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Optimal pricing and pre-sale policies for perishable product in an EOQ inventory model

LIANXIA ZHAO[1], LI ZHANG[1], JIANXIN YOU[2] AND CHUNYAN DUAN.[2]
[1]School of Management, Shanghai University, Shanghai, 200444, P.R. China (e-mail: zhaolianxia@staff.shu.edu.cn)
[2]The Economic and Management School, Tongji University, Shanghai, 200092, P.R. China

Corresponding author: Lianxia Zhao (e-mail: zhaolianxia@staff.shu.edu.cn).

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ABSTRACT This paper considers an optimal pricing and pre-sale policy of an EOQ inventory model for perishable products, in which the demand of pre-sale period is dependent on price and pre-sale time, and the demand of spot-sale period is dependent on the price and stock level. Several theoretical results are derived to demonstrate the existence and uniqueness of the solution. For comparison, we also propose a benchmark inventory model in which all the products are to be sold in spot-sale period, and the optimal policy of pricing and economic order quantity is provided. Numerical experiments are carried out and sensitivity analysis of the parameters is presented to illustrate the theoretical results for the proposed inventory model, and the comparative analysis of the two models is provided to show if the pre-sale policy is better off the spot-sale policy under some conditions, vice versa.

INDEX TERMS Inventory, deterministic demand, pre-sale policy, spot-sale policy, perishable products; pricing

I. INTRODUCTION

DETERIORATION D of products is a realistic phenomenon, perishable products such as fruits, medicines, fashion apparels, hi-tech products, etc. inevitably cause decay (expiration, devaluation) during the sale period. There are many literatures to consider the inventory policy for perishable products in recent years, see [1], [2], for comprehensive reviews. However, few articles considered pre-sale policy for perishable products, that is, perishable products will be sold before coming to the market. It is well known that the deterioration of a product results in dropping its quality and consumer's buying enthusiasm which will directly influence retailer's revenue. Therefore, it is an interesting topic for both retailers and researchers to study and design marketing policies of the perishable products.

A reasonable sale policy is necessary to speed up sale pace for perishable products. Pre-sale policy means that products are sold to customers in advance of its spot-sale time, and price discount in the pre-sale period can stimulate the consumer's purchasing desire. In reality, some people are willing to wait for some time to gain a price discount, so pre-sale policy is an effective strategy for retailers to reduce the deteriorating cost as well as holding cost, and it also reduces the uncertainty of demand, because the orders are committed in advance.

In this paper, we consider an inventory problem for perishable products with a proposed model and a benchmark model, respectively. Assume that the potential market demand is certain, and the basic demand is distributed proportional to selling time in different periods. The proposed model is composed of two periods: pre-sale period and spot-sale period, in which the total demand in the pre-sale period depends on price and pre-sale time, while the demand rate in the spot-sale period depends on price and the instantaneous stock level. The benchmark model only contains spot-sale period to decide the price of products, which can be looked as a special case of general model.

The main contributions in this paper can be summarized as follows. Firstly, we propose a division of total potential demand according to pre-sale time and spot-sale time, and also construct two demand patterns based on actual situation. Secondly, we provide the optimal conditions of the considered inventory model. Thirdly, by comparing the profit of the proposed model with that of the benchmark model, we reveal

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the fact that how the main parameters affect the optimal decisions of the retailer.

The rest of the paper is organized as follows. We review the related literature in Section 2. Section 3 provides the notations and assumptions. We formulate a two-period inventory model in Section 4. In Section 5, we provide the existence and uniqueness, and a benchmark inventory model is also considered for comparison. Section 6 presents some numerical examples to highlight the theoretical results, and managerial insights and sensitivity analysis with respect to main parameters are also provided to explain their impact on the optimal policy. A brief remarking of the paper and future research is given in final Section.

II. LITERATURE REVIEW

This paper is mainly related to the literature on pre-sale, perishable inventory management, joint optimization of pricing and economic ordering quantity. Many researchers have intensively considered inventory models for perishable product in recent years.

Firstly, we review the research on pre-sale policy. [3] modeled the decisions under advance sales discount and two-echelon trade credits by using EOQ. [4] also considered advance selling problems. They determined how the comparison of these price discrimination strategies depends on price commitment, and derived the profit maximizing selling strategy when aggregate demand is certain. Considering the uncertain supply and demand of the products, [5] examined three selling strategies of a manufacturer under uncertain supply and demand by modelling the first two strategies as single-period Stackelberg games, and modelling the last strategy as a two-period dynamic Stackelberg game. [6] studied an inventory model starting with shortage for Weibull-distributed deteriorating items, in which the shortage order are allowed partial backlogging. While, [7] considered an EOQ inventory model with pre-sale policy for deteriorating items, in which all the pre-sale orders are fully backlogged with waiting-time dependent rebate. [8] investigated a continuous review inventory model with order quantity, reorder point, backorder price discount, process quality, and lead time as decision variables. More literatures about advance selling strategy can see [9], [10] and [11], etc. However, all these above researches assumed that the demand is uncertain in supply chain environment. In this setting, in order to reduce the risk of deterioration, we consider the inventory control with pre-sale for perishable, and the actual demand depends on pre-sale time and price which is not known to the retailer in advance.

