phase machines, the 5th harmonic belongs to zero sequence. Generally, it would be eliminated in its winding set. For providing the flowing path for 5th harmonic current, the neutral point need to be connected to the middle point of the DC link capacitors, as shown in Fig. 5.

From Fig. 5, the neutral current can be expressed as:

$$i_N = i_A + i_B + i_C + i_D + i_E = 5I_5\sin(5\theta) \tag{22}$$

According to transformation matrix $T_{\alpha\beta}$, the zero sequence i_0 can be derived as:

$$i_0 = \frac{2}{5}\left(\frac{1}{2}i_A + \frac{1}{2}i_B + \frac{1}{2}i_C + \frac{1}{2}i_D + \frac{1}{2}i_E\right) = I_5\sin(5\theta) \tag{23}$$

From (6), (22) and (23)

$$i_0 = \frac{1}{5}i_N \tag{24}$$

$$i_0 = k_5 i_{q1}\sin(5\theta) \tag{25}$$

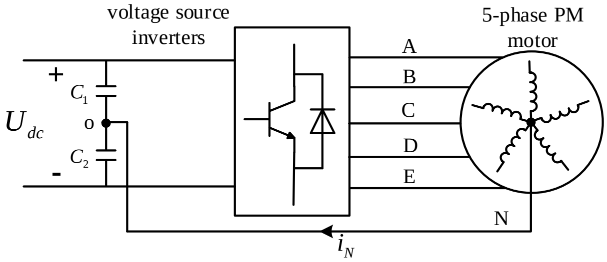


Fig. 5. Drive system for five-phase PM machine with 3rd and 5th harmonics injection.

Therefore, an extra current sensor is need to measure the feedback of the 5th harmonic current i_0. It is obvious that the conventional PI controller cannot regulate the 5th harmonic current without static error. To track the 5th harmonic current effectively, a resonant controller is adopted.

The proposed harmonic currents control block diagram is shown in Fig. 6. The i_{q3}^* is converted by i_{q1}^* directly according to (17), which is regulated by conventional proportional and integral (PI) controllers in dq3-frame. The i_0^* is calculated by (25). Different from the conventional VSD control, the component in zero sequence sub-plane is included, which is regulated by a proportional-resonant controller. It is noted that the stability of the proportional-resonant controller is determined by the cut-off frequency of the low-pass filter and the integral gain [27]. Considering the control performance, the cut-off frequency is set as 1/150 times of the resonant frequency, which is about 7 rad/s and its integral gain is consistent with the integral gain in the PI controller in $\alpha_3\beta_3$ sub-plane.

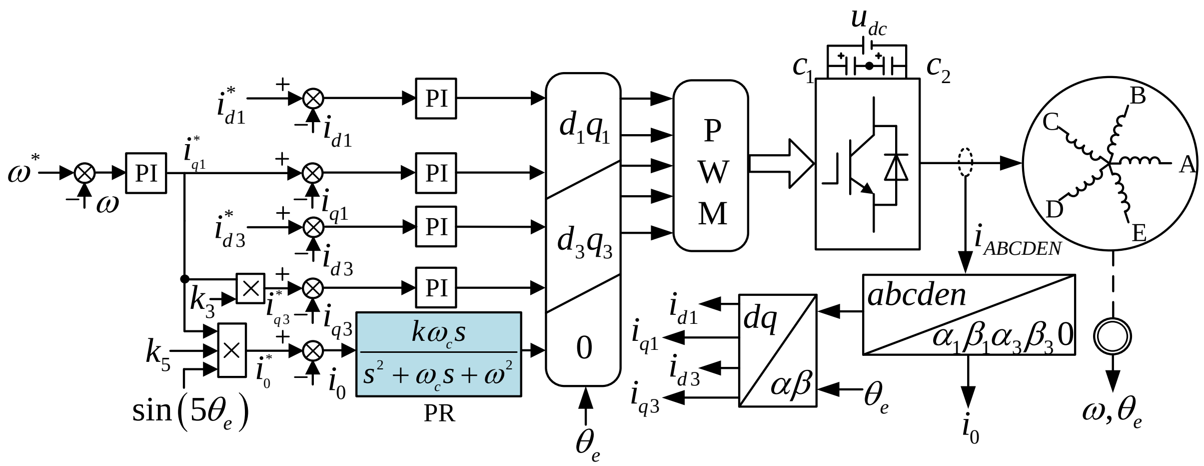


Fig. 6. VSD control with 3rd and 5th harmonic currents injection.

