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Optimization of Angle-of-Arrival Estimation Via Real-Valued Sparse Representation With Circular Array Radar

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ABSTRACT This paper suggests a real-valued sparse representation method using a unitary transformation that can convert complex-valued manifold matrices from uniform circular array into real ones. Because of this transformation, the computational complexity is modified. Simulation results confirmed the effectiveness of the proposed method with a circular array radar.

INDEX TERMS Angle of arrival (AOA), array signal processing, uniform circular array (UCA).

I. INTRODUCTION

Source localization has been an active research field due to its fundamental role in many signal processing areas ranging from radar and sonar to acoustic tracking. In array signal processing, where arrays of sensors are typically employed for the sampling of the spatial field, the problem of source localization is usually referred to as direction-of-arrival (DOA) or angle-of-arrival (AOA) estimation. The classical array processing methods can be divided into two main categories, the parametric methods which are based on the maximum likelihood paradigm and the spectral based approaches often referred to as non-parametric approaches [1]. Among them the subspace-based method of multiple signal classification (MUSIC) stands as a powerful technique to the problem of spectral analysis and system identification. However, MUSIC results in decreased performance when the incoming sources are correlated or coherent [1].

Recently, a number of sparse representation methods have emerged [2]–[5], which not only greatly improve the AOA resolution performance, but also can be applied to small number of samples and highly correlated (or coherent) signals. The concept of spatial sparsity for DOA estimation was first introduced in [2], where it was shown that the source localization problem can be cast as a sparse recovery problem in a redundant dictionary using the ℓ_1-SVD method. Under certain assumptions ℓ_1-SVD can achieve super-resolution even in the coherent sources scenario. Both the subspace-type methods and the sparse representation methods generally require complex computations: subspace-type methods need complex computations when performing the eigen-decomposition and DOA searching, while sparse representation methods require complex linear programming. Many studies have been made to reduce the computational complexity of the calculations in this manner [5]–[10]. To make the subspace-based algorithm real-valued, Huarng and Yeh [6] proposed a unitary transformation method which, however, only applies to uniform linear array (ULA). It is known that UCA has several advantages over ULA, for example, using UCA the elevation angle and azimuth over 360° can be obtained. To obtain real-valued DOA estimation algorithms in the circular array scenario [7] developed this method for the case of UCA.

In this paper, we will present a real-valued sparse representation AOA estimation method, which will significantly reduce the computational complexity at least by a factor of four without sacrificing its accuracy.

This paper is organized as follows: First, the UCA signal model is presented. Next, we introduced Real-valued transformation with UCA in Section III. Our real-valued sparse representation method for DOA estimation is developed in Section IV. In Section V, our modified results of simulation for circular array are demonstrated. Eventually, Section VI concludes these findings.

II. SIGNAL MODEL

Consider a pulse Doppler radar consists of uniform circular array (UCA) situated on an airborne platform as shown in figure 1. The circular array consists of M antenna elements, is used to transmit and receive at any one time. The antennas are distributed uniformly over a circle with radius r.

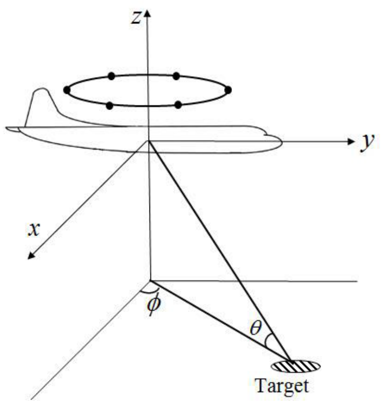

FIGURE 1. Geometry of airborne radar with uniform circular array.

Suppose the airborne radar transmits a burst of pulses in a coherent process interval. The N (N < M) narrow band far field signals impinging on the UCA of M elements from directions $(\theta_1, \phi_1), \ldots, (\theta_N, \phi_N)$, the received signal at the antenna array can be described as

$$x(t) = \sum_{n=1}^{N} a(\theta_n, \phi_n)s_n(t) + n(t) = As(t) + n(t) \qquad (1)$$

where $x(t)$ is the element space data matrix, $s(t)$ is the source matrix and $n(t)$ is the noise matrix. The noise is modeled as a stationary second-order ergodic, zero-mean spatially and temporally white circular complex Gaussian process. A is the array manifold matrix of size $M \times N$, $a(\theta_n, \phi_n) = [a_1(\theta_n, \phi_n), a_2(\theta_n, \phi_n), \ldots, a_M(\theta_n, \phi_n)]^T$ is the complex array response for a source impinging from direction $(\theta_n\ \phi_n)$, with

$$a_m(\theta_n\phi_n) = \exp((j2\pi r/\lambda)cos(\phi_n - \gamma_m)sin(\theta_n)) \qquad (2)$$

for $n = 1, 2, \ldots, N$. Here $\gamma_m = 2\pi m/M$, $m = 0, 1, \ldots, M-1$ is the sensor location, the elevation angle θ is measured down from the z-axis and ϕ is the azimuth angle measured counterclockwise from the x-axis. Since $\theta = 90°$ is fixed here, the UCA array manifold depends on the azimuth angle ϕ only, namely $a_m(\phi_n)$.