Secondly, we survey research involving demand patterns and inventory decisions for perishable products. [12] first proposed an inventory model with a time-sensitive demand rate pattern. Whereafter, there are numerous inventory control papers studied on such demand patterns. [13] developed an inventory model for deteriorating seasonal products with a ramp-type time-dependent demand rate over the season. While, relative to the constant deteriorating rate of product, [14] considered a variable deteriorating rate of perishable product. He studied an EOQ model with ramp-type demand, Weibull distribution deterioration, and partial backlogging. In addition, basing on product life cycle theory, [15], [16] considered an inventory model for deteriorating items with a trapezoidal-type time-dependent demand rate. Considering the consumer's perception, stock level/ shelf space of product is also important factor impact on consumer's desire to order. [17] presented a deterministic EOQ model with stock-dependent demand rate where a supplier gives a retailer a price discount. For perishable product, consumer also care about the freshness of product before they ordered it. [18] studied the inventory management decisions of perishable goods in a deterministic setting with age-dependent demand rate. [19] applied an EOQ model in supply chains with partial downstream delayed payment and partial upstream advance payment for a deteriorating item.

Thirdly, pricing is an important factor in inventory control for perishable product due to its deterioration. [20] introduced an inventory model for a deteriorating item with the demand to be dependent on the price of the item. While, more researchers considered the demand rate related both price and time, simultaneously. Such as, [21] investigated an inventory system with the demands depended on price and waiting time. [22] formulated an inventory model for perishable items with price- and time-dependent demand to determine the optimal pricing, order quantity. They showed that the profit function is strictly pseudo-concave and provided a method of obtaining the optimal policy. However, not all problems about inventory models have the exact optimal solution, [23] considered a production-inventory problem for a seasonal deteriorating product in which the demand is defined as price- and time-dependent. They constructed a simple algorithm to determine efficiently an optimal price for the non-concave profit function. [24] used a Tabu Search method to analyze an EOQ model that incorporates pricing and space-allocation decisions for perishable products in which the demand rate depends on the selling price and the displayed stock levels. Some other related literatures include [25], [26], [27] and [28], etc.

In summary, all the models cited above have considered the key factors in isolation. However, for the inventory model with perishable product, it is more realistic to consider the pre-sale policy, demand patterns and pricing, etc, in whole. In this paper, we construct the pricing and pre-sale model of perishable products, in which we assume that the demand is dependent on the sale time, the price and the stock level.

III. NOTATIONS AND ASSUMPTIONS

The following notations and the assumptions are used throughout the paper.

A. NOTATIONS

- T the fixed spot-sale planning horizon.
- p the selling price of a unit (decision variable).
- w the purchase price of a unit.

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- θ the deterioration rate of the item.
- c the advanced time sensitivity of price, $0 < c \leq 1$.
- c_h the holding cost per unit per unit time.
- c_d the deterioration cost per unit per unit time.
- L the length of the pre-sale horizon, and suppose $L = \epsilon T (0 \leq \epsilon \leq \frac{1}{2})$.
- $I(t)$ the instantaneous inventory level on hand at time $[0, T]$.

B. ASSUMPTIONS

(1) The potential total market demand is assumed to be 1, and ϵ and $1-\epsilon$ denotes the potential total quantity in pre-sale period and spot-sale period, respectively. Without loss of generality, we assume that $0 \leq \epsilon \leq \frac{1}{2}$

(2) We assume that the pre-sale total quantity is $Q_0 = \epsilon - \delta\epsilon p_0 - L$, where p_0 is the preferential price and $0 < \delta \leq 1$.

(3) During the spot-sale period $[0, T]$, the demand rate depends on the selling price and the instantaneous inventory level, that is, $D(t,p) = (\frac{1-\epsilon}{T} - \delta p) + \alpha I(t)$, $0 \leq t \leq T$, where α is the inventory elasticity of product and $\frac{1-\epsilon}{T}$ represents the mean demand rate.

IV. MODEL FORMULATION

According to the notations and assumptions mentioned above, we consider an inventory model for perishable product which is composed of pre-sale period and spot-sale period. The behavior of inventory system is depicted in Figure 1.

