

## RESEARCH ARTICLE

# Reliability Assessment and Profit Optimization of Multi-Unit Mixed Configured System Using ABC Algorithm Under Preventive Maintenance

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**ABSTRACT** This paper presents an arrangement of parallel and series concepts to determine the reliability features of a multifaceted system. The profit analysis examined the overall gain of the setup after maintenance, preventive maintenance, and corrective maintenance. The components' low efficiency is due to internal or external factors such as friction between the components and the corrosion of parts, and sometimes overloaded workloads slow the process of working, which can be reversed by applying proper preventive maintenance. The machine's complete breakdown can be corrected using proper corrective maintenance. From the system's compromised or malfunctioning state to its operational condition, maintenance rates are factored into the calculations in this study's computation. The notions of the Markov process and Kolmogorov's differential equations are utilized to develop the mathematical model of the planned structure. To optimize reliability indices, an artificial bee colony is used to maximize ATTF, availability, busy periods, and the expected visits of a preventive maintenance repairman. The profit is found after reliability indices. Here, we provide a mathematical illustration with graphical results presented.

**INDEX TERMS** ABC algorithm, preventive and corrective maintenance, food source, bees selection, Markov birth process.

## I. INTRODUCTION

In today's modern scenario, everything depends on the machine to complete its task very efficiently and on time with the proper accuracy and lack of risk. While purchasing any equipment, anyone who wants the system will work with low maintenance and be more and more reliable with high profit. The increasing demand for machinery work is enhancing the production of more reliable systems for developing the country. Any working system has parts configured with series, parallel, or mixed of these, ultimately contributing to overall reliability. The smooth working of

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any equipment depends upon the working of its parts, which may degrade due to some reasons like frictions between parts, corroded screws and other holding materials, long-term working of parts, overloaded work done by parts, etc. This kind of reduced situation can cause less or delay production, which is not favourable. Hence, proper oiling or eyewatch is necessary for smooth working, representing preventive maintenance. Due to long-term work, the system stopped due to a complete breakdown of parts, which required replacing them with new ones during corrective maintenance. Directly replacing units with new ones can be prohibitively expensive. To mitigate these costs, preventive maintenance can significantly boost profit margins through reduced expenditures.

To identify the parameters that control reliability, Jain and Gupta [5] identified the most suitable placement strategy and applied process transformation in conjunction with the technique of supplementary variables, while Gopalan and Venkataslam [1] estimated the reliable capacity and accessibility of a pair of server platforms using Laplace transformation and showed reliability analysis under the effect of different fault coverage. The reliability allocation problem is solved using the artificial bee colony algorithm with the assistance of a numerical instance by Yeh and Hsieh [2]. A modified variant of the ABC algorithm was put forward by Sharma et al. [3] to improve the performance and find an optimal solution by applying it to different benchmark problems. Mostofi and Safavi [4], This paper is prepared to determine the minimization of expenditure on a power plant using the ABC algorithm and the result compared with PSO using HOMER software. Garg et al. [6] focus on the dependability of manufacturing equipment using a soft computational tool like the ABC algorithm with fuzzy theory, and an example of a plant has been taken to apply the approach. The paper dealt with the system probabilistic stability evaluation of soil slopes under the ABC approach supported by vector regression by Kang and Li [7]. An improved ABC algorithm is used to speed up the result more and to get a high-quality solution in reliability optimization by Ghambari and Rahati [8]. An underwater pipeline monitoring system has been considered for study by Rykov et al. in 2021 [9]. Under the effect of preventive maintenance, a mathematical k out of n-good system model has been elaborated by using an example. Optimizing the cost of rubber plants using a nature-inspired PSO method has been discussed by Kumari et al. [10].

The BLLP-bi-level programming plan is applied to the optimization problem by Khan et al. [11], which deals with the liability of a system undergoing selective maintenance. This section addresses the 2022 gap. The machine's availability, profit, busy period, and ATTF have been carried out using the Artificial Bee Colony algorithm under preventive maintenance. Musa and Yusuf [12] presented a comprehensive study of a series-parallel system under the method of Laplace and the supplementary variable technique to find out the reliable capacity of a machine, and sensitivity analysis has also been done to measure the best solution under the copula approach. The reliability analysis of a wind turbine system under ergo ABC has been done using Weibull distribution by Han et al. [13]. Singla et al. [14] studied a deep learning process to optimize the reliability parameters to raise an industry's profit and production of a 2:3 good system. Singla et al. [15] examine a deteriorated system utilizing a genetic algorithm to determine the reliability metrics under the influence of the degraded rate and the preventive maintenance rate in conjunction with the idea of preventive maintenance. Reliability analysis and parameter estimation techniques under preventive maintenance and allocation have

been studied and proposed [16], [17], [18]. The application of neural networks in mathematics and advanced tools like artificial intelligence, machine learning, deep learning, computer graphics, etc., were discussed. Some nondeterministic polynomial-time-complete and hard problems are solved using networks by Khan et al. [19]. The fault-tolerant metric dimensions of different networks connected to each other are studied by Hassan et al. [20]. Hayat et al. [21] study the fault-tolerant dimensions of a group of butterflies and a family of honeycombs to discuss interconnection using minimal and maximal results. Based on graph theory, Imran et al. [22] computed the interconnection network and derived analytical closed results for the Butterfly and Benes networks. The atom-bond connectivity index and geometric arithmetic index are studied for silicate networks by Hayat and Imran [23]. The concept of unbounded metric dimension on convex polytopes is discussed using graph theory by Siddiqui et al. [24].

The motivation behind the current work is the need to find a novel, practical solution to the problem at hand, which is to optimize the profit of the whole industry with less consumption of cost due to maintenance. The main objective of this work is to show how to assess the maximum profit and trustworthiness of a multi-configured structure using an algorithm based on the factors of different rates of failure and repair (preventive or corrective) and how this affects the performance of systems. The paper has the following structure: Section II describes every aspect of the model, such as the state overview, assumptions, notations, and system design. Section III discusses how the mathematical model was created, and ATTF, available performing time, busy period, anticipated repairman visit for maintenance, and profit analysis are some of the major topics covered. The optimization methodology is discussed in Section IV. Section V outlines the computational simulations and findings of the cost-benefit analysis and the reliability parameter optimization. The conclusion is presented in Section VI.

## II. MODEL DETAILS AND NOTATIONS

### A. SYSTEM DESCRIPTION

In the present work, a series-parallel mixed configuration has three main units, A, B, and C, linked in a series form and arranged to represent the whole system. The unit A has two subunits,  $A_1$  and  $A_2$ , arranged in a series. Unit B has two components,  $B_1$  and  $B_2$ , connected in parallel combination, while unit C has three subcomponents,  $C_1$ ,  $C_2$ , and  $C_3$ , arranged in series. Unit A can work at low efficiency, which can be reversed to a complete efficiency condition by applying one-time preventive maintenance, but on second degradation, it stops working. On the other hand, there is no such condition for Unit C, i.e., it can completely fail, whereas Unit B failed whenever both components failed. With the concept of the Markov process and Chapman-Kolmogorov's differential

equations, a mathematical model is prepared whose working depends upon the degraded rate, failed rate, preventive maintenance rate, and corrective maintenance rate. The results obtained from the solutions of the equation are optimized using the Artificial Bee Colony algorithm to have maximum profit and maximum reliability parameters for the presented complex system.