The covariance matrix of the array response is

$$R_x = E[x(t)x^H(t)] = AR_sA^H + \sigma^2 I_M \qquad (3)$$

where $R_s = E[s(t)s^H(t)]$ is the covariance matrix of the incident signals, I is a $M \times M$ identity matrix and $(\cdot)^H$ denotes conjugate transpose.

III. REAL-VALUED TRANSFORMATION WITH UCA

In this section, we will introduce real-valued transformation. Without loss of generality, let us assume that M is even. Let $\tilde{U}$ be an $M \times M$ matrix defined by

$$\tilde{U} = \frac{1}{\sqrt{2}}\begin{bmatrix} I & J \\ jJ & -jI \end{bmatrix}$$

where $\boldsymbol{I}$ is a $M/2 \times M/2$ identity matrix, $\boldsymbol{J}$ is a $M/2 \times M/2$ permutation matrix with all its anti-diagonal elements equaling 1. With the unitary matrix $\tilde{U}$ we have the following theorem.

Theorem 1: For any $M \times M$ centro-Hermitian matrix $H, \tilde{U}H\tilde{U}^H$ is real and symmetric.

Proof: See [6].

To obtain a centro-Hermitian matrix R_x, [7] proposed a unitary transformation method in element space.

Let unitary matrix

$$\hat{U} = \begin{bmatrix} I & 0 \\ 0 & J \end{bmatrix}$$

Then we have

$$J(\hat{U}R_x\hat{U}^H)^*J = \hat{U}R_x\hat{U}^H \qquad (4)$$

where $(\cdot)^*$ represents complex conjugation and $\hat{U}R_x\hat{U}^H$ is a centro-Hermitian matrix. Let unitary matrix

$$U = \tilde{U}\hat{U} = \frac{1}{\sqrt{2}}\begin{bmatrix} I & I \\ jJ & -jJ \end{bmatrix} \qquad (5)$$

and left multiply matrix U to both sides of (1), we have

$$y(t) = \underline{A}s(t) + \underline{n}(t) \qquad (6)$$

where $\underline{n}(t)$ is a noise vector like $n(t)$.

Since the sensors are centrosymmetric, we have $\gamma_{M/2+m} = \gamma_m + \pi$ and then

$$a_m(\phi_n) = Ja^*_{M/2+m}(\phi_n) \qquad m = 1, 2, \ldots, M/2 \qquad (7)$$

Reorder these sensors as $[1, 2, \ldots, M/2M, \ldots, M/2+2, M/2+1]$, the direction vector $a(\phi_n)$ can be rewritten as

$$\begin{aligned} a(\phi_n) = [\,& a_1(\phi_n), a_2(\phi_n), \ldots, a_{M/2}(\phi_n), \\ & a_M(\phi_n), \ \ldots, \ a_{M/2+2}(\phi_n), \ a_{M/2+1}(\phi_n)]^T \end{aligned} \qquad (8)$$

and $\underline{a}(\phi_n) = Ua(\phi_n)$ is a real vector given by

$$\begin{aligned} \underline{a}(\phi_n) = \sqrt{2}[&cos(2\pi r/\lambda)cos\phi_n), \\ &cos((2\pi r/\lambda)cos(2\pi(1/M)-\phi_n)), \ldots, \\ &cos((2\pi r/\lambda)cos(2\pi(M/2)-1)/M-\phi_n)), \\ &-sin((2\pi r/\lambda)cos(2\pi(M/2)-1)/M-\phi_n)), \ldots, \\ &-sin((2\pi r/\lambda)cos(2\pi(1/M)-\phi_n)), \\ &-sin(2\pi r/\lambda)cos\phi_n)]^T \end{aligned} \qquad (9)$$

The covariance matrix of the Eq. 6 is

$$R_y = E[y(t)y^H(t)] = \tilde{U}(\hat{U}R_x\hat{U}^H)\tilde{U}^H \qquad (10)$$

Practically, the covariance matrix is, computed from finite samples. Then

$$\hat{R}_y = \frac{1}{K}\sum_{k=1}^{K} y(k)y^H(k) = \tilde{U}(\hat{U}\hat{R}_x\hat{U}^H)\tilde{U}^H \qquad (11)$$

With Theorem 1 and Eq. 5, we are able to form areal symmetric transformed covariance matrix $\hat{\underline{R}}_y$ as

$$\hat{\underline{R}}_y = \frac{1}{2}\tilde{U}[\hat{U}\hat{R}_x\hat{U}^H + J(\hat{U}\hat{R}_x\hat{U}^H)^* J]\tilde{U}^H \tag{12}$$

where FB averaging [11] is used to guarantee the validity of the unitary transformation in practice.