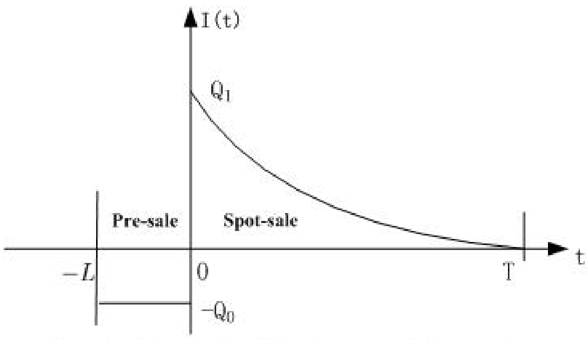

Figure 1. The graphic of the sale model during a cycle

A. PRE-SALE PERIOD

Due to the deterioration of product, retailer can reduce the deteriorating loss by adapting pre-sale policy before products come to the market. The retailer encourages customers to order the product in advance, then it will be postponed to deliver. Therefore, the total profit of the retailer in the pre-sale period is

$$\pi_1(p_0) = (p_0 - w)(\epsilon - \delta\epsilon p_0 - L) \tag{1}$$

B. SPOT-SALE PERIOD

For the perishable product, consumers are attracted to a large amount of products on the shelf, which implies freshness and popularity, meanwhile, price is an essential factor to influence the demand in an competitive market. Thus, in spot-sale period, we suppose that the demand depends on both price and inventory level. Figure 1 shows the inventory level $I(t)$ over the spot-sale period $[0, T]$. The instantaneous inventory level is given by

$$\frac{dI(t)}{dt} = -\theta I(t) - (\frac{1-\epsilon}{T} - \delta p) - \alpha I(t) \tag{2}$$

with the boundary condition $I(T) = 0$.

Then, we have

$$I(t) = \frac{1}{\alpha+\theta}(\frac{1-\epsilon}{T} - \delta p)(e^{(\theta+\alpha)(T-t)} - 1). \tag{3}$$

The total holding cost is obtained as follows.

$$\begin{aligned} HC &= c_h \int_0^T I(t)dt \\ &= \frac{c_h}{(\alpha+\theta)^2}(\frac{1-\epsilon}{T} - \delta p)[e^{(\theta+\alpha)T} - (\alpha+\theta)T - 1] \end{aligned} \tag{4}$$

In spot-sale period, the deteriorating cost is obtained

$$\begin{aligned} DC &= c_d\theta \int_0^T I(t)dt \\ &= \frac{c_d\theta}{(\alpha+\theta)^2}(\frac{1-\epsilon}{T} - \delta p)[e^{(\theta+\alpha)T} - (\alpha+\theta)T - 1] \end{aligned} \tag{5}$$

The revenue of the product sale during the spot-sale period is given as

$$\begin{aligned} TR &= p\int_0^T D(t,p)dt \\ &= p[(\frac{1-\epsilon}{T} - \delta p)T + \alpha \int_0^T I(t)dt] \end{aligned} \tag{6}$$

Therefore, the total profit of the retailer over spot-sale period is given as

$$\begin{aligned} \pi_2(p) &= (p-w)(\frac{1-\epsilon}{T} - \delta p)T \\ &+ [\alpha p - (\alpha+\theta)w - c_h - \theta c_d]\int_0^T I(t)dt \end{aligned} \tag{7}$$

Using the above discussion, the problem is to determine the optimal pre-sale time and optimal pricing policy to maximize the total profit over the sale time.

$$\begin{aligned} Max\ \pi(p_0, p) &= \pi_1(p_0) + \pi_2(p) \\ &= (p_0 - w)(\epsilon - \delta\epsilon p_0 - \epsilon T) \\ &+ [(\alpha p - (\alpha+\theta)w - c_h - \theta c_d) \\ &+ (p-w)T](\frac{1-\epsilon}{T} - \delta p) \\ s.t.\ \ w \leq p \leq \frac{1-\epsilon}{\delta T}&, w \leq p_0 \leq \frac{1-T}{\delta}. \end{aligned} \tag{8}$$

where $\phi = \frac{e^{(\theta+\alpha)T} - (\alpha+\theta)T - 1}{(\alpha+\theta)^2}$.

The total order quantity is

$$\begin{aligned} Q &= \epsilon(1 - \delta p_0 - T) \\ &+ \frac{1}{\alpha+\theta}(\frac{1-\epsilon}{T} - \delta p)(e^{(\theta+\alpha)T} - 1). \end{aligned} \tag{9}$$

V. MODEL ANALYSIS

A. BENCHMARK MODEL

We consider a basic inventory model only with spot-sale period. For potential market demand 1, the demand rate is modified as $D(t, p_b) = (\frac{1}{T} - \delta p_b) + \alpha I(t)$. Thus, the instantaneous inventory level is governed by

$$\frac{dI(t)}{dt} = -\theta I(t) - (\frac{1}{T} - \delta p_b) - \alpha I(t) \tag{10}$$

with the boundary condition $I(T) = 0$. Then, we have

$$I(t) = \frac{1}{\alpha + \theta}(\frac{1}{T} - \delta p_b)(e^{(\theta+\alpha)(T-t)} - 1), \tag{11}$$

and the order quantity is $Q_0 = \frac{1}{\alpha+\theta}(\frac{1}{T} - \delta p_b)(e^{(\theta+\alpha)T} - 1)$.