**B. ASSUMPTIONS**

- (i) At first, the system is 100% capable of carrying out its assignment precisely and effectively.
- (ii) The system becomes less effective after a deteriorated state, or its effectiveness in completing the task somewhat diminishes.
- (iii) The system functions as new after repair, with each subcomponent requiring the same amount of time to fix using the perfect preventive maintenance provided only once.
- (iv) Whenever the components completely break down even after performing preventive maintenance, they are replaced with new, fresh units, called corrective maintenance, to continue the work smoothly.
- (v) The same degradation and failure rates are assumed for subunits A<sub>1</sub> and A<sub>2</sub>. The failure rates of components C<sub>1</sub>, C<sub>2</sub>, and C<sub>3</sub> are the same as those of components B<sub>1</sub> and B<sub>2</sub>.
- (vi) It is thought that exponential distributions represent failure and degradation rates.

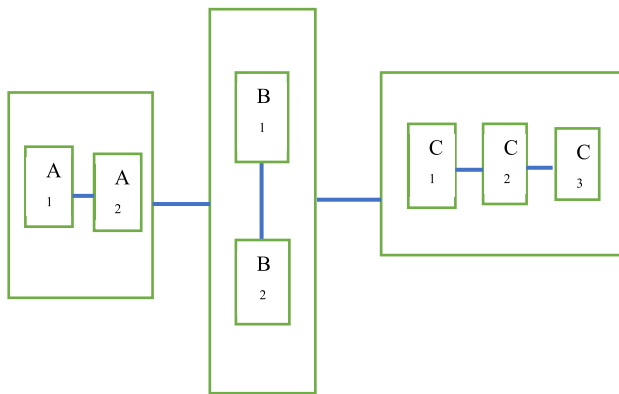


FIGURE 1. System configuration.

**C. NOTATIONS**

The potential state transition diagram for the model that is being presented can be seen as

**D. STATE DESCRIPTION**

The different state descriptions are explained in Table 2 to describe the path of the system from one state to another under various rates of failure or repair of units of the system. The system works in three primary states, i.e. fully, reduced, and failed, which are presented below with the behaviour of each unit.

TABLE 1. Various phrases pertaining to the work being presented.

ATTF	Average time to failure
$A_{v(\infty),B(\infty)}$	Availability: Busy period for repairman
$F(\infty)$	Expected Frequency of Preventive Maintenance
$U_i$	Represent transition states where $0 \leq i \leq 11$ .
A	Represent good working state of the system.
$\bar{A}$	The system unit is working in a low efficiency due to some external or internal cause and needs preventive maintenance.
A	Represent the down state, i.e. the system is completely stopped.
$\lambda_j$	failure rate from going good level to breakdown level for Units B and C or deteriorated level to breakdown level For Unit A, where $j=A, B, C$
$\alpha_A$	It is the degraded rate going from good condition to reduced condition for unit A, i.e. from A to $\bar{A}$ .
$\mu_A$	It represents the preventive maintenance rate to reverse the condition from a reduced state to a good state, which only applies once for unit A.
M	Repair rate for taking back state from failed to good working state by applying corrective maintenance
$P_i(t)$	Represent the probability of various changing states where $0 \leq i \leq 11$ .
$P(t)/\dot{P}(t)$	Represent the overall probability vector and its associated differential vector.
$X_{new}/X/X_p$	A new food source/any selected food source/any partner food source in the ABC algorithm working.
$\phi$	Any random number, $\phi \in [-1,1]$
iter	Number of iterations.

**III. MATHEMATICAL MODELLING OF THE PRESENTED MODEL**

The Markov birth process concept is considered when designing the mathematical representation of the arrangement being presented. The mnemonic rule is used to create the first-order Chapman-Kolmogorov differential equations that correspond to the transition diagram and yield the reliability parameters. The likelihood that the system will be presented in state  $U_i$  at  $t \geq 0$  is  $P_i(t)$ . In addition, let  $P(t)$  represent the probability vector at time  $t$  with a beginning condition.

$$P_i = \begin{cases} 1 & \text{if } i = 0 \\ 0 & \text{if } i \neq 0 \end{cases} \quad (1)$$

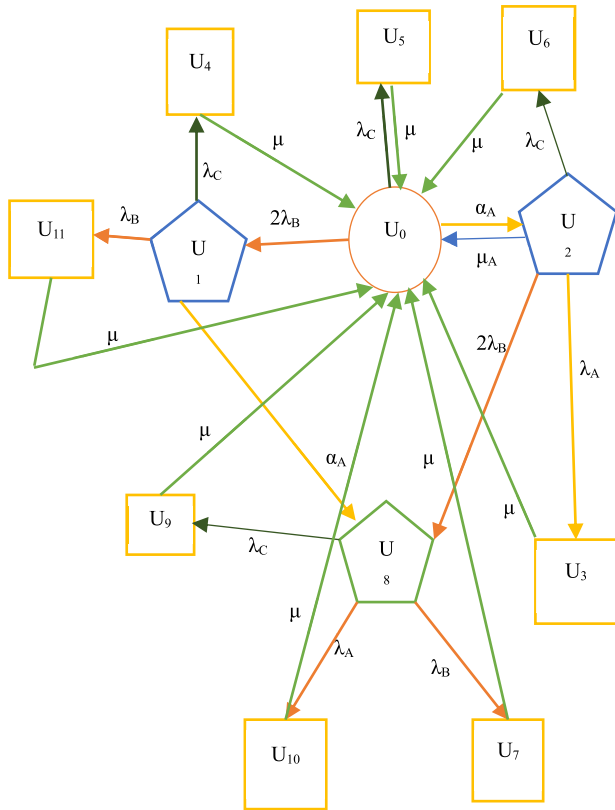


FIGURE 2. The state transition diagram shows the possibilities of working on three units: A, B, and C.

The differential equations associated with Figure 1 are:

$$P'_0 = -(\alpha_A + \lambda_C + 2\lambda_B)P_0 + \mu_A P_2 + \mu(P_3 + P_4 + P_5 + P_6 + P_7 + P_9 + P_{10} + P_{11}) \tag{2}$$

$$P'_1 = -(\alpha_A + \lambda_C + \lambda_B)P_1 + 2\lambda_B P_0 \tag{3}$$

$$P'_2 = -(\mu_A + 2\lambda_B + \lambda_A + \lambda_C)P_2 + \alpha_A P_0 \tag{4}$$

$$P'_3 = -\mu P_3 + \lambda_A P_2 \tag{5}$$

$$P'_4 = -\mu P_4 + \lambda_C P_1 \tag{6}$$

$$P'_5 = -\mu P_5 + \lambda_C P_0 \tag{7}$$

$$P'_6 = -\mu P_6 + \lambda_C P_2 \tag{8}$$

$$P'_7 = -\mu P_7 + \lambda_B P_8 \tag{9}$$

$$P'_8 = -(\lambda_A + \lambda_C + \lambda_B)P_8 + 2\lambda_B P_2 + \alpha_A P_1 \tag{10}$$

$$P'_9 = -\mu P_9 + \lambda_C P_8 \tag{11}$$

$$P'_{10} = -\mu P_{10} + \lambda_A P_8 \tag{12}$$

$$P'_{11} = -\mu P_{11} + \lambda_B P_1 \tag{13}$$

To assist with additional computations needed to obtain the necessary results, the aforementioned set of equations can be

TABLE 2. Various descriptions of states associated with the transition diagram.