IV. EXPERIMENT

A. EXPERIMENTAL SETUP

The experimental platform is built around dSPACE-1007. A five-phase PM machine is coupled with an induction generator, which is as the load. A torque sensor is installed to measure the real-time output torque. The driver consists of two three-phase voltage source inverters with a common DC link, the switching frequency is of 10 kHz. The hardware setup is shown in Fig. 7. The parameters of the prototype machine is listed in TABLE II.

B. EXPERIMENTAL VALIDATION

In order to validate the proposed control method, the prototype runs in constant phase peak current mode with the speed at 500 r/min, the reference current is 2.5A. Three conditions, I) without harmonic current injection; II) with 3rd harmonic current injection; III) with 3rd and 5th harmonic currents collaborative injection, are tested to evaluate the optimal torque per peak current control and their results are compared. The currents are recorded by current transducer, which outputting voltage signal.

In test I, the phase current without any harmonics injection are shown in Fig. 8 (a), where the amplitude is set to be 5A. Their corresponding harmonic analysis is shown in Fig. 8 (b). It is evident that there are little current harmonics in the spectrum without current harmonics injection. The peak value of fundamental current is as the same as the phase current, shown in Fig. 8 (c). The currents in $\alpha_3\beta_3$ are shown in Fig. 8 (d), which indicates the 3rd harmonic currents are restrained. In this case, the output torque is shown in Fig. 8 (e), where the average torque is 2.8 N.m.

In test II, the phase currents and the corresponding harmonics analyses with 3rd harmonic injection are shown in Fig. 9 (a) and Fig. 9 (b) respectively, where the coefficient of 3rd harmonic with respect to the fundamental is 1/6. It is consist with the theory above. The peak value of fundamental current is increased to 5.77A, as shown in Fig. 9 (c). However, the amplitude of phase current remains 5A. Compared with Fig. 8 (c), the fundamental peak current is increased by 15.4%, which results in the output torque generated by fundamental component has be increased by about 0.43N.m. The 3rd harmonic currents are shown in Fig. 9 (d). With optimal 3rd current injection, the output torque is shown in Fig. 9 (e), where the average torque is 3.4N.m. Compared with Fig. 8 (e), the average torque is improved by 0.6N.m, about 21.4%. Therefore, it is can be concluded that the torque contributed by 3rd harmonic component is approximately 0.17 N.m.

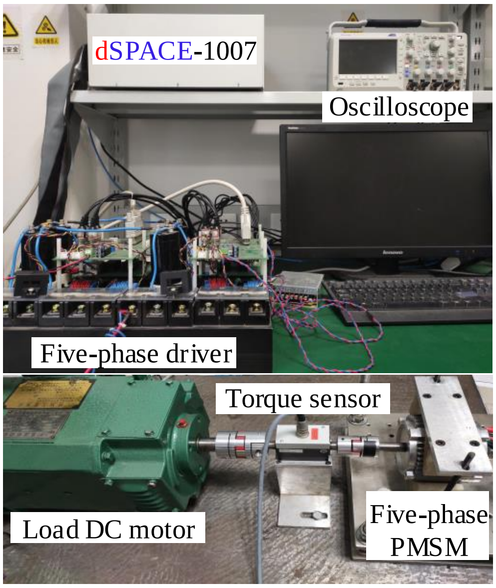


Fig. 7. Experimental setup.