Similar to the above section, we get the following proposition.

Proposition 1: For benchmark model, $\pi(p_b)$ is a concave function with respect to p_1 and the optimal price is $p_b^* = \frac{T+\alpha\phi+\delta T[wT+(\theta c_d+c_h+\alpha w)\phi]}{2\delta T(\alpha\phi+T)}$.

Proof 1: Similar to (4)-(7) and (12), we can obtain the total profit function

$$\begin{aligned}\pi(p_b) =& \frac{(1-\delta p_b T)\left[(\alpha p_b - (\alpha+\theta)w - c_d\theta - c_h)\phi\right]}{T} \\ &+ (1-\delta p_b T)(p_b - w)\end{aligned} \tag{12}$$

Since the necessary condition to find the optimal price p_b^* is

$$\begin{aligned}\frac{d\pi(p_b)}{dp_b} =& \frac{\alpha\phi + T}{T} \\ &+ \delta[(c_d\theta + c_h + (\alpha+\theta)w)\phi - 2p_b(\alpha\phi + T) + Tw].\end{aligned} \tag{13}$$

where $\phi = \frac{e^{(\alpha+\theta)T} - (\alpha+\theta)T - 1}{(\alpha+\theta)^2}$.

Taking the second derivative of $\pi(p_b)$ with respect to p_b, we have $\frac{d^2\pi(p_b)}{dp_b^2} = -2(\alpha\phi + T)\delta < 0$, which means that $\pi(p_b)$ is a concave function with respect to p_b.

Let

$$\begin{aligned}f(p_b) =& \alpha\phi + T \\ &+ \delta T[(c_d\theta + c_h + (\alpha+\theta)w)\phi - 2p_b(\alpha\phi+T) + Tw]\end{aligned}$$

then, we have

$$\frac{df(p_b)}{dp_b} = -2\delta T(\alpha\phi + T) < 0,$$

and

$$\begin{aligned}f(w) =& (1 - \delta Tw)(\alpha\phi + T) + \delta\phi T(c_d\theta + c_h + (\theta+\theta)w) \\ &> 0\end{aligned}$$

Thus, from $\frac{d\pi(p_b)}{dp_b} = 0$, we have

$$p_b^* = \frac{T + \alpha\phi + \delta T[wT + (\theta c_d + c_h + (\alpha+\theta)w)\phi]}{2\delta T(\alpha\phi + T)}.$$

Substituting the optimal price p_b^* into (13), the total profit of the retailer can be obtained as

$$\pi_{p_b^*} = \frac{[\alpha\phi + T - \delta w T^2 - \delta T(c_d\theta + c_h + (\alpha+\theta)w)\phi]^2}{4\delta T^2(\alpha\phi + T)}.$$

B. PROPOSED MODEL

We consider a more general model combining with pre-sale strategy to reduce deteriorating cost of the product. we prove that the considered total profit function is strictly concave with respect to price, and find the optimal pricing of the proposed model.

Proposition 2: For the inventory model with pre-sale stage, we have (i) the total profit $\pi(p_0, p)$ is concave in p_0 and p, and (ii) there exist optimal prices as follows: $p_0^* = \frac{1-T+\delta w}{2\delta}$ and $p^* = \frac{(1-\epsilon)(T+\alpha\phi)+\delta T[wT+(c_d\theta+c_h+(\alpha+\theta)w)\phi]}{2\delta T(T+\alpha\phi)}$.

Proof 2: (i) Taking the first-order partial derivative of $\pi(p_0, p)$ with respect to p_0 and p, respectively, we have

$$\frac{\partial\pi(p_0,p)}{\partial p_0} = (1-T)\epsilon - 2\delta p_0 + \delta w \tag{14}$$

$$\begin{aligned}\frac{\partial\pi(p_0,p)}{\partial p} =& \delta[\phi(c_d\theta + c_h + \alpha w) - 2p(\alpha\phi + T) + Tw] \\ &+ \frac{(1-\epsilon)(\alpha\phi + T)}{T}\end{aligned} \tag{15}$$

Taking the second-order partial derivative of $\pi(p_0, p)$ with respect to p_0 and p, respectively, we have

$$\frac{\partial^2\pi(p_0,p)}{\partial p^2} = -2\delta(T + \alpha\phi) < 0,$$

$$\frac{\partial^2\pi(p_0,p)}{\partial p_0^2} = -2\delta\epsilon < 0,$$

$$\frac{\partial^2\pi(p_0,p)}{\partial p_0 \partial p} = 0$$

Thus, the Hessian matrix of $\pi(p_0, p)$ is

$$H_\pi = \begin{bmatrix} -2\delta(T + \alpha\phi) & 0 \\ 0 & -2\delta\epsilon \end{bmatrix}$$

Obviously, H_π is negative definite and $\pi(p_0, p)$ is concave in p_0 and p.

