This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2917588, IEEE Access

CODAS method for Pythagorean 2-tuple linguistic multiple attribute group decision making

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Abstract: The evaluation of the financial performance of universities is conducive to the sustainable development of universities. Based on this, we extend the traditional CODAS (COmbinative Distance-based ASsessment) method to the Pythagorean 2-tuple linguistic fuzzy environment and propose a P2TL-CODAS model to evaluate the financial management performance of universities. Firstly, we briefly describe the definition, the score function, accuracy function, operational laws and the distance calculating method of P2TLSs. Next, two aggregation operators of P2TL including Pythagorean 2-tuple linguistic weighted averaging (P2TLWA) operator and Pythagorean 2-tuple linguistic weighted geometric (P2TLWG) operator are also introduced to fuse overall Pythagorean 2-tuple linguistic evaluation information. Then the steps of CODAS method are depicted briefly. Moreover, we use linguistic Pythagorean 2-tuple linguistic fuzzy numbers to extend the CODAS method. The P2TL-CODAS model is established and all computing steps are simply presented. Furthermore, we apply the proposed method to evaluate the financial management performance about five universities. Finally, a comparison between P2TL-CODAS method and P2TL-TODIM method is made to demonstrate the stability of the new method. The results show that the proposed method has unique advantages.

Keywords: multiple attribute decision making (MADM); Pythagorean 2-tuple linguistic sets (P2TLSs); Pythagorean 2-tuple linguistic weighted averaging (P2TLWA) operator; Pythagorean 2-tuple linguistic weighted geometric (P2TLWG) operator; CODAS method; P2TL-CODAS model; University Financial Performance; Performance Evaluation

1. Introduction

The exact mathematical set uses the numbers 0 and 1 to express the "no" and "yes" of the description of the thing, but in the description of the real world there is often an ambiguous state. Based on this, Professor Zadeh [1] proposed fuzzy set theory, using membership degree to express the ambiguity of the description of things, but the theory can't describe both support and opposition attitudes, so Professor Atanassov [2] based on fuzzy sets, the theory of intuitionistic fuzzy sets is proposed. This theory expresses support and opposition to things with membership and non-membership, and can more comprehensively contain decision information. In the intuitionistic fuzzy set, the sum of the membership value and the non-membership value is less than or equal to 1, and literatures [3, 4] extend the intuitionistic fuzzy set with the Pythagorean fuzzy set to make the sum of the squares of the degrees of the membership value and non-membership less than or equal to 1, thus extending the description range of the intuitionistic fuzzy set, and its practical range is wider. Pythagorean fuzzy sets have a wide range of applications in multi-attribute decision making (MADM) and multi-attribute group decision making (MAGDM) [5-13].The Pythagorean fuzzy set [3, 4], like the intuitionistic fuzzy set, uses the membership degree and the non-membership degree to express the pros and cons of the object, respectively, the difference is the description range of intuitionistic fuzzy number and Pythagorean fuzzy number, the sum of membership degree and non-membership degree of intuitionistic fuzzy number is less than 1, while the sum of the square of membership degree and non-membership degree of Pythagoras fuzzy number is less than 1.If a decision information can be expressed by the intuitionistic fuzzy number, it can also be expressed by the Pythagorean fuzzy number, and vice versa. Therefore, the Pythagorean fuzzy theory can be established in more environments and thus has better practicability. Based on the Pythagorean fuzzy set, Reformat and Yager [5] proposed a new collaborative-based recommendation system, which recommended the customer's data for the customer, and proposed a multi-user based ranking method. Netflix's database data is used to generate a list of movie recommendations. The Pythagorean fuzzy set has developed addition, multiplication, power, and multiple arithmetic, and in order to better understand the Pythagorean fuzzy set, Peng and Yang [6]also proposed the division and subtraction algorithms of Pythagorean fuzzy sets and give the proof of process and related properties, and also develop a fuzzy superiority and inferior ranking

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method based on the distance formula to solve the multi-attribute decision problem. Garg [7] combined the Pythagorean fuzzy set with the Einstein operator to derive the Pythagorean fuzzy Einstein weighted averaging operator and the Pythagorean fuzzy Einstein ordered weighted averaging operator, and applied these operators to solve multi-attribute decision problems. Ren, et al. [8] extended the TODIM (Portuguese acronym for Interactive Multi-Criteria Decision Making) method based on the prospect theory and applied the method to the multi-attribute decision making problem in the Pythagorean fuzzy environment. Zeng, et al. [9] combined the distance measure with the Pythagorean fuzzy set to propose the Pythagoras ordered weighted averaging distance operator, taking into account the importance of each element in the set, and This method is combined with the TOPSIS operator to establish a mathematical model for dealing with multi-attribute decision problems. Zhang [10] proposed the Pythagorean weighted operator and the Pythagorean weighted ordered operator, and also proposed a decision method based on similarity measure. Finally, he solved the problem of multi-attribute decision making under the Pythagorean fuzzy environment with the given method. Garg [11] introduced the statistical concept of confidence level into the Pythagorean fuzzy set, performed confidence level operation on each Pythagorean fuzzy number, and finally defined two kinds of confidence Pythagorean averaging operators. Zhang, et al. [12] combined the generalized Bonferroni average and the bilateral Bonferroni average with the Pythagorean fuzzy set, respectively, and proposed a Pythagorean fuzzy Bonferroni average calculation model, which was applied to the MADM problem. Wei and Lu [13] applied the Pythagorean fuzzy set to the clustering analysis of information, and combined the Maclaurin symmetric mean (MSM) operator with the Pythagorean fuzzy set to derive the Pythagorean fuzzy Maclaurin symmetric mean (MSM) operator. Zhang, et al. [14] developed the Pythagorean fuzzy rough set model by combining the coarse-grained multi-granular rough set with the Pythagorean fuzzy set, and demonstrated the basic definition and attributes of the model. A general model for solving the M&A problem is also given. Peng [15] combined the intuitionistic fuzzy soft set with the Pythagorean fuzzy set to solve the problem that the sum of the membership degree and the non-membership degree cannot be greater than 1. The basic operations and set operations of Pythagorean fuzzy soft sets are discussed. Zhang [16] extended the Pythagorean fuzzy set to the interval value form, and explored the algorithm and some characteristics of the interval Pythagorean fuzzy set. It also introduced its sorting method and distance measure. A Pythagorean method based on the intimacy index is used to solve the hierarchical multi-criteria decision problem. Garg [17] proposed two sets of interval Pythagorean weighted averaging operator and weighted geometric mean operator for the Pythagorean environment. They also proposed a new accurate function calculation method, which not only includes the membership degree and non-membership degree, but also considers the degree of hesitation of the unknown. Finally, the new method is compared with the previous method. [18]An improved scoring function calculation method is proposed. Based on the TOPSIS method, a multi-attribute decision problem solving method in Pythagorean environment is proposed. In the present case, The law of the exponential operation of the interval Pythagorean fuzzy set is widely accepted, but Garg [19] developed another index algorithm, which is the base of the previous interval Pythagorean fuzzy number, and the real number is not indexed. He uses the real number or the interval number as the base to use the interval Pythagorean fuzzy number as the index. According to the new law, the interval value Pythagorean fuzzy weighted index averaging operator and the double interval value Pythagorean fuzzy weighted index averaging operator are introduced, and the method is verified by numerical examples. Zhang and Jiang [20] introduced the concept of entropy into the Pythagorean fuzzy environment, and combed the relationship between entropy and similarity measure. So that the calculation of entropy and similarity measure can be transformed into each other, and finally some formulas for calculating entropy and similarity are derived. Wei [21] presented some Pythagorean fuzzy interaction aggregation operators for MADM. Wei and Lu [22] introduced some Pythagorean fuzzy power aggregation operators, such as Pythagorean fuzzy power average (PFPA) operator, Pythagorean fuzzy power geometric (PFPG) operator, Pythagorean fuzzy power weighted average (PFPWA) operator, Pythagorean fuzzy power weighted geometric (PFPWG) operator, Pythagorean fuzzy power ordered weighted average (PFPOWA) operator, Pythagorean fuzzy power ordered weighted geometric (PFPOWG) operator, Pythagorean fuzzy power hybrid average (PFPHA) operator and Pythagorean fuzzy power hybrid geometric (PFPHG) operator in multiple attribute decision making. Wei and Lu [23] developed some dual hesitant Pythagorean fuzzy Hamacher aggregation operators in MADM. Wei, et al. [24] defined the concept of Pythagorean 2-tuple linguistic sets and developed some Pythagorean 2-tuple