U <sub>0</sub>	(A, B, C)	This is a fresh state, i.e., all parts are in perfect condition to perform their work with 100% efficiency.
U <sub>1</sub>	(A,B(b),C)	One of the components of B has failed but is still working due to the parallel configuration of B <sub>1</sub> and B <sub>2</sub> .
U <sub>2</sub>	( $\bar{A}$ , B, C)	There is a degradation in the efficiency of A by going from A to $\bar{A}$ and we need preventive maintenance here.
U <sub>3</sub>	(a, B, C)	Representing a breakdown state due to the total demise of Unit A from going $\bar{A}$ to a.
U <sub>4</sub>	(A,B(b),c)	One of the components of B failed, but it is in a down state as Unit C was demoted.
U <sub>5</sub>	(A, B,c)	Represent a breakdown state as because Unit C is demised.
U <sub>6</sub>	( $\bar{A}$ , B,c)	Unit A is in a degraded state, and Unit C is completely failed; hence, the whole state is a failed state.
U <sub>7</sub>	( $\bar{A}$ ,b,c)	Unit A is in a degraded state, and Unit B is completely failed due to the failure of both components of B and; hence, the whole state is a failed state.
U <sub>8</sub>	( $\bar{A}$ ,B(b),C)	There is a degradation in the efficiency of A, and One of the components of B has failed. As a result the whole condition refers to a reduced state.
U <sub>9</sub>	( $\bar{A}$ ,B(b),c)	Unit A is in a reduced state, and one of the components of Unit B has failed, but it is in a state of shutdown resulting from the malfunction of Unit C.
U <sub>10</sub>	(a,b(b),C)	Represent a shutdown state due to the malfunction of Unit A.
U <sub>11</sub>	(A,b,C)	Represent a shutdown state due to the malfunction of Unit B.

represented as a matrix system.

$$P'(t) = AP(t)$$

A

$$= \begin{bmatrix} -U & 0 & \mu_A & \mu & \mu & \mu & \mu & \mu & 0 & \mu & \mu & \mu \\ 2\lambda_B & -V & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_A & 0 & -W & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_A & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_C & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_C & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_C & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & \lambda_B & 0 & 0 & 0 \\ 0 & \alpha_A & 2\lambda_B & 0 & 0 & 0 & 0 & 0 & -X & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_C & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_A & 0 & -\mu & 0 \\ 0 & \lambda_B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu \end{bmatrix} \tag{14}$$

where

$$U = \alpha_A + \lambda_C + 2\lambda_B, V = \alpha_A + \lambda_C + \lambda_B,$$

$$W = \mu_A + 2\lambda_B + \lambda_A + \lambda_C, X = \lambda_A + \lambda_C + \lambda_B$$

**A. AVERAGE TIME TO SYSTEM FAILURE**

Matrix D is obtained by transposing the aforementioned matrix, which removes the barriers to evaluating ATTF by limiting the elimination of the absorbing states, i.e.

$$ATTF = P(0)(-D^{-1}) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (15)$$

where  $D = \begin{bmatrix} -U & 2\lambda_B & \alpha_A & 0 \\ 0 & -V & 0 & \alpha_A \\ \mu_A & 0 & -W & 2\lambda_B \\ 0 & 0 & 0 & -X \end{bmatrix}$

$$ATTF = N_1/D_1 \quad (16)$$

where  $N_1 = (\alpha_A + \lambda_C + \lambda_B)(\lambda_A + \lambda_C + \lambda_B)(\mu_A + \lambda_C + 2\lambda_B + \lambda_A) + 2\lambda_B(\lambda_A + \lambda_C + \lambda_B)(\mu_A + \lambda_C + 2\lambda_B + \lambda_A) + \alpha_A(\alpha_A + \lambda_C + \lambda_B)(\lambda_A + \lambda_C + \lambda_B) + 2\alpha_A\lambda_B \times (\mu_A + 2\lambda_C + 3\lambda_B + \lambda_A + \alpha_A)$   
 $D_1 = (\alpha_A + \lambda_C + \lambda_B)(\lambda_A + \lambda_C + \lambda_B)(2\alpha_A\lambda_B + \alpha_A\lambda_C + \alpha_A\lambda_A + 4\lambda_B\lambda_B + 4\lambda_C\lambda_B + 2\lambda_A\lambda_B + \lambda_C\lambda_C + \lambda_C\lambda_A + \lambda_C\mu_A + 2\mu_A\lambda_B)$

**B. OPERATING TIME(AVAILABILITY)**

The system is available to operate at full efficiency or at a reduced level, depending on its current state. These states are the sole determinants of a system’s availability.

$$Availability = P_0 + P_1 + P_2 + P_8 \quad (17)$$

$$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} + P_{11} = 1 \quad (18)$$

With the normalized condition (18) and setting the derivative of (2) to (13) equal to 0.

And figure out the equation for finding the value of  $P_i$  ( $i=0$  to 11)

$$Availability(Av(\infty)) = 1 + (\mu_A\alpha_A/(\mu_A + 2\lambda_B + \lambda_A + \lambda_C) - (\lambda_C + 2\lambda_B + \alpha_A))(P_0/\mu) \quad (19)$$

where  $P_0 = \mu / [(\mu + 2\lambda_B\mu/(\alpha_A + \lambda_C + \lambda_B) + \alpha_A\mu/(\mu_A + 2\lambda_B + \lambda_A + \lambda_C) + (\lambda_C + 2\lambda_B + \alpha_A) - \mu_A\alpha_A/(\mu_A + 2\lambda_B + \lambda_A + \lambda_C) + 2\lambda_B\alpha_A\mu [1/(\mu_A + 2\lambda_B + \lambda_A + \lambda_C) + 1/(\alpha_A + \lambda_C + \lambda_B)] (\lambda_B + \lambda_A + \lambda_C)^{-1}]$ .

**C. BUSY PERIOD FOR REPAIRMAN**

The progression of maintenance, whether it be corrective or preventive, can be expressed as:

$$\begin{aligned} \text{Busy period of time} &: -B(\infty) \\ &= 1 - -(P_0(0) + P_1(\infty) + P_8(\infty)) \\ &= 1 - Av + \alpha_AP_0/(\mu_A + 2\lambda_B + \lambda_A + \lambda_C) \end{aligned} \quad (20)$$

where  $P_0 = \mu / [(\mu + 2\lambda_B\mu/(\alpha_A + \lambda_C + \lambda_B) + \alpha_A\mu/(\mu_A + 2\lambda_B + \lambda_A + \lambda_C) + (\lambda_C + 2\lambda_B + \alpha_A) - \mu_A\alpha_A/(\mu_A + 2\lambda_B + \lambda_A + \lambda_C) + 2\lambda_B\alpha_A\mu [1/(\mu_A + 2\lambda_B + \lambda_A + \lambda_C) + 1/(\alpha_A + \lambda_C + \lambda_B)] (\lambda_B + \lambda_A + \lambda_C)^{-1}]$ .