From $\frac{\partial\pi(p_0,p)}{\partial p_0} = 0$ and $\frac{\partial\pi(p_0,p)}{\partial p} = 0$, we have

$$p_0^* = \frac{1 - T + \delta w}{2\delta},$$

$$p^* = \frac{(1-\epsilon)(T+\alpha\phi) + \delta T[wT + (c_d\theta + c_h + (\alpha+\theta)w)\phi]}{2\delta T(T + \alpha\phi)}.$$

Based on Proposition 2, the total profit of the inventory system is given as

$$\begin{aligned}\pi(p_0^*, p^*) =& \frac{\epsilon[T - 1 + \delta w]^2}{4\delta} \\ &+ \frac{[(\alpha\phi + T)(1-\epsilon) - \delta\phi T(c_d\theta + c_h + \alpha w + \theta w) - \delta T^2 w]^2}{4\delta T^2(\alpha\phi + T)}.\end{aligned} \tag{16}$$

C. COMPARISON OF THE PRE-SALE AND SPOT-SALE SETTINGS

We discuss the optimization due to pre-sale and spot-sale settings which will impact on retailer's decision.

Proposition 3: When the retailer adopts pre-sale strategy, the optimal prices of the different stages satisfy $p_0^* < p^* < p_b^*$.

Proof 3: From above analysis and the assumption $0 \leq \epsilon \leq \frac{1}{2}$, we have

$$p^* - p_0^* = \frac{1-\epsilon-\epsilon T+\epsilon T^2}{2\delta T} + \frac{\phi(c_d\theta + c_h + \theta w)}{2(\alpha\phi + T)} > 0$$

and

$$p^* - p_b^* = -\frac{\epsilon}{2\delta T} < 0.$$

Thus, we have $p_0^* < p^* < p_b^*$.

Proposition 3 indicates that retailer adopts either pre-sale policy or only spot-sale policy, the price of the product in different stage satisfied with $p_0^* < p^* < p_b^*$.

Proposition 4: The retailer's strategy depends on market sale ϵ: i) when $\epsilon > \epsilon_0$, the retailer chooses pre-sale policy; ii) when $\epsilon \leq \epsilon_0$, the retailer chooses spot-sale policy, where $\epsilon_0 = 2 - \frac{2\delta T(\theta c_d + c_h + \alpha w + \theta w)\phi + 2\delta w T^2 + T^2(T-1+\delta w)^2}{\alpha\phi + T}$.

Proof 4: From above analysis, we have

$$\pi_{p_0^*,p^*} - \pi_{p_b^*} = \frac{\epsilon(\alpha\phi+T)(2\delta wT+\epsilon-2)}{4\delta T^2} + \frac{\epsilon[2\delta\phi(c_d\theta+c_h+\theta w)+T(T+\delta w-1)^2]}{4\delta T}.$$

Solving the equation $\pi_{p_0^*,p^*} - \pi_{p_b^*} = 0$ with respect to ϵ, we have

$$\epsilon_0 = 2 - \frac{2\delta T(\theta c_d + c_h + \alpha w + \theta w)\phi}{\alpha\phi + T} - \frac{2\delta w T^2 + T^2(T-1+\delta w)^2}{\alpha\phi + T}.$$

Obviously, if $\epsilon > \epsilon_0$, then $\pi_{p_0^*,p^*} > \pi_{p_b^*}$ which means that the retailer chooses pre-sale policy, and if $\epsilon \leq \epsilon_0$, then $\pi_{p_0^*,p^*} \leq \pi_{p_b^*}$ which means that the retailer chooses spot-sale policy.

Proposition 5: The retailer's strategy depends on the deteriorating rate θ: i) when $\theta > \theta_0$, the retailer chooses pre-sale and spot-sale policies; ii) when $\theta \leq \theta_0$, the retailer chooses spot-sale policy, where $\theta_0 = \frac{(2+\alpha T)(2-\epsilon-2\delta wT)-2T(T+\delta w-1)^2-\delta T^2 c_h}{2\delta T^2(c_d+w)}$.