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linguistic aggregation operators: Pythagorean 2-tuple linguistic weighted average (P2TLWA) operator, Pythagorean 2-tuple linguistic weighted geometric (P2TLWG) operator, Pythagorean 2-tuple linguistic ordered weighted average (P2TLOWA) operator, Pythagorean 2-tuple linguistic ordered weighted geometric (P2TLOWG) operator, Pythagorean 2-tuple linguistic hybrid average (P2TLHA) operator and Pythagorean 2-tuple linguistic hybrid geometric (P2TLHG) operator.

The CODAS (COmbinative Distance-based ASsessment)method was proposed by Keshavarz Ghorabaee [25]. This is a new and stable method used to deal with multi-criteria decision-making problems by computing the Euclidean distance and Hamming distance to determine the desirability of an alternative .In the current research, Ghorabaee, et al. [26] proposed a fuzzy extension of the CODAS method to select the best suppliers. In their study, they combined the linguistic variables and trapezoidal fuzzy numbers with the CODAS method and put the extending method into a numerical example of a shoe company. The results prove the applicability of their method in multi-criteria market segment evaluation and selection. Thereafter, they made a comparison and sensitivity analysis to show the availability and stability of the proposed fuzzy CODAS method. Pamucar, et al. [27] introduced new LNN PW-CODAS model that they integrated linguistic Neutrosophic Numbers (LNN) with LNN Pairwise (LNN PW) used for determining the weight coefficients of the criteria. This model can eliminate subjective qualitative assessments and assumptions by decision makers in complex decision-making conditions. They also provided a case study of the selection of optimal Power-Generation Technology (PGT) in Libya to test the LNN PW-CODAS model. A sensitivity analysis was carried out to show a high degree of stability of the model. By comparing with other LNN extensions , the results were validated . Panchal [28] applied an integrated MCDM framework based on analytical hierarchy process (AHP) and a fuzzy CODAS approach for solving the maintenance decision problem in a process industry. Badi [29] used CODAS method to select the best location of desalination plant in the northwestern coast of Libya.

However, the above research did not concern about the MADM problem with P2TLNs based on CODAS method. In this study, we use the linguistic variables and Pythagorean 2-tuple linguistic numbers to extend the CODAS method and introduce a new multi-criteria group decision-making approach. A case study is utilized to show the applicability of the proposed model. To demonstrate the stability of the P2TL-CODAS model, we compare the results with the results of P2TL-CODAS method and P2TL-TODIM method. The results show that the proposed method is with stability and validity.

The remainder of this paper is organized as follows. In Section 2 and 3, we give an overview of some basic concepts and definitions of P2TLNs and MABAC method. In Section 5, the extending CODAS method is presented to handle MAGDM. In Section 5, we give a numerical example for financial management performance evaluation to show the effectiveness of the proposed approaches. The paper is concluded in Section 6.

2. Preliminaries

2.1 Pythagorean 2-tuple linguistic sets

Wei, et al. [24] proposed the Pythagorean 2-tuple linguistic sets (P2TLSs) based on the PFSs [13, 21] and 2-tuple linguistic [30-38].

Definition 1[24]. A the Pythagorean 2-tuple linguistic set A in X is given

$$P=\left\{\left(s_{\beta(x)},\varphi\right),\left(u_p\left(x\right),v_p\left(x\right)\right),x\in X\right\} \tag{1}$$

Where $s_{\beta(x)}\in S$, $\varphi\in\left[-0.5,0.5\right)$, $u_p\left(x\right)\in[0,1]$ and $v_p\left(x\right)\in[0,1]$, with the condition $0\le\left(u_p\left(x\right)\right)^2+\left(v_p\left(x\right)\right)^2\le 1,\forall x\in X$. The numbers $u_p\left(x\right),v_p\left(x\right)$ represent, respectively, the degree of membership and degree of non-membership of the element x to linguistic variable $\left(s_{\beta(x)},\varphi\right)$.

Wei et al.[24] call $P=\left\langle\left(s_\beta,\varphi\right),\left(u_p,v_p\right)\right\rangle$ a Pythagorean 2-tuple linguistic number (P2TLN).

Definition 2[24]. Suppose that $p=\left\langle\left(s_\beta,\varphi\right),\left(u_p,v_p\right)\right\rangle$ is a P2TLN, the score function of P2TLN can be represented as follows:

$$S(p)=\Delta\left(\Delta^{-1}\left(s_{\beta(p)},\varphi\right)\frac{1+\left(u_p\right)^2-\left(v_p\right)^2}{2}\right), \tag{2}$$

$$\Delta^{-1}\left(S\left(p\right)\right)\in\left[0,L\right].$$

Definition 3[24]. Suppose that $p=\left\langle\left(s_\beta,\varphi\right),\left(u_p,v_p\right)\right\rangle$ is a P2TLN, the accuracy function of P2TLN can be represented as follows:

$$H(p)=\Delta\left(\Delta^{-1}\left(s_{\beta(p)},\varphi\right)\frac{\left(u_p\right)^2+\left(v_p\right)^2}{2}\right), \quad (3)$$

$\Delta^{-1}\left(H\left(p\right)\right)\in\left[0,L\right]$.