**D. EXPECTED FREQUENCY OF PREVENTIVE MAINTENANCE,  $F(\infty)$**

Prior to a unit failing, the idea of preventive maintenance helps maximize availability and minimize the worst effects of total failure, providing profit. Thus, it is possible to determine the anticipated frequency of preventive maintenance per unit of time as follows:

$$F(\infty) = P_2(\infty) = \alpha_AP_0/(\mu_A + 2\lambda_B + \lambda_A + \lambda_C) \quad (21)$$

where  $P_0 = \mu / [(\mu + 2\lambda_B\mu/(\alpha_A + \lambda_C + \lambda_B) + \alpha_A\mu/(\mu_A + 2\lambda_B + \lambda_A + \lambda_C) + (\lambda_C + 2\lambda_B + \alpha_A) - \mu_A\alpha_A/(\mu_A + 2\lambda_B + \lambda_A + \lambda_C) + 2\lambda_B\alpha_A\mu [1/(\mu_A + 2\lambda_B + \lambda_A + \lambda_C) + 1/(\alpha_A + \lambda_C + \lambda_B)] (\lambda_B + \lambda_A + \lambda_C)^{-1}]$ .

**E. PROFIT ANALYSIS**

The model presented uses preventive and corrective maintenance measures for units to have higher productivity and profit in states 1 to 11. Let  $C_0$ ,  $C_1$ , and  $C_2$  represent the revenue generated if the system is operating correctly, the loss incurred if it malfunctions, and the costs associated with each repair (corrective maintenance) and overhaul (preventive maintenance), respectively, for profit analysis. The system’s anticipated total revenue over time is.

Profit = Total revenue received – repair man’s costs for performing preventive maintenance and fixing malfunctioning units.

$$\text{Profit Analysis} : -C_0Av(\infty) - C_1B(\infty) - C_2F(\infty) \quad (22)$$

where

- $C_0$ : the system’s revenue per unit uptime.
- $C_1$ : is the cost per unit time the system is under repair.
- $C_2$ : is the cost due to preventive maintenance.

**F. SPECIAL CASE**

Let us consider that all the failure chances for each state are the same, i.e.,  $\lambda_A = \lambda_B = \lambda_C = \lambda$ . By putting these values in the above-solved reliability parameters, i.e., in (16),(19),(20) and (21), we have the following metrics:

**ATTF**

$$\begin{aligned} &= [3(\alpha_A + 2\lambda) (\mu_A + 4\lambda) + 6\lambda (\mu_A + 4\lambda) + 3\alpha_A(\alpha_A + 2\lambda) \\ &\quad + 2\alpha_A (\mu_A + 6\lambda + \alpha_A)]/3(\alpha_A + 2\lambda)(4\alpha_A\lambda + 12\lambda^2 + 3\mu_A\lambda) \end{aligned} \quad (23)$$

**Availability(Av)**

$$\begin{aligned} &= 1 - 3[3\lambda (\mu_A + 4\lambda) + 4\alpha_A\lambda](\alpha_A + 2\lambda)/ \\ &\quad (\lambda^2 (72\lambda + 48\mu + 60\alpha_A + 18\mu_A) \\ &\quad + \lambda (30\mu\alpha_A + 12\mu\mu_A + 12\alpha_A\alpha_A + 9\alpha_A\mu_A) \\ &\quad + 5\alpha_A\mu_A\mu + 5\alpha_A\alpha_A\mu) \end{aligned} \quad (24)$$

**Busy period(B)**

$$\begin{aligned} &= 3[3\lambda (\mu_A + 4\lambda) + 4\alpha_A\lambda + \alpha_A\mu](\alpha_A + 2\lambda)/ \\ &\quad (\lambda^2 (72\lambda + 48\mu + 60\alpha_A + 18\mu_A) \\ &\quad + \lambda (30\mu\alpha_A + 12\mu\mu_A + 12\alpha_A\alpha_A + 9\alpha_A\mu_A) + 5\alpha_A\mu_A\mu \\ &\quad + 5\alpha_A\alpha_A\mu) \end{aligned} \quad (25)$$

**Expected Frequency of Preventive Maintenance(F)**

$$= 3\alpha_A\mu(\alpha_A + 2\lambda)/(\lambda^2 (72\lambda + 48\mu + 60\alpha_A + 18\mu_A) + \lambda (30\mu\alpha_A + 12\mu\mu_A + 12\alpha_A\alpha_A + 9\alpha_A\mu_A) + 5\alpha_A\mu_A\mu + 5\alpha_A\alpha_A\mu) \quad (26)$$

**Profit Analysis**

$$: -C_0Av(\infty) - C_1B(\infty) - C_2F(\infty) \quad (27)$$

**IV. METHODOLOGY****A. INTRODUCTION**

One of Dervis Karaboga's most popular algorithms in 2005 was Artificial Bee Colony (ABC), and from then on, the algorithm spread widely. It is entirely influenced by how honey bees act. The goal of the artificial bees, using ABC as an optimization tool, is to find foods that contain significant amounts of sweetness and, ultimately, the most sweetness. In this process, the artificial bees gradually alter each individual's food position. In the ABC model, the movement of algorithms depends upon the working of bees, which is recorded in three phases:

- 1) Employed phase
- 2) Onlookers phase
- 3) Scouts phase

Within this system, the artificial bees hover in a complex search environment, while some onlookers and employed bees modify their positions and select foodstuffs according to their own and their nestmates' experiences. Not relying on past experience, certain scouting bees fly and select foodstuffs at random. If the new supply of nectar is more vivid in their memory than the amount from the old one, they will remember the newly established location and forget the previous one.

When employed, bees visit their food source, return to the hive, and dance here. The working bee evolves into a scout and begins looking for novel nourishment supplies after its food supplies are abandoned. Onlookers watch employed bees perform dances; they select their food sources based on the dances.

**B. ABC BASIC ALGORITHM**

The following are the stages of the fundamental ABC algorithm:

- For each employed bee, the initial food supplies are produced.

**REPEAT**

- After utilizing her recollection to locate a nearby food supply and estimating how much nectar it holds, each worker bee dances around the colony.
- In the onlooker phase, after witnessing the working bees dance, viewers visit one of their sources of information based on the decisions they made. She picks up a nearby neighbour and measures the amount of nectar.
- Scouts replace abandoned food sources with newly discovered ones after identifying which ones have been abandoned.

- The most promising food source that has so far been found has been noted.
- Until all prerequisites are satisfied.

A food supply's location within the community-driven algorithm ABC represents a possible approach to the optimization issue, and the amount of food available indicates how appropriate (fitness) that solution is. There are exactly as many employed bees as there are solutions in the population. Initially, an initial crowd (positions of food sources) is created haphazardly. Then, the population is initialized and allowed to go through the cycles of the employed, onlooker, and scout bees' search processes again. After finding a new location for the foodstuffs, an employed bee adapts the source perspective in her recollection.

The bee remembers the location of the fresh origin and disregards the original one as long as the amount of honey from the new supplier is more vivid than that from the previous supplier; if not, she recalls the prior source of honey. The places where the sources are located are shared with the onlookers in the dancing area once all employed bees have exhausted their efforts. Depending on the amount of sweetness in each source, each onlooker selects a food source after analyzing the honey data collected from all working bees. She modifies the source location in her memory, just like the employed bee does, and verifies the quantity of nectar it contains. The bee keeps the new location and disregards the old one as long as the sweetness is more plentiful than the previous one. Artificial scouts replace the randomly generated new sources, which are determined to be abandoned. To try to balance the processes of exploration and exploitation, the ABC system thus integrates neighbourhood searching tactics, handled by employed and onlooker bees, along with global search tactics, overseen by scouts. The whole process can be understood using Figure 3.

**C. ABC OPTIMIZATION ALGORITHM**

- Establish the test-case populations.