Then, $\hat{\underline{R}}_y$ could be summarized to

$$\hat{\underline{R}}_y = \underline{A}S + \sigma^2 I_M. \tag{13}$$

IV. REAL-VALUED SPARSE REPRESENTATION DOA ESTIMATION

In the literature, the existing sparse representation DOA estimation methods generally require a complex linear programming. For example the computational complexity of ℓ_1-SVD is $O((N \times N_\theta)^3)$. It is higher than the cost of MUSIC, where the main complexity is in the subspace decomposition of the covariance matrix, which is $O(M^3)$ [2]. It was realized that complex multiplication costs four times as much as that of real multiplication [5], [6], and therefore a considerable amount of computations can be saved if we transform the complex-valued into a real-valued one. This motivates us to propose a simple real valued sparse representation method in the following part of the section. This sparse representation method is exactly the same as the ℓ_1-SVD method proposed in [2]. However, since we use the real manifold $\underline{a}(\phi)$ instead of $a(\phi)$, the computational complexity will be decreased by a factor of at least four.

The eigen-decomposition of $\hat{\underline{R}}_y$ can be written in the form of

$$\hat{\underline{R}}_y = \hat{Q}_s\Lambda_s\hat{Q}_s^H + \hat{Q}_n\Lambda_n\hat{Q}_n^H \tag{14}$$

Where $\hat{Q}_s \epsilon \mathbb{R}^{M\times N}$ is a real matrix whose columns are the eigenvectors corresponding to the N largest eigenvalues, while the columns of $\hat{Q}_n \epsilon \mathbb{R}^{M\times(M-N)}$ are the eigenvectors corresponding to the $M-N$ smallest eigenvalues.

To derive a reduced $M \times N$ dimensional signal space, we introduce a new matrix $\hat{\underline{R}}_y^{EV} = \hat{\underline{R}}_y Q_s$. In addition, let $S^{EV} = SQ_s$ and $N^{EV} = \sigma^2 Q_s$ to obtain

$$\hat{\underline{R}}_y^{EV} = \underline{A}S^{EV} + N^{EV} \tag{15}$$

In order to cast the AOA estimation problem as a sparse representation one, let Ω denote the set of possible locations, and let θ be a generic location parameter. Also, let $\{\theta_i\}_{i=1}^{N_{\hat{\theta}}}$ denote a grid that covers Ω. If the grid is fine enough such that the true AOAs lie on (or, practically, close to) the grid, we can use the following nonparametric model for $\hat{\underline{R}}_y^{EV}$:

$$\hat{\underline{R}}_y^{EV} = \underline{A}_{\hat{\theta}}S_{\hat{\theta}} + N^{EV} \tag{16}$$

where $S_{\hat{\theta}}$ is an $N_{\hat{\theta}} \times N$ matrix, whose the ith row is nonzero and corresponds to the signal impinging on the array from a possible source at $\hat{\theta}_i$.

In order to impose sparsity in $S_{\hat{\theta}}$ spatially (in term of the rows, rather than the columns), we try to find the spatial spectrum of $S_{\hat{\theta}}$ by minimizing the following optimization problem:

$$\min_{S_{\hat{\theta}}} \left\| \hat{\underline{R}}_y^{EV} - \underline{A}_{\hat{\theta}}S_{\hat{\theta}} \right\|_F^2 + \mu \left\| S_{\hat{\theta}} \right\|_{2,1} \tag{17}$$

Where μ is a regularization parameter [2]. Moreover, it is convenient to consider the constrained optimization problem instead:

$$\min_{S_{\hat{\theta}}} \left\| S_{\hat{\theta}} \right\|_{2,1}, \text{ subject to } \left\| \hat{\underline{R}}_y^{EV} - \underline{A}_{\hat{\theta}}S_{\hat{\theta}} \right\|_F^2 \le \varepsilon \tag{18}$$

Note that the way to choose ε has been described in [2]. In particular, if the noise n is i.i.d Gaussian, $\|nQ_s\|_2^2$ has approximately a χ^2 distribution with M degrees of freedom upon normalization by variance σ^2. Then, the upper value of $\|nQ_s\|_2$ with a 99% confidence interval is used as a choice for ε.