Definition 4[24]. Suppose that $p_1=\left\langle\left(s_{\beta(p_1)},\varphi_1\right),\left(u_{p_1},v_{p_1}\right)\right\rangle$ and $p_2=\left\langle\left(s_{\beta(p_2)},\varphi_1\right),\left(u_{p_2},v_{p_2}\right)\right\rangle$ are two P2TLNs, $S\left(p_1\right)=\Delta\left(\Delta^{-1}\left(s_{\beta(p_1)},\varphi_1\right)\cdot\frac{1+\left(u_{p_1}\right)^2-\left(v_{p_1}\right)^2}{2}\right)$ and $S\left(p_2\right)=\Delta\left(\Delta^{-1}\left(s_{\beta(p_2)},\varphi_2\right)\cdot\frac{1+\left(u_{p_2}\right)^2-\left(v_{p_2}\right)^2}{2}\right)$ be the scores of p_1 and p_2, respectively, and let $H\left(p_1\right)=\Delta\left(\Delta^{-1}\left(s_{\beta(p_1)},\varphi_1\right)\cdot\frac{\left(u_{p_1}\right)^2+\left(v_{p_1}\right)^2}{2}\right)$ and $H\left(p_2\right)=\Delta\left(\Delta^{-1}\left(s_{\beta(p_2)},\varphi_2\right)\cdot\frac{\left(u_{p_2}\right)^2+\left(v_{p_2}\right)^2}{2}\right)$ be the accuracy degrees of p_1 and p_2, then

$(1)\, if\ S\left(p_1\right)<S\left(p_2\right), p_1<p_2;$

$(2)\, if\ S\left(p_1\right)>S\left(p_2\right), p_1>p_2;$

$(3)\, if\ S\left(p_1\right)=S\left(p_2\right), H\left(p_1\right)<H\left(p_2\right), then\ p_1<p_2;$

$(3)\, if\ S\left(p_1\right)=S\left(p_2\right), H\left(p_1\right)>H\left(p_2\right), then\ p_1>p_2;$

$(3)\, if\ S\left(p_1\right)=S\left(p_2\right), H\left(p_1\right)=H\left(p_2\right), then\ p_1=p_2.$

Wei et al. [34] defined some operational laws of P2TLNs.

Definition 5[39]. Suppose that $p_1=\left\langle\left(s_{\beta(p_1)},\varphi_1\right),\left(u_{p_1},v_{p_1}\right)\right\rangle$ and $p_2=\left\langle\left(s_{\beta(p_2)},\varphi_1\right),\left(u_{p_2},v_{p_2}\right)\right\rangle$ are two P2TLNs,the normalized Hamming distance between p_1 and p_2 can be represented as follows:

$$H_d\left(p_1,p_2\right)=\frac{1}{2L}\left[\left|\begin{array}{l}\left(1+\left(u_{p_1}\right)^2-\left(v_{p_1}\right)^2\right)\bullet\Delta^{-1}\left(s_{\beta(p_1)},\varphi_1\right)-\\ \left(1+\left(u_{p_2}\right)^2-\left(v_{p_2}\right)^2\right)\bullet\Delta^{-1}\left(s_{\beta(p_2)},\varphi_2\right)\end{array}\right|\right] \quad (4)$$

where L is a numerical value that represents the length of the language scale.

Definition 6[39]. Suppose that $p_1=\left\langle\left(s_{\beta(p_1)},\varphi_1\right),\left(u_{p_1},v_{p_1}\right)\right\rangle$ and $p_2=\left\langle\left(s_{\beta(p_2)},\varphi_1\right),\left(u_{p_2},v_{p_2}\right)\right\rangle$ are two P2TLNs,the normalized Euclidean distance between p_1 and p_2 can be represented as follows:

$$E_d\left(p_1,p_2\right)=\sqrt{\left(\begin{array}{l}\left(1+\left(u_{p_1}\right)^2-\left(v_{p_1}\right)^2\right)\bullet\Delta^{-1}\left(s_{\beta(p_1)},\varphi_1\right)-\\ \left(1+\left(u_{p_2}\right)^2-\left(v_{p_2}\right)^2\right)\bullet\Delta^{-1}\left(s_{\beta(p_2)},\varphi_2\right)\end{array}\right)^2\Big/2L} \quad (5)$$

where L is a numerical value that represents the length of the language scale.

Definition 7[24]. Suppose that $p_1=\left\langle\left(s_{\beta(p_1)},\varphi_1\right),\left(u_{p_1},v_{p_1}\right)\right\rangle$ and $p_2=\left\langle\left(s_{\beta(p_2)},\varphi_1\right),\left(u_{p_2},v_{p_2}\right)\right\rangle$ are two P2TLNs, then

$$p_1\oplus p_2=\left\langle\Delta\left(\Delta^{-1}\left(s_{\beta(p_1)},\varphi_1\right)+\Delta^{-1}\left(s_{\beta(p_2)},\varphi_2\right)\right),\left(\sqrt{\left(u_{p_1}\right)^2+\left(u_{p_2}\right)^2-\left(u_{p_1}\right)^2\left(u_{p_2}\right)^2},v_{p_1}v_{p_2}\right)\right\rangle;$$

$$p_1\otimes p_2=\left\langle\Delta\left(\Delta^{-1}\left(s_{\beta(p_1)},\varphi_1\right)\cdot\Delta^{-1}\left(s_{\beta(p_2)},\varphi_2\right)\right),\left(u_{p_1}u_{p_2},\sqrt{\left(v_{p_1}\right)^2+\left(v_{p_2}\right)^2-\left(v_{p_1}\right)^2\left(v_{p_2}\right)^2}\right)\right\rangle;$$

$$\lambda p_1=\left\langle\Delta\left(\lambda\Delta^{-1}\left(s_{\beta(p_1)},\varphi_1\right)\right),\left(\sqrt{1-\left(1-\left(u_{p_1}\right)^2\right)^{\lambda}},\left(v_{p_1}\right)^{\lambda}\right)\right\rangle;$$

$$\left(p_1\right)^{\lambda}=\left\langle\Delta\left(\left(\Delta^{-1}\left(s_{\beta(p_1)},\varphi_1\right)\right)^{\lambda}\right),\left(\left(u_{p_1}\right)^{\lambda},\sqrt{1-\left(1-\left(v_{p_1}\right)^2\right)^{\lambda}}\right)\right\rangle.$$

Theorem 1[24]. For any two Pythagorean 2-tuple linguistic numbers $p_1=\left\langle\left(s_{\beta(p_1)},\varphi_1\right),\left(u_{p_1},v_{p_1}\right)\right\rangle$ and $p_2=\left\langle\left(s_{\beta(p_2)},\varphi_1\right),\left(u_{p_2},v_{p_2}\right)\right\rangle$, According to the Definition 6, it's clear that the operation laws have the following properties.

(1) $p_1 \oplus p_2 = p_2 \oplus p_1$

(2) $p_1 \otimes p_2 = p_2 \otimes p_1$

(3) $k\left(p_1 \oplus p_2\right) = kp_1 \oplus kp_2, 0 \le k \le 1$

(4) $k_1 p_1 \oplus k_2 p_1 = \left(k_1 \oplus k_2\right) p_1, 0 \le k_1, k_2, k_1 + k_2 \le 1$

(5) $p_1^{k_1} \otimes p_1^{k_2} = \left(p_1\right)^{k_1+k_2}, 0 \le k_1, k_2, k_1 + k_2 \le 1$

(6) $p_1^{k_1} \otimes p_2^{k_1} = \left(p_1 \otimes p_2\right)^{k_1}, k_1 \ge 0$

(7) $\left(\left(p_1\right)^{k_1}\right)^{k_2} = \left(p_1\right)^{k_1 k_2}$

2.2 Pythagorean 2-tuple linguistic arithmetic aggregation operators

In this section, we put forward some arithmetic aggregation operators with Pythagorean 2-tuple linguistic information, such as Pythagorean 2-tuple linguistic weighted averaging (P2TLWA) operator, and Pythagorean 2-tuple linguistic weighted geometric (P2TLWG) operator.