**Employed phase**

- Create an innovative solution.
- The fitness value will then be calculated after that.
- Make use of greedy selection, i.e., comparing the previous and current solutions, i.e., one that is beneficial or detrimental.

**Onlooker phase**

- Calculate the probabilities.
- Create an entirely novel solution based on probability.
- Compute your updated level of fitness.
- Apply greedy selection.

**Scout Phase:**

- Using the limit's value as a guide, identify the abandoned solution.
- Randomly generate an alternative solution to replace them.

Working can be seen in Figure 4.

**D. WORKING OF ABC ALGORITHM**

As part of the population-based approach that is being suggested, every food source is associated with a potential nutrition source that can be used to address an optimization problem. The quality of the corresponding solution is gauged by its fitness value.

Figure 4 illustrates the proposed approach’s operating principle. In the initial phase, a population with a random distribution, i.e., randomly selected food sources, is generated between the provided boundaries using a standard equation, and the number of food sources is equal to the number of employed bees or the number of onlooker bees, which is again half of the population size. After the initial setup, the population is forced to go through the search process again. One by one, placing the food sources reveals the function (f) worth that needs to be optimized. Having a maximum value is the primary goal of using the ABC algorithm; a minimum value will be determined by matching that fitness value. The fitness value is calculated using the equation:

Fitness value corresponds to each food source.

$$= \begin{cases} \frac{1}{1+f}; & f \geq 0 \\ 1 + \text{absolute value of } f; & f < 0 \end{cases} \quad (28)$$

where *f* is the function which is going to be optimized.

Now update the trial value; if the solution obtained is improved, it is set to 0; otherwise, it is reset to 1. The employed bee phase will then get to work. This allows us to update every food source. First, a food source, say *X*, is chosen, and in addition to *Ist*, any other partner food source, say *X<sub>p</sub>*, is also chosen. A new food location is generated by using an equation: new food source = *X<sub>new</sub>* = *X* + (*X*-*X<sub>p</sub>*), ∈ [-1,1].

Now, find the value of function *f* and its fitness value and apply greedy selection, in which the old fitness value corresponds to *X* and the fitness value corresponds to *X<sub>new</sub>*. The food source is updated, and the trial value is set to 0 if it better meets our requirements; if not, the original value is preserved per food source *X*, and the trial value is set to 1. For improved values, repeat this cycle for each food source and update the trial, function value, fitness values, and food sources; hence, the employed phase will be completed.

The next phase, the onlooker phase, will come into play, and its primary purpose is to update the solution. Initial probabilities regarding the fitness value are found using the gernal formula. The solutions in this phase will update only when they satisfy the particular condition, i.e., the selected random number is less than the probability for each fitness value, and if the condition is satisfied, the complete procedure of the employed phase will perform. Also, the trial value is updated to 0 if the solution is updated; otherwise, it is set to the next number, 2. If the condition is not satisfied, the solution will not be updated. This onlooker bee will perform continuously to have the next trial number, or 0. In this phase, a new solution may or may not be updated, as its working

depends on a selected random number and the probabilities of the food source.

The next task is to memorize the best answer. The scout phase is now active. First, we check whether the scout phase will be implemented, which is checked by the trial value and limit. If the trial value exceeds the limit value, then the scout phase will start; otherwise, it will not apply to the food source that does not satisfy the above condition. If a food source meets the above criteria, the value of that food source is updated to the new food source, and thus, the solution. In this case, there is no greedy selection. This whole process is for iteration to get maximum value, and similarly, this process will perform for a given number of iterations to maximize results in each iteration.

**E. PSEUDO CODE**

We have preselected Randomly Chosen variables: population size, number of iterations, dimension of the problem, limit, number of employed bees = number of onlooker bees = food sources = half of the population size, upper and lower bounds for the variables of the problem.

Procedure:

Initialize the population using the equation.

$$X_i = L + rand.(U - L); \quad (29)$$

where *X<sub>i</sub>* =*i*th food sources, *U* =upper bound, *L* = lower bound, *i* = 1,2,3,..., (population/2), and rand is any random number between 0 and 1.

**iteration =1;**

**while** iteration<= Maximum number of iterations **do**

1. Produce a new solution for employed bees using a formula *X<sub>new</sub>*=*X* + (*X*-*X<sub>p</sub>*), ∈ [1, -1]; and evaluate them.
2. Apply greedy selection for the employed bee phase.
3. Estimate the probabilities for the responses using the fitness value.
4. Produce a new solution for the selected solution for the onlookers depending upon their probability and assessment.
5. Make greedy choices for the onlooker phase.
6. If the scout bee phase’s abandoned solution exists, identify it and use an equation to replace it with a newly generated, randomly generated solution (*X*).  
*X*= *L* + rand (*U*-*L*)
7. Memorize the best solution, i.e., the maximum value in our case.
8. Iteration=iteration + 1

**end while**

**F. FLOW CHART FOR PRESENTED METHODOLOGY**

The provided flow chart illustrates how each loop in the entire algorithm works to describe the entire working process. The three stages of the ABC optimization algorithm’s operation are described in this flow chart. The flow chart provides a clear understanding of how an algorithm operates when implemented.