TABLE II. PARAMETERS OF DUAL THREE-PHASE PM MACHINE

Parameter	Value
Resistance (Ω)	*0.46*
Stator inductance (mH)	*3.75*
Rated current (A)	*5*
Rated speed (r/min)	*500*
PM flux linkage (Wb)	*0.0646*
Pole pairs	*4*
DC-link voltage (V)	*50*

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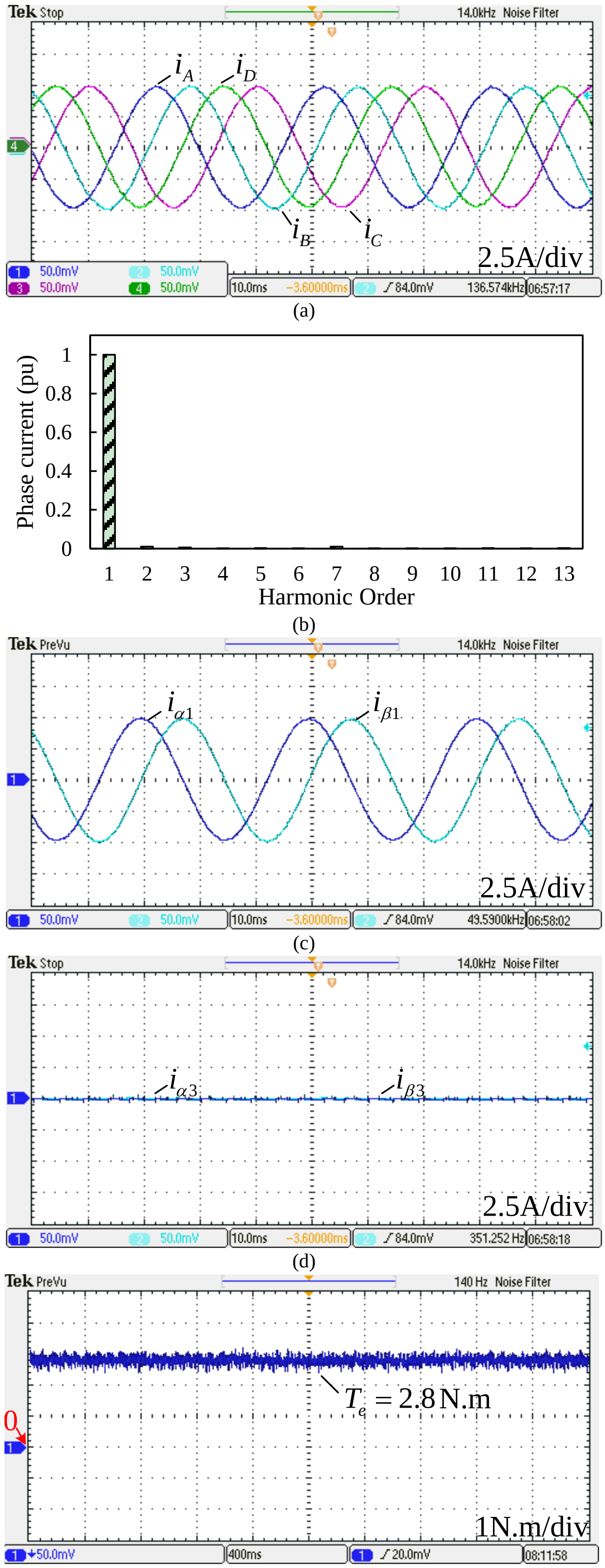

Fig. 8. Experimental results during test I, when prototype runs at 500rpm without harmonic injection, for (a) Phase currents, (b) Harmonics analysis of phase A current, (c) Fundamental currents, (d) 3rd harmonic currents, (e) Output torque.