Definition 8[24]. Suppose that $p=\left\langle\left(s_{\beta(p_j)},\varphi_j\right),\left(u_{p_j},v_{p_j}\right)\right\rangle (j=1,2,...,n)$ is a collection of Pythagorean 2-tuple linguistic numbers (P2TLNs). P2TLWA operator can be defined as follows:

$$\text{P2TLWA}_{\omega}\left(p_1, p_2, ..., p_n\right) = \mathop{\oplus}_{j=1}^{n}\left(\omega_j p_j\right) = \left\langle \begin{array}{l} \Delta\left(\sum_{j=1}^{n} \omega_j \Delta^{-1}\left(s_{\beta(p_j)},\varphi_j\right)\right), \\ \left(\sqrt{1-\prod_{j=1}^{n}\left(1-\left(u_{p_j}\right)^2\right)^{\omega_j}}, \prod_{j=1}^{n}\left(v_{p_j}\right)^{\omega_j}\right) \end{array} \right\rangle \tag{6}$$

where $\omega=\left(\omega_1,\omega_2,...,\omega_n\right)^T$ be the weight vector of $p_j\left(j=1,2,...,n\right)$ and $\omega_j > 0, \sum_{j=1}^{n}\omega_j = 1.$

Definition 9[24]. Suppose that $p=\left\langle\left(s_{\beta(p_j)},\varphi_j\right),\left(u_{p_j},v_{p_j}\right)\right\rangle (j=1,2,...,n)$ is a collection of Pythagorean 2-tuple linguistic numbers (P2TLNs). P2TLWG operator can be defined as follows:

$$\text{P2TLWG}_{\omega}\left(p_1, p_2, ..., p_n\right) = \mathop{\otimes}_{j=1}^{n}\left(\omega_j p_j\right) = \left\langle \begin{array}{l} \Delta\left(\prod_{j=1}^{n} \Delta^{-1}\left(s_{\beta(p_j)},\varphi_j\right)^{\omega_j}\right), \\ \left(\prod_{j=1}^{n}\left(u_{p_j}\right)^{\omega_j}, \sqrt{1-\prod_{j=1}^{n}\left(1-\left(v_{p_j}\right)^2\right)^{\omega_j}}\right) \end{array} \right\rangle \tag{7}$$

where $\omega=\left(\omega_1,\omega_2,...,\omega_n\right)^T$ be the weight vector of $p_j\left(j=1,2,...,n\right)$ and $\omega_j > 0, \sum_{j=1}^{n}\omega_j = 1.$

3. The CODAS Method

Keshavarz Ghorabaee [25]recently introduced a new and efficient MCDM method，that is the CODAS method , which is used the Euclidean distance and Hamming distance [39]to determine the desirability of an alternative . Suppose that we have m alternatives and n criteria and l decision-makers (DMs). The steps of crisp CODAS are presented as follows.

Step 1: Establish the fuzzy decision matrix (R)，$R=\left[r_{ij}\right]_{m\times n}, i=1,2,\ldots,m, j=1,2,\ldots,n$ which can be shown as follows.

$$R=\left[r_{ij}\right]_{m\times n}=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \tag{8}$$

where r_{ij} denotes the fuzzy performance value of i-th alternative ($i=1,2,\ldots,m$)with respect to j-th criterion($j=1,2,\ldots,n$) .

Step 2. Calculate average fuzzy weights according to the fuzzy weight of each criterion from each decision-maker as in the following equation.

$$W_t = \left[w_{jt}\right]_{1\times n} \tag{9}$$

$$W = \left[w_j\right]_{1\times n} \tag{10}$$

$$w_j = \mathop{\oplus}_{t=1}^{l} w_{jt} \tag{11}$$

where w_{jt} represents the fuzzy weight of j-th criterion

($j=1,2,...,n$) with respect to t-th ($t=1,2,...,l$) decision-maker, and w_j denotes the average fuzzy weight of j-th criterion.

Step 3: Compute the normalized matrix using linear normalization, the equations are given in the as follows:

$$\tilde{r}_{ij}=\begin{cases}\dfrac{r_{ij}}{\max_i r_{ij}} & if\ \eta_j\in \mathrm{B},\\ \dfrac{\max_i r_{ij}}{r_{ij}} & if\ \eta_j\in \mathrm{C}.\end{cases} \quad (12)$$

where B and C represent the sets of benefit and cost cr iteria, respectively.

Step 4: Calculate the weighted normalized matrix. The weighted normalized performance values are calculated as given in the following equation:

$$t_{ij}=w_j\tilde{r}_{ij} \quad (13)$$

where $w_j\left(0<w_j<1\right)$ denotes the weight of j-th criterion, and $\sum_{j=1}^{n}w_j=1$

Step 5: Determine the negative ideal solution using the following equations:

$$NS=\left[NS_j\right]_{1\times n}, \quad (14)$$

$$NS_j=\min_i t_{ij} \quad (15)$$

Step 6. Calculate the fuzzy weighted Euclidean distance(E_{d_i})and Hamming distance(H_{d_i}) of alternatives from the fuzzy negative-ideal solution as follows:

$$E_{d_i}=\sum_{j=1}^{n}d_E\left(t_{ij},NS_j\right); \quad (16)$$

$$H_{d_i}=\sum_{j=1}^{n}d_H\left(t_{ij},NS_j\right). \quad (17)$$

Step 7: Construct the relative assessment matrix as in the following equations:

$$RA=\left[h_{ik}\right]_{m\times m} \quad (18)$$

$$h_{ik}=\left(E_{d_i}-E_{d_k}\right)+\left(\lambda\left(E_{d_i}-E_{d_k}\right)\times\left(H_{d_i}-H_{d_k}\right)\right), \quad (19)$$

Where $k=1,2,...,n$ and λ represents a threshold function to recognize the equality of the Euclidean distances of two alternatives, and is given as in the following equation:

$$\lambda(k)=\begin{cases}1 & if\ |x|\geq\tau\\ 0 & if\ |x|<\tau\end{cases}, \quad (20)$$

where τ is a threshold parameter set by DMs ,and its value is between 0.01 and 0.05. In this study, we use τ=0.02 for the calculations.

Step 8: Compute the assessment score of each alternative with equation (21):

$$AS_i=\sum_{k=1}^{m}h_{ik}. \quad (21)$$

Step 9: Rank the alternatives according to the decreasing values of assessment score (AS_i), the alternative with maximum value is the best choice.

4. The CODAS Method with Pythagorean 2-tuple Linguistic

In this section, an extended CODAS method will be proposed，and P2TL-CODAS is a new model for handling the multi-criteria group decision-making (MCGDM) problems. The computing steps of our proposed model are given below.