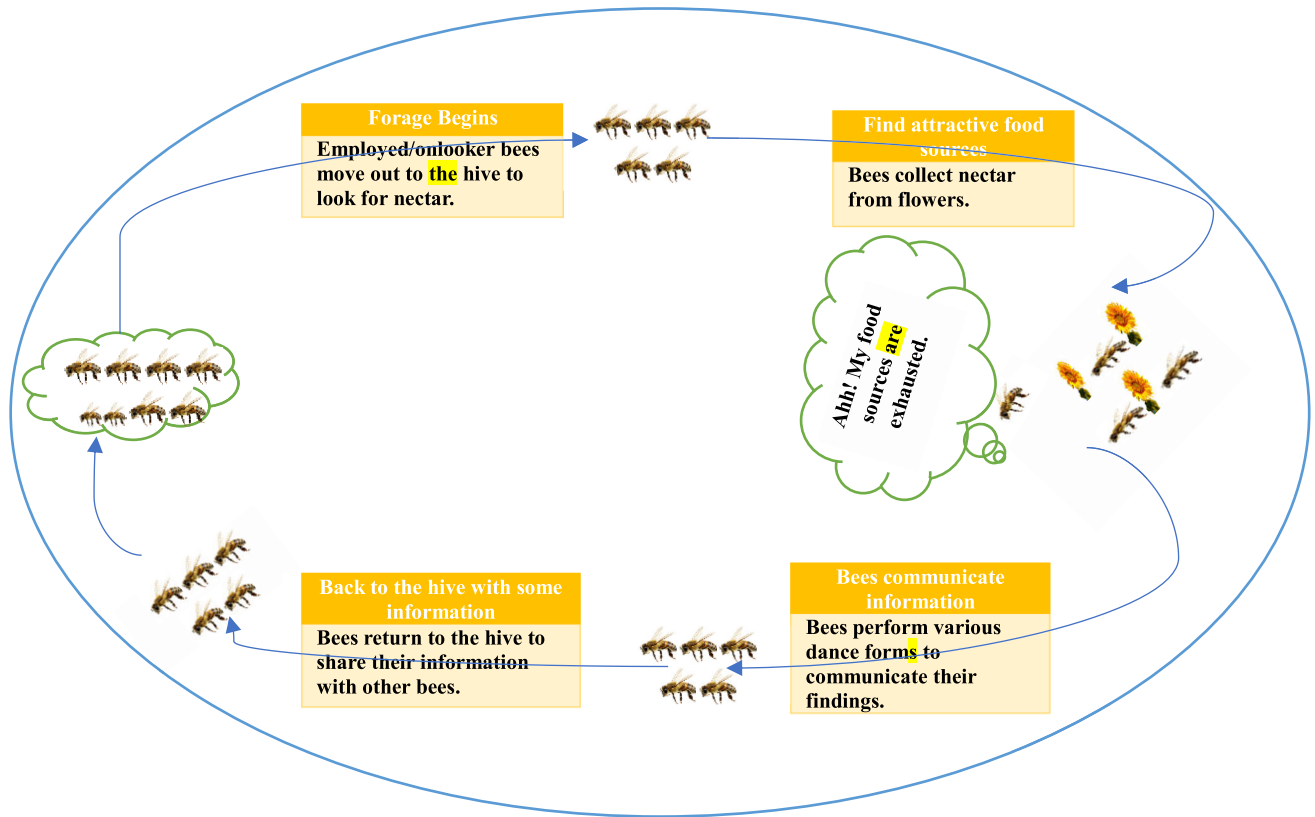


FIGURE 3. Simple illustrations of how honey bees strive to extract the most nectar.

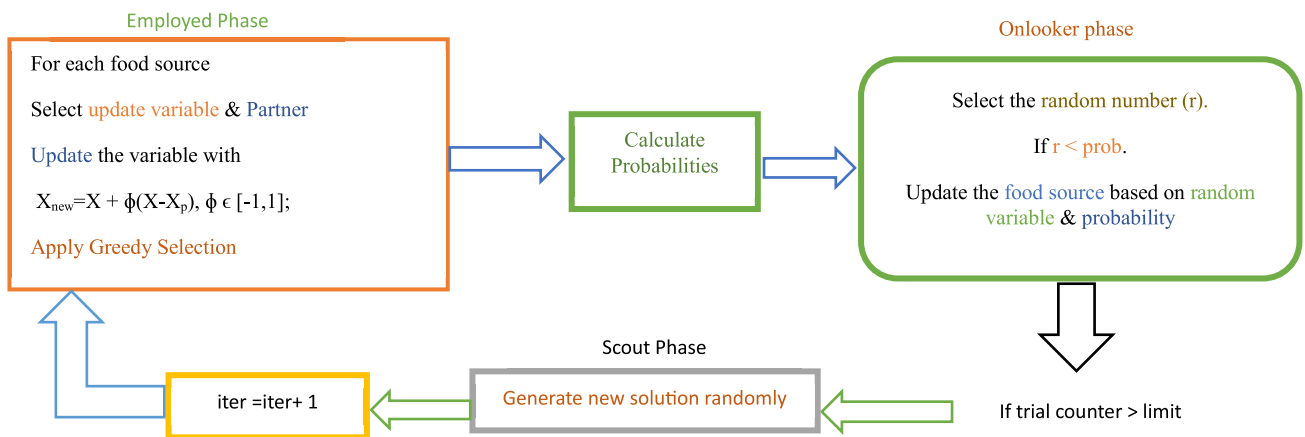


FIGURE 4. The calculations working of ABC algorithm.

## V. NUMERICAL SIMULATIONS AND RESULT

### A. AVERAGE TIME TO SYSTEM FAILURE

Table 3 and Figure 5 illustrate how the ATTF varies in relation to the rates of degradation, failure, and preventive maintenance. Employing the initial parameter bounds, i.e., the ATTF, is obtaining the maximum value across a range of parameter values when using the lower and upper bounds for the three variables,  $\alpha_A$ ,  $\mu_A$ , and  $\lambda$ , and the ABC optimization algorithm with 14 iterations. Given the available

data, the maximum optimized value for ATTF is 6.0215. The bounds for variables are  $\alpha_A \in [0.1, 0.5]$ ,  $\mu_A \in [0.1, 0.5]$  and  $\lambda \in [0.1, 0.9]$ .

### B. OPERATING TIME(AVAILABILITY)

Table 4 and Figure 6 illustrate how availability varies in relation to the rates of degradation, failure, repair and preventive maintenance. Employing the initial parameter bounds, i.e., the availability, is obtaining the maximum value across



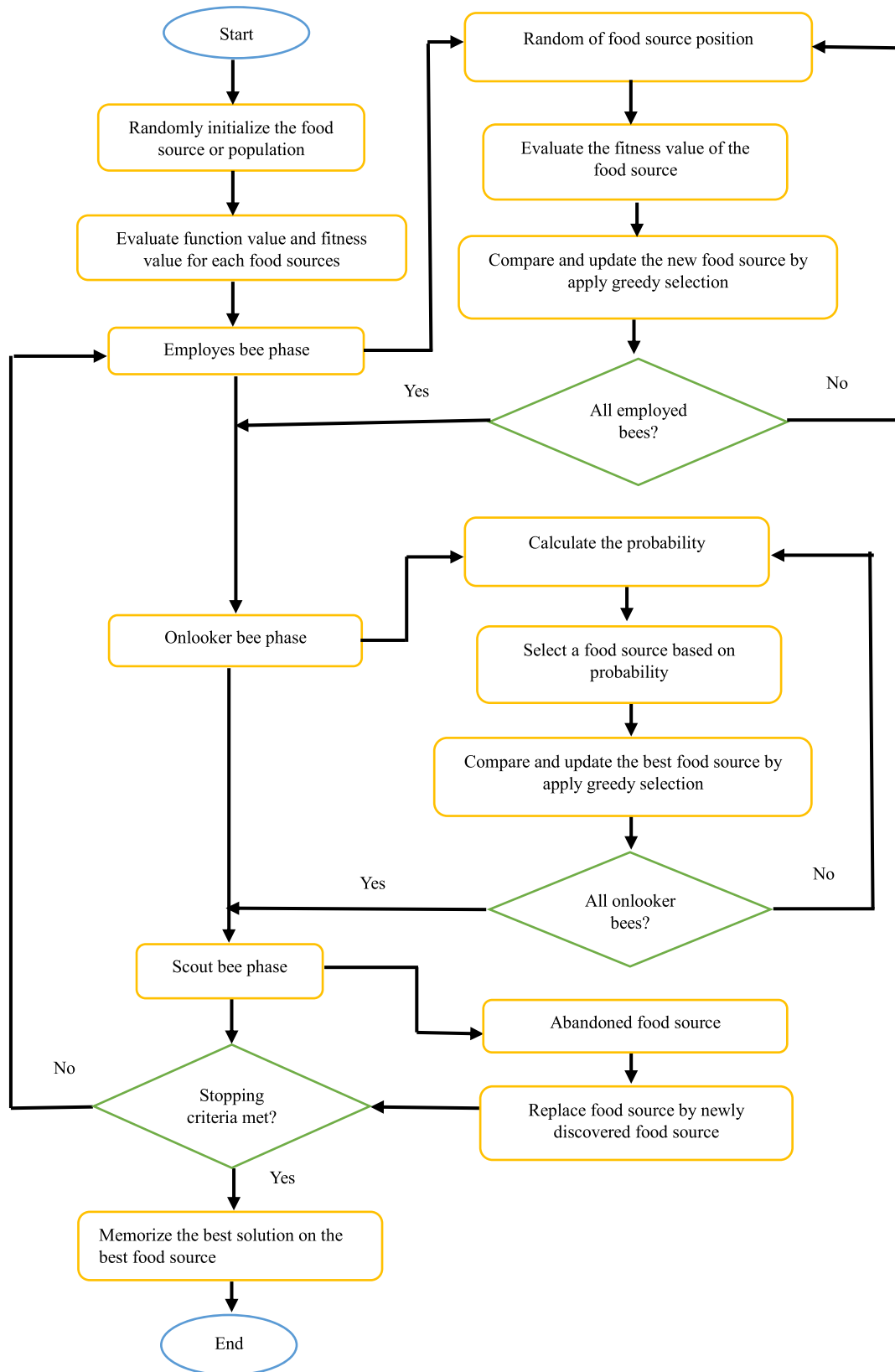


FIGURE 5. Flow chart for the proposed methodology.