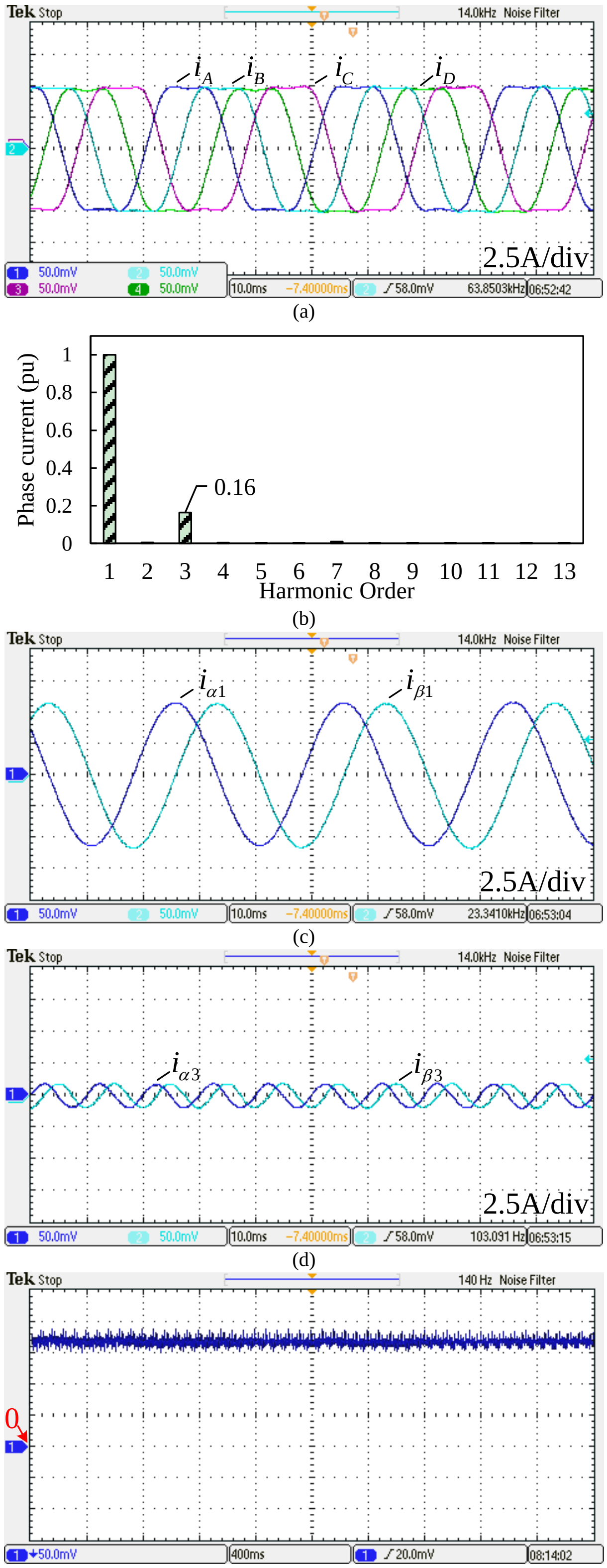

Fig. 9. Experimental results during test II, when prototype runs at 500rpm with 3rd harmonic current injection, for (a) Phase currents, (b) Harmonics analysis of phase A current, (c) Fundamental currents, (d) 3rd harmonic currents, (e) Output torque.