Suppose that m alternatives $\{K_1,K_2,\ldots K_m\}$, n attributes $\{A_1,A_2,\ldots A_n\}$ and l experts $\{D_1,D_2,\ldots D_l\}$, let $\{\psi_1,\psi_2,\ldots\psi_l\}$ be the expert's weighting vector which satisfy $\psi_t\in[0,1]$ and $\sum_{t=1}^{l}\psi_t=1$. Then:

Step 1. Construct a panel of DMs, select the best attributes for the performance measurements of alternatives, and finally form the Pythagorean 2-tuple linguistic fuzzy decision matrix $R^{(l)}=\left(r_{ij}^l\right)_{m\times n}$ of each decision maker.

$$R^{(l)}=\left[r_{ij}^l\right]_{m\times n}=\begin{bmatrix} r_{11}^l & r_{12}^l & \cdots & r_{1n}^l\\ r_{21}^l & r_{22}^l & \cdots & r_{2n}^l\\ \vdots & \vdots & \vdots & \vdots\\ r_{m1}^l & r_{m2}^l & \cdots & r_{mn}^l\end{bmatrix} \quad (22)$$

where r_{ij}^l represents the fuzzy performance value of i-th alternative ($i=1,2,\ldots,m$)with respect to j-th criterion($j=1,2,\ldots,n$) and t-th decision-maker($t=1,2,\ldots,l$).

Step 2. Based on Pythagorean 2-tuple linguistic weighted average (P2TLWA) operator or Pythagorean 2-tuple linguistic weighted geometric (P2TLWG) operator, we can obtain the group Pythagorean 2-tuple linguistic fuzzy decision matrix $R=\left(r_{ij}\right)_{m\times n}$.

$$R=\left[r_{ij}\right]_{m\times n}=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \quad (23)$$

$$r_{ij}=\overset{l}{\underset{t=1}{\oplus}} r_{ij}^{l}=\text{P2TLWA}(r_{ij}^{1},r_{ij}^{2},...,r_{ij}^{l}) =\left\langle \begin{array}{l} \Delta\left(\sum_{t=1}^{l}\psi_t\Delta^{-1}\left(s_{\beta\left(p_{ij}\right)},\varphi_{ij}\right)\right), \\ \left(\sqrt{1-\prod_{t=1}^{l}\left(1-\left(u_{p_{ij}}^{l}\right)^{2}\right)^{\psi_t}},\prod_{t=1}^{l}\left(v_{p_{ij}}^{l}\right)^{\psi_t}\right) \end{array}\right\rangle \quad (24)$$

Or

$$r_{ij}=\overset{l}{\underset{t=1}{\otimes}} r_{ij}^{l}=\text{P2TLWG}(r_{ij}^{1},r_{ij}^{2},...,r_{ij}^{l}) =\left\langle \begin{array}{l} \Delta\left(\prod_{t=1}^{l}\Delta^{-1}\left(s_{\beta\left(p_{ij}\right)},\varphi_{ij}\right)^{\psi_t}\right), \\ \left(\prod_{t=1}^{l}\left(u_{p_{ij}}^{l}\right)^{\psi_t},\sqrt{1-\prod_{t=1}^{l}\left(1-\left(v_{p_{ij}}^{l}\right)^{2}\right)^{\psi_t}}\right) \end{array}\right\rangle \quad (25)$$

where r_{ij} denotes the average fuzzy performance value of ith alternative with respect to j-th criterion.

Step 3. Calculate average fuzzy weights based on the fuzzy weight of each criterion from each decision-maker as follows:

$$W_t=\left[w_{jt}\right]_{1\times n} \quad (26)$$

$$W=\left[w_j\right]_{1\times n} \quad (27)$$

$$w_j=\overset{l}{\underset{t=1}{\oplus}} w_{jt} \quad (28)$$

Step 4. Normalize the decision matrix $R=\left(r_{ij}\right)_{m\times n}$ into $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}$

$$\tilde{r}_{ij}=\begin{cases} \left\langle\Delta\left(\Delta^{-1}\left(s_{\beta\left(p_{ij}\right)},\varphi_{ij}\right)\right),\left(u_{p_{ij}}, v_{p_{ij}}\right)\right\rangle & if\ A_j \text{ is benefit at} \\ \left\langle\Delta\left(T-\Delta^{-1}\left(s_{\beta\left(p_{ij}\right)},\varphi_{ij}\right)\right),\left(v_{p_{ij}},u_{p_{ij}}\right)\right\rangle & if\ A_j \text{ is cost attribute.} \end{cases} \quad (29)$$

Step 5: Calculate the Pythagorean 2-tuple linguistic fuzzy weighted normalized matrix as in the following equation:

$$t_{ij}=w_j\tilde{r}_{ij} \quad (30)$$

where $w_j\left(0<w_j<1\right)$ is the weight of j-th criterion,

and $\sum_{j=1}^{n}w_j=1$.

Step 6: Obtain the negative ideal solution based on the following equations:

$$NS=\left[NS_j\right]_{1\times n}, \quad (31)$$

$$NS_j=\min_i t_{ij} \quad (32)$$

Step 7. Determine the fuzzy weighted Euclidean distance(E_{d_i})and Hamming distance(H_{d_i})as follows:

$$E_{d_i}=\sum_{j=1}^{n}d_E\left(t_{ij},NS_j\right); \quad (33)$$

$$H_{d_i}=\sum_{j=1}^{n}d_H\left(t_{ij},NS_j\right). \quad (34)$$

Step 8: The relative assessment matrix can be constructed as in the following equations:

$$RA=\left[h_{ik}\right]_{m\times m} \quad (35)$$

$$h_{ik}=\left(E_{d_i}-E_{d_k}\right)+\left(\lambda\left(E_{d_i}-E_{d_k}\right)\times\left(H_{d_i}-H_{d_k}\right)\right), \quad (36)$$

where $k\in\{1, 2, \ldots, n\}$ and λ denotes a threshold function which is defined as in the following equation:

$$\lambda\left(k\right)=\begin{cases}1 & if\ |x|\geq\tau \\ 0 & if\ |x|<\tau\end{cases}, \quad (37)$$

where τ is a threshold parameter set by DMs with a value between 0.01 and 0.05. In this study, τ=0.02.

Step 9: Calculate the value of AS_i using equation (38).

$$AS_i = \sum_{k=1}^{m} h_{ik} . \quad (38)$$

Step 10: We can rank all the alternatives based on the calculating results of AS_i .The alternative with the highest assessment score is the best.