TABLE 3. The variation in ATTF with respect to various rates.

$\alpha_A$	$\mu_A$	$\lambda$	ATTF
0.1000	0.5000	0.5404	1.194
0.4546	0.3568	0.1857	2.899
0.2454	0.4022	0.1773	3.241
0.1086	0.4895	0.1606	3.8201
0.3196	0.4239	0.1248	4.3684
0.1233	0.5000	0.1074	5.5347
0.1000	0.4430	0.1068	5.6413
0.1000	0.5000	0.1057	5.7117
0.1018	0.5000	0.1019	5.9076
0.1004	0.5000	0.1013	5.9452
0.1000	0.4088	0.1000	5.9948
0.1010	0.4951	0.1000	6.0155
0.1000	0.4971	0.1000	6.0207
0.1000	0.5000	0.1000	6.0215

TABLE 4. The variation in availability with respect to various rates.

$\alpha_A$	$\mu_A$	$\lambda$	$\mu$	Availability
0.1657	0.1036	0.6080	0.5180	0.3600
0.1122	0.1036	0.6037	0.9000	0.4938
0.3701	0.1562	0.3635	0.6091	0.5010
0.2036	0.3607	0.2858	0.9000	0.6540
0.4027	0.2034	0.1847	0.7042	0.6720
0.1141	0.1000	0.1862	0.6726	0.6886
0.1000	0.1000	0.1308	0.8626	0.7980
0.1349	0.3838	0.1058	0.8323	0.8181
0.1112	0.1000	0.1000	0.7823	0.8191
0.1061	0.5000	0.1091	0.9000	0.8291
0.1368	0.4571	0.1000	0.9000	0.8369
0.1000	0.1006	0.1000	0.9000	0.8403
0.1000	0.2774	0.1000	0.9000	0.8407
0.1000	0.5000	0.1000	0.9000	0.8410

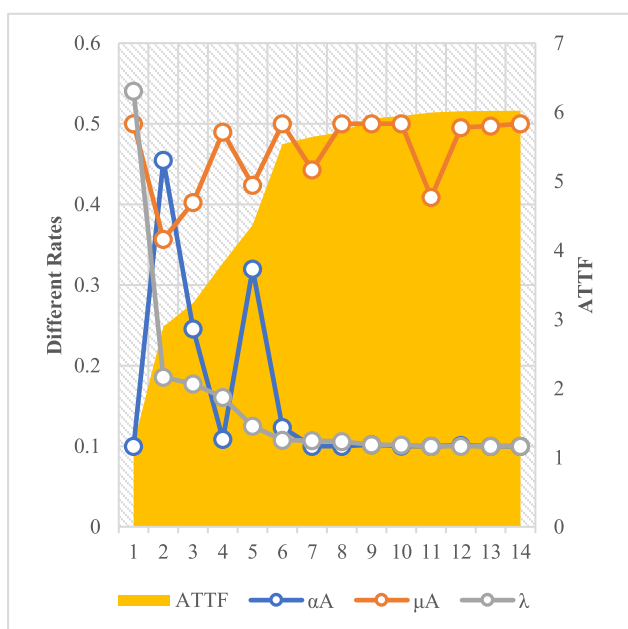


FIGURE 6. ATTF vs. Different rates.

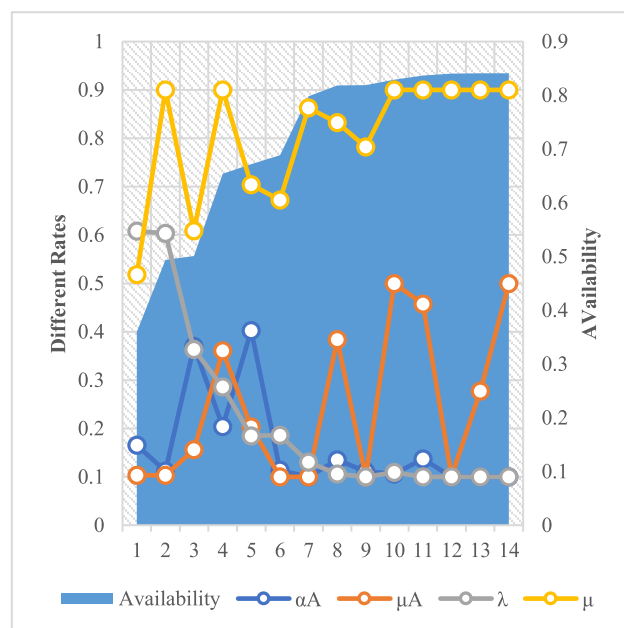


FIGURE 7. Availability vs Different rates.

a range of parameter values when using the lower and upper bounds for the four variables,  $\alpha_A$ ,  $\mu_A$ ,  $\lambda$ , and  $\mu$ , and the ABC optimization algorithm with 14 iterations. Given the available data, the maximum optimized value for availability is 0.8410. The bounds for variables are  $\alpha_A \in [0.1, 0.5]$ ,  $\mu_A \in [0.1, 0.5]$ ,  $\lambda \in [0.1, 0.9]$ , and  $\mu \in [0.1, 0.9]$ .

**C. BUSY PERIOD FOR REPAIRMAN**

The relationship between the degradation, failure, repair, and preventive maintenance rates and the busy period is demonstrated in Table 5 and Figure 7. Making use of the initial parameter bounds, i.e., the challenging part of the process, is finding the minimum value—our need to have lower expenditure and repair costs—across a range of parameter values by

applying the 14-iteration ABC optimization algorithm with lower and upper bounds for the four variables,  $\alpha_A$ ,  $\mu_A$ ,  $\lambda$ , and  $\mu$ . Given the available data, the minimum optimized value for a busy period is 0.2051. The bounds for variables are  $\alpha_A \in [0.1, 0.5]$ ,  $\mu_A \in [0.1, 0.5]$ ,  $\lambda \in [0.1, 0.9]$ , and  $\mu \in [0.1, 0.9]$ .

**D. EXPECTED FREQUENCY OF PREVENTIVE MAINTENANCE**

The relationship between the rates of degradation, failure, repair, and preventive maintenance and the expected frequency of preventive maintenance is demonstrated in Table 6 and Figure 8. Making use of the initial parameter bounds, i.e., the challenging part of the process, is finding the minimum value—our need to have lower expenditure and repair

TABLE 5. The variation in busy periods with respect to various rates.