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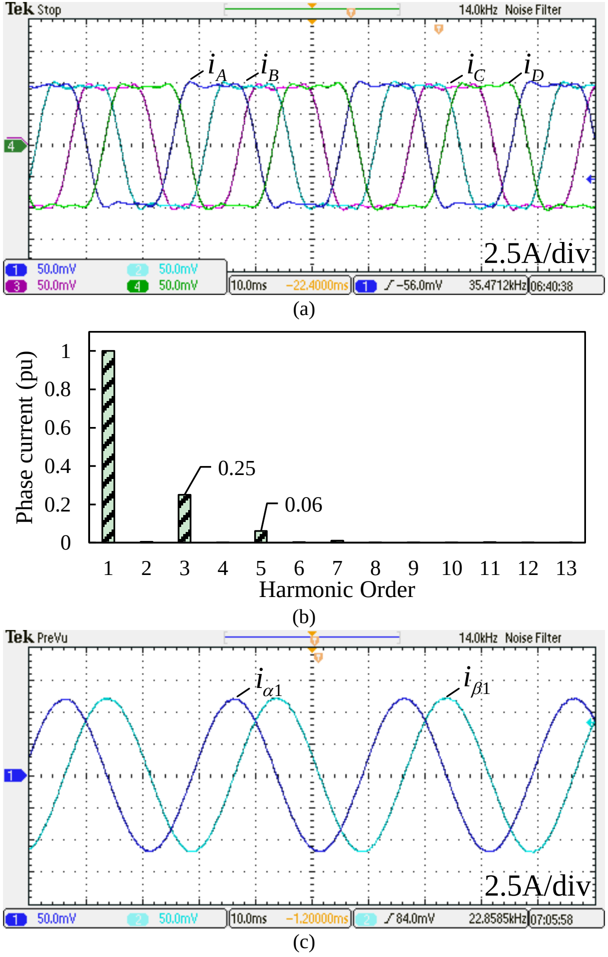
In test III, the phase current with 3rd and 5th harmonics collaborative injection are shown in Fig. 10 (a), where the coefficients of 3rd and 5th harmonics with respect to the fundamental are consistent with (17). Their corresponding harmonic analysis is shown in Fig. 10 (b). It is can be seen that the 3rd harmonic component is 0.52, and the 5th one is 0.06. According to Fig. 10 (c), the amplitude of fundamental current is increased to 6A. In other words, the average torque generated by fundamental component is improved by 20%. Meanwhile, the phase peak current keeps unchanged. The harmonic currents are shown in Fig. 10 (d). The 3rd harmonic current is increased to 1.5A. There are obvious 5th harmonic current, which is in accordance with the optimal injection coefficients in (17). It is evident that the zero sequence is controllable. Meanwhile, the average torque is around 3.7 N.m, as shown in Fig. 10 (e), which is 1.32 times of that one in test I. Furthermore, the torque ripple remains similar to that of the one without harmonic injection.



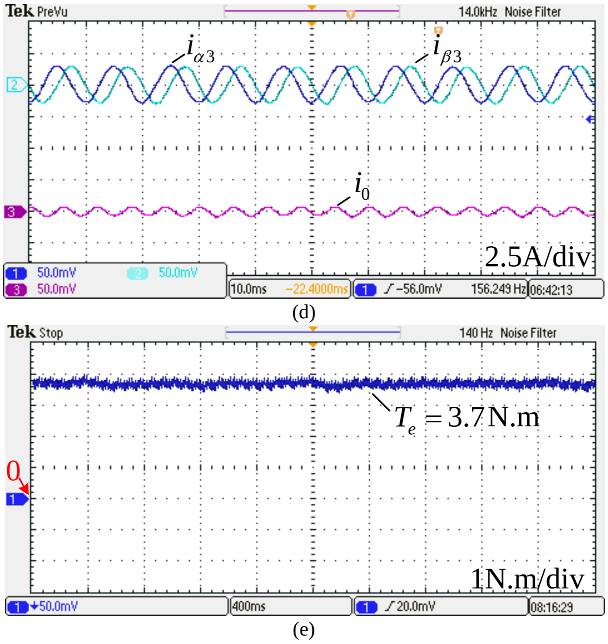


Fig. 10. Experimental results during test III, when prototype runs at 500rpm with 3rd & 5th harmonic currents injection, for (a) Phase currents, (b) Harmonics analysis of phase A current, (c) Fundamental currents, (d) 3rd and 5th harmonic currents, (e) Output torque.

V. CONCLUSION

A multi-harmonic currents injection control strategy is proposed in this paper to improve the torque capability of a five-phase PM machine. The optimal torque per peak current profile of phase current is derived and the hardware modification is needed for providing the 5th harmonic current with the flowing path. A current sensor is added to measure the neutral current, which ensures effective closed-loop control of the zero sequence current. This method is validated through the tests on a prototype machine. It is found that although the 5th harmonic component makes little contribution to the output torque, the average torque can be increased by 32.1%. This is a result of the 5th harmonic current increasing peak currents of the fundamental and 3rd harmonic. It can be concluded that the torque capability of the five-phase PM machine can be improved effectively within the same phase peak current limit by 3rd and 5th harmonics injection.