5. Application

In this section, we shall give a numerical example of multi-criteria evaluation of the financial management performance in five universities (K_1 to K_5) to select best school with the best financial performance by using CODAS model with Pythagorean 2-tuple linguistic fuzzy information. This paper constructs the financial management performance evaluation index system from four aspects (A_1 to A_4): teaching performance, scientific research performance, self-financing performance and asset performance. From the four aspects, we select 15 indicators for financial management performance evaluation to establish an index system. The decision-makers are all 5 experts (D_1 to D_5) who are familiar with the financial and scientific research status of the school. Because of the different knowledge backgrounds and experience of each expert, the weights of the five experts are $\psi = (0.20, 0.22, 0.17, 0.25, 0.16)$. Then the decision-making group selected the most important evaluation criteria (sub-criteria) shown in Table 1. To make this evaluation, the decision-makers use linguistic variables to express their assessments .The linguistic variables for weighting criteria and the linguistics variables for rating alternatives are given in Table 2. In the following, the steps of using the proposed P2TL-CODAS method for the evaluation of university financial performance are presented:

Table 1. Evaluation indicators

Criteria	A_{ij}	Sub-criteria
Teaching performance(A_1)	A_{11}	Teacher-student ratio
	A_{12}	The proportion of full-time teachers and faculty
	A_{13}	Student per capita training costs
	A_{14}	Per capita funding for teachers
	A_{15}	Teaching staff per capita teaching activity income
Research performance(A_2)	A_{21}	Teacher per capita research funding
	A_{22}	Yield rate of scientific research achievements
	A_{23}	Research expenditure as a percentage of career expenditure
Self-financing(A_3)	A_{31}	Self-financing income as a percentage of total income
	A_{32}	Self-financing infrastructure funds account for the proportion of infrastructure funds
	A_{33}	Financial education funds account for the proportion of total financial income
	A_{34}	School self-financing annual growth rate
Asset performance(A_4)	A_{41}	Each student occupies an area
	A_{42}	Growth rate of fixed assets
	A_{43}	Per capita teaching and research equipment asset value

Table 2 .Linguistic variables and their fuzzy numbers

Linguistic variable	Pythagorean 2-tuple linguistic numbers
Very low (VL)	$((s_0,0),(0.1,0.8))$
Low (L)	$((s_1,0),(0.3,0.7))$
Medium low (ML)	$((s_2,0),(0.4,0.6))$
Medium (M)	$((s_3,0),(0.5,0.5))$
Medium high (MH)	$((s_4,0),(0.6,0.4))$
High (H)	$((s_5,0),(0.7,0.3)$
Very high (VH)	$((s_6,0),(0.8,0.1))$

Step 1. According to these 15 indicators, five experts will evaluate and score the financial performance. The scoring form adopts Pythagorean 2-tuple linguistic numbers using the linguistic variables presented in Table 2.The

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2917588, IEEE Access

assessments of decision makers are shown in Table 3. Based on this table and Eqs. (24), the average fuzzy decision matrix is calculated. The results are shown in Table 4.

Table 3. Rating alternatives on each criterion by each DM

		A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{21}	A_{22}	A_{23}	A_{31}	A_{32}	A_{33}	A_{34}	A_{41}	A_{42}	A_{43}
D_1	K_1	L	H	ML	MH	VH	M	VL	M	VL	H	MH	ML	M	H	VH
	K_2	ML	VH	MH	VH	M	VL	H	MH	VH	L	VL	ML	H	M	ML
	K_3	H	M	ML	H	H	L	ML	M	M	H	ML	MH	VL	M	ML
	K_4	L	H	H	H	VL	L	ML	ML	H	MH	ML	ML	VL	H	L
	K_5	MH	L	H	ML	MH	VH	L	ML	VL	H	VH	MH	M	M	VL
D_2	K_1	H	L	H	MH	M	M	ML	L	L	ML	VL	H	VH	ML	MH
	K_2	L	ML	MH	MH	L	VL	H	VH	M	ML	ML	VL	L	MH	ML
	K_3	ML	M	M	MH	H	H	VH	L	VL	M	MH	H	VH	L	MH
	K_4	MH	M	ML	VL	MH	L	H	VL	M	M	ML	L	MH	H	VH
	K_5	H	MH	H	L	L	ML	VL	H	VH	M	M	ML	H	MH	L
D_3	K_1	L	ML	ML	M	H	MH	L	M	ML	MH	M	VH	L	ML	M
	K_2	H	MH	VH	M	L	VL	H	MH	VL	M	M	ML	H	MH	VH
	K_3	H	VL	L	ML	H	MH	L	M	H	ML	L	VL	MH	VH	VL
	K_4	MH	ML	H	L	ML	VL	ML	L	M	H	VH	VL	ML	MH	H
	K_5	M	ML	L	MH	ML	L	VL	ML	VH	MH	VL	MH	ML	L	MH
D_4	K_1	L	VL	ML	H	MH	ML	VH	MH	L	ML	M	M	ML	MH	VH
	K_2	H	MH	VH	VL	L	ML	M	MH	VH	L	ML	MH	H	VH	L
	K_3	VL	MH	M	MH	VL	L	ML	H	VH	L	M	MH	ML	VH	H
	K_4	L	H	M	MH	VH	L	ML	H	VH	VL	M	ML	H	M	VL
	K_5	ML	MH	H	M	MH	MH	ML	ML	VH	H	VL	L	MH	VL	H
D_5	K_1	M	H	ML	VH	VL	H	ML	H	ML	ML	M	H	VH	L	MH
	K_2	MH	H	M	L	VL	ML	MH	VH	L	H	L	ML	H	VL	H
	K_3	ML	H	MH	L	VL	H	L	MH	VL	ML	H	M	H	MH	MH
	K_4	L	ML	M	M	ML	MH	M	H	MH	L	ML	VL	H	VH	MH
	K_5	L	MH	ML	H	MH	VH	H	L	VL	ML	H	M	ML	H	VH

Table 4. Weighted average fuzzy decision matrix

	K_1	K_2	K_3
A_{11}	<(s_2,0.2),(0.4725,0.5505)>	<(s_3,0.36),(0.5799,0.4348)>	<(s_3,-0.39),(0.5219,0.4989)>
A_{12}	<(s_2,0.36),(0.5057,0.5197)>	<(s_4,0.12),(0.645,0.3165)>	<(s_3,0.06),(0.5381,0.472)>
A_{13}	<(s_3,-0.34),(0.4973,0.5151)>	<(s_5,-0.32),(0.696,0.2316)>	<(s_3,-0.38),(0.4765,0.5298)>
A_{14}	<(s_4,0.4),(0.6584,0.3097)>	<(s_3,-0.25),(0.5555,0.4095)>	<(s_3,0.38),(0.5678,0.4425)>
A_{15}	<(s_4,-0.29),(0.625,0.3388)>	<(s_1,0.24),(0.3356,0.6686)>	<(s_3,-0.05),(0.575,0.4485)>
A_{21}	<(s_3,0.24),(0.5421,0.4643)>	<(s_1,-0.18),(0.2729,0.711)>	<(s_3,0.03),(0.5587,0.4613)>
A_{22}	<(s_2,0.43),(0.5367,0.4169)>	<(s_4,0.34),(0.6462,0.3569)>	<(s_3,-0.45),(0.5331,0.4256)>
A_{23}	<(s_3,0.13),(0.5411,0.4692)>	<(s_5,-0.24),(0.6969,0.2362)>	<(s_3,0.22),(0.5542,0.4572)>
A_{31}	<(s_1,0.13),(0.3141,0.6833)>	<(s_4,-0.48),(0.6459,0.277)>	<(s_3,-0.05),(0.5919,0.3665)>
A_{32}	<(s_3,-0.06),(0.5236,0.4875)>	<(s_2,0.2),(0.4597,0.558)>	<(s_3,-0.43),(0.4935,0.5215)>
A_{33}	<(s_3,-0.46),(0.4771,0.5303)>	<(s_2,-0.39),(0.3708,0.6315)>	<(s_3,0),(0.5293,0.4818)>
A_{34}	<(s_4,0.07),(0.6443,0.3248)>	<(s_2,0.06),(0.4316,0.5776)>	<(s_3,0.38),(0.5722,0.4378)>
A_{41}	<(s_4,-0.45),(0.6299,0.3006)>	<(s_4,0.12),(0.6486,0.3615)>	<(s_3,0.3),(0.6041,0.358)>
A_{42}	<(s_3,-0.06),(0.5297,0.4838)>	<(s_4,-0.34),(0.6217,0.3304)>	<(s_4,-0.02),(0.6629,0.2643)>