$\alpha_A$	$\mu_A$	$\lambda$	$\mu$	Busy period
0.4854	0.4438	0.3574	0.8906	0.4825
0.1250	0.3864	0.1000	0.2522	0.4691
0.3401	0.2251	0.1543	0.8164	0.3814
0.1030	0.4108	0.1934	0.6921	0.3433
0.1060	0.4575	0.1999	0.8455	0.3086
0.1092	0.5000	0.1964	0.8988	0.2946
0.1117	0.4245	0.1492	0.8621	0.2660
0.1000	0.3344	0.1000	0.7173	0.2457
0.1000	0.1334	0.1020	0.9000	0.2327
0.1112	0.5000	0.1122	0.8939	0.2238
0.1000	0.5000	0.1157	0.9000	0.2203
0.1000	0.5000	0.1138	0.9000	0.2185
0.1026	0.4027	0.1000	0.9000	0.2117
0.1000	0.5000	0.1000	0.9000	0.2051

TABLE 6. The variation in expected frequency of preventive maintenance with respect to various rates.

$\alpha_A$	$\mu_A$	$\lambda$	$\mu$	Expected frequency
0.2147	0.3737	0.1555	0.3038	0.0531
0.2639	0.1641	0.347	0.7711	0.0437
0.4165	0.1953	0.6527	0.4543	0.0198
0.2837	0.4563	0.6202	0.6781	0.0185
0.3552	0.3816	0.3957	0.1133	0.0119
0.2475	0.5000	0.9000	0.7147	0.0098
0.1893	0.5000	0.835	0.5549	0.0071
0.2047	0.4582	0.7131	0.1323	0.0031
0.1681	0.3460	0.8719	0.1000	0.0015
0.1040	0.4492	0.9000	0.1000	0.0009
0.1000	0.4992	0.9000	0.1000	0.0008
0.1000	0.5000	0.9000	0.1000	0.0008
0.1000	0.3669	0.9000	0.1000	0.0008
0.1000	0.5000	0.9000	0.1048	0.0008

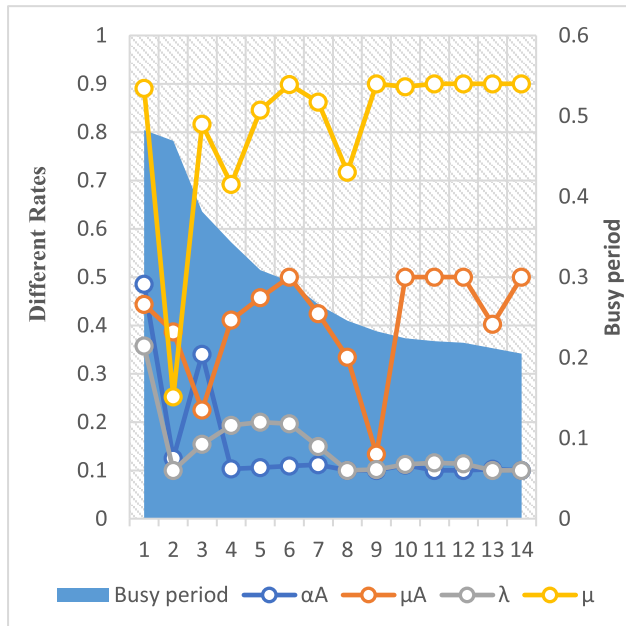


FIGURE 8. Busy period vs Different rates.

costs—across a range of parameter values by applying the 14-iteration ABC optimization algorithm with lower and upper bounds for the four variables,  $\alpha_A$ ,  $\mu_A$ ,  $\lambda$ , and  $\mu$ . Given the available data, the minimum optimized value for the expected frequency of preventive maintenance is 0.0008. The bounds for variables are  $\alpha_A \in [0.1,0.5]$ ,  $\mu_A \in [0.1,0.5]$ ,  $\lambda \in [0.1,0.9]$ , and  $\mu \in [0.1,0.9]$ .

E. PROFIT ANALYSIS

Since profit is the foundation of any industry, all relevant factors and requirements must be taken into account in order to maximize profit. To satisfy our needs, the cost of industry work, including maintenance, should be minimal. By taking the values of  $C_0 = 1000$ ,  $C_1 = 100$

and  $C_2 = 100$  with maximum availability, minimum busy period and minimum expected frequency of preventive maintenance, i.e.,  $Av(\infty) = 0.8410$ ,  $B(\infty) = 0.2051$  and  $F(\infty) = 0.0008$ , we have.

Profit = Total revenue received – repair man’s costs for performing preventive maintenance – costs for fixing malfunctioning units.

$$\text{Profit Analysis} = C_0Av(\infty) - C_1B(\infty) - C_2F(\infty) = 820.41$$

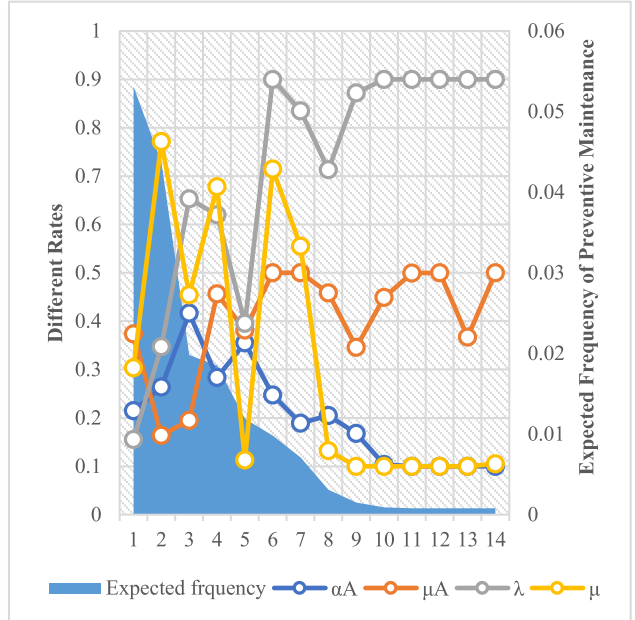


FIGURE 9. Expected frequency of preventive maintenance vs Different rates.

VI. CONCLUSION

The presented work aims to assess the profit and reliability parameters analysis of a series-parallel mixed configuration system that incorporates both complete and degraded component failure. This work has used the Chapman-Kolmogorov differential equations approach and

the Markov birth process. The Artificial Bee Colony algorithm has successfully optimized the mathematical simulation results, achieving the maximum profit value while meeting all required conditions, as demonstrated in Tables 3, 4, and 6. Figures 6, 7, 8, and 9 illustrate the impact of different rates on optimizing reliability parameters, showcasing the system's overall performance. Although there is a wealth of literature on mixed configurations, preventive maintenance combined with optimization through ABC is not as widely applied. The study's conclusions demonstrated the importance of preventive maintenance in lowering machine part maintenance costs and boosting machine dependability and, ultimately, profit.

The application of appropriate preventive maintenance significantly impacts the system, according to the study's overall findings. It is also evident that the failure and degradation rates directly contribute to the system's decreased availability and profit. Engineers and designers can create a highly reliable and profitable system with the ABC algorithm's results.

The proposed study helps concentrate on preventive maintenance to avoid the cost used to maintain the system components in future studies, and different algorithms can also be helpful to assist engineers and designers in implementing more profitable and low-maintenance systems. In order to create cost-effective systems, authors can create mathematical models in the future that will maximize reliability while minimizing cost. One more factor encouraging the designers to have lower service costs is the study's application of preventive maintenance. A meta-heuristic approach can be used to optimize reliability and other aspects of the system by developing the model with the help of the reliability function.

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