Appendix A

$$f(x,k_3,k_5) := \sin(x) + k_3\sin(3x) + k_5\sin(5x)$$

$$h(x,k_3,k_5) := \left| f(x,k_3,k_5) \right|$$

$$g(x,k_3,k_5) := \frac{d}{dx} f(x,k_3,k_5)$$

$$y(x,k_3,k_5) := \frac{d}{dx} g(x,k_3,k_5)$$

$$k_3 := 0.0005$$

$$k_5 := 0.0005$$

$$x := 1$$

$$Given$$

$$g(x,k_3,k_5) = 0$$

$$k_3 < 0.3$$

$$k_5 < 0.1$$

$$0 < x < 1.5$$

$$y(x,k_3,k_5) < 0$$

$$Minimize(f, x, k_3, k_5) =$$

Appendix B

According to VSD coordinate transformation, the five-phase currents in real frame can be decompose into three orthogonal sub-planes, $\alpha_1\beta_1$, $\alpha_3\beta_3$ and 0. The fundamental and $(10\mathrm{k}\pm 1)^{\mathrm{th}}$, $\mathrm{k}\in(1,2,3,\cdots)$ harmonic currents are mapped to $\alpha_1\beta_1$ sub-plane; the $(5\mathrm{k}\pm 2)^{\mathrm{th}}$, $\mathrm{k}\in(1,3,5,\cdots)$ harmonic currents belong to $\alpha_3\beta_3$ sub-plane; the $5\mathrm{k}^{\mathrm{th}}$, $\mathrm{k}\in(1,3,5,\cdots)$ harmonic currents are involved in zero sequence sub-plane. The transformation matrix is obtained as

$$\begin{bmatrix} i_{\alpha 1} \\ i_{\beta 1} \\ i_{\alpha 3} \\ i_{\beta 3} \\ i_0 \end{bmatrix} = \frac{2}{5}\begin{bmatrix} 1 & \cos\alpha & \cos 2\alpha & \cos 3\alpha & \cos 4\alpha \\ 0 & \sin\alpha & \sin 2\alpha & \sin 3\alpha & \sin 4\alpha \\ 1 & \cos 3\alpha & \cos 6\alpha & \cos 9\alpha & \cos 12\alpha \\ 0 & \sin 3\alpha & \sin 6\alpha & \sin 9\alpha & \sin 12\alpha \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \\ i_E \end{bmatrix} \quad (26)$$

where $\alpha = \dfrac{2\pi}{5}$.

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J. Y. Zhang received the B.S. degree from Nanjing University of Information Science & Technology, Nanjing, China, in 2009, and the Master's degree from Zhejiang Sci-Tech University of Technology, Hangzhou, China, in 2016, both in electrical engineering. He is currently working toward the Ph.D. degree in multi-phase motor drives from Nanjing University of Aeronautics and Astronautics, Nanjing, China.

S. J. He received the B.E degree from Brock University, Canada, in 2006, and the MBA degree in Zhejiang University of Technology in 2010. He is currently CEO of Ningbo Haibo Group Co., Ltd.

K. Wang (M'13–SM'14) received the B.Eng. degree from China Jiliang University, Hangzhou, China, in 2004, and the Ph.D. degree from Zhejiang University, Hangzhou, China, in 2009. From 2009 to 2010, he was with the Memorial University of Newfoundland, St. John's, NL, Canada, as a Postdoctoral Fellow. From 2010 to 2013, he was with The University of Sheffield, Sheffield, U.K. From 2013 to 2015, he was a Research Associate with the Sheffield Siemens Wind Power Research Centre, Sheffield, U.K., and a Research and Development Engineer with Ansys Inc., Canonsburg, PA, USA. Since 2015, he has been with the College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, China. His current research interests include the design and control of permanent-magnet machines.