A_{43}	<(s_5,-0.27),(0.7018,0.2226)>	<(s_3,-0.09),(0.5612,0.4116)>	<(s_3,0.17),(0.5589,0.4542)>

	K_4	K_5
A_{11}	<(s_2,0.17),(0.4547,0.5627)>	<(s_3,0.07),(0.5402,0.472)>
A_{12}	<(s_4,-0.43),(0.5878,0.422)>	<(s_3,0.06),(0.5299,0.4793)>
A_{13}	<(s_4,-0.48),(0.5773,0.4308)>	<(s_4,-0.16),(0.6249,0.3871)>
A_{14}	<(s_3,-0.35),(0.5167,0.5013)>	<(s_3,-0.15),(0.5171,0.4954)>
A_{15}	<(s_3,0.04),(0.5817,0.3714)>	<(s_3,0),(0.5251,0.4847)>
A_{21}	<(s_1,0.31),(0.357,0.6547)>	<(s_4,-0.23),(0.6431,0.292)>
A_{22}	<(s_3,-0.18),(0.5107,0.5003)>	<(s_2,-0.5),(0.3997,0.6196)>
A_{23}	<(s_3,-0.38),(0.5296,0.4939)>	<(s_3,-0.5),(0.4875,0.528)>
A_{31}	<(s_4,0.31),(0.6607,0.2913)>	<(s_4,-0.16),(0.6941,0.2114)>
A_{32}	<(s_2,0.47),(0.4977,0.5203)>	<(s_4,-0.09),(0.6124,0.3939)>
A_{33}	<(s_3,-0.07),(0.5413,0.4227)>	<(s_3,-0.34),(0.562,0.4068)>
A_{34}	<(s_1,0.12),(0.3122,0.6825)>	<(s_3,-0.35),(0.4889,0.5213)>
A_{41}	<(s_3,0.27),(0.5776,0.4375)>	<(s_3,0.36),(0.5591,0.4488)>
A_{42}	<(s_4,0.49),(0.6693,0.3003)>	<(s_2,0.45),(0.4957,0.5224)>
A_{43}	<(s_3,0.01),(0.5923,0.3734)>	<(s_3,0.11),(0.5914,0.3874)>

Step 2. Calculate the average fuzzy weight of each attribute based on the evaluations of DMs using linguistic variables and Equation (28), The calculation results are shown in Table 5.

Table 5. Weight of each attribute

		D_1	D_2	D_3	D_4	D_5	Average
A_1	A_{11}	M	L	MH	ML	MH	<(s_3,-0.2),(0.5016,0.5073)>
	A_{12}	MH	MH	M	VH	M	<(s_4,0),(0.6263,0.3314)>
	A13	H	ML	L	MH	H	<(s_3,0.4),(0.5813,0.4324)>
	A_{14}	L	H	VH	H	ML	<(s_4,-0.2),(0.6402,0.3277)>
	A_{15}	VL	VL	H	L	H	<(s_2,0.2),(0.5034,0.5261)>
A_2	A_{21}	L	L	L	M	M	<(s_2,-0.2),(0.3972,0.6119)>
	A_{22}	H	ML	H	ML	VL	<(s_3,-0.2),(0.5376,0.4816)>
	A_{23}	MH	MH	M	L	H	<(s_3,0.4),(0.568,0.4416)>
A_3	A_{31}	VL	L	MH	H	VH	<(s_3,0.2),(0.6015,0.3677)>
	A_{32}	L	M	VH	H	ML	<(s_3,0.4),(0.6021,0.363)>
	A_{33}	M	L	ML	M	MH	<(s_3,-0.4),(0.4769,0.5305)>
	A_{34}	MH	ML	MH	ML	ML	<(s_3,-0.2),(0.4966,0.5102)>
A_4	A_{41}	VL	VL	VL	H	M	<(s_2,-0.4),(0.4241,0.5985)>
	A_{42}	MH	M	MH	L	VL	<(s_2,0.4),(0.476,0.5373)>
	A_{43}	VL	ML	M	H	L	<(s_2,0.2),(0.4686,0.5502)>

Steps 3 to 6. Determine the fuzzy normalized decision matrix, and then compute the Pythagorean 2-tuple linguistic fuzzy weighted normalized matrix using equation(30)according to Step 3and Step 4.Afterwards, the fuzzy negative-ideal solution can be calculated based on equation(31) and (32). The results of these steps are presented in Table 6.

Table 6. P2TL Fuzzy weighted normalized matrix and negative-ideal solution

	K_1	K_2	K_3
A_{11}	<(s_2,-0.2267),(0.2761,0.6505)>	<(s_1,0.232),(0.2181,0.7121)>	<(s_2,-0.418),(0.2502,0.6779)>
A_{12}	<(s_2,-0.4267),(0.3167,0.5918)>	<(s_3,-0.2533),(0.4039,0.4461)>	<(s_2,0.04),(0.337,0.5551)>
A_{13}	<(s_2,-0.4927),(0.2891,0.6346)>	<(s_3,-0.348),(0.4046,0.4802)>	<(s_1,0.4847),(0.277,0.6444)>
A_{14}	<(s_3,-0.2133),(0.4215,0.4393)>	<(s_2,-0.2583),(0.3556,0.507)>	<(s_2,0.1407),(0.3635,0.5312)>