Proof 5: Since $\phi = \frac{e^{(\theta+\alpha)T}-(\alpha+\theta)T-1}{(\alpha+\theta)^2}$, we use $\frac{T^2}{2}$ to approximately instead of ϕ. From above analysis, we have

$$\pi_{p_0^*,p^*} - \pi_{p_b^*} = \frac{\epsilon(\alpha T+2)(2\delta wT+\epsilon-2)}{8\delta T} + \frac{\epsilon}{4\delta}[\delta T(c_d\theta+c_h+\theta w)+(T+\delta w-1)^2]$$

Solving the equation $\pi_{p_0^*,p^*} - \pi_{p_b^*} = 0$ with respect to θ, we have

$$\theta_0 = \frac{(2+\alpha T)(2-\epsilon-2\delta wT)-2T(T+\delta w-1)^2-\delta T^2 c_h}{2\delta T^2(c_d+w)}.$$

Therefore, if $\theta > \theta_0$, then $\pi_{p_0^*,p^*} > \pi_{p_b^*}$ which means that the retailer chooses pre-sale policy; and if $\theta \leq \theta_0$, then $\pi_{p_0^*,p^*} \leq \pi_{p_b^*}$ which means that the retailer chooses spot-sale policy.

From Proposition 4 and 5, we know that the retailer's sale strategy depends on the market sale and the deteriorating rate. If the market sale ϵ (the deteriorating rate, θ) more than a threshold, he will adopt both the pre-sale and spot-sale policies; Otherwise, he only adopt spot-sale policy.

VI. NUMERICAL STUDY

We illustrate the theoretical results and propose some managerial insights on main parameters involving the inventory model. We also will examine the impact of the coefficients ϵ, θ, δ and T on the retailer's optimal decision. All results are based on values of some parameters (w = \$3.0/unit, $\alpha = 0.3$, c_d = \$0.4 and c_h = \$0.25). The other parameters are adjusted according to their sensitivity analysis.

Figure 1 shows a comparison of the profits and prices, respectively, under the pre-sale policy and spot-sale policy at different potential market portion. We observe that there exists a threshold of market portion value ϵ_0 such that if $\epsilon \leq \epsilon_0$, the retailer would like to adopt spot-sale policy and if $\epsilon > \epsilon_0$, the retailer would like to adopt pre-sale policy so as to maximize his system profit. Under pre-sale policy, when ϵ increases, the retailer's profit increases, synchronously. It means that if the retailer can obtain more advance order, he should positively adopt pre-sale policy for perishable product. Although the price (p) in normal sale stage decreases with increasing of ϵ, the total profit still increases.

Figure 2 shows a comparison of the profits and prices, respectively, under the pre-sale policy and spot-sale policy at different deteriorating rate. We observe that there exists a threshold of deteriorating rate θ_0 such that if $\theta \leq \theta_0$, the retailer would like to adopt spot-sale policy and if $\theta > \theta_0$, the retailer would like to adopt pre-sale policy so as to maximize his system profit. For both sale policies, when θ increases, the retailer's profit decreases, synchronously. However, the retailer's profit has more sensitivity to the change of θ under spot-sale policy. We also find that the prices (p, p_b) increase with increasing of θ either pre-sale policy or spot-sale policy. It means that the retailer should adopt pre-sale policy for the highly perishable product to obtain much more profit.

Figure 3 shows a comparison of the profits and prices, respectively, under the pre-sale policy and spot-sale policy at different price sensitivity parameter δ. We observe that there exists a threshold of deteriorating rate δ_0 such that if $\delta \leq \delta_0$, the retailer would like to adopt spot-sale policy and if $\delta > \delta_0$, the retailer would like to adopt pre-sale policy so as to maximize his system profit. For both sale policies, when δ increases, the retailer's profit decreases, synchronously. However, the retailer's profit has more sensitivity to the change of δ under pre-sale policy. We also find that the prices (p_0, p, p_b) decrease with increasing of δ either pre-sale policy or spot-sale policy, and the price p_0 has more sensitivity to the change of δ for low δ. It means that the retailer should adopt pre-sale policy to obtain much more profit for high δ, and adopt spot-sale policy to obtain much more profit for low δ.

Figure 4 shows a comparison of the profits and prices, respectively, under the pre-sale policy and spot-sale policy at different planing horizon, T. We observe that there exists a threshold of planning horizon T_0 such that $T \leq T_0$, the retailer would like to adopt spot-sale policy and if $T > T_0$, the retailer would like to adopt pre-sale policy so as to maximize his system profit. For both sale policies, when