V. SIMULATION RESULTS

The simulation results in this section illustrate the performance of our proposed method using UCA. Since the computation reduction is obvious due to the real-valued transformation, we only represent the accuracy evaluation of AOA estimation in this section.

We compared our method with UCA to ℓ_1-SVD with ULA [2] and ℓ_1-SVD with UCA. For comparison of the method we suggested with UCA to ℓ_1-SVD with ULA [2] and that with UCA, we showed our system model for UCA and ULA in Fig. 2. In this figure r and d represent radius and distance (spacing between the linear array elements) respectively, and both are equivalent to $\lambda/2$.

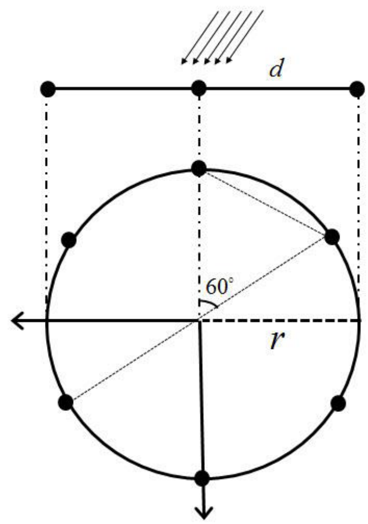

FIGURE 2. System model for the UCA and ULA assumed in this letter.

In this experiment we considered two zero mean narrowband far-field sources with equal power levels arriving on the arrays from directions 68.2° and 87.5°. Initially, the number of time snapshots was K = 50. The additive noise at the sensors was white gaussian and the noise level varied from 0 dB to 30 dB with a step size of 5 dB.

Fig. 3 illustrates the average RMSE (root mean-squared error) of the DOA estimation problem against the noise level for the K = 5 snapshots. The results have been averaged over 200 trials.

Fig. 4 shows the RMSE of DOA estimate against the number of snapshots computed via 200 runs for each snapshot and SNR = 10 dB for each run.

Evidently, our real-valued method with UCA performs better than the original ℓ_1-SVD method with ULA and UCA.

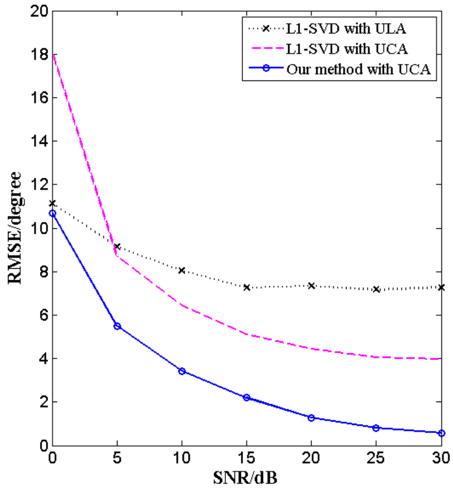

FIGURE 3. Average RMSE of AOA estimation against SNR(dB) with 200 runs for K = 50.

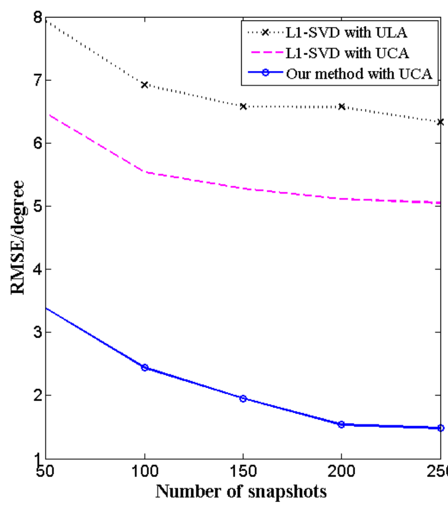

FIGURE 4. Average RMSE of AOA estimation against the number of snapshots with 200 runs for SNR = 10(dB).

VI. CONCLUSION

In this paper, we proposed a real-valued sparse representation method for AOA estimation with UCA. We converted the complex optimization problem into a real one by applying a unitary transformation and subsequently reduced the computational complexity at a considerable rate.

Simulations validate the effectiveness of our algorithm with UCA and suggest that the performance of the AOA estimation is more modified than the original ℓ_1-SVD method with ULA.

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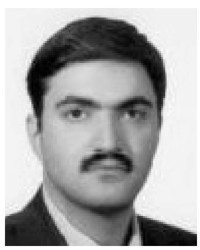

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