A_{15}	$<(s_1,0.3603),(0.3146,0.5998)>$	$<(s_0,0.4547),(0.1689,0.7746)>$	$<(s_1,0.0817),(0.2894,0.6498)>$
A_{21}	$<(s_1,-0.028),(0.2153,0.7136)>$	$<(s_0,0.246),(0.1084,0.831)>$	$<(s_1,-0.091),(0.2219,0.7124)>$
A_{22}	$<(s_1,0.134),(0.2885,0.6045)>$	$<(s_2,0.0253),(0.3474,0.5743)>$	$<(s_1,0.19),(0.2866,0.6092)>$
A_{23}	$<(s_2,-0.3737),(0.2665,0.6563)>$	$<(s_1,-0.2973),(0.1342,0.7655)>$	$<(s_2,-0.4247),(0.2597,0.665)>$
A_{31}	$<(s_1,-0.3973),(0.1889,0.7341)>$	$<(s_2,-0.1227),(0.3885,0.449)>$	$<(s_2,-0.4267),(0.356,0.5014)>$
A_{32}	$<(s_2,-0.334),(0.3153,0.5815)>$	$<(s_1,0.2467),(0.2768,0.6341)>$	$<(s_1,0.4563),(0.2971,0.6065)>$
A_{33}	$<(s_1,0.4993),(0.2529,0.6671)>$	$<(s_2,-0.0977),(0.3012,0.6166)>$	$<(s_1,0.3),(0.2298,0.6948)>$
A_{34}	$<(s_2,-0.1007),(0.3199,0.5817)>$	$<(s_1,-0.0387),(0.2143,0.7121)>$	$<(s_2,-0.4227),(0.2842,0.6341)>$
A_{41}	$<(s_1,-0.0533),(0.2671,0.6451)>$	$<(s_1,0.0987),(0.2751,0.6649)>$	$<(s_1,-0.12),(0.2562,0.6637)>$
A_{42}	$<(s_1,0.176),(0.2521,0.6747)>$	$<(s_1,0.464),(0.2959,0.6053)>$	$<(s_2,-0.408),(0.3155,0.5817)>$
A_{43}	$<(s_2,-0.2657),(0.3289,0.5807)>$	$<(s_1,0.067),(0.263,0.6487)>$	$<(s_1,0.1623),(0.2619,0.6682)>$
	K_4	K_5	NS
A_{11}	$<(s_2,-0.2127),(0.2823,0.641)>$	$<(s_1,0.3673),(0.2368,0.6885)>$	$<(s_1,0.232),(0.2181,0.7121)>$
A_{12}	$<(s_2,0.38),(0.3681,0.518)>$	$<(s_2,0.04),(0.3319,0.5607)>$	$<(s_2,-0.4267),(0.3167,0.5918)>$
A_{13}	$<(s_2,-0.0053),(0.3356,0.5813)>$	$<(s_2,0.176),(0.3632,0.5557)>$	$<(s_1,0.4847),(0.277,0.6444)>$
A_{14}	$<(s_2,-0.3217),(0.3308,0.576)>$	$<(s_2,-0.195),(0.331,0.5714)>$	$<(s_2,-0.3217),(0.3308,0.576)>$
A_{15}	$<(s_1,0.1147),(0.2928,0.6137)>$	$<(s_1,0.1),(0.2643,0.6684)>$	$<(s_0,0.4547),(0.1689,0.7746)>$
A_{21}	$<(s_0,0.393),(0.1418,0.8016)>$	$<(s_1,0.131),(0.2554,0.654)>$	$<(s_0,0.246),(0.1084,0.831)>$
A_{22}	$<(s_1,0.316),(0.2746,0.6513)>$	$<(s_1,-0.3),(0.2149,0.7258)>$	$<(s_1,-0.3),(0.2149,0.7258)>$
A_{23}	$<(s_2,-0.0847),(0.2805,0.6487)>$	$<(s_2,-0.0167),(0.2999,0.6215)>$	$<(s_1,-0.2973),(0.1342,0.7655)>$
A_{31}	$<(s_2,0.2987),(0.3974,0.4567)>$	$<(s_2,0.048),(0.4175,0.4169)>$	$<(s_1,-0.3973),(0.1889,0.7341)>$
A_{32}	$<(s_1,0.3997),(0.2997,0.6057)>$	$<(s_2,0.2157),(0.3687,0.5162)>$	$<(s_1,0.2467),(0.2768,0.6341)>$
A_{33}	$<(s_1,0.3303),(0.2016,0.7014)>$	$<(s_1,0.4473),(0.194,0.713)>$	$<(s_1,0.3303),(0.2016,0.7014)>$
A_{34}	$<(s_1,-0.4773),(0.155,0.7777)>$	$<(s_1,0.2367),(0.2427,0.6792)>$	$<(s_1,-0.4773),(0.155,0.7777)>$
A_{41}	$<(s_1,-0.128),(0.2449,0.6936)>$	$<(s_1,-0.104),(0.2371,0.6982)>$	$<(s_1,-0.128),(0.2449,0.6936)>$
A_{42}	$<(s_2,-0.204),(0.3186,0.594)>$	$<(s_1,-0.02),(0.236,0.6949)>$	$<(s_1,-0.02),(0.236,0.6949)>$
A_{43}	$<(s_1,0.1037),(0.2776,0.6324)>$	$<(s_1,0.1403),(0.2771,0.6382)>$	$<(s_1,0.067),(0.263,0.6487)>$

Step 7. Calculate the fuzzy weighted Euclidean distance (E_{d_i}) and Hamming distance (H_{d_i}) based on the above results and results are as follows:

E_{d1}=2.0753, E_{d2}=2.0753, E_{d3}=1.8011, E_{d4}=2.0161, E_{d5}=2.0736, H_{d1}=0.5991, H_{d2}=0.6248, H_{d3}=0.5199, H_{d4}=0.5820, H_{d5}=0.5986

Step 8: The relative assessment matrix can be calculated by using equation (35) to (37). As is shown in Table 8.

Table 7. Relative assessment matrix (RA)

	K1	K2	K3	K4	K5
K1	0.0000	0.1145	-0.3535	-0.0763	-0.0017
K2	-0.1145	0.0000	-0.4680	-0.1909	-0.1168
K3	0.3535	0.4680	0.0000	0.2771	0.3512
K4	0.0763	0.1909	-0.2771	0.0000	0.0741
K5	0.0017	0.1168	-0.3512	-0.0741	0.0000

Step 9: Calculate the value of AS_i by using equation (38).

AS_1=-0.3170, AS_2=-0.8902, AS_3=1.4498,
AS_4= 0.0641, AS_5=-0.3068

Step 10: According to the calculating results of AS_i ,we can rank all the alternatives. Obviously, the rank is $K_3 > K_4 > K_5 > K_1 > K_2$ and the best alternative among five alternatives is K_3 .

In order to show the validity of this method, we compare it with the result of P2TL-TODIM method. The rank of P2TL-TODIM is also $K_3 > K_4 > K_5 > K_1 > K_2$ .As can be seen, the ranking result of the fuzzy CODAS is completely consistent with the results of the fuzzy TODIM methods. Moreover, P2TL-CODAS model uses combination of two distance formulas, and is more accurate than the single distance formula.

6. Conclusion

In this research, we combine the CODAS method with P2TLNs information to deal with multi-criteria decision-making (MCDM) problems under uncertain environment. Firstly, we have a brief overview of the definition, operational laws, the distance and aggregation operators of P2TLSs.Then, the calculating steps of the CODAS model were given simply. Thereafter, we list the calculation steps of the extended CODAS model. This method uses linguistic variables which are defined by Pythagorean 2-tuple linguistic numbers to express DMs' assessments. A combination of Euclidean distances and Hamming distances is used to determine the desirability of alternatives with respect to a negative-ideal solution. Finally, a numerical example for financial management performance evaluation is proposed to prove the effectiveness of this method. The comparative analysis with P2TL-TODIM shows that the extension of CODAS method is efficient and consistent with the other methods. Further studies will carry out the application of the proposed model in many other uncertain and fuzzy environments [40-45].

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that they have no conflicts of interest.

Acknowledgments

This paper is supported by the Natural Science Foundation of China (No. 71571128).

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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2019.2917588, IEEE Access

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