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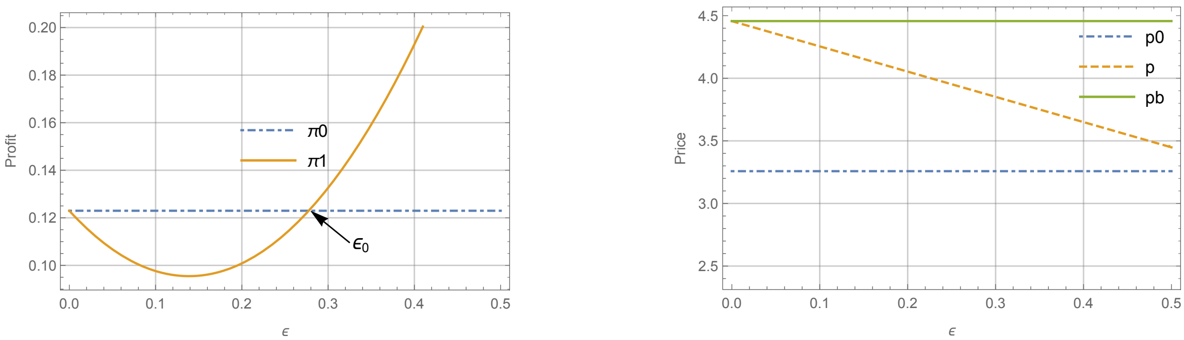

Fig. 1. Comparison of the profits and prices at different portions. ($\theta = 0.04$, $\alpha = 0.3$, $\delta = 0.32$, $T = 0.6$).

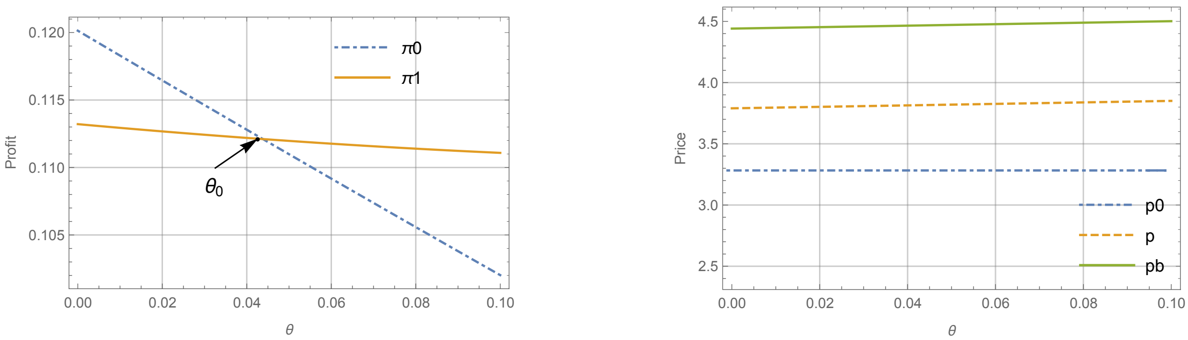

Fig. 2. Comparison of the profits and prices at different deteriorating rate ($\theta = 0.04$, $\alpha = 0.3$, $\delta = 0.32$, $T = 0.6$).

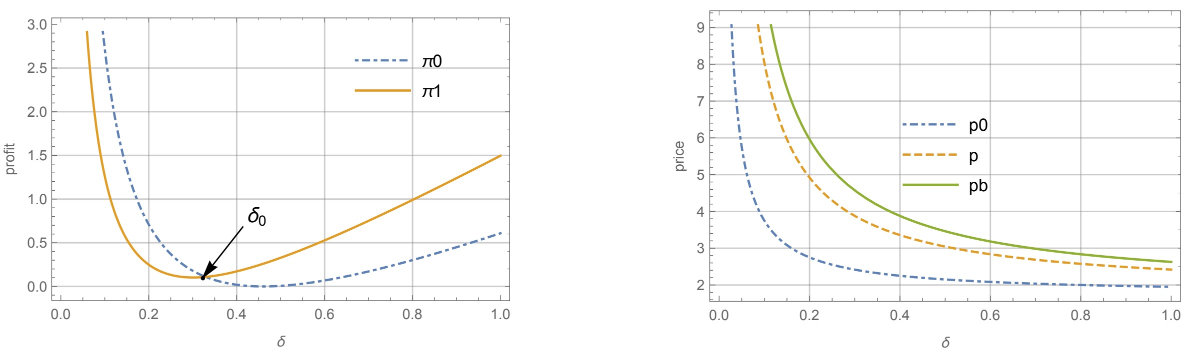

Fig. 3. Comparison of the profits and prices at different price sensitivity($\theta = 0.04$, $\alpha = 0.3$, $\delta = 0.32$, $T = 0.6$).

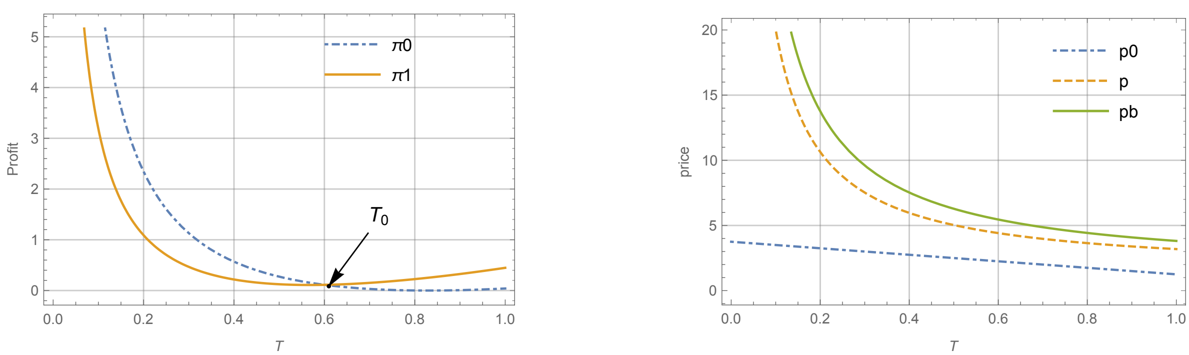

Fig. 4. Comparison of the profits and prices at different planning horizon ($\theta = 0.04$, $\alpha = 0.3$, $\delta = 0.32$, $T = 0.6$).

T increases, the retailer's profit decreases, synchronously. However, the retailer's profit has more sensitive to the change of T under spot-sale policy and the profit increasingly trends for long T. We also find that the price (p_0, p, p_b) decrease with increasing of T either pre-sale policy or spot-sale policy, however, p, p_b has more sensitive to the change of T. It means that the retailer should adopt spot-sale policy to obtain much more profit for short T, and adopt pre-sale policy to obtain much more profit for long T.

VII. CONCLUSION

In this paper, we have proposed an optimal pricing and pre-sale policy for an EOQ inventory model with perishable products in which the inventory system is composed of a pre-sale period and a spot-sale period. We assume that the total demand quantity depends on product's price and pre-sale time in pre-sale period, while the demand rate of spot-sale period depends on product's price and stock level. As for comparison, we propose a benchmark inventory model which only includes spot-sale period. Several theoretical results have been derived to demonstrate the existence and uniqueness of the solution. We also obtain that the retailer's optimal sale strategy for different influent parameters (ϵ, δ, θ and T). Numerical experiments of the main parameters are also carried out to illustrate the theoretical results. Based on the statistical numerical experiments, we can find that potential demand portion ϵ, price-elasticity coefficient δ, deteriorating rate θ and planning horizon T have a significant impact on total average profit of the retailer's inventory system.

In this setting, we only considered an inventory model with a perishable product, single cycle and determined demand rate. However, there are some problems to be considered in the future, such as, for considering multiple products, multiple cycle and stochastic demand in inventory setting, etc.

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LIANXIA ZHAO was born in Yinshui County, Shandong Province, China in 1977. She received the B.S. degree in mathematics and information science from Yantai University, Shandong, in 2007, M.S. degree in applied mathematics from Shanghai University, Shanghai, in 2004 and the Ph.D. degree in management science and engineering from Shanghai University, Shanghai, in 2014.

She is now an associate professor in the School of Management, Shanghai University. She has published more than 10 articles, and has undertaken 1 fund from national natural science foundation of China. Her research interests include supply chain management, operations management.

LI ZHANG was born in Longnan City, Gansu Province, China in 1993. He received the B.S. degree in information management and information system from Northwest Normal University, Lanzhou, in 2017. From 2017 to now, he was a postgraduate in management science and engineering from Shanghai University, Shanghai.

JIANXIN YOU was born in Suzhou , Jiangsu Province, China in 1962. He received his B.S. degree, M.S. degree Ph.D. degree in management science and engineering from Tongji University, in 1984,1992 and1999 respectively.

He is now a professor in the school of Economics and management, Tongji University. He has published more than 100 articles, and has undertaken more than 30 provincial or ministerial funds. His research interests include management theories and methods, supply chain management, operations management, industrial engineering and quality management.

CHUNYAN DUAN was born in Shangcai County, Henan Province, China in 1987. She received the B.S. in Information and Computation Science from Henan University of Technology, Zhengzhou, in 2009. She was being a postgraduate from September 2010 to January 2012 and was recommended to be a Ph.D. candidate in March 2012 in advance. She received the Ph.D. degree in Management Science and Engineering from Tongji University, Shanghai, in 2016.

She was being a senior fellow from February 2017 to February 2018 and a visiting PhD student from August 2014 to August 2015 at University of Washington, USA. She is currently a joint postdoctoral fellow at the School of Economics and Management, Tongji University, China, and the Department of Industrial Engineering, University of Arkansas, USA. She is the author of more than 10 articles, and she has hosted a National Natural Science Foundation in China and an International Postdoctoral Exchange Fellowship Project. Her research interests include data analysis, machine learning, mathematics modeling and management decision.